Calculus for Biology and Medicine 4th edition Neuhauser Solutions Manual

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Chapter 2

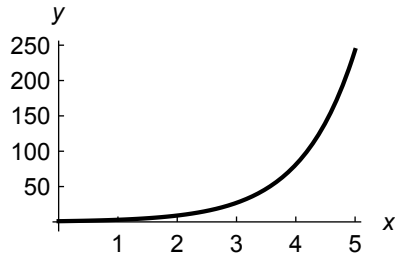
Discrete-Time Models, Sequences, and Difference Equations

2.1 Exponential Growth and Decay

■ 2.1.1

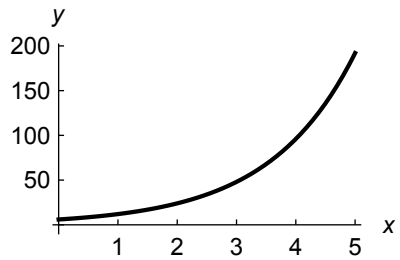
1.

t	0	1	2	3	4	5
3^t	1	3	9	27	81	243



2.

t	0	1	2	3	4	5
$6 \cdot 2^t$	6	12	24	48	96	192

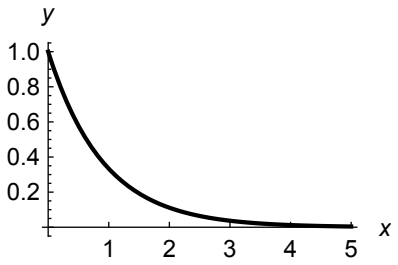


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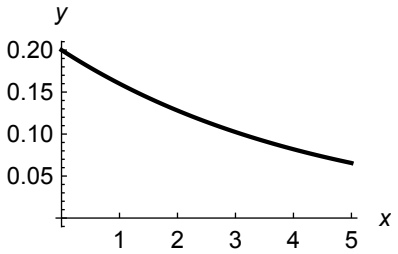
3.

t	0	1	2	3	4	5
$\left(\frac{1}{3}\right)^t$	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$



4.

t	0	1	2	3	4	5
$0.2(0.8)^t$	0.2	0.16	0.128	0.1024	0.081 92	0.065 536



5. Since the population doubles every 20 minutes, and 20 minutes is the unit of time, the formula is $N_t = N_0 \cdot 2^t$. Since there are initially $N_0 = 2$ individuals, we get $N_t = 2 \cdot 2^t = 2^{t+1}$.

6. Since the population doubles every 40 minutes, and 40 minutes is the unit of time, the formula is $N_t = N_0 \cdot 2^t$. Since there are initially $N_0 = 4$ individuals, we get $N_t = 4 \cdot 2^t = 2^{t+2}$.

7. Since the population doubles every 40 minutes, but 80 minutes is the unit of time, the population will quadruple in one time unit, so that the formula is $N_t = N_0 \cdot 4^t$. Since there is initially $N_0 = 1$ individual, we get $N_t = 4^t$.

8. At $t = 0$ the population is $20 \cdot 4^0 = 20$; we want to determine t so that $20 \cdot 4^t = 2 \cdot 20$, so that we want $4^t = 2$. Thus $t = \frac{1}{2}$, so the population doubles after $\frac{1}{2}$ time units, or 1.5 hours.

9. At $t = 0$ the population is $100 \cdot 2^t = 100$; we want to determine t so that $100 \cdot 2^t = 3 \cdot 100$, so that we want $2^t = 3$. Thus $t = \frac{\ln 3}{\ln 2} \approx 1.585$, so the population triples after about 1.585 time units, or $1.585 \cdot 2 \approx 3.17$ hours, or about 190 minutes.

10. The initial population is 10, and the population triples in size every time unit, so the formula is $N_t = 10 \cdot 3^t$.

11. Since each cell splits in two each hour, the population doubles each hour. Since there was one cell at time $t = 0$, the formula is $N_t = 2^t$:

t	0	1	2	3	4	5
2^t	1	2	4	8	16	32

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12. Since each cell splits in two every 30 minutes, the population doubles every 30 minutes. Letting the time unit be 30 minutes, since there was one cell at time $t = 0$, the formula is $N_t = 2^t$. We want to know the population after 1, 2, 3, 4, and 5 hours; these corresponds to $t = 2, 4, 6, 8, 10$:

t	0	2	4	6	8	10
2^t	1	4	16	64	256	1024

One could alternatively take the time unit as one hour, in which case the formula would be $N_t = 4^t$.

13. Let the unit of time be 42 minutes; then since the doubling time is one unit, starting from one bacterium the formula is $N_t = 2^t$. Then $N_t = 1024$ when $2^t = 1024$, so when $t = 10$. Since one time unit represents 42 minutes, this is 420 minutes = 7 hrs.

14. Let the unit of time be 24 minutes; then since the doubling time is one unit, starting from one bacterium the formula is $N_t = 2^t$. Then $N_t = 512$ when $2^t = 512$, so when $t = 9$. Since one time unit represents 24 minutes, this is $24 \cdot 9 = 216$ minutes = 3 hrs 36 min.

15. Let the unit of time be 10 minutes; then since the doubling time is one unit, starting from 5 bacteria the formula is $N_t = 5 \cdot 2^t$. Then $N_t = 320$ when $5 \cdot 2^t = 320$, or $2^t = 64$, so when $t = 6$. Since one time unit represents 10 minutes, this is 60 minutes = 1 hr.

16. Let the unit of time be 12 minutes; then since the doubling time is one unit, starting from 10 bacteria the formula is $N_t = 10 \cdot 2^t$. Then $N_t = 160$ when $10 \cdot 2^t = 160$, or $2^t = 16$, so when $t = 4$. Since one time unit represents 12 minutes, this is 48 minutes.

17. Since the population doubles every unit of time, the equation is $N_t = N_0 \cdot 2^t$; since initially there are 40 individuals, we have $N_0 = 40$ so that $N_t = 40 \cdot 2^t$.

18. Since the population halves in size every unit of time, the equation is $N_t = N_0 \cdot \left(\frac{1}{2}\right)^t$; since initially there are 1024 individuals, we have $N_0 = 1024$ so that $N_t = 1024 \cdot \left(\frac{1}{2}\right)^t = 2^{10} \cdot 2^{-t} = 2^{10-t}$. Of course, this is valid only up to $t = 10$, since after that we are counting fractional individuals.

19. The reproductive rate is 4, so we have

$$4 = \frac{N_{i+1}}{N_i} - 1 \quad \Rightarrow \quad 5 = \frac{N_{i+1}}{N_i} \quad \Rightarrow \quad N_{i+1} = 5N_i.$$

Therefore the population quintuples every unit of time, so the equation is $N_t = N_0 \cdot 5^t$; since initially there are 20 individuals, the equation is $N_t = 20 \cdot 5^t$.

20. Since the population triples every unit of time, the equation is $N_t = N_0 \cdot 3^t$; since initially there are 72 individuals, we have $N_0 = 72$ so that $N_t = 72 \cdot 3^t$.

21. Since the population quadruples every unit of time, the equation is $N_t = N_0 \cdot 4^t$; since initially there are 5 individuals, we have $N_0 = 5$ so that $N_t = 5 \cdot 4^t$.

22. Since the population grows by 50%, or $\frac{3}{2}$, every unit of time, the equation is $N_t = N_0 \cdot \left(\frac{3}{2}\right)^t$; since initially there are 32 individuals, we have $N_0 = 32$ so that $N_t = 32 \cdot \left(\frac{3}{2}\right)^t = 2^5 \cdot 3^t \cdot 2^{-t} = 3^t \cdot 2^{5-t}$.

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2.1.2

23. Since the population doubles every unit of time, the recursion is $N_{t+1} = 2N_t$. (The corresponding equation is $N_t = N_0 \cdot 2^t$; since initially there are 11 individuals, we have $N_0 = 11$ so that $N_t = 11 \cdot 2^t$.)

24. Since the population triples every unit of time, the recursion is $N_{t+1} = 3N_t$. (The corresponding equation is $N_t = N_0 \cdot 3^t$; since initially there are 6 individuals, we have $N_0 = 6$ so that $N_t = 6 \cdot 3^t$.)

25. Since the population quadruples every unit of time, the recursion is $N_{t+1} = 4N_t$. (The corresponding equation is $N_t = N_0 \cdot 4^t$; since initially there are 30 individuals, we have $N_0 = 30$ so that $N_t = 30 \cdot 4^t$.)

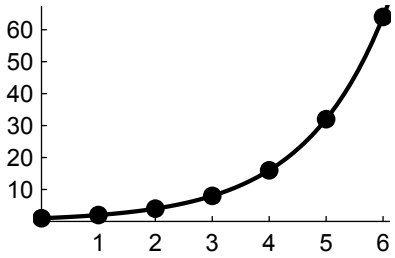
26. The reproductive rate is $\frac{1}{3}$, so we have

$$\frac{1}{3} = \frac{N_{i+1}}{N_i} - 1 \quad \Rightarrow \quad \frac{4}{3} = \frac{N_{i+1}}{N_i} \quad \Rightarrow \quad N_{i+1} = \frac{4}{3} N_i.$$

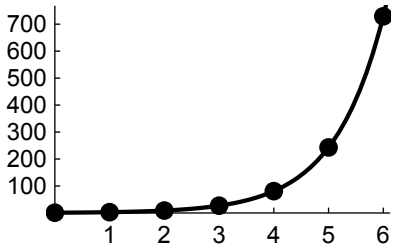
The recursion is $N_{t+1} = \frac{4}{3} N_t$. (The corresponding equation is $N_t = N_0 \cdot \left(\frac{4}{3}\right)^t$; since initially there are 63 individuals, the equation is $N_t = 63 \cdot \left(\frac{4}{3}\right)^t$.)

2.1.3

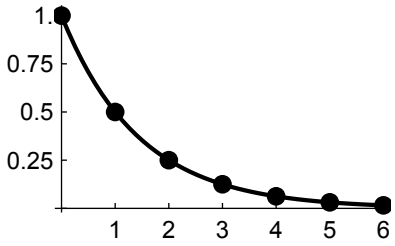
27. The curve is plotted below with the points superimposed on it:



28. The curve is plotted below with the points superimposed on it:



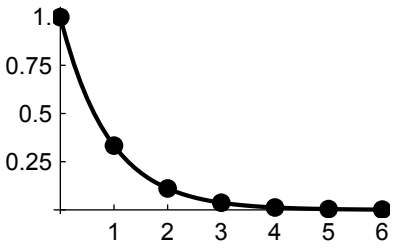
29. The curve is plotted below with the points superimposed on it:



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30. The curve is plotted below with the points superimposed on it:



31. $N_{t+1} = 2N_t$ with $N_0 = 3$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	3	6	12	24	48	96

In general, $N_t = 2^t N_0$; since $N_0 = 3$ we get $N_t = 3 \cdot 2^t$.

32. $N_{t+1} = 2N_t$ with $N_0 = 5$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	5	10	20	40	80	160

In general, $N_t = 2^t N_0$; since $N_0 = 5$ we get $N_t = 5 \cdot 2^t$.

33. $N_{t+1} = 3N_t$ with $N_0 = 2$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	2	6	18	54	162	486

In general, $N_t = 3^t N_0$; since $N_0 = 2$ we get $N_t = 2 \cdot 3^t$.

34. $N_{t+1} = 3N_t$ with $N_0 = 7$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	7	21	63	189	567	1701

In general, $N_t = 3^t N_0$; since $N_0 = 7$ we get $N_t = 7 \cdot 3^t$.

35. $N_{t+1} = 5N_t$ with $N_0 = 1$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	1	5	25	125	625	3125

In general, $N_t = 5^t N_0$; since $N_0 = 1$ we get $N_t = 1 \cdot 5^t = 5^t$.

36. $N_{t+1} = 7N_t$ with $N_0 = 4$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	4	28	196	1372	9604	67228

In general, $N_t = 7^t N_0$; since $N_0 = 4$ we get $N_t = 4 \cdot 7^t$.

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37. $N_{t+1} = \frac{1}{2}N_t$ with $N_0 = 640$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
N_t	640	320	160	80	40	20

In general, $N_t = \left(\frac{1}{2}\right)^t N_0$; since $N_0 = 640$ we get $N_t = 640 \cdot \left(\frac{1}{2}\right)^t$.

38. $N_{t+1} = \frac{3}{2}N_t$ with $N_0 = 32$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
N_t	32	48	72	108	162	243

In general, $N_t = \left(\frac{3}{2}\right)^t N_0$; since $N_0 = 32$ we get $N_t = 32 \cdot \left(\frac{3}{2}\right)^t$.

39. $N_{t+1} = \frac{1}{3}N_t$ with $N_0 = 1215$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
N_t	1215	405	135	45	15	5

In general, $N_t = \left(\frac{1}{3}\right)^t N_0$; since $N_0 = 1215$ we get $N_t = 1215 \cdot \left(\frac{1}{3}\right)^t$.

40. $N_{t+1} = \frac{1}{3}N_t$ with $N_0 = 2430$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
N_t	2430	810	270	90	30	10

In general, $N_t = \left(\frac{1}{3}\right)^t N_0$; since $N_0 = 2430$ we get $N_t = 2430 \cdot \left(\frac{1}{3}\right)^t$.

41. $N_{t+1} = \frac{1}{5}N_t$ with $N_0 = 31250$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	31250	6250	1250	250	50	10

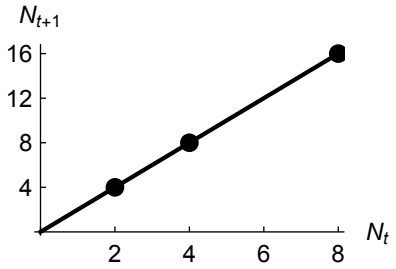
In general, $N_t = \left(\frac{1}{5}\right)^t N_0$; since $N_0 = 31250$ we get $N_t = 31250 \cdot \left(\frac{1}{5}\right)^t$.

42. $N_{t+1} = \frac{1}{4}N_t$ with $N_0 = 8192$, for $t = 0, 1, 2, 3, 4, 5$.

t	0	1	2	3	4	5
$N(t)$	8192	2048	512	218	32	8

In general, $N_t = \left(\frac{1}{4}\right)^t N_0$; since $N_0 = 8192$ we get $N_t = 8192 \cdot \left(\frac{1}{4}\right)^t = \left(\frac{1}{2}\right)^{2t-13}$.

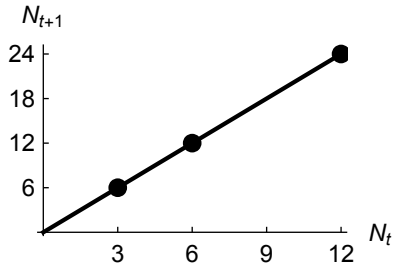
43. Since $N_{t+1} = RN_t = 2N_t$ and $N_0 = 2$, the formula is $N_t = 2 \cdot 2^t = 2^{t+1}$. A graph of $N_{t+1} = 2N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



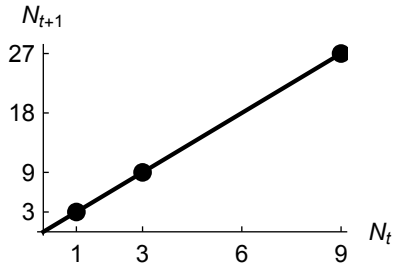
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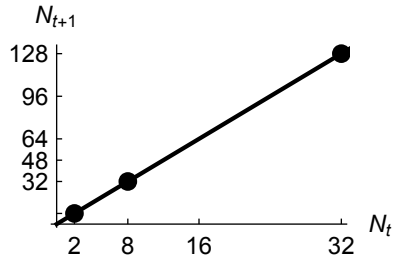
44. Since $N_{t+1} = RN_t = 2N_t$ and $N_0 = 3$, the formula is $N_t = 3 \cdot 2^t$. A graph of $N_{t+1} = 2N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



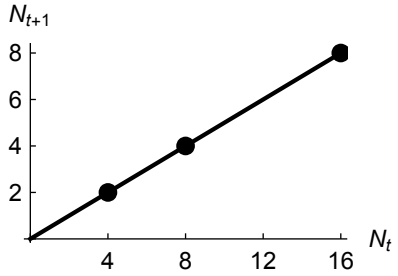
45. Since $N_{t+1} = RN_t = 3N_t$ and $N_0 = 1$, the formula is $N_t = 3^t$. A graph of $N_{t+1} = 3N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



46. Since $N_{t+1} = RN_t = 4N_t$ and $N_0 = 2$, the formula is $N_t = 2 \cdot 4^t$. A graph of $N_{t+1} = 4N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



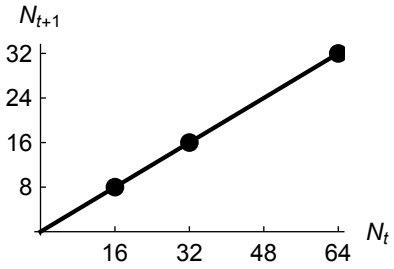
47. Since $N_{t+1} = RN_t = \frac{1}{2}N_t$ and $N_0 = 16$, the formula is $N_t = 16 \cdot \left(\frac{1}{2}\right)^t$. A graph of $N_{t+1} = \frac{1}{2}N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



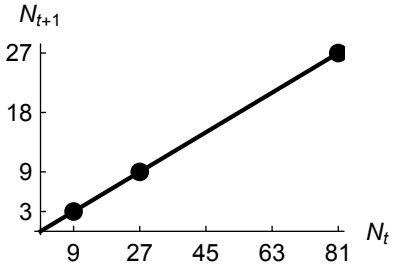
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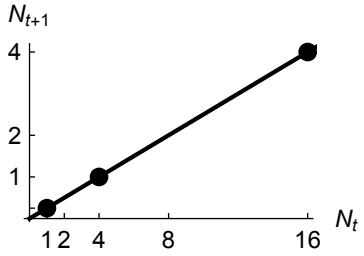
48. Since $N_{t+1} = RN_t = \frac{1}{2}N_t$ and $N_0 = 64$, the formula is $N_t = 64 \cdot \left(\frac{1}{2}\right)^t$. A graph of $N_{t+1} = \frac{1}{2}N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



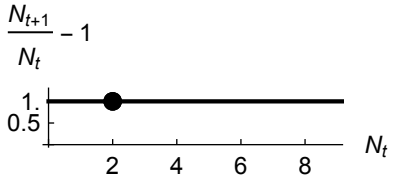
49. Since $N_{t+1} = RN_t = \frac{1}{3}N_t$ and $N_0 = 81$, the formula is $N_t = 81 \cdot \left(\frac{1}{3}\right)^t$. A graph of $N_{t+1} = \frac{1}{3}N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



50. Since $N_{t+1} = RN_t = \frac{1}{4}N_t$ and $N_0 = 16$, the formula is $N_t = 16 \cdot \left(\frac{1}{4}\right)^t$. A graph of $N_{t+1} = \frac{1}{4}N_t$ in the N_t-N_{t+1} plane, with the points (N_t, N_{t+1}) superimposed on it for $t = 0, 1, 2$, is below:



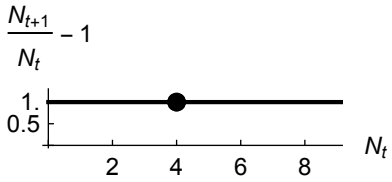
51. Since $N_{t+1} = RN_t = 2N_t$ and $N_0 = 2$, the formula is $N_t = 2 \cdot 2^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 1$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t + 1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 1$.



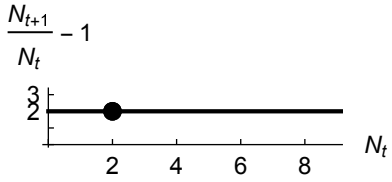
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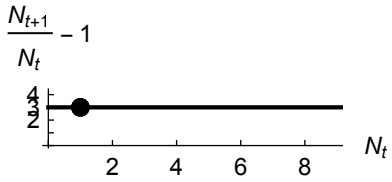
52. Since $N_{t+1} = RN_t = 2N_t$ and $N_0 = 4$, the formula is $N_t = 4 \cdot 2^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 1$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 1$.



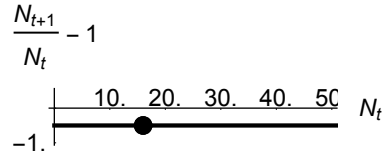
53. Since $N_{t+1} = RN_t = 3N_t$ and $N_0 = 2$, the formula is $N_t = 2 \cdot 3^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 2$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 2$.



54. Since $N_{t+1} = RN_t = 4N_t$ and $N_0 = 1$, the formula is $N_t = 4^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 3$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = 3$.



55. Since $N_{t+1} = RN_t = \frac{1}{2}N_t$ and $N_0 = 16$, the formula is $N_t = 16 \cdot \left(\frac{1}{2}\right)^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{1}{2}$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{1}{2}$.

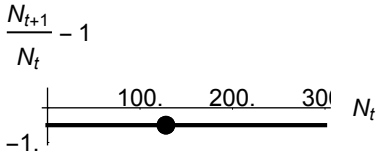


56. Since $N_{t+1} = RN_t = \frac{1}{2}N_t$ and $N_0 = 128$, the formula is $N_t = 128 \cdot \left(\frac{1}{2}\right)^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{1}{2}$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below.

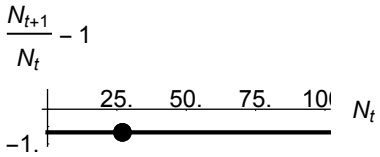
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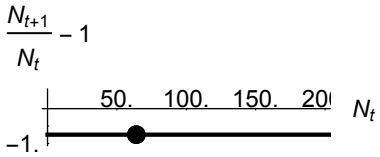
The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{1}{2}$.



57. Since $N_{t+1} = RN_t = \frac{1}{3}N_t$ and $N_0 = 27$, the formula is $N_t = 27 \cdot \left(\frac{1}{3}\right)^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{2}{3}$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{2}{3}$.



58. Since $N_{t+1} = RN_t = \frac{1}{4}N_t$ and $N_0 = 64$, the formula is $N_t = 64 \cdot \left(\frac{1}{4}\right)^t$. A graph of $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{3}{4}$ in the N_t-$\frac{N_{t+1}}{N_t} - 1$ plane, with the point $\left(N_0, \frac{N_1}{N_0} - 1\right)$ superimposed on it, is below. The reproductive rate is the number of organisms added per parent from time t to time $t+1$, which is $\frac{N_{t+1}}{N_t} - 1 = R - 1 = -\frac{3}{4}$.



For exercises 59—61, which refer to Figure 2.9, note that the first graph in Figure 2.9 describes a population with a constant reproductive rate, so it represents exponential growth. In the second population, the reproductive rate decreases towards zero, so that $\frac{N_{t+1}}{N_t} - 1$ decreases to zero. This means that $\frac{N_{t+1}}{N_t}$ decreases towards 1, so that the population eventually becomes constant (at the carrying capacity). In the third graph, $\frac{N_{t+1}}{N_t} - 1$ increases to some constant, so that $\frac{N_{t+1}}{N_t}$ does as well. This reflects a population that eventually grows exponentially since the reproductive rate is eventually essentially constant.

59. **(a)** The population will grow quickly at first (high reproductive rate), but as it grows, there will be less sugar for each individual, so the growth rate will slow and eventually stop; the population will stabilize. This is the second graph.

(b) As the habitat is large, the population will grow exponentially (in the absence of predators), at least until some environmental constraints are encountered. This is the first graph.

(c) In this situation, the reproductive rate is higher with increased population; this is the third graph.

60. **(a)** This population will exhibit exponential growth since (at least in the short term) there are no environmental constraints. This is the first graph.

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(b) While there are a small number of lions so that prey is plentiful, the growth rate will be high. As the population increases, however, the amount of prey available per individual decreases, so the population levels off at the carrying capacity. This is the second graph.

(c) As the colony outcompetes other colonies, the resources available to it expand and it can continue to grow. This is the third graph.

61. **(a)** As the population of cells gets larger, they can acquire additional resources through metastasis, so can continue to grow. This is the third graph.

(b) The number of sick individuals first expands rapidly as there is a large pool of uninfected individuals. As those resources (well individuals) become scarce, the growth rate slows down and eventually stops. This is graph 2.

(c) There are no obvious growth constraints in the short term, so the population grows exponentially. This is the first graph.

62. **(a)** The recursion is $N_{t+1} = \frac{3}{2}N_t$, since the population grows by 50% each year. Thus the formula is $N_t = N_1\left(\frac{3}{2}\right)^{t-1}$ (note that we use $t-1$ rather than t since at time 1 we want to have N_1 individuals). Further, $N_1 = 16$. so the formula is $N_t = 16\left(\frac{3}{2}\right)^{t-1}$.

(b) The population size is 100 when $100 = 16\left(\frac{3}{2}\right)^{t-1}$, so that $\left(\frac{3}{2}\right)^{t-1} = \frac{25}{4}$. This gives

$$(t-1)\ln\frac{3}{2} = \ln\frac{25}{4} \quad \Rightarrow \quad t-1 = \frac{\ln 25 - \ln 4}{\ln 3 - \ln 2} \quad \Rightarrow \quad t = 1 + \frac{\ln 25 - \ln 4}{\ln 3 - \ln 2} \approx 5.52 \text{ years.}$$

(c) The population size is 1000 when $1000 = 16\left(\frac{3}{2}\right)^{t-1}$, so that $\left(\frac{3}{2}\right)^{t-1} = \frac{125}{2}$. This gives

$$(t-1)\ln\frac{3}{2} = \ln\frac{125}{2} \quad \Rightarrow \quad t-1 = \frac{\ln 125 - \ln 2}{\ln 3 - \ln 2} \quad \Rightarrow \quad t = 1 + \frac{\ln 125 - \ln 2}{\ln 3 - \ln 2} \approx 11.2 \text{ years.}$$

(d) According to the model, the population size is 1 000 000 when $1\,000\,000 = 16\left(\frac{3}{2}\right)^{t-1}$, so that $\left(\frac{3}{2}\right)^{t-1} = 62\,500$. This gives

$$(t-1)\ln\frac{3}{2} = \ln 62\,500 \quad \Rightarrow \quad t-1 = \frac{\ln 62\,500}{\ln 3 - \ln 2} \quad \Rightarrow \quad t = 1 + \frac{\ln 62\,500}{\ln 3 - \ln 2} \approx 28.2 \text{ years.}$$

The model is unlikely to be valid for such a large population unless the island is very large or the birds require little space (and food).

2.2 Sequences

■ 2.2.1

1. With $a_n = n + 1$, we get

n	0	1	2	3	4	5
a_n	1	2	3	4	5	6

2. With $a_n = 3n^2$, we get

n	0	1	2	3	4	5
a_n	0	3	12	27	48	75

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3. With $a_n = \frac{n+2}{n}$, we get

n	0	1	2	3	4	5
a_n	undefined	3	2	$\frac{5}{3}$	$\frac{3}{2}$	$\frac{7}{5}$

4. With $a_n = \frac{n}{n+2}$, we get

n	0	1	2	3	4	5
a_n	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{5}{7}$

5. WIth $f(n) = \frac{1}{(1+n)^2}$, we have

u	0	1	2	3	4	5
$f(n)$	1	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$	$\frac{1}{25}$	$\frac{1}{36}$

6. With $a_n = \frac{1}{\sqrt{n+1}}$, we get

n	0	1	2	3	4	5
$f(n)$	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{6}}$

7. With $f(n) = (n+1)^2$, we have

n	0	1	2	3	4	5
$f(n)$	1	4	9	16	25	36

8. With $f(n) = (n+4)^{1/3}$, we get

n	0	1	2	3	4	5
$f(n)$	$4^{1/3}$	$5^{1/3}$	$6^{1/3}$	$7^{1/3}$	2	$9^{1/3}$

9. With $a_n = (-1)^n + (-1)^{n+1}$, note that if k is even then $(-1)^k = 1$, while if k is odd then $(-1)^k = -1$. Since exactly one of n and $n+1$ is even and the other is odd, the sum is zero, so that $a_n = 0$ for all n.

10. With $a_n = (-1)^n + 1$, note that if n is even then $(-1)^n = 1$, while if n is odd then $(-1)^n = -1$. It follows that $a_n = 1 + 1 = 2$ if n is even and $a_n = -1 + 1 = 0$ if n is odd:

n	0	1	2	3	4	5
a_n	2	0	2	0	2	0

11. With $a_n = \frac{n^2}{n+1}$, we get

n	0	1	2	3	4	5
a_n	0	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{9}{4}$	$\frac{16}{5}$	$\frac{25}{6}$

12. With $a_n = n^3\sqrt{n+1}$, we get

n	0	1	2	3	4	5
a_n	0	$\sqrt{2}$	$8\sqrt{3}$	54	$64\sqrt{5}$	$125\sqrt{6}$

13. With $f(n) = e^{n/2}$, we get

n	0	1	2	3	4	5
$f(n)$	1	$\sqrt{e}$	e	$e^{3/2}$	e^2	$e^{5/2}$

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14. With $f(n) = \log(n+1)$, we get

n	0	1	2	3	4	5
$f(n)$	0	$\log 2$	$\log 3$	$\log 4$	$\log 5$	$\log 6$

15. With $f(n) = \left(\frac{1}{3}\right)^n$, we have

n	0	1	2	3	4	5
$f(n)$	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$	$\frac{1}{243}$

16. With $f(n) = 2^{0.2n}$, we get

n	0	1	2	3	4	5
$f(n)$	1	$2^{0.2}$	$2^{0.4}$	$2^{0.6}$	$2^{0.8}$	2

17. This appears to be the sequence of squares of integers, $a_n = (n+1)^2$ for $n = 0, 1, 2, \ldots$. Then $a_5 = 36$, $a_6 = 49$, $a_7 = 64$, and $a_8 = 81$.

18. This appears to be the sequence $a_n = \sqrt{n}$, for $n = 0, 1, 2, 3, 4$. Thus for $n = 5, 6, 7, 8$ we have $a_n = \sqrt{5},\ \sqrt{6},\ \sqrt{7},\ \sqrt{8}$.

19. This appears to be the sequence $a_n = \frac{1}{n+1}$ for $n = 0, 1, 2, \ldots$. Then $a_5 = \frac{1}{6}$, $a_6 = \frac{1}{7}$, $a_7 = \frac{1}{8}$, $a_8 = \frac{1}{9}$.

20. This appears to be the sequence $a_n = \frac{(-1)^{n+1}}{(n+1)^2}$, for $n = 0, 1, 2, 3, 4$. Thus for $n = 5, 6, 7, 8$ we have $a_n = \frac{1}{36}, -\frac{1}{49}, \frac{1}{64}, -\frac{1}{81}$.

21. This appears to be the sequence $a_n = \frac{n+1}{n+2}$, for $n = 0, 1, 2, 3, 4$. Thus for $n = 5, 6, 7, 8$ we have $a_n = \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}$.

22. Note that each term is one more than a square. This appears to be the sequence $a_n = (n+2)^2 + 1 = n^2 + 4n + 5$. Thus for $n = 5, 6, 7, 8$ we would get 50, 65, 82, and 101.

23. This appears to be the sequence $a_n = \sqrt{(n+1) + e^{n+1}}$, for $n = 0, 1, 2, 3, 4$. Thus for $n = 5, 6, 7, 8$ we have $a_n = \sqrt{6 + e^6},\ \sqrt{7 + e^7},\ \sqrt{8 + e^8},\ \sqrt{9 + e^9}$.

24. This appears to be successive powers of 3: $a_n = 3^n$ for $n = 0, 1, 2, \ldots$. Then $a_5 = 243$, $a_6 = 729$, $a_7 = 2187$, and $a_8 = 6561$.

25. The given sequence is $a_0 = 0,\ a_1 = 1,\ a_2 = 2,\ a_3 = 3$ and $a_4 = 4$. Thus we can guess the expression to be $a_n = n$ for $n = 0, 1, 2, \ldots$

26. The given sequence is $a_0 = 0,\ a_1 = 2,\ a_2 = 4,\ a_3 = 6$ and $a_4 = 8$. Thus we can guess the expression to be $a_n = 2n$ for $n = 0, 1, 2, \ldots$

27. The given sequence is $a_0 = 1,\ a_1 = 2,\ a_2 = 4,\ a_3 = 8$ and $a_4 = 16$. Thus we can guess the expression to be $a_n = 2^n$ for $n = 0, 1, 2, \ldots$

28. The given sequence is $a_0 = 1,\ a_1 = 3,\ a_2 = 5,\ a_3 = 7$ and $a_4 = 9$. Thus we can guess the expression to be $a_n = 2n + 1$ for $n = 0, 1, 2, \ldots$.

29. The given sequence is $a_0 = 1,\ a_1 = \frac{1}{3},\ a_2 = \frac{1}{9},\ a_3 = \frac{1}{27}$ and $a_4 = \frac{1}{81}$. Thus we can guess the expression to be $a_n = \frac{1}{3^n}$ for $n = 0, 1, 2, \ldots$.

30. The given sequence is $a_0 = \frac{1}{3},\ a_1 = \frac{2}{5},\ a_2 = \frac{3}{7},\ a_3 = \frac{4}{9}$ and $a_4 = \frac{5}{11}$. Thus we can guess the expression to be $a_n = \frac{n+1}{2n+3}$ for $n = 0, 1, 2, \ldots$.

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31. The given sequence is $a_0 = -1,\ a_1 = 2,\ a_2 = -3,\ a_3 = 4$ and $a_4 = -5$. Thus we can guess the expression to be $a_n = (-1)^{n+1}(n+1)$ for $n = 0, 1, 2, \ldots$.

32. The given sequence is $a_0 = 9$, $a_1 = 16$, $a_2 = 25$, $a_3 = 36$, and $a_4 = 49$. We can guess the expression to be $a_n = (n+3)^2$ for $n = 0, 1, 2, \ldots$.

33. The given sequence is $a_0 = 5$, $a_1 = 7$, $a_2 = 9$, $a_3 = 11$, and $a_4 = 13$. We can guess the expression to be $a_n = 2n + 5$ for $n = 0, 1, 2, \ldots$.

34. The given sequence is $a_0 = 8$, $a_1 = 18$, $a_2 = 32$, $a_3 = 50$, and $a_4 = 72$. Note that $\frac{1}{2}a_n = (n+2)^2$ for the given values of n, so we guess the sequence to be $a_n = 2(n+2)^2$ for $n = 0, 1, 2, \ldots$.

35. The given sequence is $a_0 = 2,\ a_1 = 0,\ a_2 = 2,\ a_3 = 0$, and $a_4 = 2$. Recalling exercise 10 in this section, we can write this as $a_n = (-1)^n + 1$.

36. The given sequence is repetitions of the sequence $0, 1, 2$. One way of defining a_n is that it is the remainder upon dividing n by 3.

■ 2.2.2

37. We get the following sequence for $a_{n+1} = \sqrt{a_n + 1}$ and $a_0 = 1$:

n	0	1	2	3	4	5	6	7	8	9	10	11
a_n	1	1.414	1.554	1.598	1.612	1.616	1.617	1.618	1.618	1.618	1.618	1.618

38. We get the following sequence for $a_{n+1} = \frac{1}{a_n+1}$ and $a_0 = 2$:

n	0	1	2	3	4	5	6	7	8	9	10
a_n	2	0.3333	0.75	0.5714	0.6364	0.6111	0.6207	0.6170	0.6184	0.6179	0.6181

n	11	12	13
a_n	0.6180	0.6180	0.6180

39. We get the following sequence for $a_{n+1} = a_n - \frac{1}{a_n}$ and $a_0 = 3$:

n	0	1	2	3	4	5	6	7	8
a_n	3	2.667	2.292	1.855	1.316	0.557	−1.240	−0.434	1.873

n	9	10	11
a_n	1.339	0.592	−1.095

40. We get the following sequence for $a_{n+1} = a_n + \frac{1}{a_n}$ and $a_0 = 2$:

n	0	1	2	3	4	5	6	7	8	9	10
a_n	1	2	2.5	2.9	3.245	3.553	3.834	4.095	4.339	4.570	4.789

n	11	12	13
a_n	4.998	5.198	5.390

41. We get the following sequence for $a_{n+1} = \sqrt{\sqrt{a_n} + 1}$ and $a_0 = 6$:

n	0	1	2	3	4	5	6	7	8	9	10
a_n	6.000	1.857	1.537	1.497	1.491	1.490	1.490	1.490	1.490	1.490	1.490

n	11	12
a_n	1.490	1.490

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42. We get the following sequence for $a_{n+1} = a_n + \frac{1}{a_n^2}$ and $a_0 = 1$:

n	0	1	2	3	4	5	6	7	8	9	10
a_n	1	2	2.25	2.448	2.614	2.761	2.892	3.012	3.122	3.224	3.321
n	11	12									
a_n	3.411	3.497									

43. We get the following sequence for $a_{n+1} = \frac{1}{4}a_n + 1$ and $a_0 = 0$:

n	0	1	2	3	4	5	6	7	8	9	10
a_n	0	1	1.25	1.313	1.328	1.332	1.333	1.333	1.333	1.333	1.333
n	11	12	13	14							
a_n	1.333	1.333	1.333	1.333							

44. We get the following sequence for $a_{n+1} = \sqrt{a_n^2 + 1}$ and $a_0 = 1$:

n	0	1	2	3	4	5	6	7	8	9	10
a_n	1	1.414	1.732	2	2.236	2.449	2.646	2.828	3.000	3.162	3.317
n	11	12	13	14	15	16					
a_n	3.464	3.606	3.742	3.873	4	4.123					

■ 2.2.3

45. We get $a_0 = 1$, $a_1 = \frac{1}{2}$, $a_2 = \frac{1}{3}$, $a_3 = \frac{1}{4}$, $a_4 = \frac{1}{5}$, and $a_5 = \frac{1}{6}$. By limit law 9, since $\lim_{n\to\infty} 1 = 1$ is finite and $\lim_{n\to\infty}(n+1) = \infty$, we know that

$$\lim_{n\to\infty} \frac{1}{n+1} = 0.$$

46. We get $a_0 = 1$, $a_1 = \frac{3}{4}$, $a_2 = \frac{3}{5}$, $a_3 = \frac{1}{2}$, $a_4 = \frac{3}{7}$, and $a_5 = \frac{3}{8}$. By limit law 9, since $\lim_{n\to\infty} 3 = 3$ is finite and $\lim_{n\to\infty}(n+3) = \infty$, we know that

$$\lim_{n\to\infty} \frac{3}{n+3} = 0.$$

47. We get $a_0 = 0$, $a_1 = \frac{1}{2}$, $a_2 = \frac{2}{3}$, $a_3 = \frac{3}{4}$, $a_4 = \frac{4}{5}$, and $a_5 = \frac{5}{6}$. The text showed that $\lim_{n\to\infty} \frac{n+1}{n} = 1$, so by limit law 4,

$$\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{\frac{n+1}{n}} = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} \frac{n+1}{n}} = \frac{1}{1} = 1.$$

48. We get $a_0 = 0$, $a_1 = \frac{2}{9}$, $a_2 = \frac{1}{4}$, $a_3 = \frac{6}{25}$, $a_4 = \frac{2}{9}$, and $a_5 = \frac{10}{49}$. Now,

$$\lim_{n\to\infty} \frac{(n+2)^2}{2n} = \lim_{n\to\infty} \frac{n^2+4n+4}{2n} = \lim_{n\to\infty} \frac{n}{2} + \lim_{n\to\infty} 2 + \lim_{n\to\infty} \frac{2}{n} = \infty.$$

To see the final equality, note that $\lim_{n\to\infty} 2 = 2$ and by limit law 9, $\lim_{n\to\infty} \frac{2}{n} = 0$. Since both these limits exist and are finite, limit law 5 then implies the result. Finally, by limit law 9,

$$\lim_{n\to\infty} \frac{2n}{(n+2)^2} = \lim_{n\to\infty} \frac{1}{\frac{(n+2)^2}{2n}} = 0.$$

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49. We get $a_0 = 5$, $a_1 = 3$, $a_2 = \frac{9}{5}$, $a_3 = \frac{7}{5}$, $a_4 = \frac{21}{17}$, and $a_5 = \frac{15}{13}$. Now,

$$\lim_{n\to\infty} \frac{n^2+5}{n^2+1} = \lim_{n\to\infty} \frac{n^2+1+4}{n^2+1} = \lim_{n\to\infty} \left(1 + \frac{4}{n^2+1}\right).$$

By limit law 9, $\lim_{n\to\infty} \frac{4}{n^2+1} = 0$, so we can break up the limit above by limit law 1:

$$\lim_{n\to\infty} \left(1 + \frac{4}{n^2+1}\right) = \lim_{n\to\infty} 1 + \lim_{n\to\infty} \frac{4}{n^2+1} = 1 + 0 = 1.$$

50. We get $a_0 = 1$, $a_1 = \frac{1}{\sqrt{2}}$, $a_2 = \frac{1}{\sqrt{3}}$, $a_3 = \frac{1}{2}$, $a_4 = \frac{1}{\sqrt{5}}$, and $a_5 = \frac{1}{\sqrt{6}}$. Since $\lim_{n\to\infty} 1 = 1$ and $\lim_{n\to\infty} \sqrt{n+1} = \infty$, by limit law 9 we get

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = 0.$$

51. a_0 is undefined, and $a_1 = -1$, $a_2 = \frac{1}{2}$, $a_3 = -\frac{1}{3}$, $a_4 = \frac{1}{4}$, and $a_5 = -\frac{1}{5}$. Note that $|a_n| = \frac{1}{n}$, and that by limit law 9, $\lim_{n\to\infty} |a_n| = 0$. Since the a_n have limit zero in absolute value, the a_n themselves must also have limit zero.

52. We get $a_0 = \frac{1}{3}$, $a_1 = -\frac{1}{4}$, $a_2 = \frac{1}{11}$, $a_3 = -\frac{1}{30}$, $a_4 = \frac{1}{67}$, and $a_5 = -\frac{1}{128}$. Note that $|a_n| = \frac{1}{n^3+3}$, and that by limit law 9, $\lim_{n\to\infty} |a_n| = 0$. Since the a_n have limit zero in absolute value, the a_n themselves must also have limit zero.

53. We have $a_0 = 0$, $a_1 = \frac{1}{2}$, $a_2 = \frac{4}{3}$, $a_3 = \frac{9}{4}$, and $a_4 = \frac{16}{5}$. Now,

$$\lim_{n\to\infty} \frac{n^2}{n+1} = \lim_{n\to\infty} \frac{n^2+2n+1-2n-1}{n+1} = \lim_{n\to\infty} \frac{(n+1)^2-2n-1}{n+1} = \lim_{n\to\infty} \left(n+1-\frac{2n+1}{n+1}\right).$$

By an argument similar to that given in the text for finding $\lim_{n\to\infty} \frac{n+1}{n}$ we see that the latter term has limit 2, while $\lim_{n\to\infty}(n+1) = \infty$, so the limit of the sequence is ∞.

54. We have $a_0 = \frac{1}{2}$, $a_1 = \frac{2}{3}$, $a_2 = \frac{3}{4}$, $a_3 = \frac{4}{5}$, and $a_4 = \frac{5}{6}$. Now,

$$\lim_{n\to\infty} \frac{n+1}{n+2} = \lim_{n\to\infty} \frac{(n+2)-1}{n+2} = \lim_{n\to\infty} \left(1 - \frac{1}{n+2}\right).$$

Applying laws 9 and 1 gives $\lim_{n\to\infty} \left(1 - \frac{1}{n+2}\right) = \lim_{n\to\infty} 1 = 1$.

55. We have $a_0 = 0$, $a_1 = 1$, $a_2 = \sqrt{2}$, $a_3 = \sqrt{3}$, and $a_4 = 2$. Since the terms grow without bound as $n \to \infty$, we see that $\lim_{n\to\infty} \sqrt{n} = \infty$.

56. We have $a_0 = 3$, $a_1 = 4$, $a_2 = 7$, $a_3 = 12$, and $a_4 = 19$. Since the terms grow without bound as $n \to \infty$, we see that $\lim_{n\to\infty}(n^2+3) = \infty$.

57. We have $a_0 = 1$, $a_1 = 2$, $a_2 = 4$, $a_3 = 8$, and $a_4 = 16$. Since the terms grow without bound as $n \to \infty$, we see that $\lim_{n\to\infty} 2^n = \infty$.

58. We have $a_0 = 8$, $a_1 = 16$, $a_2 = 32$, $a_3 = 64$, and $a_4 = 128$. Since the terms grow without bound as $n \to \infty$, we see that $\lim_{n\to\infty} 2^{n+3} = \infty$.

59. We have $a_0 = 1$, $a_1 = 3$, $a_2 = 9$, $a_3 = 27$, and $a_4 = 81$. Since the terms grow without bound as $n \to \infty$, we see that $\lim_{n\to\infty} 3^n = \infty$.

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60. We have $a_0 = 1$, $a_1 = \frac{1}{9}$, $a_2 = \frac{1}{81}$, $a_3 = \frac{1}{729}$, and $a_4 = \frac{1}{6561}$. Further,

$$\lim_{n\to\infty} 3^{-2n} = \lim_{n\to\infty} \frac{1}{9^n};$$

since 9^n grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} 9^n = \infty$, so by limit law 9, $\lim_{n\to\infty} 3^{-2n} = 0$.

61. With $a_n = \frac{1}{n}$, since n grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that $\left|\frac{1}{n} - 0\right| < \epsilon = 0.01$ whenever $n > N$. Solving the inequality $\left|\frac{1}{n} - 0\right| < 0.01$ for positive n, we get $\frac{1}{n} < 0.01$ or $n > \frac{1}{0.01} = 100$. Thus the smallest value of N that we can choose is $N = 100$. And indeed

$$a_{101} = \frac{1}{101} \approx 0.00990 < 0.01.$$

62. With $a_n = \frac{1}{n}$, since n grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that $\left|\frac{1}{n} - 0\right| < \epsilon = 0.02$ whenever $n > N$. Solving the inequality $\left|\frac{1}{n} - 0\right| < 0.02$ for positive n, we get $\frac{1}{n} < 0.02$ or $n > \frac{1}{0.02} = 50$. Thus the smallest value of N that we can choose is $N = 50$. And indeed

$$a_{51} = \frac{1}{51} \approx 0.0196 < 0.02.$$

63. With $a_n = \frac{1}{n^2}$, since n^2 grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that $\left|\frac{1}{n^2} - 0\right| < \epsilon = 0.01$ whenever $n > N$. Solving the inequality $\left|\frac{1}{n^2} - 0\right| < 0.01$ we get $\frac{1}{n^2} < 0.01$ or $n^2 > \frac{1}{0.01} = 100$. This gives $n > 10$, so the smallest value of N that we can choose is $N = 10$. And indeed

$$a_{11} = \frac{1}{121} \approx 0.008 < 0.01.$$

64. With $a_n = \frac{1}{n^2}$, since n^2 grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that $\left|\frac{1}{n^2} - 0\right| < \epsilon = 0.001$ whenever $n > N$. Solving the inequality $\left|\frac{1}{n^2} - 0\right| < 0.001$ we get $\frac{1}{n^2} < 0.001$ or $n^2 > \frac{1}{0.001} = 1000$. This gives $n > \sqrt{1000} \approx 31.62$, so the smallest value of N that we can choose is $N = 31$. And indeed

$$a_{32} = \frac{1}{32^2} = \frac{1}{1024} \approx 0.0009766 < 0.001.$$

65. With $a_n = \frac{1}{\sqrt{n}}$, since $\sqrt{n}$ grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that $\left|\frac{1}{\sqrt{n}} - 0\right| < \epsilon = 0.1$ whenever $n > N$. Solving the inequality $\left|\frac{1}{\sqrt{n}} - 0\right| < 0.1$ we get $\frac{1}{\sqrt{n}} < 0.1$ or $\sqrt{n} > \frac{1}{0.1} = 10$. This gives $n > 100$, so the smallest value of N that we can choose is $N = 100$. And indeed

$$a_{101} = \frac{1}{\sqrt{101}} \approx 0.0995 < 0.1.$$

66. With $a_n = \frac{1}{\sqrt{n}}$, since $\sqrt{n}$ grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that $\left|\frac{1}{\sqrt{n}} - 0\right| < \epsilon = 0.05$ whenever $n > N$. Solving the inequality $\left|\frac{1}{\sqrt{n}} - 0\right| < 0.05$ we get $\frac{1}{\sqrt{n}} < 0.05$ or $\sqrt{n} > \frac{1}{0.05} = 20$. This gives $n > 400$, so the smallest value of N that we can choose is $N = 400$. And indeed

$$a_{401} = \frac{1}{\sqrt{401}} \approx 0.04994 < 0.05.$$

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67. With $a_n = \frac{(-1)^n}{n}$, since n grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Now, we need to find an integer N such that

$$\left|\frac{(-1)^n}{n} - 0\right| = \left|\frac{(-1)^n}{n}\right| = \frac{1}{n} < \epsilon = 0.01 \text{ whenever } n > N.$$

Solving the inequality $\frac{1}{n} < 0.01$ we get $n > 100$, so the smallest value of N that we can choose is $N = 100$. And indeed

$$|a_{101} - 0| = \left|\frac{(-1)^{11}}{11}\right| \approx 0.0099 < 0.01.$$

68. With $a_n = e^{-n} = \frac{1}{e^n}$, since e^n grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Thus we want to find an integer N such that $|e^{-n} - 0| = |e^{-n}| < \epsilon = 0.01$ whenever $n > N$. Since e^{-n} is always positive, this is equivalent to saying that $e^{-n} < 0.01$ for $n > N$. Taking natural logs gives $-n < \ln 0.01 \approx -4.61$, so that $n > 4.62$. So taking $N = 4$ (that is, $n \geq 5$) will suffice. And indeed $e^{-5} \approx 0.0067 < 0.01$.

69. With $a_n = e^{-3n} = \frac{1}{e^{3n}}$, since e^{3n} grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Thus we want to find an integer N such that $\left|e^{-3n} - 0\right| = \left|e^{-3n}\right| < \epsilon = 0.001$ whenever $n > N$. Since e^{-3n} is always positive, this is equivalent to saying that $e^{-3n} < 0.001$ for $n > N$. Taking natural logs gives $-3n < \ln \sum 0.001 \approx -6.91$, so that $n > \frac{6.91}{3} \approx 2.30$. So taking $N = 2$ (that is, $n \geq 3$) will suffice. And indeed $e^{-3\cdot 3} \approx 0.000\,12 < 0.001$.

70. We have $a_n = \ln\left(1 + \frac{1}{n}\right)$. Since $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right) = 1$ by limit laws 9 and 1, it follows that $\lim_{n\to\infty} \ln\left(1 + \frac{1}{n}\right) = \lim_{n\to\infty} \ln 1 = 0$. Thus we want to find an integer N such that

$$\left|\ln\left(1 + \frac{1}{n}\right) - 0\right| = \left|\ln\left(1 + \frac{1}{n}\right)\right| < \epsilon = 0.1 \text{ whenever } n > N.$$

Since $1 + \frac{1}{n} > 1$ for $n > 0$, it follows that $\ln\left(1 + \frac{1}{n}\right) > 0$, so we can remove the absolute value signs and solve $\ln\left(1 + \frac{1}{n}\right) < 0.1$. Exponentiate both sides, giving $1 + \frac{1}{n} = e^{0.1}$, so that $n = \frac{1}{e^{0.1} - 1} \approx 9.51$. So taking $N = 9$ (that is, $n \geq 10$) will suffice. And indeed $\ln\left(1 + \frac{1}{10}\right) \approx 0.095 < 0.1$.

71. With $a_n = 2^{-n} = \frac{1}{2^n}$, since 2^n grows without bound as $n \to \infty$, it follows that $\lim_{n\to\infty} a_n = 0$. Thus we want to find an integer N such that $|2^{-n} - 0| = |2^{-n}| < \epsilon = 0.01$ whenever $n > N$. Since 2^{-n} is always positive, this is equivalent to saying that $2^{-n} < 0.01$ for $n > N$. Taking natural logs gives $-n \ln 2 < \ln 0.01$, so that $n > \frac{\ln 100}{\ln 2} \approx 6.64$. So taking $N = 6$ (that is, $n \geq 7$) will suffice. And indeed $2^{-7} \approx 0.007 < 0.01$.

72. We have $a_n = \log\left(1 + \frac{2}{n^2}\right)$. Since $\lim_{n\to\infty}\left(1 + \frac{2}{n^2}\right) = 1$ by limit laws 9 and 1, it follows that $\lim_{n\to\infty} \log\left(1 + \frac{2}{n^2}\right) = \lim_{n\to\infty} \log 1 = 0$. Thus we want to find an integer N such that

$$\left|\log\left(1 + \frac{2}{n^2}\right) - 0\right| = \left|\log\left(1 + \frac{2}{n^2}\right)\right| < \epsilon = 0.05 \text{ whenever } n > N.$$

Since $1 + \frac{2}{n^2} > 1$ for $n > 0$, it follows that $\log\left(1 + \frac{2}{n^2}\right) > 0$, so we can remove the absolute value signs and solve $\log\left(1 + \frac{2}{n^2}\right) < 0.05$. Exponentiate both sides, giving $1 + \frac{2}{n^2} = 10^{0.05}$, so that $\frac{n^2}{2} = \frac{1}{10^{0.05} - 1}$. Solving for n gives $n \approx 4.05$. So taking $N = 4$ (that is, $n \geq 5$) will suffice. And indeed $\log\left(1 + \frac{2}{25}\right) \approx 0.033 < 0.05$.

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73. We must show that for every $\epsilon > 0$ we can find N such that

$$\left|\frac{3}{n} - 0\right| = \left|\frac{3}{n}\right| < \epsilon \text{ whenever } n > N.$$

To find a candidate for N, we solve $\left|\frac{3}{n}\right| < \epsilon$ for positive n. This gives $\frac{3}{n} < \epsilon$, or $n > \frac{3}{\epsilon}$. Thus $N = \frac{3}{\epsilon}$ gives the required inequality.

74. We must show that for every $\epsilon > 0$ we can find N such that

$$\left|\frac{1}{n+1} - 0\right| = \left|\frac{1}{n+1}\right| < \epsilon \text{ whenever } n > N.$$

To find a candidate for N, we solve $\left|\frac{1}{n+1}\right| < \epsilon$ for positive n. This gives $\frac{1}{n+1} < \epsilon$, or $n + 1 > \frac{1}{\epsilon}$, so that $n > \frac{1}{\epsilon} - 1$. Thus we may choose $N = \frac{1}{\epsilon}$, since this is clearly greater than $\frac{1}{\epsilon} - 1$. If $n > N$ then $\left|\frac{1}{n+1}\right| < \frac{1}{n} < \frac{1}{N} = \epsilon$.

75. We must show that for every $\epsilon > 0$ we can find N such that

$$\left|\frac{1}{n^2} - 0\right| = \left|\frac{1}{n^2}\right| < \epsilon \text{ whenever } n > N.$$

To find a candidate for N, we solve $\left|\frac{1}{n^2}\right| = \frac{1}{n^2} < \epsilon$ for positive n. This gives $n^2 > \frac{1}{\epsilon}$, or $n > \frac{1}{\sqrt{\epsilon}}$. Thus $N = \frac{1}{\sqrt{\epsilon}}$ gives the required inequality.

76. We must show that for every $\epsilon > 0$ we can find N such that

$$\left|e^{-2n} - 0\right| = \left|e^{-2n}\right| < \epsilon \text{ whenever } n > N.$$

To find a candidate for N, we solve $\left|e^{-2n}\right| < \epsilon$ for n. Since $e^{-2n} > 0$ for all n, we can drop the absolute value signs, giving $e^{-2n} < \epsilon$. Now take natural logs, giving $-2n < \ln \epsilon$, so that $n > -\frac{\ln \epsilon}{2} = \frac{1}{2} \ln \frac{1}{\epsilon}$. Thus $N = \frac{1}{2} \ln \frac{1}{\epsilon}$ gives the required inequality.

77. We must show that for every $\epsilon > 0$ we can find N such that

$$\left|2^{-3n} - 0\right| = \left|2^{-3n}\right| < \epsilon \text{ whenever } n > N.$$

To find a candidate for N, we solve $\left|2^{-3n}\right| < \epsilon$ for n. Since $2^{-3n} > 0$ for all n, we can drop the absolute value signs, giving $2^{-3n} < \epsilon$. Now take natural logs, giving $-3n \ln 2 < \ln \epsilon$, so that $n > -\frac{\ln \epsilon}{3 \ln 2} = \frac{1}{3 \ln 2} \ln \frac{1}{\epsilon}$. Thus $N = \frac{1}{3 \ln 2} \ln \frac{1}{\epsilon}$ gives the required inequality.

78. We must show that for every $\epsilon > 0$ we can find N such that

$$\left|\frac{n}{n+1} - 1\right| = \left|-\frac{1}{n+1}\right| < \epsilon \text{ whenever } n > N.$$

To find a candidate for N, we solve $\left|-\frac{1}{n+1}\right| < \epsilon$ for positive n. This gives $\frac{1}{n+1} < \epsilon$, or $n + 1 > \frac{1}{\epsilon}$, so that $n > \frac{1}{\epsilon} - 1$. Thus we may choose $N = \frac{1}{\epsilon}$, since this is clearly greater than $\frac{1}{\epsilon} - 1$. If $n > N$ then $\left|-\frac{1}{n+1}\right| < \frac{1}{n} < \frac{1}{N} = \epsilon$.

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79. Since $\lim_{n\to\infty} \frac{1}{n} = 0$ and $\lim_{n\to\infty} \frac{2}{n^2} = 0$ by limit law 9 (since $\lim_{n\to\infty} n = \infty$ and $\lim_{n\to\infty} n^2 = \infty$), limit law 1 implies that

$$\lim_{n\to\infty} \left(\frac{1}{n} + \frac{2}{n^2}\right) = \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} \frac{2}{n^2} = 0 + 0 = 0.$$

80. Since $\lim_{n\to\infty} \frac{2}{n} = 0$ and $\lim_{n\to\infty} \frac{3}{n^2+1} = 0$ by limit law 9 (since $\lim_{n\to\infty} n = \infty$ and $\lim_{n\to\infty} n^2 + 1 = \infty$), limit law 1 implies that

$$\lim_{n\to\infty} \left(\frac{2}{n} - \frac{3}{n^2+1}\right) = \lim_{n\to\infty} \frac{2}{n} - \lim_{n\to\infty} \frac{3}{n^2+1} = 0 - 0 = 0.$$

81. We can write $\frac{n+1}{n}$ as $1 + 1/n$. Since both $\lim_{n\to\infty} 1$ and $\lim_{n\to\infty} \frac{1}{n}$ exist and are equal to 1 and 0 respectively, we have by limit law 1 that

$$\lim_{n\to\infty} \frac{n+1}{n} = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right) = \lim_{n\to\infty} 1 + \lim_{n\to\infty} \frac{1}{n} = 1 + 0 = 1.$$

82. We have

$$\frac{2n-3}{\sqrt{n}} = 2n^{1/2} - \frac{3}{\sqrt{n}}.$$

Since $2n^{1/2}$ increases without bound as $n \to \infty$, we have $\lim_{n\to\infty}(2n^{1/2}) = \infty$. Also, since $\sqrt{n}$ increases without bound as $n \to \infty$, we have $\lim_{n\to\infty} \sqrt{n} = \infty$, so by limit law 9, $\lim_{n\to\infty} \frac{3}{\sqrt{n}} = 0$. Thus by limit law 5, the given limit is ∞.

83. We can write $\frac{n^2+1}{n^2}$ as $1 + \frac{1}{n^2}$. Since both $\lim_{n\to\infty} 1$ exists and is equal to 1, and by limit law 9 $\lim_{n\to\infty} \frac{1}{n^2}$ exists and is equal to 0, we have by limit law 1 that

$$\lim_{n\to\infty} \frac{n^2+1}{n^2} = \lim_{n\to\infty} \left(1 + \tfrac{1}{n^2}\right) = \lim_{n\to\infty} 1 + \lim_{n\to\infty} \frac{1}{n^2} = 1 + 0 = 1.$$

84. We have

$$\frac{3n^2-5}{n} = \frac{3n^2}{n} - \frac{5}{n} = 3n - \frac{5}{n}.$$

Since $3n$ increases without bound as $n \to \infty$, we have $\lim_{n\to\infty} 3n = \infty$. Also, since n increases without bound as $n \to \infty$, we have $\lim_{n\to\infty} n = \infty$, so by limit law 9, $\lim_{n\to\infty} \frac{5}{n} = 0$. Thus by limit law 5, the given limit is

$$\lim_{n\to\infty} \frac{3n^2-5}{n} = \lim_{n\to\infty} \left(3n - \frac{5}{n}\right) = \infty.$$

85. Factor the denominator of $\frac{n+1}{n^2-1}$, so that

$$\frac{n+1}{n^2-1} = \frac{n+1}{(n+1)(n-1)} = \frac{1}{n-1}.$$

(Note that this is valid only if $n \neq -1$, but since we are interested in the behavior as $n \to \infty$, we may certainly assume that $n \neq -1$.) By limit law 9, since $n-1$ increases without bound as $n \to \infty$, we see that $\lim_{n\to\infty} \frac{1}{n-1}$ exists and is equal to 0. Thus by limit law 9

$$\lim_{n\to\infty} \frac{n+1}{n^2-1} = \lim_{n\to\infty} \frac{1}{n-1} = 0.$$

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86. We have

$$\frac{n^2}{n^2+4} = \frac{1}{\frac{n^2+4}{n^2}} = \frac{1}{1+\frac{4}{n^2}}.$$

Since n^2 increases without bound as $n \to \infty$, we have $\lim_{n\to\infty} n^2 = \infty$, so by limit law 9, $\lim_{n\to\infty} \frac{4}{n^2} = 0$, and then limit law 1 implies that $\lim_{n\to\infty} \left(1+\frac{4}{n^2}\right) = 1$. Since $\lim_{n\to\infty} 1 = 1$, limit law 4 applies, giving

$$\lim_{n\to\infty} \frac{n^2}{n^2+4} = \lim_{n\to\infty} \frac{1}{1+\frac{4}{n^2}} = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty}\left(1+\frac{4}{n^2}\right)} = \frac{1}{1} = 1.$$

87. Note that $\left(\frac{1}{3}\right)^n = \frac{1}{3^n}$, and that 3^n increases without bound as $n \to \infty$. Thus $\lim_{n\to\infty} 3^n = \infty$, so that $\lim_{n\to\infty} \left(\frac{1}{3}\right)^n = 0$ by limit law 9. Further, $\lim_{n\to\infty} 2^n = \infty$. Thus by limit law 5,

$$\lim_{n\to\infty} \left(\left(\frac{1}{3}\right)^n + 2^n\right) = \infty.$$

88. First, notice that the expression $3^{-n} - 4^{-n}$ can also be written as $\left(\frac{1}{3}\right)^n - \left(\frac{1}{4}\right)^n$. By Example 12, we know that both $\lim_{n\to\infty} \left(\frac{1}{3}\right)^n$ and $\lim_{n\to\infty} \left(\frac{1}{4}\right)^n$ exist and are equal to 0. Thus by limit law 1 we have

$$\lim_{n\to\infty} \left[\left(\frac{1}{3}\right)^n - \left(\frac{1}{4}\right)^n\right] = \lim_{n\to\infty} \left(\frac{1}{3}\right)^n - \lim_{n\to\infty} \left(\frac{1}{4}\right)^n = 0 - 0 = 0.$$

89. First, notice that the expression $(n+2^{-n})/n$ can also be written as $1 + \left(\frac{1}{2}\right)^n \cdot \frac{1}{n}$. Since the limits $\lim_{n\to\infty} 1$, $\lim_{n\to\infty} \left(\frac{1}{2}\right)^n$ and $\lim_{n\to\infty} \frac{1}{n}$ exist and are equal to 1, 0 and 0 respectively, we have that

$$\begin{aligned}
\lim_{n\to\infty} \frac{n+2^{-n}}{n} &= \lim_{n\to\infty} \left[1 + \left(\frac{1}{2}\right)^n \cdot \frac{1}{n}\right] \\
&= \lim_{n\to\infty} 1 + \lim_{n\to\infty} \left[\left(\frac{1}{2}\right)^n \cdot \frac{1}{n}\right] \\
&= \lim_{n\to\infty} 1 + \lim_{n\to\infty} \left(\frac{1}{2}\right)^n \cdot \lim_{n\to\infty} \frac{1}{n} \\
&= 1 + 0 \cdot 0 = 1.
\end{aligned}$$

90. Note that

$$\frac{1+e^{-n}}{n} = \frac{1}{n} + \frac{1}{ne^n}.$$

Since $\lim_{n\to\infty} n = \lim_{n\to\infty} e^n = \infty$, it follows from limit law 9 that $\lim_{n\to\infty} \frac{1}{n} = \lim_{n\to\infty} \frac{1}{ne^n} = 0$, so that

$$\lim_{n\to\infty} \frac{1+e^{-n}}{n} = 0.$$

■ 2.2.4

91. By repeatedly applying the recursion to the equation $a_{n+1} = 2a_n$ with $a_0 = 1$, we have

$$\begin{aligned}
a_1 &= 2a_0 = 2 \cdot 1 = 2 \\
a_2 &= 2a_1 = 2 \cdot 2 = 4 \\
a_3 &= 2a_2 = 2 \cdot 4 = 8 \\
a_4 &= 2a_3 = 2 \cdot 8 = 16 \\
a_5 &= 2a_4 = 2 \cdot 16 = 32.
\end{aligned}$$

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92. By repeatedly applying the recursion to the equation $a_{n+1} = 2a_n$ with $a_0 = 3$, we have

$$\begin{aligned} a_1 &= 2a_0 = 2 \cdot 3 = 6 \\ a_2 &= 2a_1 = 2 \cdot 6 = 12 \\ a_3 &= 2a_2 = 2 \cdot 12 = 24 \\ a_4 &= 2a_3 = 2 \cdot 24 = 48 \\ a_5 &= 2a_4 = 2 \cdot 48 = 96. \end{aligned}$$

93. By repeatedly applying the recursion to the equation $a_{n+1} = -2a_n$ with $a_0 = 1$, we have

$$\begin{aligned} a_1 &= -2a_0 = -2 \\ a_2 &= -2a_1 = 4 \\ a_3 &= -2a_2 = -8 \\ a_4 &= -2a_3 = 16 \\ a_5 &= -2a_4 = -32. \end{aligned}$$

94. By repeatedly applying the recursion to the equation $a_{n+1} = -2a_n$ with $a_0 = 2$, we have

$$\begin{aligned} a_1 &= -2a_0 = -4 \\ a_2 &= -2a_1 = 8 \\ a_3 &= -2a_2 = -16 \\ a_4 &= -2a_3 = 32 \\ a_5 &= -2a_4 = -64. \end{aligned}$$

95. By repeatedly applying the recursion to the equation $a_{n+1} = 1 + 2a_n$ with $a_0 = 0$, we have

$$\begin{aligned} a_1 &= 1 + 2a_0 = 1 + 0 = 1 \\ a_2 &= 1 + 2a_1 = 1 + 2 \cdot 1 = 3 \\ a_3 &= 1 + 2a_2 = 1 + 2 \cdot 3 = 7 \\ a_4 &= 1 + 2a_3 = 1 + 2 \cdot 7 = 15 \\ a_5 &= 1 + 2a_4 = 1 + 2 \cdot 15 = 31. \end{aligned}$$

96. By repeatedly applying the recursion to the equation $a_{n+1} = 4 - 2a_n$ with $a_0 = \frac{4}{3}$, we have

$$a_1 = 4 - 2a_0 = 4 - 2 \cdot \frac{4}{3} = \frac{4}{3}.$$

Since $a_1 = a_0$, it follows that $a_2 = a_1$ and that $a_n = a_0 = \frac{4}{3}$ for all n.

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97. By repeatedly applying the recursion to the equation $a_{n+1} = \frac{a_n}{1+a_n}$ with $a_0 = 1$, we have

$$a_1 = \frac{a_0}{1+a_0} = \frac{1}{1+1} = \frac{1}{2}$$
$$a_2 = \frac{a_1}{1+a_1} = \frac{1/2}{3/2} = \frac{1}{3}$$
$$a_3 = \frac{a_2}{1+a_2} = \frac{1/3}{4/3} = \frac{1}{4}$$
$$a_4 = \frac{a_3}{1+a_3} = \frac{1/4}{5/4} = \frac{1}{5}$$
$$a_5 = \frac{a_4}{1+a_4} = \frac{1/5}{6/5} = \frac{1}{6}.$$

98. By repeatedly applying the recursion to the equation $a_{n+1} = \sqrt{a_n}$ with $a_0 = 16$, we have

$$a_1 = \sqrt{a_0} = \sqrt{16} = 4$$
$$a_2 = \sqrt{a_1} = \sqrt{4} = 2$$
$$a_3 = \sqrt{a_2} = \sqrt{2}$$
$$a_4 = \sqrt{a_3} = \sqrt{\sqrt{2}} = 2^{1/4}$$
$$a_5 = \sqrt{a_4} = \sqrt{2^{1/4}} = 2^{1/8}.$$

99. By repeatedly applying the recursion to the equation $a_{n+1} = a_n + \frac{1}{a_n}$ with $a_0 = 1$, we have

$$a_1 = a_0 + \frac{1}{a_0} = 1 + \frac{1}{2} = 2$$
$$a_2 = a_1 + \frac{1}{a_0} = 2 + \frac{1}{2} = \frac{5}{2}$$
$$a_3 = a_2 + \frac{1}{a_2} = \frac{5}{2} + \frac{2}{5} = \frac{29}{10}$$
$$a_4 = a_3 + \frac{1}{a_3} = \frac{29}{10} + \frac{10}{29} = \frac{941}{290}$$
$$a_5 = a_4 + \frac{1}{a_4} = \frac{941}{290} + \frac{290}{941} = \frac{969581}{272890}.$$

100. By repeatedly applying the recursion to the equation $a_{n+1} = 2a_n^2$ with $a_0 = 1$, we have

$$a_1 = 2a_0^2 = 2 \cdot 1^2 = 2$$
$$a_2 = 2a_1^2 = 2 \cdot 2^2 = 8$$
$$a_3 = 2a_2^2 = 2 \cdot 8^2 = 2^7$$
$$a_4 = 2a_3^2 = 2 \cdot \left(2^7\right)^2 = 2^{15}$$
$$a_5 = 2a_4^2 = 2 \cdot \left(2^{15}\right)^2 = 2^{31}.$$

101. Following the method of Example 14, here we have $f(a) = \frac{1}{2}a + 2$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. Then the only fixed point is $a = 4$:

$$a = \frac{1}{2}a + 2 \quad \Rightarrow \quad \frac{1}{2}a = 2 \quad \Rightarrow \quad a = 4.$$

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102. Following the method of Example 14, here we have $f(a) = \frac{1}{3}a + \frac{4}{3}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \frac{1}{3}a + \frac{4}{3} \quad \Rightarrow \quad \frac{2}{3}a = \frac{4}{3} \quad \Rightarrow \quad a = 2.$$

Thus the only fixed point is $a = 2$.

103. Following the method of Example 14, we have $f(a) = \frac{5}{2} - \frac{1}{2}a$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \frac{5}{2} - \frac{1}{2}a \quad \Rightarrow \quad \frac{3}{2}a = \frac{5}{2} \quad \Rightarrow \quad a = \frac{5}{3}.$$

The only fixed point is $a = \frac{5}{3}$.

104. Following the method of Example 14, we have $f(a) = a^2 - a$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = a^2 - a \quad \Rightarrow \quad a^2 - 2a = 0 \quad \Rightarrow \quad a = 0 \text{ or } a = 2.$$

The two fixed points are $a = 0$ and $a = 2$.

105. Following the method of Example 14, here we have $f(a) = \frac{4}{a}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \frac{4}{a} \quad \Rightarrow \quad a^2 = 4 \quad \Rightarrow \quad a = \pm 2.$$

Thus there are two fixed points, namely $a = 2$ and $a = -2$.

106. Following the method of Example 14, we have $f(a) = \frac{4}{a-3}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \frac{4}{a-3} \quad \Rightarrow \quad a^2 - 3a = 4 \quad \Rightarrow \quad a^2 - 3a - 4 = 0 \quad \Rightarrow \quad a = -1 \text{ or } a = 4.$$

The two fixed points are $a = -1$ and $a = 4$.

107. Following the method of Example 14, here we have $f(a) = \frac{2}{a+2}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \frac{2}{a+2} \quad \Rightarrow \quad a(a+2) = 2 \quad \Rightarrow \quad a^2 + 2a - 2 = 0 \quad \Rightarrow \quad a = -1 \pm \sqrt{3}.$$

Thus there are two fixed points, namely $a = -1 + \sqrt{3}$ and $a = -1 - \sqrt{3}$.

108. Following the method of Example 14, we have $f(a) = \frac{8}{\sqrt{a}}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \frac{8}{\sqrt{a}} \quad \Rightarrow \quad a^{3/2} = 8 \quad \Rightarrow \quad a = 8^{2/3} = 4.$$

The only fixed point is $a = 4$.

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109. Following the method of Example 14, here we have $f(a) = \sqrt{5a}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \sqrt{5a} \quad \Rightarrow \quad a^2 = 5a \quad \Rightarrow \quad a(a-5) = 0 \quad \Rightarrow \quad a = 0 \text{ or } 5.$$

Thus there are two fixed points, namely $a = 0$ and $a = 5$.

110. Following the method of Example 14, we have $f(a) = \sqrt{a+2}$. If a is a fixed point, then it must satisfy the equation $a = f(a)$. That is,

$$a = \sqrt{a+2} \quad \Rightarrow \quad a^2 = a+2 \quad \Rightarrow \quad a^2 - a - 2 = 0 \quad \Rightarrow \quad a = -1 \text{ or } a = 2.$$

However, note that $\sqrt{a+2} \geq 0$, so that $a = -1$ cannot be a fixed point. The only fixed point is $a = 2$.

111. We first compute the fixed points. As in Example 14, we solve the equation $a = \frac{1}{3}a + \frac{4}{3}$, giving $a = 2$, so we guess that the limit is 2. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5	6
a_n	0	1.333	1.778	1.926	1.975	1.992	1.997

112. We first compute the fixed points. As in Example 14, we solve the equation $a = \frac{1}{3}\left(a + \frac{1}{9}\right)$ to find that $\frac{2a}{3} = \frac{1}{27}$, or that $a = \frac{1}{18}$. Now when $a_0 = 1$, then $a_n < 1$ for all $n = 1, 2, 3, \ldots$, and so we could conclude that $\lim_{n\to\infty} a_n = \frac{1}{18} \approx 0.05556$. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5	6
a_n	1	0.37037	0.16049	0.09054	0.06722	0.05944	0.05685

113. We first compute the fixed points. As in Example 14, we solve the equation $a = \sqrt{2a}$ to find that $a(a-2) = 0$, which means that $a = 0$ or $a = 2$. Now when $a_0 = 1$, then $a_n > 1$ for all $n = 1, 2, 3, \ldots$ and so we could conclude that $\lim_{n\to\infty} a_n = 2$. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5	6
a_n	1	$2^{1/2}$	$2^{3/4}$	$2^{7/8}$	$2^{15/16}$	$2^{31/32}$	$2^{63/64}$

114. We first compute the fixed points. As in Example 14, we solve the equation $a = \frac{3}{a+2}$, giving $a = 1$ or $a = -3$. Since a_1 is positive, succeeding terms are positive as well, so we guess that the limit is 1. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5	6
a_n	0	1.5	0.857	1.05	0.984	1.005	0.998

115. We first compute the fixed points. As in Example 14, we solve the equation $a = 2a(1-a)$ to find that $a(2a-1) = 0$, which means that $a = 0$ or $a = \frac{1}{2}$. Now when $a_0 = 0.1$, then $a_n > 0.1$ for all $n = 1, 2, 3, \ldots$ and so we might conclude that $\lim_{n\to\infty} a_n = \frac{1}{2}$. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5	6
a_n	0.1	0.18	0.2952	0.41611	0.48592	0.4996	0.49999

116. Note that if $a_0 = 0$, then $a_n = 0$ for all n, since each a_i is a multiple of the previous one. Thus $\lim_{n\to\infty} a_n = 0$.

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117. We first compute the fixed points. As in Example 14, we solve the equation $a = \frac{1}{3}\left(a + \frac{2}{a}\right)$, giving $a = 1$ or $a = -1$. Since a_0 is positive, succeeding terms are positive as well, so we guess that the limit is 1. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5	6
a_n	3	1.222	0.953	1.017	0.994	1.002	0.999

118. We first compute the fixed points. As in Example 14, we solve the equation $a = \frac{1}{2}\left(a + \frac{9}{a}\right)$ to find that $a = \frac{9}{a}$, or that $a = \pm 3$. Now when $a_0 = -1$, then $a_n < -1$ for all $n = 1, 2, 3, \ldots$ so we guess that $\lim_{n\to\infty} a_n = -3$. Evaluating a_n for various values of n supports that conclusion:

n	0	1	2	3	4	5
a_n	-1	-5	-3.4	-3.02353	-3.00009	-3.00000

■ 2.2.5

119. $\displaystyle\sum_{k=1}^{4} \sqrt{k} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}.$

120. $\displaystyle\sum_{k=3}^{5} (k-1)^2 = (3-1)^2 + (4-1)^2 + (5-1)^2 = 2^2 + 3^2 + 4^2.$

121. $\displaystyle\sum_{k=2}^{6} 3^k = 3^2 + 3^3 + 3^4 + 3^5 + 3^6.$

122. $\displaystyle\sum_{k=1}^{3} \frac{k^2}{k^2+1} = \frac{1^2}{1^2+1} + \frac{2^2}{2^2+1} + \frac{3^2}{3^2+1} = \frac{1}{2} + \frac{4}{5} + \frac{9}{10}.$

123. $\displaystyle\sum_{n=0}^{3} a_n = a_0 + a_1 + a_2 + a_3 = a_0 + 2a_0 + 4a_0 + 8a_0 = 1 + 2 + 4 + 8.$

124. $\displaystyle\sum_{n=0}^{4} a_n = a_0 + a_1 + a_2 + a_3 + a_4 = 2 + (2+2) + (2+2+2) + (2+2+2+2) + (2+2+2+2+2) = 2 + 4 + 6 + 8 + 10.$

125. $2 + 4 + 6 + 8 + \cdots + 2n = \displaystyle\sum_{k=1}^{n} 2k.$

126. $\dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} = \displaystyle\sum_{k=1}^{4} \frac{1}{\sqrt{k}}.$

127. $\ln 2 + \ln 3 + \ln 4 + \ln 5 = \displaystyle\sum_{k=2}^{5} \ln k.$

128. $\dfrac{3}{5} + \dfrac{4}{6} + \dfrac{5}{7} + \dfrac{6}{8} + \dfrac{7}{9} = \displaystyle\sum_{k=3}^{7} \frac{k}{k+2}$. Another possibility is $\displaystyle\sum_{k=1}^{5} \frac{k+2}{k+4}$.

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129. Note that each denominator is five greater than the corresponding numerator, so one way to write this is $-\frac{1}{4}+\frac{1}{6}+\frac{2}{7}+\frac{3}{8}=\sum_{k=-1}^{3}\frac{k}{k+5}$.

130. $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots+\frac{1}{2^n}=\sum_{k=0}^{n}\frac{1}{2^k}$.

131. $1+q+q^2+q^3+q^4+\cdots+q^{n-1}=\sum_{k=0}^{n-1}q^k$.

132. $1-a+a^2-a^3+a^4-a^5+\cdots+(-1)^n a^n=\sum_{k=0}^{n}(-1)^k a^k$.

2.3 Modeling with Recursion Relations

■ 2.3.1

1.

$$\begin{aligned} N_{t+1} = N_t + &\text{ number of cod fish born during the year} \\ - &\text{ number of cod fish dying of old age during the year} \\ - &\text{ number of cod fish killed by predators during the year} \\ - &\text{ number of cod fish removed by fishing boats during the year} \end{aligned}$$

2.

$$\begin{aligned} N_{t+1} = N_t + &\text{ number of children born during the year} \\ + &\text{ number of people moving into the town from other towns during the year} \\ - &\text{ number of people dying from any cause during the year} \\ - &\text{ number of people leaving the town to live in other towns during the year} \end{aligned}$$

3.

$$\begin{aligned} N_{t+1} = N_t + &\text{ number of kakapo births in the wild during the year} \\ + &\text{ number of kakapo reintroduced into the wild from captive breeding during a year} \\ - &\text{ number of kakapo removed for captive breeding during the year} \\ - &\text{ number of kakapo killed by predators during the year} \\ - &\text{ number of kakapo deaths from disease during the year} \end{aligned}$$

4.

$$\begin{aligned} N_{t+1} = N_t + &\text{ number of trees seeded in the wild during the year} \\ + &\text{ number of trees planted by people during the year} \\ - &\text{ number of trees killed by disease during the year} \\ - &\text{ number of trees cut down by loggers during the year} \end{aligned}$$

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5.

$$\begin{aligned} N_{t+1} = N_t + & \text{ area of reef restored or rebuilt during the year} \\ - & \text{ area killed by ocean acidification during the year} \\ - & \text{ area killed by fishing during the year} \end{aligned}$$

6. Note that the number of rhinos born in other parks during the year does not impact the population in the national park under consideration. Neither does the number of ill or injured rhinos, the number of female rhinos, or the number that become pregnant (though the latter two categories are certainly interesting for population consideration, they do not directly affect the population — only the number of births does).

$$\begin{aligned} N_{t+1} = N_t + & \text{ number of rhinos born in the park during the year} \\ + & \text{ rhinos introduced into the park from captive breeding programs during the year} \\ + & \text{ rhinos relocated to this park from other parks during the year} \\ - & \text{ rhinos moved out of this park to other parks during the year} \\ - & \text{ rhinos that die during the year} \end{aligned}$$

7. Since we are modeling the population of amœba, not the population of bacteria, the number of bacteria on the plate, the number that are eaten in one hour, and the number that divide into two cells in one hour are irrelevant to the model. They certainly affect the amœba population by modifying the food supply, but they are not direct inputs to the model.

$$\begin{aligned} N_{t+1} = N_t + & \text{ number of amœba cells that divide into two cells in one hour} \\ - & \text{ number of amœba that die in one hour} \end{aligned}$$

8. The total number of students on campus, and the number who are not sick, are not direct inputs to the model (although they do have an effect on the rate at which the disease spreads). The number of doctors or nurses is also not a direct input, though it too has an effect on the number of students with the flu.

$$\begin{aligned} N_{t+1} = N_t + & \text{ number of students who catch the flu} \\ - & \text{ number of students who recover from the flu in one day} \\ - & \text{ number of students who return home to recuperate from the flu in one day} \end{aligned}$$

9. (a) (i) If there are N_t individuals, then $N_t \cdot 0.5$ of those are female. Of those, one quarter lays an egg this year (since on average each female lays once every four years), so there are $N_t \cdot 0.5 \cdot 0.25$ eggs. Of those, only $29\% = 0.29$ survive the first year, so the number of births that survive the first year is $N_t \cdot 0.5 \cdot 0.25 \cdot 0.29 = 0.03625N_t$.

(ii) Since one in fifty will die in a given year, the deaths reduce the population by $\frac{1}{50}N_t = 0.02N_t$.

(iii) The recursion formula, from the above calculations, is $N_{t+1} = N_t + 0.03625N_t - 0.02N_t = 1.01625N_t$. In the first five years, the population is given by the table below:

t	0	1	2	3	4	5
N_t	50	50.8	51.6	52.5	53.3	54.2

Of course, in the real world, there would always be an integer number of individuals, but keeping the fractions during the computation prevents errors from building up too quickly.

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(iv) The formula in part (iii) represents exponential growth; since $N_0 = 50$ the formula is $N_t = 50 \cdot 1.01625^t$. The population reaches n individuals when

$$n = 50 \cdot 1.01625^t \quad \Rightarrow \quad \frac{n}{50} = 1.01625^t \quad \Rightarrow \quad \ln \frac{n}{50} = t \ln 1.01625 \quad \Rightarrow t = \frac{\ln {}^{n}/_{50}}{\ln 1.01625}.$$

So the population reaches $n = 100$ and $n = 200$ at the following times:

$$n = 100 \quad t = \frac{\ln {}^{100}/_{50}}{\ln 1.01625} = \frac{\ln 2}{\ln 1.01625} \approx 43 \text{ years}$$
$$n = 200 \quad t = \frac{\ln {}^{200}/_{50}}{\ln 1.01625} = \frac{\ln 4}{\ln 1.01625} \approx 86 \text{ years}$$

(b) (i) If strategy 1 is implemented, then the number of births surviving the first year becomes $N_t \cdot 0.5 \cdot 0.5 \cdot 0.29 = 0.0725N_t$, since on average half the females will lay each year. So the recurrence becomes $N_{t+1} = N_t + 0.0725N_t - 0.02N_t = 1.0525N_t$. The population in the next five years is as follows:

t	0	1	2	3	4	5
N_t	50	52.6	55.4	58.3	61.4	64.6

(ii) If strategy 2 is implemented, then the number of births surviving the first year becomes $N_t \cdot 0.5 \cdot 0.25 \cdot 0.75 = 0.093\,75N_t$, since three quarters of the eggs will survive their first year. So the recurrence becomes $N_{t+1} = N_t + 0.093\,75N_t - 0.02N_t = 1.073\,75N_t$. The population in the next five years is as follows:

t	0	1	2	3	4	5
N_t	50	53.7	57.6	61.9	66.5	71.4

(iii) Clearly the second strategy causes the population to increase more rapidly.

10. (a) (i) If there are N_t individuals, $50\% = 0.5$ of them are female, so there are $0.5 \cdot N_t$ females. Of those, $75\% = 0.75$ of them are females of reproductive age, so there are $0.75 \cdot 0.5 \cdot N_t$ females of reproductive age. Finally, of those, $22\% = 0.22$ will give birth in a given year, so there are $0.75 \cdot 0.5 \cdot 0.22 \cdot N_t = 0.0825N_t$ births each year.

(ii) $4.5\% = 0.045$ of the N_t individuals will die, so the number of deaths is $0.045N_t$.

(iii) From the previous parts, the recursion relation is $N_{t+1} = N_t + 0.0825N_t - 0.045N_t = 1.0375N_t$. This represents exponential growth; $N_t = N_0 \cdot 1.0375^t$; if there are 300 gorillas initially, the formula is $N_t = 300 \cdot 1.0375^t$.

(iv) Using the formula above, we get

t	0	1	2	5	10
N_t	300	311.2	322.9	360.6	433.5

(v) The population will double to 600 when $600 = 300 \cdot 1.0375^t$, or $1.0375^t = 2$. Taking natural logs and solving for t gives

$$t = \frac{\ln 2}{\ln 1.0375} \approx 18.8 \text{ years}.$$

(b) (i) Since the birth rate is 0.0825, in order for the population to remain stagnant, the death rate would have to be the same, or $0.0825 = 8.25\%$.

(ii) Since the number of deaths is $0.045N_t$, in order for the population to remain stagnant, the number of births would have to be the same, or $0.045N_t$. So if r is the birth rate for females of reproductive age, we must have $0.5 \cdot 0.25 \cdot r = 0.045$, or $r = 0.36 = 36\%$. The birth rate for females of reproductive age would have to be 36%.

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(iii) The recursion is $N_{t+1} = 1.0375N_t$; is r gorillas leave each year, the recursion would be $N_{t+1} = 1.0375N_t - r$. We want $N_{t+1} = N_t$, so solving $N_t = 1.0375N_t - r$ gives $r = 0.0375N_t$. Thus 3.75% of the gorillas would have to leave the park each year for the population to remain constant. Since there are 300 gorillas, this is $0.0375 \cdot 300 = 11.25$. So on average 11.25 gorillas would have to leave each year.

11. In this model, $R_0 = 4$ and $a = \frac{1}{30}$, so the fixed points are $N = 0$ and $N = \frac{R_0 - 1}{a} = \frac{3}{1/30} = 90$.

12. In this model, $R_0 = 2$ and $a = \frac{1}{60}$, so the fixed points are $N = 0$ and $N = \frac{R_0 - 1}{a} = \frac{1}{1/60} = 60$.

13. In this model, $R_0 = 2$ and $a = \frac{1}{90}$, so the fixed points are $N = 0$ and $N = \frac{R_0 - 1}{a} = \frac{1}{1/90} = 90$.

14. In this model, $R_0 = 3$ and $a = \frac{1}{100}$, so the fixed points are $N = 0$ and $N = \frac{R_0 - 1}{a} = \frac{2}{1/100} = 200$.

15. In this model, $R_0 = 3$ and $a = \frac{1}{30}$, so the fixed points are $N = 0$ and $N = \frac{R_0 - 1}{a} = \frac{2}{1/30} = 60$.

16. In this model, $R_0 = 5$ and $a = \frac{1}{240}$, so the fixed points are $N = 0$ and $N = \frac{R_0 - 1}{a} = \frac{4}{1/240} = 960$.

17. The population sizes are given by $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t = \frac{2}{1+0.01N_t}N_t$, $N_0 = 2$:

t	0	1	2	3	4	5
N_t	2	3.92	7.55	14.04	24.62	39.51

Since $N_0 > 0$, the limiting value is given by

$$\lim_{t\to\infty} N_t = \frac{R_0 - 1}{a} = 100.$$

18. The population sizes are given by $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t = \frac{2}{1+0.1N_t}N_t$, $N_0 = 2$:

t	0	1	2	3	4	5
N_t	2	3.33	5	6.67	8	8.89

Since $N_0 > 0$, the limiting value is given by

$$\lim_{t\to\infty} N_t = \frac{R_0 - 1}{a} = 10.$$

19. The population sizes are given by $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t = \frac{3}{1+1/20N_t}N_t$, $N_0 = 7$:

t	0	1	2	3	4	5
N_t	7	15.56	26.25	34.05	37.8	39.24

Since $N_0 > 0$, the limiting value is given by

$$\lim_{t\to\infty} N_t = \frac{R_0 - 1}{a} = 40.$$

20. The population sizes are given by $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t = \frac{3}{1+1/10N_t}N_t$, $N_0 = 3$:

t	0	1	2	3	4	5
N_t	3	6.92	12.27	16.53	18.69	19.54

Since $N_0 > 0$, the limiting value is given by

$$\lim_{t\to\infty} N_t = \frac{R_0 - 1}{a} = 20.$$

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21. The population sizes are given by $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t = \frac{4}{1+1/40N_t}N_t$, $N_0 = 2$:

t	0	1	2	3	4	5
N_t	2	7.62	25.6	62.44	97.52	113.46

Since $N_0 > 0$, the limiting value is given by

$$\lim_{t\to\infty} N_t = \frac{R_0 - 1}{a} = 120.$$

22. The population sizes are given by $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t = \frac{4}{1+1/60N_t}N_t$, $N_0 = 2$:

t	0	1	2	3	4	5
N_t	2	7.74	27.43	75.29	133.57	165.61

Since $N_0 > 0$, the limiting value is given by

$$\lim_{t\to\infty} N_t = \frac{R_0 - 1}{a} = 180.$$

23. Since $\lim_{t\to\infty} N_t = \frac{R_0-1}{a}$, we know that $100 = \frac{3-1}{a} = \frac{2}{a}$, so $a = 0.02$.

24. Since $\lim_{t\to\infty} N_t = \frac{R_0-1}{a}$, we know that $200 = \frac{5-1}{a} = \frac{4}{a}$, so $a = 0.02$.

25. Since $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t$, substituting the given values we get

$$30 = \frac{2}{1+20a}\cdot 20 = \frac{40}{1+20a} \quad \Rightarrow \quad 600a + 30 = 40 \quad \Rightarrow \quad a = \frac{1}{60}.$$

26. Since $N_{t+1} = R(N_t)N_t = \frac{R_0}{1+aN_t}N_t$, substituting the given values we get

$$40 = \frac{4}{1+50a}\cdot 50 = \frac{200}{1+50a} \quad \Rightarrow \quad 2000a + 40 = 200 \quad \Rightarrow \quad a = 0.08.$$

■ 2.3.3

27. From the discussion in the text, $x_{t+1} = R_0x_t(1-x_t) = x_t(1-x_t)$ with $R_0 = 1$. Since $x_t = \frac{N_t}{R_0/b} = \frac{N_tb}{R_0}$, we have $x_t = \frac{N_t 1/10}{1} = \frac{N_t}{10}$.

28. From the discussion in the text, $x_{t+1} = R_0x_t(1-x_t) = x_t(1-x_t)$ with $R_0 = 1$. Since $x_t = \frac{N_t}{R_0/b} = \frac{N_tb}{R_0}$, we have $x_t = \frac{N_t 1/20}{1} = \frac{N_t}{20}$.

29. From the discussion in the text, $x_{t+1} = R_0x_t(1-x_t) = 2x_t(1-x_t)$ with $R_0 = 2$. Since $x_t = \frac{N_t}{R_0/b} = \frac{N_tb}{R_0}$, we have $x_t = \frac{N_t 1/15}{2} = \frac{N_t}{30}$.

30. From the discussion in the text, $x_{t+1} = R_0x_t(1-x_t) = 2x_t(1-x_t)$ with $R_0 = 2$. Since $x_t = \frac{N_t}{R_0/b} = \frac{N_tb}{R_0}$, we have $x_t = \frac{N_t 1/20}{2} = \frac{N_t}{40}$.

31. From the discussion in the text, $x_{t+1} = R_0x_t(1-x_t) = 2.5x_t(1-x_t)$ with $R_0 = 2$. Since $x_t = \frac{N_t}{R_0/b} = \frac{N_tb}{R_0}$, we have $x_t = \frac{N_t 1/30}{2.5} = \frac{N_t}{75}$.

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32. From the discussion in the text, $x_{t+1} = R_0 x_t(1 - x_t) = 2.5x_t(1 - x_t)$ with $R_0 = 2$. Since $x_t = \frac{N_t}{R_0/b} = \frac{N_t b}{R_0}$, we have $x_t = \frac{N_t 1/50}{2.5} = \frac{N_t}{125}$.

33. A fixed point occurs when $N_{t+1} = N_t = N$, so we get $N = R_0 \cdot N - b \cdot N^2$, or $bN^2 + (1 - R_0)N = N(bN + 1 - R_0) = 0$. The fixed points are therefore $N = 0$ and $N = \frac{R_0 - 1}{b}$.

34. Since $N_t \to 50$, $N = 50$ is a fixed point, so that $50 = \frac{R_0 - 1}{b} = \frac{1}{b}$ and therefore $b = \frac{1}{50}$.

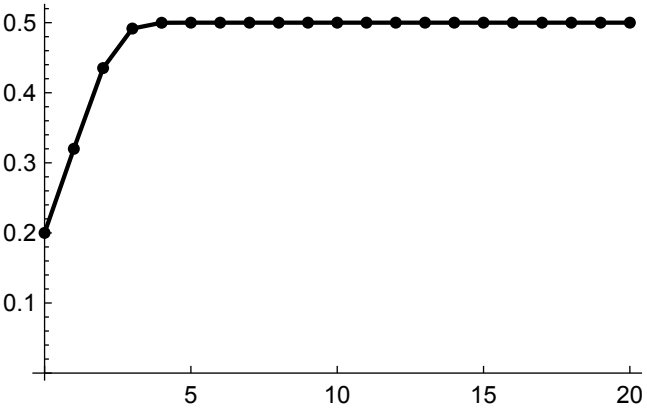
35. Since $N_t \to 40$, $N = 40$ is a fixed point, so that $40 = \frac{R_0 - 1}{b} = \frac{1.5}{b}$ and therefore $b = \frac{1.5}{40} = \frac{3}{80}$.

36. We have $N_{t+1} = R_0 \cdot N_t - b \cdot N_t^2$. Substituting the given values gives $15 = 2 \cdot 10 - b \cdot 10^2 = 20 - 100b$, so that $b = \frac{5}{100} = \frac{1}{20}$.

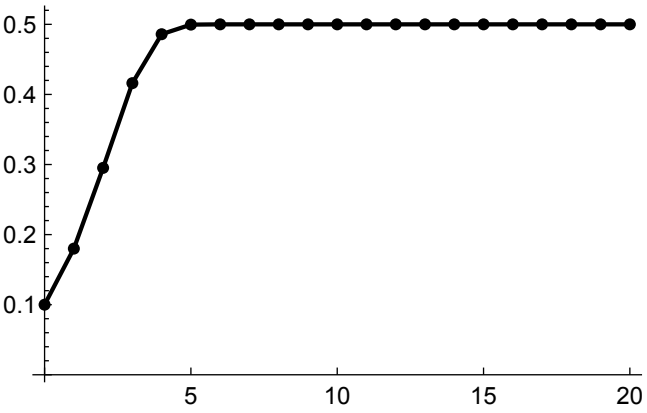
37. We have $N_{t+1} = R_0 \cdot N_t - b \cdot N_t^2$. Substituting the given values gives $30 = 3 \cdot 15 - b \cdot 15^2 = 45 - 225b$, so that $b = \frac{15}{225} = \frac{1}{15}$.

38. We have $N_{t+1} = R_0 \cdot N_t - b \cdot N_t^2$. Substituting the given values gives $20 = 15R_0 - \frac{1}{10} \cdot 15^2 = 15R_0 - 22.5$, so that $R_0 \approx 2.833$.

39.



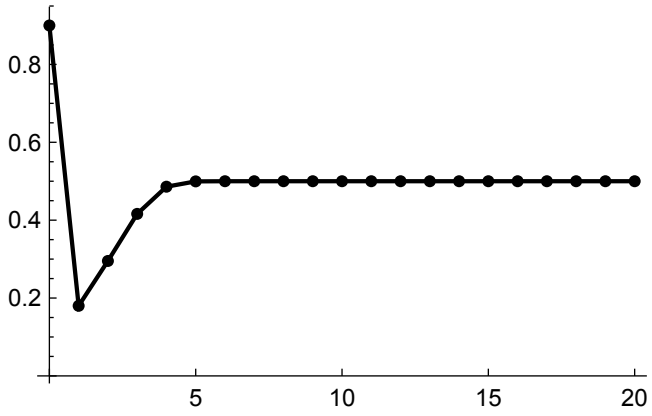
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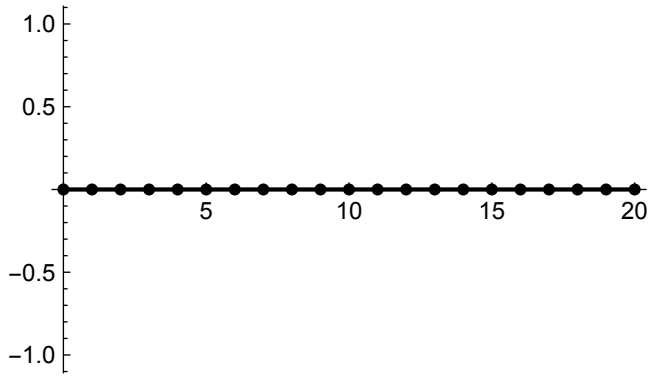
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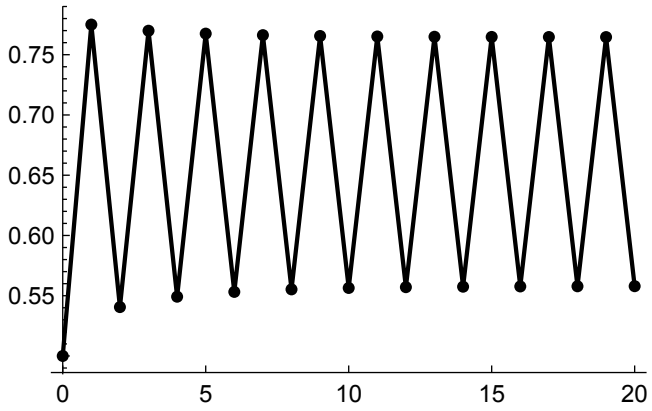
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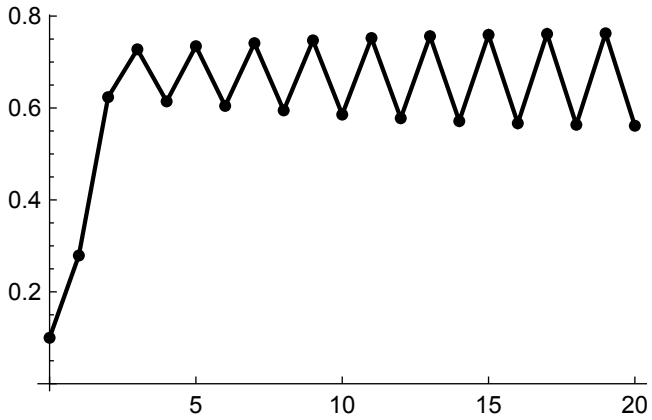
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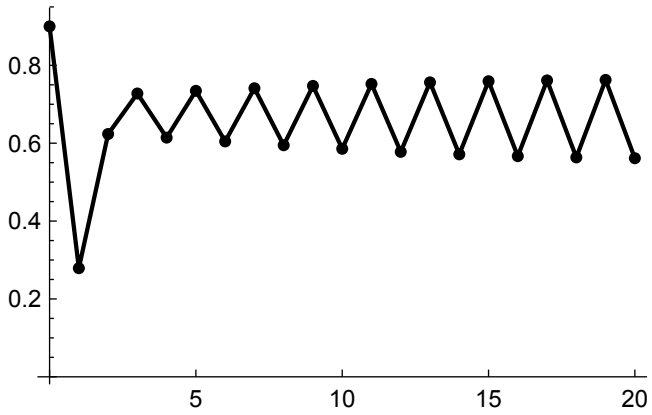
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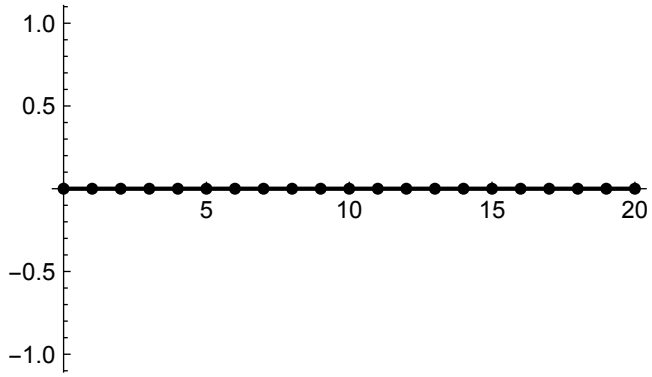
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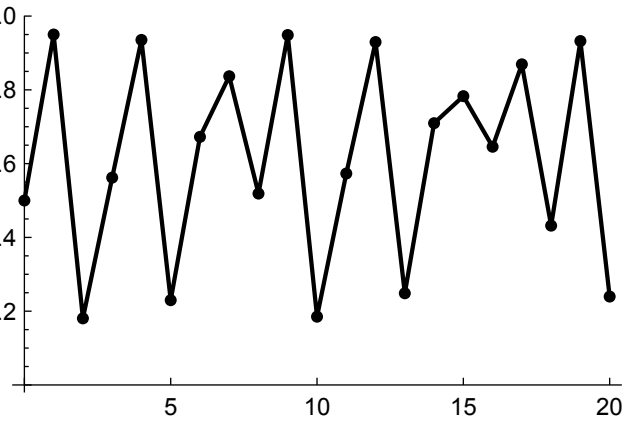
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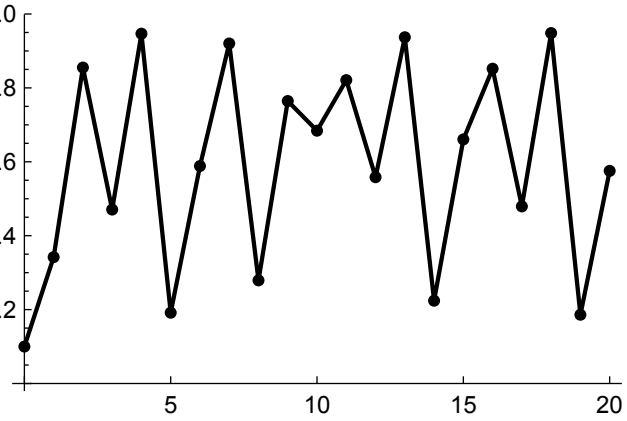
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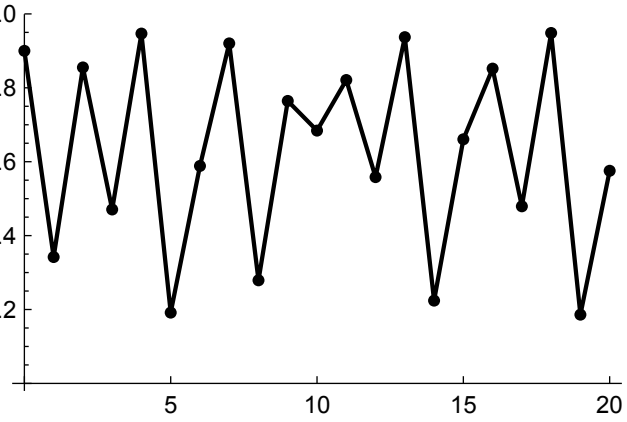
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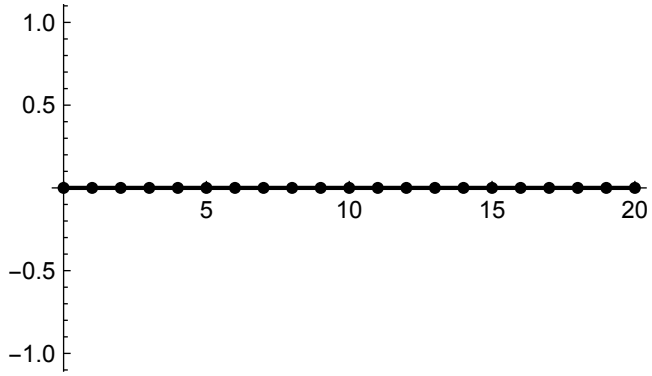
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50.



■ 2.3.4

51. **(a)** Since $7.7\% = 0.077$ of the drug is eliminated each hour, we have $C_{t+1} = C_t - 0.077C_t = 0.923C_t$.

(b) The recurrence relation expresses exponential decay: $C_t = C_0 \cdot 0.923^t$. Since $C_0 = 33.8\,\text{ng/ml}$, we have $C_t = 33.8 \cdot 0.923^t\,\text{ng/ml}$.

(c) We want to find t such that $C_t = 0.1$; then $0.1 = 33.8 \cdot 0.923^t$, so that $\frac{0.1}{33.8} = 0.923^t$. Taking natural logs of both sides and simplifying gives

$$t = \frac{\ln 0.1 - \ln 33.8}{\ln 0.923} \approx 72.67\,\text{hrs}.$$

52. **(a)** Given the absorption rate, and since $10\% = 0.1$ of the drug is eliminated each hour, we have

$$a_{t+1} = a_t + 10 \cdot 0.4^t - 0.1a_t = 0.9a_t + 10 \cdot 0.4^t.$$

(b) Starting with $a_0 = 0$ we get

t	0	1	2	3	4	5	6
a_t	0	10	13	13.3	12.61	11.61	10.55

(c) The maximum amount of drug is ≈ 6.38, at $t = 5$.

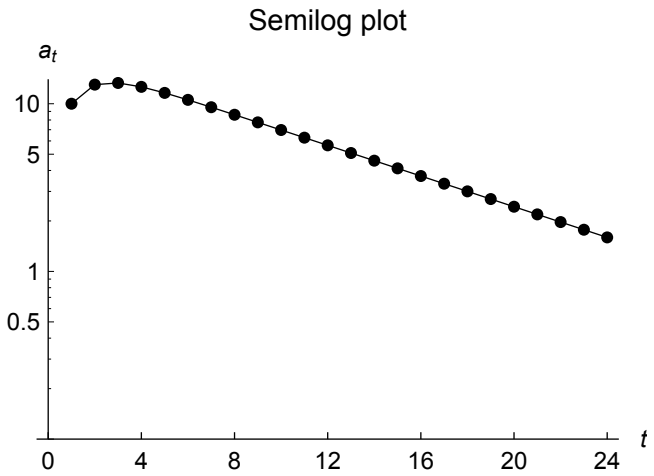
(d) Continuing the table from part (b) through $t = 24$ gives

t	7	8	9	10	11	12	13	14	15
a_t	9.53	8.60	7.74	6.97	6.28	5.65	5.08	4.58	4.12
t	16	17	18	19	20	21	22	23	24
a_t	3.71	3.34	3.00	2.70	2.43	2.19	1.97	1.77	1.60

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(e) Plotting a_t against t using a semilog plot gives the graph below:



For t larger than about 4, the graph is roughly linear, indicating exponential decay.

53. **(a)** Given the absorption rate, and since $40\% = 0.4$ of the drug is eliminated each hour, we have

$$a_{t+1} = a_t + 20 \cdot 0.2^t - 0.4a_t = 0.6a_t + 20 \cdot 0.2^t.$$

(b) Starting with $a_0 = 0$ we get

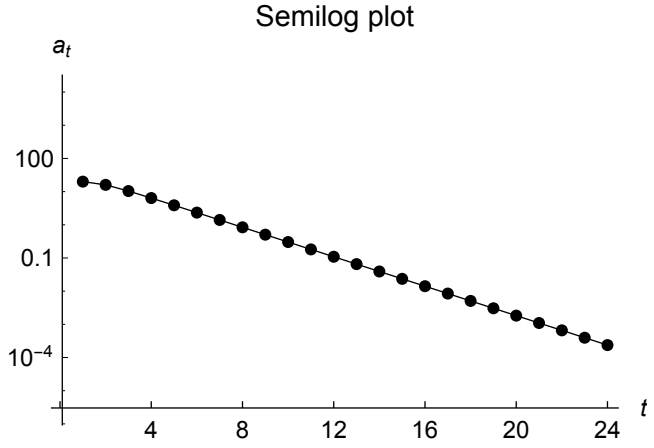
t	0	1	2	3	4	5	6
a_t	0	20.00	16.00	10.40	6.40	3.87	2.33

(c) The maximum amount of drug is 20, at $t = 1$.

(d) Continuing the table from part (b) through $t = 24$ gives (to three decimal places)

t	7	8	9	10	11	12	13	14	15
a_t	1.40	0.84	0.50	0.30	0.18	0.11	0.07	0.04	0.02
t	16	17	18	19	20	21	22	23	24
a_t	0.01	0.01	0.01	0	0	0	0	0	0

(e) Plotting a_t against t using a semilog plot gives the graph below. For t larger than about 1, the graph is roughly linear, indicating exponential decay.



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54. **(a)** Since the drug has zeroth order elimination kinetics, the amount removed each hour is $20 - 14\,\text{mg} = 6\,\text{mg}$.

(b) Each hour, 6 mg is removed, so we get $a_{t+1} = a_t - 6$.

(c) Since 6 mg is removed each hour, after t hours we have removed $6t$ mg, so the explicit formula is $a_t = 20 - 6t$.

(d) The amount of drug present drops to zero when $20 - 6t = 0$, so when $t = \frac{10}{3} \approx 3.33$ hours.

55. **(a)** The percentage removed between $t = 0$ and $t = 1$ is $\frac{20-14}{20} = \frac{6}{20} = 0.3 = 30\%$. Since the drug has first order elimination kinetics, this percentage is removed each hour.

(b) $a_{t+1} = a_t - 0.3a_t = 0.7a_t$.

(c) The recursion relation expresses exponential decay, so $a_t = a_0 \cdot 0.7^t = 20 \cdot 0.7^t$.

(d) Since $0.7^t > 0$ for all values of t, the amount of drug present will never be zero according to the model.

56. **(a)** If the drug has zeroth order kinetics, the amount eliminated each hour is $40 - 32 = 8\,\text{mg/ml}$.

(b) Since 8 mg/ml is eliminated each hour, we have $c_{t+1} = c_t - 8$, so c_2 should be $c_1 - 8 = 24$.

(c) If the drug has first order kinetics, the percentage eliminated each hour is $\frac{40-32}{40} = 0.2 = 20\%$.

(d) Since 20% is eliminated each hour, we get $c_{t+1} = c_t - 0.2c_t = 0.8c_t$. Then we would expect $c_2 = 0.8c_1 = 25.6$.

(e) Since the amount found matches the amount in part (d), we conclude that the drug has first order kinetics.

57. **(a)** If the drug has zeroth order kinetics, the amount eliminated each hour is $50 - 35 = 15\,\text{mg/ml}$.

(b) Since 15 mg/ml is eliminated each hour, we have $c_{t+1} = c_t - 15$, so c_2 should be $c_1 - 15 = 20$.

(c) If the drug has first order kinetics, the percentage eliminated each hour is $\frac{50-35}{50} = 0.3 = 30\%$.

(d) Since 30% is eliminated each hour, we get $c_{t+1} = c_t - 0.3c_t = 0.7c_t$. Then we would expect $c_2 = 0.7c_1 = 24.5$.

(e) Since the amount found matches the amount in part (b), we conclude that the drug has zeroth order kinetics.

58. **(a)** Since 23% of the drug is eliminated each hour, we have $c_{t+1} = c_t - 0.23c_t = 0.77c_t$.

(b) This recursion expresses exponential decay; since the initial concentration is $15\,\mu\text{g/ml}$ at $t = 1$, the formula is $c_t = 15 \cdot 0.77^{t-1}$.

(c) From the formula in part (b), $c_4 = 15 \cdot 0.77^3 \approx 6.85\,\mu\text{g/ml}$.

(d) During that hour, $15\,\mu\text{g/ml}$ enter the bloodstream, and 0.23 of the amount present at time $t = 4$ leaves the bloodstream, so the amount in the bloodstream at $t = 5$ is

$$c_5 = c_4 + 15 - 0.23c_4 = 6.85 + 15 - 1.58 \approx 20.3\,\mu\text{g/ml}.$$

(e) Using the recursion relation $c_{t+1} = 0.77c_t$, we get

$$\begin{aligned} c_6 &= 0.77c_5 = 0.77 \cdot 20.3 \approx 15.63 \\ c_7 &= 0.77c_6 = 0.77 \cdot 15.63 \approx 12.04 \\ c_8 &= 0.77c_7 = 0.77 \cdot 12.04 \approx 9.27. \end{aligned}$$

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(f) During the ninth hour, $15\,\mu$g/ml enters the bloodstream, and $0.23c_8$ leaves, so the amount at $t = 9$ is

$$c_9 = c_8 + 15 - 0.23c_8 = 9.27 + 15 - 2.13 \approx 22.14\,\mu\text{g/ml}.$$

(g) One hour after the n^{th} pill is taken, the amount in the bloodstream is 0.77 times the amount when the pill was taken (accounting for elimination) plus $15\,\mu$g/ml. The amount in the bloodstream when the pill was taken is 0.77^3 times the amount one hour after the previous pill was taken, since three hours elapse. So the amount is the bloodstream one hour after the n^{th} pill is taken is

$$C_n = 0.77^3 C_{n-1} \cdot 0.77 + 15 = 0.77^4 C_{n-1} + 15 \approx 15 + 0.35C_{n-1}.$$

(h) Since pills are taken at $t = 1, 5, 9, \ldots$, C_1 corresponds to $t = 1$, C_2 to $t = 5$, C_3 to $t = 9$, and in general C_n corresponds to t_{4n-3}.

(i) See the discussion in part (g) for the recursion. $c_1 = 15$ since that is the concentration one hour after the first pill was taken.

(j) Using the recursion from part (i), we get

t	1	2	3	4	5	6
C_t	15	20.273	22.127	22.778	23.007	23.088

(k) The amount of increase in each C_n appears to be decreasing, so it seems likely that C_n converges.

(l) A fixed point of the recursion relation is found by solving $C = 0.77^4 C + 15 = 0.35C + 15$, so that $0.65C = 15$ and $C \approx 23.1\,\mu$g/ml.

59. Since the absolute amount by which the concentration decreases changes each hour (it decreases), this cannot be zeroth order kinetics, so it must be first order kinetics. Indeed, computing the percentage decrease each hour we get

$$\frac{16-12}{16} = 0.25, \quad \frac{12-9}{12} = 0.25, \quad \frac{9-6.75}{9} = 0.25,$$

so that 25% of the drug is eliminated each hour.

60. Since $2\,\mu$g/ml is eliminated each hour, this drug has zeroth order kinetics.

61. Since $4\,\mu$g/ml is eliminated each hour, this drug has zeroth order kinetics.

62. Since the absolute amount by which the concentration decreases changes each hour (it decreases), this cannot be zeroth order kinetics, so it must be first order kinetics. Indeed, computing the percentage decrease each hour we get

$$\frac{40-36}{40} = 0.1, \quad \frac{36-32.4}{36} = 0.1, \quad \frac{32.4-29.16}{32.4} = 0.1,$$

so that 10% of the drug is eliminated each hour.

63. **(a)** $a_{t+1} = a_t +$ amount added to the blood in t^{th} day $-$ amount eliminated in t^{th} day.

(b) The amount added each day is $20\,\mu$g, and the amount removed each day is 4% of the amount present (at the start of the day). Thus

$$a_{t+1} = a_t + 20 - 0.04a_t = 0.96a_t + 20.$$

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(c) Applying the recursion we get

t	0	1	2	3	4	5	6
a_t	0	20	39.2	57.632	75.327	92.314	108.621

(d) Solving $a = 0.96a + 20$ gives $0.04a = 20$, so that $a = 500\,\mu$g.

64. **(a)** $a_{t+1} = a_t +$ amount added to the blood in t^{th} day $-$ amount eliminated in t^{th} day.

(b) On the t^{th} day, $10t + 10$ mg is added, and each day half of the existing amount is removed; substituting this information gives

$$a_{t+1} = a_t + 10t + 10 - 0.5a_t = 0.5a_t + 10(t+1) + 10\,\text{mg}.$$

(c) Applying the recursion we get

t	0	1	2	3	4	5
a_t	10	25	42.5	61.25	80.625	100.312

(d) a_t reaches (and barely exceeds) 100 at the start of the sixth day.

Chapter 2 Review

1. The expression 2^{-n} can also be written as $\left(\frac{1}{2}\right)^n$. Thus, from Example 12 in Section 2.2.2, we conclude that

$$\lim_{n\to\infty} 2^{-n} = \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$

2. For successive values of n, the values $1, 3, 9, 27, 81, 243, \ldots$ of 3^n indicate that the terms continue to grow. Thus 3^n goes to infinity as $n \to \infty$, and we can write $\lim_{n\to\infty} 3^n = \infty$. This can also be seen using Example 12 in Section 2.2.2, from which we conclude that since $R = 3 > 1$,

$$\lim_{n\to\infty} 3^n = \infty.$$

3. The expression $40(1 - 4^{-n})$ can be written as $40 - 40\left(\frac{1}{4}\right)^n$. From Example 12 in Section 2.2.2, we know that $\lim_{n\to\infty}\left(\frac{1}{4}\right)^n = 0$. Also, it is obvious that $\lim_{n\to\infty} 40 = 40$. Thus

$$\lim_{n\to\infty} 40(1 - 4^{-n}) = \lim_{n\to\infty} 40 - 40 \lim_{n\to\infty} \left(\frac{1}{4}\right)^n = 40 - 40 \cdot 0 = 40.$$

4. The expression $\frac{2}{1+2^{-n}}$ can be written as $\frac{2}{1+(1/2)^n}$. From Example 12 in Section 2.2.2, we know that $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n$ exists and is equal to 0. Thus

$$\lim_{n\to\infty} \frac{2}{1 - 2^{-n}} = \frac{\lim_{n\to\infty} 2}{\lim_{n\to\infty}(1 - (1/2)^n)} = \frac{\lim_{n\to\infty} 2}{\lim_{n\to\infty} 1 - \lim_{n\to\infty}(1/2)^n} = \frac{2}{1-0} = 2.$$

5. Example 12 in Section 2.2 states without proof that this limit is ∞. To see that this is true, choose any real number $K > 0$, and let $x = a - 1$. Since $a > 1$, we see that $x > 0$, so there is some positive integer n such that $nx > K - 1$. Now, using the binomial expansion of $(1 + x)^n$, we get

$$a^n = (1 + x)^n \geq 1 + nx > 1 + (K - 1) = K,$$

so that $a^n > K$. This proves that no matter what real number K we choose, we can find an n such that $a^n > K$, so that a^n grows without bound as $n \to \infty$, and thus $\lim_{n\to\infty} a_n = \infty$.

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6. Let $b = \frac{1}{a}$. Since $0 < a < 1$, we have $b > 1$, so that $\lim_{n\to\infty} a^n = \lim_{n\to\infty} \frac{1}{b^n}$. By the previous exercise, $\lim_{n\to\infty} b^n = \infty$, so that by the limit rules $\lim_{n\to\infty} a^n = 0$.

7. Note that the expression $\frac{n(n+1)}{n^2-1}$ can be written as $\frac{n(n+1)}{(n+1)(n-1)}$, which is the same as $\frac{n}{n-1}$ if $n \neq -1$. Since we are interested in the behavior of a_n as $n \to \infty$, we may assume that $n \neq -1$. Since $\lim_{n\to\infty} \frac{1}{n} = 0$ we have

$$\lim_{n\to\infty} \frac{n(n+1)}{n^2-1} = \lim_{n\to\infty} \frac{n(n+1)}{(n-1)(n+1)} = \lim_{n\to\infty} \frac{n}{n-1} = \lim_{n\to\infty} \frac{1}{1-1/n}$$
$$= \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} 1 - \lim_{n\to\infty} 1/n} = \frac{1}{1-0} = 1.$$

8. Note that the expression $\frac{n^2+n-6}{n-2}$ can be written as $\frac{(n-2)(n+3)}{n-2}$, which is the same as $n+3$ if $n \neq 2$. Since we are interested in the behavior of a_n as $n \to \infty$, we may assume that $n > 2$. Recalling that $\lim_{n\to\infty} n$ does not exist, since successive values of n grow without bound, we have

$$\lim_{n\to\infty} \frac{n^2+n-6}{n-2} = \lim_{n\to\infty} \frac{(n-2)(n+3)}{n-2} = \lim_{n\to\infty} (n+3) = \lim_{n\to\infty} n + \lim_{n\to\infty} 3 = \infty.$$

9. Note that by dividing numerator and denominator by n, the expression $\frac{\sqrt{n}}{n+1}$ can be written as $\frac{1/\sqrt{n}}{1+1/n}$. Recalling that both $\lim_{n\to\infty} 1/n$ and $\lim_{n\to\infty} 1/\sqrt{n}$ exist and are equal to 0, we have

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = \lim_{n\to\infty} \frac{1/\sqrt{n}}{1+1/n} = \frac{\lim_{n\to\infty} 1/\sqrt{n}}{\lim_{n\to\infty}(1+1/n)} = \frac{0}{1+0} = 0.$$

10. We first note that the expression $\frac{n+1}{\sqrt{n}}$ can be written as $\sqrt{n} + \frac{1}{\sqrt{n}}$. Now, $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$, but the successive terms $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \ldots$ of $\sqrt{n}$ are clearly growing without bound. That is, $\lim_{n\to\infty} \sqrt{n} = \infty$. Then

$$\lim_{n\to\infty} \frac{n+1}{\sqrt{n}} = \lim_{n\to\infty} \left(\sqrt{n} + \frac{1}{\sqrt{n}}\right) = \lim_{n\to\infty} \sqrt{n} + \lim_{n\to\infty} \left(\frac{1}{\sqrt{n}}\right) = \lim_{n\to\infty} \sqrt{n} + 0 = \lim_{n\to\infty} \sqrt{n} = \infty.$$

11. Looking at the sequence, we can guess the next terms, namely, $\frac{11}{12}$, $\frac{13}{14}$, $\frac{15}{16}$, $\frac{17}{18}$, $\frac{19}{20}$ and so on. We thus find

$$a_n = \frac{2n+1}{2n+2} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

12. Note that the numerator of a_n is the sum of the first $n+1$ even numbers, starting with 2, so it is twice the sum of the first $n+1$ positive integers; this sum is $2 \cdot \frac{(n+1)(n+2)}{2} = (n+1)(n+2)$. The denominators are powers of two. So we can guess that the sequence is

$$a_n = \frac{(n+1)(n+2)}{2^{n+1}} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

So the next few terms of the sequence would be $\frac{42}{64}$, $\frac{56}{128}$, $\frac{72}{256}$, $\frac{90}{512}$, and $\frac{110}{1024}$.

13. The numerator of a_n appears to be $n+1$, while the denominator is $(n+1)^2+1$, so we can guess that the sequence is

$$a_n = \frac{n+1}{(n+1)^2+1} = \frac{n+1}{n^2+2n+2} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

So the next few terms of the sequence would be $\frac{6}{37}$, $\frac{7}{50}$, $\frac{8}{65}$, $\frac{9}{82}$, and $\frac{10}{101}$.

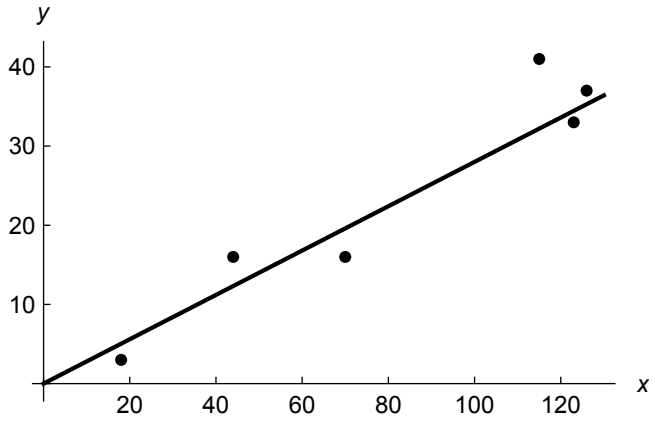
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14. Looking at the sequence, we can guess the next terms, namely, $\frac{5}{7}$, $\frac{6}{8}$, $\frac{7}{9}$, $\frac{8}{10}$,, $\frac{9}{11}$ and so on. We thus find

$$a_n = \frac{n}{n+2} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

15. **(a)** **(i)** A plot of the points together with a plot of the number of pups born as a function of the population size N_t is below:



The line is a reasonably good approximation to the data.

(ii) The number of wolves that die is $0.22N_t$, so putting that together with the approximation from part (i), we get

$$N_{t+1} = N_t + 0.28N_t - 0.22N_t = 1.06N_t.$$

(iii) Applying the recursion we get

t	0	1	2	3	4	5	6	7	8	9	10
N_t	130	137.8	146.1	154.8	164.1	174.	184.4	195.5	207.2	219.6	232.8

(iv) According to the table above, the population size of 220 will be reached some time in the tenth year.

(b) **(i)** The population increases by the number of pups born, which is unchanged at $0.28N_t$, and also by the number of new wolves introduced, which is r. It decreases by the death rate, which is now only $0.30N_t$. Putting this together gives

$$N_{t+1} = N_t + 0.28 \cdot N_t + r - 0.30 \cdot N_t = 0.98N_t + r.$$

(ii) For $r = 5$, $N_0 = 130$, applying the recursion gives

t	0	1	2	3	4	5	6	7	8	9	10
N_t	130	132.4	134.8	137.1	139.3	141.5	143.7	145.8	147.9	150	152

If we use $r = 10$, we get

t	0	1	2	3	4	5	6	7	8	9	10
N_t	130	137.4	144.7	151.8	158.7	165.5	172.2	178.8	185.2	191.5	197.7

(c) Assuming the original recursion relation, from part (a), but adding $r = 10$ pups each year, we get for a recursion

$$N_{t+1} = 1.06N_t + 10.$$

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Computing this relation for $N_0 = 130$ gives

t	0	1	2	3	4	5	6	7	8	9	10
N_t	130	147.8	166.7	186.7	207.9	230.3	254.2	279.4	306.2	334.5	364.6

A population of 220 is reached under this strategy in the fifth year. However, the population at the start of the fifth year in part (b) is 165.5, which is farther from 220 than the population of 230.3 in this part; we expect 220 to be reached sooner using the strategy in this part.

16. **(a)** $N_2 = N_1 + N_0 = 1 + 1 = 2$.

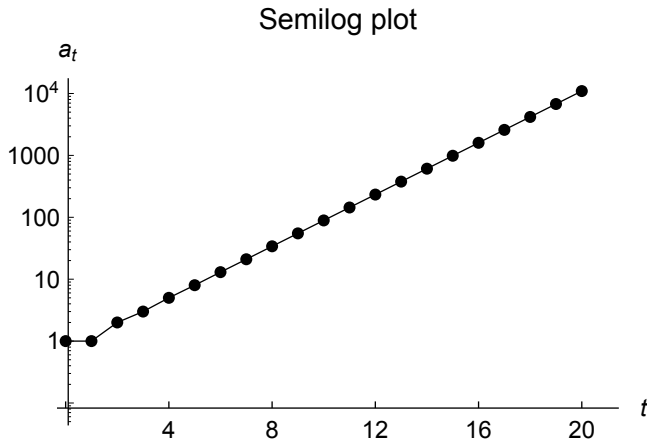
(b) $N_3 = N_2 + N_1 = 2 + 1 = 3$, $N_4 = N_3 + N_2 = 3 + 2 = 5$, $N_5 = N_4 + N_3 = 5 + 3 = 8$, and $N_6 = N_5 + N_4 = 8 + 5 = 13$.

(c) Values of N_k for k up to 20 are below:

k	0	1	2	3	4	5	6	7	8	9	10
N_k	1	1	2	3	5	8	13	21	34	55	89

k	11	12	13	14	15	16	17	18	19	20
N_k	144	233	377	610	987	1597	2584	4181	6765	10946

(d) The required semilog plot is below:



The points are approximately linear, so the Fibonacci sequence appears to grow exponentially.

17. **(a)** Calculating the recurrence gives

k	0	1	2	3	4
N_k	10	19.5	37.1	67.3	112

(b) Fixed points for the relation are given by solving $N = 2N - \frac{1}{200}N^2$, or $N^2 - 200N = 0$. This gives $N = 0$ or $N = 200$; since the population is increasing, we see that $\lim_{t\to\infty} N_t = 200$.

(c) We need to find fixed points of $N = 2N - \frac{1}{200}N^2 - pN$. Simplifying gives

$$N^2 + 200(p-1)N = 0 \quad \to \quad N(N + 200(p-1)) = 0 \quad \to \quad N = 0 \text{ or } N = 200(1-p).$$

The limiting population is $200(1-p)$.

(d) Since the observed population limit is 160, solving $160 = 200(1-p)$ gives $1 - p = 0.8$, so that $p = 0.2$.

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18. **(a)** Since 42% of the drug in the gut leaves the gut (into the blood), we get a recursion relation $a_{t+1} = a_t - 0.42a_t = 0.58a_t$, $a_0 = 10$.

(b) The recursion expresses exponential decay with an initial value of 10: $a_t = 10 \cdot 0.58^t$.

(c) Solving $10 \cdot 0.58^t = 0.01 \cdot 10 = 0.1$ gives $0.58^t = 0.001$; taking logs and simplifying gives $t = \frac{\ln 0.001}{\ln 0.58} \approx 12.7$ hours.

(d) The amount of drug entering the blood at time t is $0.42a_t$, and the amount leaving is the amount eliminated, which is $0.06b_t$, so

$$b_{t+1} = b_t + 0.42a_t - 0.06b_t = 0.94b_t + 0.42a_t, \quad b_0 = 0.$$

(e) Substituting the explicit formula for a_t into the above, we get

$$b_{t+1} = 0.94b_t + 4.2 \cdot 0.58^t, \quad b_0 = 0.$$

Calculating values gives

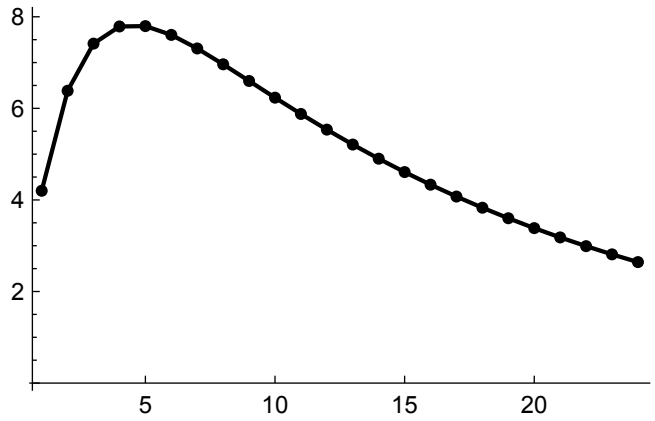
k	0	1	2	3	4	5	6
b_k	0	4.20	6.38	7.41	7.79	7.80	7.60

The maximum amount of drug in the blood is about 7.80, in the fifth or sixth hour.

(f) Continuing the table from the previous part, we get

k	7	8	9	10	11	12	13	14	15
b_k	7.31	6.96	6.60	6.23	5.88	5.54	5.21	4.90	4.61
k	16	17	18	19	20	21	22	23	24
b_k	4.33	4.07	3.83	3.60	3.38	3.18	2.99	2.81	2.64

A plot of these values is below:



(g) **(i)** If the drug immediately entered the bloodstream, then no more would come in after the initial bolus, but each hour 6% of it would be eliminated, so that we would have $b_{t+1} = b_t - 0.06b_t = 0.94b_t$.

(ii) The model above is exponential decay with initial value 10, so we have $b_t = 10 \cdot 0.94^t$. Setting $t = 24$ gives $b_{24} \approx 2.265$ mg.

(iii) The amount in the blood 24 hours later is lower with immediate absorption.

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