Linear Algebra with Applications 10th edition Leon Solutions Manual

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INSTRUCTOR'S SOLUTIONS MANUAL

LINEAR ALGEBRA WITH APPLICATIONS

TENTH EDITION

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Preface

This solutions manual is designed to accompany the tenth edition of *Linear Algebra with Applications* by Steven J. Leon and Lisette de Pillis. The manual contains the complete solutions to all of the nonroutine exercises and Chapter test questions in the first seven chapters the book. Each of those chapters also includes a set of MATLAB computer exercises. Most of the MATLAB computations are straightforward. and consequently the computational results are not included in this manual. However, the MATLAB Exercises also include questions related to the computations. The purpose of the questions is to emphasize the significance of the computations. This manual does provide the answers to most of these questions.

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Chapter 1

Matrices and Systems of Equations

1 SYSTEMS OF LINEAR EQUATIONS

2. (d) $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & -2 & 1 \\ 0 & 0 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

5. (a) $\begin{aligned} 3x_1 + 2x_2 &= 8 \\ x_1 + 5x_2 &= 7 \end{aligned}$

(b) $\begin{aligned} 5x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 + 3x_2 - 4x_3 &= 0 \end{aligned}$

(c) $\begin{aligned} 2x_1 + x_2 + 4x_3 &= -1 \\ 4x_1 - 2x_2 + 3x_3 &= 4 \\ 5x_1 + 2x_2 + 6x_2 &= -1 \end{aligned}$

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(d) $\begin{aligned} 4x_1 - 3x_2 + \ x_3 + 2x_4 &= 4 \\ 3x_1 + \ x_2 - 5x_3 + 6x_4 &= 5 \\ x_1 + \ x_2 + 2x_3 + 4x_4 &= 8 \\ 5x_1 + \ x_2 + 3x_3 - 2x_4 &= 7 \end{aligned}$

9. Given the system

$$\begin{aligned} -m_1x_1 + x_2 &= b_1 \\ -m_2x_1 + x_2 &= b_2 \end{aligned}$$

one can eliminate the variable x_2 by subtracting the first row from the second. One then obtains the equivalent system

$$\begin{aligned} -m_1x_1 + x_2 &= b_1 \\ (m_1 - m_2)x_1 &= b_2 - b_1 \end{aligned}$$

(a) If $m_1 \neq m_2$, then one can solve the second equation for x_1

$$x_1 = \frac{b_2 - b_1}{m_1 - m_2}$$

One can then plug this value of x_1 into the first equation and solve for x_2. Thus, if $m_1 \neq m_2$, there will be a unique ordered pair (x_1, x_2) that satisfies the two equations.

(b) If $m_1 = m_2$, then the x_1 term drops out in the second equation

$$0 = b_2 - b_1$$

This is possible if and only if $b_1 = b_2$.

(c) If $m_1 \neq m_2$, then the two equations represent lines in the plane with different slopes. Two nonparallel lines intersect in a point. That point will be the unique solution to the system. If $m_1 = m_2$ and $b_1 = b_2$, then both equations represent the same line and consequently every point on that line will satisfy both equations. If $m_1 = m_2$ and $b_1 \neq b_2$, then the equations represent parallel lines. Since parallel lines do not intersect, there is no point on both lines and hence no solution to the system.

10. The system must be consistent since $(0, 0)$ is a solution.

11. A linear equation in 3 unknowns represents a plane in three space. The solution set to a 3×3 linear system would be the set of all points that lie on all three planes. If the planes are parallel or one plane is parallel to the line of intersection of the other two, then the solution set will be empty. The three equations could represent the same plane or the three planes could all intersect in a line. In either case the solution set will contain infinitely many points. If the three planes intersect in a point, then the solution set will contain only that point.

2 ROW ECHELON FORM

2. (b) The system is consistent with a unique solution $(4, -1)$.

4. (b) x_1 and x_3 are lead variables and x_2 is a free variable.
(d) x_1 and x_3 are lead variables and x_2 and x_4 are free variables.
(f) x_2 and x_3 are lead variables and x_1 is a free variable.

5. (l) The solution is $(0, -1.5, -3.5)$.

6. (c) The solution set consists of all ordered triples of the form $(0, -\alpha, \alpha)$.

7. A homogeneous linear equation in 3 unknowns corresponds to a plane that passes through the origin in 3-space. Two such equations would correspond to two planes through the origin. If one equation is a multiple of the other, then both represent the same plane through the origin and every point on that plane will be a solution to the system. If one equation is not a multiple of the other, then we have two distinct planes that intersect in a line through the

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origin. Every point on the line of intersection will be a solution to the linear system. So in either case the system must have infinitely many solutions.

In the case of a nonhomogeneous 2×3 linear system, the equations correspond to planes that do not both pass through the origin. If one equation is a multiple of the other, then both represent the same plane and there are infinitely many solutions. If the equations represent planes that are parallel, then they do not intersect and hence the system will not have any solutions. If the equations represent distinct planes that are not parallel, then they must intersect in a line and hence there will be infinitely many solutions. So the only possibilities for a nonhomogeneous 2×3 linear system are 0 or infinitely many solutions.

9. (a) Since the system is homogeneous it must be consistent.

13. A homogeneous system is always consistent since it has the trivial solution $(0, \ldots, 0)$. If the reduced row echelon form of the coefficient matrix involves free variables, then there will be infinitely many solutions. If there are no free variables, then the trivial solution will be the only solution.

14. A nonhomogeneous system could be inconsistent in which case there would be no solutions. If the system is consistent and underdetermined, then there will be free variables and this would imply that we will have infinitely many solutions.

16. At each intersection, the number of vehicles entering must equal the number of vehicles leaving in order for the traffic to flow. This condition leads to the following system of equations

$$\begin{aligned} x_1 + a_1 &= x_2 + b_1 \\ x_2 + a_2 &= x_3 + b_2 \\ x_3 + a_3 &= x_4 + b_3 \\ x_4 + a_4 &= x_1 + b_4 \end{aligned}$$

If we add all four equations, we get

$$x_1 + x_2 + x_3 + x_4 + a_1 + a_2 + a_3 + a_4 = x_1 + x_2 + x_3 + x_4 + b_1 + b_2 + b_3 + b_4$$

and hence

$$a_1 + a_2 + a_3 + a_4 = b_1 + b_2 + b_3 + b_4$$

17. If (c_1, c_2) is a solution, then

$$\begin{aligned} a_{11}c_1 + a_{12}c_2 &= 0 \\ a_{21}c_1 + a_{22}c_2 &= 0 \end{aligned}$$

Multiplying both equations through by α, one obtains

$$\begin{aligned} a_{11}(\alpha c_1) + a_{12}(\alpha c_2) &= \alpha \cdot 0 = 0 \\ a_{21}(\alpha c_1) + a_{22}(\alpha c_2) &= \alpha \cdot 0 = 0 \end{aligned}$$

Thus $(\alpha c_1, \alpha c_2)$ is also a solution.

18. (a) If $x_4 = 0$, then x_1, x_2, and x_3 will all be 0. Thus if no glucose is produced, then there is no reaction. $(0, 0, 0, 0)$ is the trivial solution in the sense that if there are no molecules of carbon dioxide and water, then there will be no reaction.
(b) If we choose another value of x_4, say $x_4 = 2$, then we end up with solution $x_1 = 12$, $x_2 = 12$, $x_3 = 12$, $x_4 = 2$. Note the ratios are still 6:6:6:1.

3 MATRIX ARITHMETIC

1. (e) $\begin{pmatrix} 8 & -15 & 11 \\ 0 & -4 & -3 \\ -1 & -6 & 6 \end{pmatrix}$

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(g) $\begin{pmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{pmatrix}$

2. (d) $\begin{pmatrix} 36 & 10 & 56 \\ 10 & 3 & 16 \end{pmatrix}$

5. (a) $5A = \begin{pmatrix} 15 & 20 \\ 5 & 5 \\ 10 & 35 \end{pmatrix}$

$2A + 3A = \begin{pmatrix} 6 & 8 \\ 2 & 2 \\ 4 & 14 \end{pmatrix} + \begin{pmatrix} 9 & 12 \\ 3 & 3 \\ 6 & 21 \end{pmatrix} = \begin{pmatrix} 15 & 20 \\ 5 & 5 \\ 10 & 35 \end{pmatrix}$

(b) $6A = \begin{pmatrix} 18 & 24 \\ 6 & 6 \\ 12 & 42 \end{pmatrix}$

$3(2A) = 3\begin{pmatrix} 6 & 8 \\ 2 & 2 \\ 4 & 14 \end{pmatrix} = \begin{pmatrix} 18 & 24 \\ 6 & 6 \\ 12 & 42 \end{pmatrix}$

(c) $A^T = \begin{pmatrix} 3 & 1 & 2 \\ 4 & 1 & 7 \end{pmatrix}$

$(A^T)^T = \begin{pmatrix} 3 & 1 & 2 \\ 4 & 1 & 7 \end{pmatrix}^T = \begin{pmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{pmatrix} = A$

6. (a) $A + B = \begin{pmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{pmatrix} = B + A$

(b) $3(A + B) = 3\begin{pmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{pmatrix} = \begin{pmatrix} 15 & 12 & 18 \\ 0 & 15 & 3 \end{pmatrix}$

$3A + 3B = \begin{pmatrix} 12 & 3 & 18 \\ 6 & 9 & 15 \end{pmatrix} + \begin{pmatrix} 3 & 9 & 0 \\ -6 & 6 & -12 \end{pmatrix}$

$= \begin{pmatrix} 15 & 12 & 18 \\ 0 & 15 & 3 \end{pmatrix}$

(c) $(A + B)^T = \begin{pmatrix} 5 & 4 & 6 \\ 0 & 5 & 1 \end{pmatrix}^T = \begin{pmatrix} 5 & 0 \\ 4 & 5 \\ 6 & 1 \end{pmatrix}$

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$$A^T + B^T = \begin{pmatrix} 4 & 2 \\ 1 & 3 \\ 6 & 5 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ 3 & 2 \\ 0 & -4 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 4 & 5 \\ 6 & 1 \end{pmatrix}$$

7. (a) $3(AB) = 3\begin{pmatrix} 5 & 14 \\ 15 & 42 \\ 0 & 16 \end{pmatrix} = \begin{pmatrix} 15 & 42 \\ 45 & 126 \\ 0 & 48 \end{pmatrix}$

$$(3A)B = \begin{pmatrix} 6 & 3 \\ 18 & 9 \\ -6 & 12 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 6 \end{pmatrix} = \begin{pmatrix} 15 & 42 \\ 45 & 126 \\ 0 & 48 \end{pmatrix}$$

$$A(3B) = \begin{pmatrix} 2 & 1 \\ 6 & 3 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 6 & 12 \\ 3 & 18 \end{pmatrix} = \begin{pmatrix} 15 & 42 \\ 45 & 126 \\ 0 & 48 \end{pmatrix}$$

(b) $(AB)^T = \begin{pmatrix} 5 & 14 \\ 15 & 42 \\ 0 & 16 \end{pmatrix}^T = \begin{pmatrix} 5 & 15 & 0 \\ 14 & 42 & 16 \end{pmatrix}$

$$B^T A^T = \begin{pmatrix} 2 & 1 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} 2 & 6 & -2 \\ 1 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 15 & 0 \\ 14 & 42 & 16 \end{pmatrix}$$

8. (a) $(A+B)+C = \begin{pmatrix} 0 & 5 \\ 1 & 7 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 3 & 8 \end{pmatrix}$

$$A+(B+C) = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 3 & 8 \end{pmatrix}$$

(b) $(AB)C = \begin{pmatrix} -4 & 18 \\ -2 & 13 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 24 & 14 \\ 20 & 11 \end{pmatrix}$

$$A(BC) = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} -4 & -1 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 24 & 14 \\ 20 & 11 \end{pmatrix}$$

(c) $A(B+C) = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 24 \\ 7 & 17 \end{pmatrix}$

$$AB + AC = \begin{pmatrix} -4 & 18 \\ -2 & 13 \end{pmatrix} + \begin{pmatrix} 14 & 6 \\ 9 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 24 \\ 7 & 17 \end{pmatrix}$$

(d) $(A+B)C = \begin{pmatrix} 0 & 5 \\ 1 & 7 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 17 & 8 \end{pmatrix}$

$$AC + BC = \begin{pmatrix} 14 & 6 \\ 9 & 4 \end{pmatrix} + \begin{pmatrix} -4 & -1 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 17 & 8 \end{pmatrix}$$

9. (b) $\mathbf{x} = (2,1)^T$ is a solution since $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$. There are no other solutions since the echelon form of A is strictly triangular.
(c) The solution to $A\mathbf{x} = \mathbf{c}$ is $\mathbf{x} = (-\frac{5}{2}, -\frac{1}{4})^T$. Therefore $\mathbf{c} = -\frac{5}{2}\mathbf{a}_1 - \frac{1}{4}\mathbf{a}_2$.

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11. The given information implies that

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

are both solutions to the system. So the system is consistent and since there is more than one solution, the row echelon form of A must involve a free variable. A consistent system with a free variable has infinitely many solutions.

12. The system is consistent since $\mathbf{x} = (1, 1, 1, 1)^T$ is a solution. The system can have at most 3 lead variables since A only has 3 rows. Therefore, there must be at least one free variable. A consistent system with a free variable has infinitely many solutions.

13. (a) It follows from the reduced row echelon form that the free variables are x_2, x_4, x_5. If we set $x_2 = a$, $x_4 = b$, $x_5 = c$, then

$$\begin{aligned} x_1 &= -2 - 2a - 3b - c \\ x_3 &= 5 - 2b - 4c \end{aligned}$$

and hence the solution consists of all vectors of the form

$$\mathbf{x} = (-2 - 2a - 3b - c,\ a,\ 5 - 2b - 4c,\ b,\ c)^T$$

(b) If we set the free variables equal to 0, then $\mathbf{x}_0 = (-2, 0, 5, 0, 0)^T$ is a solution to $A\mathbf{x} = \mathbf{b}$ and hence

$$\mathbf{b} = A\mathbf{x}_0 = -2\mathbf{a}_1 + 5\mathbf{a}_3 = (8, -7, -1, 7)^T$$

14. If w_3 is the weight given to professional activities, then the weights for research and teaching should be $w_1 = 3w_3$ and $w_2 = 2w_3$. Note that

$$1.5w_2 = 3w_3 = w_1,$$

so the weight given to research is 1.5 times the weight given to teaching. Since the weights must all add up to 1, we have

$$1 = w_1 + w_2 + w_3 = 3w_3 + 2w_3 + w_3 = 6w_3$$

and hence it follows that $w_3 = \frac{1}{6}$, $w_2 = \frac{1}{3}$, $w_1 = \frac{1}{2}$. If C is the matrix in the example problem from the Analytic Hierarchy Process Application, then the rating vector $\mathbf{r}$ is computed by multiplying C times the weight vector $\mathbf{w}$.

$$\mathbf{r} = C\mathbf{w} = \begin{pmatrix} \frac{1}{2} & \frac{1}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{10} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{6} \end{pmatrix} = \begin{pmatrix} \frac{43}{120} \\ \frac{45}{120} \\ \frac{32}{120} \end{pmatrix}$$

15. A^T is an $n \times m$ matrix. Since A^T has m columns and A has m rows, the multiplication A^TA is possible. The multiplication AA^T is possible since A has n columns and A^T has n rows.

16. If A is skew-symmetric, then $A^T = -A$. Since the (j, j) entry of A^T is a_{jj} and the (j, j) entry of $-A$ is $-a_{jj}$, it follows that $a_{jj} = -a_{jj}$ for each j and hence the diagonal entries of A must all be 0.

17. The search vector is $\mathbf{x} = (1, 0, 1, 0, 1, 0)^T$. The search result is given by the vector

$$\mathbf{y} = A^T\mathbf{x} = (1, 2, 2, 1, 1, 2, 1)^T$$

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The ith entry of $\mathbf{y}$ is equal to the number of search words in the title of the ith book.

18. If $\alpha = a_{21}/a_{11}$, then

$$\begin{pmatrix} 1 & 0 \\ \alpha & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ 0 & b \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ \alpha a_{11} & \alpha a_{12} + b \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & \alpha a_{12} + b \end{pmatrix}$$

The product will equal A provided

$$\alpha a_{12} + b = a_{22}$$

Thus we must choose

$$b = a_{22} - \alpha a_{12} = a_{22} - \frac{a_{21}a_{12}}{a_{11}}$$

4 MATRIX ALGEBRA

1. (a) $(A+B)^2 = (A+B)(A+B) = (A+B)A + (A+B)B = A^2 + BA + AB + B^2$
For real numbers, $ab + ba = 2ab$; however, with matrices $AB + BA$ is generally not equal to $2AB$.

(b)

$$\begin{aligned} (A+B)(A-B) &= (A+B)(A-B) \\ &= (A+B)A - (A+B)B \\ &= A^2 + BA - AB - B^2 \end{aligned}$$

For real numbers, $ab - ba = 0$; however, with matrices $AB - BA$ is generally not equal to O.

2. If we replace a by A and b by the identity matrix, I, then both rules will work, since

$$(A+I)^2 = A^2 + IA + AI + B^2 = A^2 + AI + AI + B^2 = A^2 + 2AI + B^2$$

and

$$(A+I)(A-I) = A^2 + IA - AI - I^2 = A^2 + A - A - I^2 = A^2 - I^2$$

3. There are many possible choices for A and B. For example, one could choose

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

More generally if

$$A = \begin{pmatrix} a & b \\ ca & cb \end{pmatrix} \qquad B = \begin{pmatrix} db & eb \\ -da & -ea \end{pmatrix}$$

then $AB = O$ for any choice of the scalars a, b, c, d, e.

4. To construct nonzero matrices A, B, C with the desired properties, first find nonzero matrices C and D such that $DC = O$ (see Exercise 3). Next, for any nonzero matrix A, set $B = A + D$. It follows that

$$BC = (A+D)C = AC + DC = AC + O = AC$$

5. A 2×2 symmetric matrix is one of the form

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

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Thus

$$A^2 = \begin{pmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{pmatrix}$$

If $A^2 = O$, then its diagonal entries must be 0.

$$a^2+b^2=0 \qquad \text{and} \qquad b^2+c^2=0$$

Thus $a=b=c=0$ and hence $A=O$.

6. Let

$$D=(AB)C=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{pmatrix}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

It follows that

$$\begin{aligned} d_{11} &= (a_{11}b_{11}+a_{12}b_{21})c_{11}+(a_{11}b_{12}+a_{12}b_{22})c_{21} \\ &= a_{11}b_{11}c_{11}+a_{12}b_{21}c_{11}+a_{11}b_{12}c_{21}+a_{12}b_{22}c_{21} \\ d_{12} &= (a_{11}b_{11}+a_{12}b_{21})c_{12}+(a_{11}b_{12}+a_{12}b_{22})c_{22} \\ &= a_{11}b_{11}c_{12}+a_{12}b_{21}c_{12}+a_{11}b_{12}c_{22}+a_{12}b_{22}c_{22} \\ d_{21} &= (a_{21}b_{11}+a_{22}b_{21})c_{11}+(a_{21}b_{12}+a_{22}b_{22})c_{21} \\ &= a_{21}b_{11}c_{11}+a_{22}b_{21}c_{11}+a_{21}b_{12}c_{21}+a_{22}b_{22}c_{21} \\ d_{22} &= (a_{21}b_{11}+a_{22}b_{21})c_{12}+(a_{21}b_{12}+a_{22}b_{22})c_{22} \\ &= a_{21}b_{11}c_{12}+a_{22}b_{21}c_{12}+a_{21}b_{12}c_{22}+a_{22}b_{22}c_{22} \end{aligned}$$

If we set

$$E=A(BC)=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11}c_{11}+b_{12}c_{21} & b_{11}c_{12}+b_{12}c_{22} \\ b_{21}c_{11}+b_{22}c_{21} & b_{21}c_{12}+b_{22}c_{22} \end{pmatrix}$$

then it follows that

$$\begin{aligned} e_{11} &= a_{11}(b_{11}c_{11}+b_{12}c_{21})+a_{12}(b_{21}c_{11}+b_{22}c_{21}) \\ &= a_{11}b_{11}c_{11}+a_{11}b_{12}c_{21}+a_{12}b_{21}c_{11}+a_{12}b_{22}c_{21} \\ e_{12} &= a_{11}(b_{11}c_{12}+b_{12}c_{22})+a_{12}(b_{21}c_{12}+b_{22}c_{22}) \\ &= a_{11}b_{11}c_{12}+a_{11}b_{12}c_{22}+a_{12}b_{21}c_{12}+a_{12}b_{22}c_{22} \\ e_{21} &= a_{21}(b_{11}c_{11}+b_{12}c_{21})+a_{22}(b_{21}c_{11}+b_{22}c_{21}) \\ &= a_{21}b_{11}c_{11}+a_{21}b_{12}c_{21}+a_{22}b_{21}c_{11}+a_{22}b_{22}c_{21} \\ e_{22} &= a_{21}(b_{11}c_{12}+b_{12}c_{22})+a_{22}(b_{21}c_{12}+b_{22}c_{22}) \\ &= a_{21}b_{11}c_{12}+a_{21}b_{12}c_{22}+a_{22}b_{21}c_{12}+a_{22}b_{22}c_{22} \end{aligned}$$

Thus

$$d_{11}=e_{11} \qquad d_{12}=e_{12} \qquad d_{21}=e_{21} \qquad d_{22}=e_{22}$$

and hence

$$(AB)C=D=E=A(BC)$$

9.

$$A^2=\begin{pmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix} \qquad A^3=\begin{pmatrix} 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{pmatrix}$$

and $A^4=O$. If $n>4$, then

$$A^n=A^{n-4}A^4=A^{n-4}O=O$$

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10. (a) The matrix C is symmetric since

$$C^T = (A+B)^T = A^T + B^T = A + B = C$$

(b) The matrix D is symmetric since

$$D^T = (AA)^T = A^T A^T = A^2 = D$$

(c) The matrix $E = AB$ is not symmetric since

$$E^T = (AB)^T = B^T A^T = BA$$

and in general, $AB \neq BA$.

(d) The matrix F is symmetric since

$$F^T = (ABA)^T = A^T B^T A^T = ABA = F$$

(e) The matrix G is symmetric since

$$G^T = (AB+BA)^T = (AB)^T + (BA)^T = B^T A^T + A^T B^T = BA + AB = G$$

(f) The matrix H is not symmetric since

$$H^T = (AB-BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB = -H$$

11. (a) The matrix A is symmetric since

$$A^T = (C+C^T)^T = C^T + (C^T)^T = C^T + C = A$$

(b) The matrix B is not symmetric since

$$B^T = (C-C^T)^T = C^T - (C^T)^T = C^T - C = -B$$

(c) The matrix D is symmetric since

$$A^T = (C^T C)^T = C^T (C^T)^T = C^T C = D$$

(d) The matrix E is symmetric since

$$\begin{aligned} E^T &= (C^T C - CC^T)^T = (C^T C)^T - (CC^T)^T \\ &= C^T (C^T)^T - (C^T)^T C^T = C^T C - CC^T = E \end{aligned}$$

(e) The matrix F is symmetric since

$$F^T = ((I+C)(I+C^T))^T = (I+C^T)^T (I+C)^T = (I+C)(I+C^T) = F$$

(e) The matrix G is not symmetric.

$$\begin{aligned} F &= (I+C)(I-C^T) = I + C - C^T - CC^T \\ F^T &= ((I+C)(I-C^T))^T = (I-C^T)^T (I+C)^T \\ &= (I-C)(I+C^T) = I - C + C^T - CC^T \end{aligned}$$

F and F^T are not the same. The two middle terms $C - C^T$ and $-C + C^T$ do not agree.

12. If $d = a_{11}a_{22} - a_{21}a_{12} \neq 0$, then

$$\frac{1}{d}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \frac{a_{11}a_{22}-a_{12}a_{21}}{d} & 0 \\ 0 & \frac{a_{11}a_{22}-a_{12}a_{21}}{d} \end{pmatrix} = I$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\left[\frac{1}{d}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}\right] = \begin{pmatrix} \frac{a_{11}a_{22}-a_{12}a_{21}}{d} & 0 \\ 0 & \frac{a_{11}a_{22}-a_{12}a_{21}}{d} \end{pmatrix} = I$$

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Therefore

$$\frac{1}{d}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = A^{-1}$$

13. (b) $\begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix}$

14. If A were nonsingular and $AB = A$, then it would follow that $A^{-1}AB = A^{-1}A$ and hence that $B = I$. So if $B \neq I$, then A must be singular.

15. Since

$$A^{-1}A = AA^{-1} = I$$

it follows from the definition that A^{-1} is nonsingular and its inverse is A.

16. Since

$$\begin{aligned} A^T(A^{-1})^T &= (A^{-1}A)^T = I \\ (A^{-1})^T A^T &= (AA^{-1})^T = I \end{aligned}$$

it follows that

$$(A^{-1})^T = (A^T)^{-1}$$

17. If $A\mathbf{x} = A\mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$, then A must be singular, for if A were nonsingular, then we could multiply by A^{-1} and get

$$\begin{aligned} A^{-1}A\mathbf{x} &= A^{-1}A\mathbf{y} \\ \mathbf{x} &= \mathbf{y} \end{aligned}$$

18. For $m = 1$,

$$(A^1)^{-1} = A^{-1} = (A^{-1})^1$$

Assume the result holds in the case $m = k$, that is,

$$(A^k)^{-1} = (A^{-1})^k$$

It follows that

$$(A^{-1})^{k+1}A^{k+1} = A^{-1}(A^{-1})^k A^k A = A^{-1}A = I$$

and

$$A^{k+1}(A^{-1})^{k+1} = AA^k(A^{-1})^k A^{-1} = AA^{-1} = I$$

Therefore

$$(A^{-1})^{k+1} = (A^{k+1})^{-1}$$

and the result follows by mathematical induction.

19. If $A^2 = O$, then

$$(I + A)(I - A) = I + A - A + A^2 = I$$

and

$$(I - A)(I + A) = I - A + A + A^2 = I$$

Therefore $I - A$ is nonsingular and $(I - A)^{-1} = I + A$.

20. If $A^{k+1} = O$, then

$$\begin{aligned} (I + A + \cdots + A^k)(I - A) &= (I + A + \cdots + A^k) - (A + A^2 + \cdots + A^{k+1}) \\ &= I - A^{k+1} = I \end{aligned}$$

and

$$\begin{aligned} (I - A)(I + A + \cdots + A^k) &= (I + A + \cdots + A^k) - (A + A^2 + \cdots + A^{k+1}) \\ &= I - A^{k+1} = I \end{aligned}$$

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Therefore $I - A$ is nonsingular and $(I - A)^{-1} = I + A + A^2 + \cdots + A^k$.

21. Since

$$R^T R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$RR^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

it follows that R is nonsingular and $R^{-1} = R^T$

22.

$$G^2 = \begin{pmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{pmatrix} = I$$

23.

$$\begin{aligned} H^2 = (I - 2\mathbf{u}\mathbf{u}^T)^2 &= I - 4\mathbf{u}\mathbf{u}^T + 4\mathbf{u}\mathbf{u}^T\mathbf{u}\mathbf{u}^T \\ &= I - 4\mathbf{u}\mathbf{u}^T + 4\mathbf{u}(\mathbf{u}^T\mathbf{u})\mathbf{u}^T \\ &= I - 4\mathbf{u}\mathbf{u}^T + 4\mathbf{u}\mathbf{u}^T = I \quad (\text{since } \mathbf{u}^T\mathbf{u} = 1) \end{aligned}$$

24. In each case, if you square the given matrix, you will end up with the same matrix.

25. (a) If $A^2 = A$, then

$$(I - A)^2 = I - 2A + A^2 = I - 2A + A = I - A$$

(b) If $A^2 = A$, then

$$(I - \frac{1}{2}A)(I + A) = I - \frac{1}{2}A + A - \frac{1}{2}A^2 = I - \frac{1}{2}A + A - \frac{1}{2}A = I$$

and

$$(I + A)(I - \frac{1}{2}A) = I + A - \frac{1}{2}A - \frac{1}{2}A^2 = I + A - \frac{1}{2}A - \frac{1}{2}A = I$$

Therefore $I + A$ is nonsingular and $(I + A)^{-1} = I - \frac{1}{2}A$.

26. (a)

$$D^2 = \begin{pmatrix} d_{11}^2 & 0 & \cdots & 0 \\ 0 & d_{22}^2 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & d_{nn}^2 \end{pmatrix}$$

Since each diagonal entry of D is equal to either 0 or 1, it follows that $d_{jj}^2 = d_{jj}$, for $j = 1, \ldots, n$ and hence $D^2 = D$.

(b) If $A = XDX^{-1}$, then

$$A^2 = (XDX^{-1})(XDX^{-1}) = XD(X^{-1}X)DX^{-1} = XDX^{-1} = A$$

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27. If A is an involution, then $A^2 = I$ and it follows that

$$B^2 = \frac{1}{4}(I+A)^2 = \frac{1}{4}(I+2A+A^2) = \frac{1}{4}(2I+2A) = \frac{1}{2}(I+A) = B$$
$$C^2 = \frac{1}{4}(I-A)^2 = \frac{1}{4}(I-2A+A^2) = \frac{1}{4}(2I-2A) = \frac{1}{2}(I-A) = C$$

So B and C are both idempotent.

$$BC = \frac{1}{4}(I+A)(I-A) = \frac{1}{4}(I+A-A-A^2) = \frac{1}{4}(I+A-A-I) = O$$

28. $(A^TA)^T = A^T(A^T)^T = A^TA$
$(AA^T)^T = (A^T)^TA^T = AA^T$

29. Let A and B be symmetric $n \times n$ matrices. If $(AB)^T = AB$, then

$$BA = B^TA^T = (AB)^T = AB$$

Conversely, if $BA = AB$, then

$$(AB)^T = B^TA^T = BA = AB$$

30. (a)

$$B^T = (A+A^T)^T = A^T + (A^T)^T = A^T + A = B$$
$$C^T = (A-A^T)^T = A^T - (A^T)^T = A^T - A = -C$$

(b) $A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T)$

34. False. For example, if

$$A = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

then

$$A\mathbf{x} = B\mathbf{x} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$$

however, $A \neq B$.

35. False. For example, if

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

then it is easy to see that both A and B must be singular, however, $A + B = I$, which is nonsingular.

36. True. If A and B are nonsingular, then their product AB must also be nonsingular. Using the result from Exercise 23, we have that $(AB)^T$ is nonsingular and $((AB)^T)^{-1} = ((AB)^{-1})^T$. It follows then that

$$((AB)^T)^{-1} = ((AB)^{-1})^T = (B^{-1}A^{-1})^T = (A^{-1})^T(B^{-1})^T$$

5 ELEMENTARY MATRICES

2. (a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, type I

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(b) The given matrix is not an elementary matrix. Its inverse is given by

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$, type III

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, type II

5. (c) Since

$$C = FB = FEA$$

where F and E are elementary matrices, it follows that C is row equivalent to A.

6. (b) $E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$, $E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

The product $L = E_1^{-1}E_2^{-1}E_3^{-1}$ is lower triangular.

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$$

7. A can be reduced to the identity matrix using three row operations

$$\begin{pmatrix} 2 & 1 \\ 6 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The elementary matrices corresponding to the three row operations are

$$E_1 = \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \quad E_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

So

$$E_3E_2E_1A = I$$

and hence

$$A = E_1^{-1}E_3^{-1}E_3^{-1} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

and $A^{-1} = E_3E_2E_1$.

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8. (b) $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 0 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix}$

9. (a) $\begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & -3 \\ -1 & 1 & -1 \\ 0 & -2 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 4 \\ 2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

10. (e) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

12. (b) $XA + B = C$
$X = (C - B)A^{-1}$
$= \begin{pmatrix} 8 & -14 \\ -13 & 19 \end{pmatrix}$

(d) $XA + C = X$
$XA - XI = -C$
$X(A - I) = -C$
$X = -C(A - I)^{-1}$
$= \begin{pmatrix} 2 & -4 \\ -3 & 6 \end{pmatrix}$

13. (a) If E is an elementary matrix of type I or type II, then E is symmetric. Thus $E^T = E$ is an elementary matrix of the same type. If E is the elementary matrix of type III formed by adding α times the ith row of the identity matrix to the jth row, then E^T is the elementary matrix of type III formed from the identity matrix by adding α times the jth row to the ith row.

(b) In general, the product of two elementary matrices will not be an elementary matrix. Generally, the product of two elementary matrices will be a matrix formed from the identity matrix by the performance of two row operations. For example, if

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

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then E_1 and E_2 are elementary matrices, but

$$E_1E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

is not an elementary matrix.

14. If $T = UR$, then

$$t_{ij} = \sum_{k=1}^{n} u_{ik} r_{kj}$$

Since U and R are upper triangular

$$\begin{aligned} u_{i1} &= u_{i2} = \cdots = u_{i,i-1} = 0 \\ r_{j+1,j} &= r_{j+2,j} = \cdots - r_{nj} = 0 \end{aligned}$$

If $i > j$, then

$$\begin{aligned} t_{ij} &= \sum_{k=1}^{j} u_{ik} r_{kj} + \sum_{k=j+1}^{n} u_{ik} r_{kj} \\ &= \sum_{k=1}^{j} 0\, r_{kj} + \sum_{k=j+1}^{n} u_{ik} 0 \\ &= 0 \end{aligned}$$

Therefore T is upper triangular.

If $i = j$, then

$$\begin{aligned} t_{jj} = t_{ij} &= \sum_{k=1}^{i-1} u_{ik} r_{kj} + u_{jj} r_{jj} + \sum_{k=j+1}^{n} u_{ik} r_{kj} \\ &= \sum_{k=1}^{i-1} 0\, r_{kj} + u_{jj} r_{jj} + \sum_{k=j+1}^{n} u_{ik} 0 \\ &= u_{jj} r_{jj} \end{aligned}$$

Therefore

$$t_{jj} = u_{jj} r_{jj} \qquad j = 1, \ldots, n$$

15. If we set $\mathbf{x} = (2, 1 - 4)^T$, then

$$A\mathbf{x} = 2\mathbf{a}_1 + 1\mathbf{a}_2 - 4\mathbf{a}_3 = \mathbf{0}$$

Thus $\mathbf{x}$ is a nonzero solution to the system $A\mathbf{x} = \mathbf{0}$. But if a homogeneous system has a nonzero solution, then it must have infinitely many solutions. In particular, if c is any scalar, then $c\mathbf{x}$ is also a solution to the system since

$$A(c\mathbf{x}) = cA\mathbf{x} = c\mathbf{0} = \mathbf{0}$$

Since $A\mathbf{x} = \mathbf{0}$ and $\mathbf{x} \neq \mathbf{0}$, it follows that the matrix A must be singular. (See Theorem 1.5.2)

16. If $\mathbf{a}_1 = 3\mathbf{a}_2 - 2\mathbf{a}_3$, then

$$\mathbf{a}_1 - 3\mathbf{a}_2 + 2\mathbf{a}_3 = \mathbf{0}$$

Therefore $\mathbf{x} = (1, -3, 2)^T$ is a nontrivial solution to $A\mathbf{x} = \mathbf{0}$. It follows from Theorem 1.5.2 that A must be singular.

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17. If $\mathbf{x}_0 \neq \mathbf{0}$ and $A\mathbf{x}_0 = B\mathbf{x}_0$, then $C\mathbf{x}_0 = \mathbf{0}$ and it follows from Theorem 1.5.2 that C must be singular.

18. If B is singular, then it follows from Theorem 1.5.2 that there exists a nonzero vector $\mathbf{x}$ such that $B\mathbf{x} = \mathbf{0}$. If $C = AB$, then

$$C\mathbf{x} = AB\mathbf{x} = A\mathbf{0} = \mathbf{0}$$

Thus, by Theorem 1.5.2, C must also be singular.

19. (a) If U is upper triangular with nonzero diagonal entries, then using row operation II, U can be transformed into an upper triangular matrix with 1's on the diagonal. Row operation III can then be used to eliminate all of the entries above the diagonal. Thus, U is row equivalent to I and hence is nonsingular.

(b) The same row operations that were used to reduce U to the identity matrix will transform I into U^{-1}. Row operation II applied to I will just change the values of the diagonal entries. When the row operation III steps referred to in part (a) are applied to a diagonal matrix, the entries above the diagonal are filled in. The resulting matrix, U^{-1}, will be upper triangular.

20. Since A is nonsingular it is row equivalent to I. Hence, there exist elementary matrices $E_1, E_2, \ldots, E_k$ such that

$$E_k \cdots E_1 A = I$$

It follows that

$$A^{-1} = E_k \cdots E_1$$

and

$$E_k \cdots E_1 B = A^{-1}B = C$$

The same row operations that reduce A to I, will transform B to C. Therefore, the reduced row echelon form of $(A \mid B)$ will be $(I \mid C)$.

21. (a) If the diagonal entries of D_1 are $\alpha_1, \alpha_2, \ldots, \alpha_n$ and the diagonal entries of D_2 are $\beta_1, \beta_2, \ldots, \beta_n$, then D_1D_2 will be a diagonal matrix with diagonal entries $\alpha_1\beta_1, \ldots, \alpha_n\beta_n$ and D_2D_1 will be a diagonal matrix with diagonal entries $\beta_1\alpha_1, \beta_2\alpha_2, \ldots, \beta_n\alpha_n$. Since the two have the same diagonal entries, it follows that $D_1D_2 = D_2D_1$.

(b)

$$\begin{aligned} AB &= A(a_0I + a_1A + \cdots + a_kA^k) \\ &= a_0A + a_1A^2 + \cdots + a_kA^{k+1} \\ &= (a_0I + a_1A + \cdots + a_kA^k)A \\ &= BA \end{aligned}$$

22. If A is symmetric and nonsingular, then

$$(A^{-1})^T = (A^{-1})^T(AA^{-1}) = ((A^{-1})^TA^T)A^{-1} = A^{-1}$$

23. If A is row equivalent to B, then there exist elementary matrices $E_1, E_2, \ldots, E_k$ such that

$$A = E_kE_{k-1} \cdots E_1B$$

Each of the E_i's is invertible and E_i^{-1} is also an elementary matrix (Theorem 1.4.1). Thus

$$B = E_1^{-1}E_2^{-1} \cdots E_k^{-1}A$$

and hence B is row equivalent to A.

24. (a) If A is row equivalent to B, then there exist elementary matrices $E_1, E_2, \ldots, E_k$ such that

$$A = E_kE_{k-1} \cdots E_1B$$

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Since B is row equivalent to C, there exist elementary matrices $H_1, H_2, \ldots, H_j$ such that

$$B = H_j H_{j-1} \cdots H_1 C$$

Thus

$$A = E_k E_{k-1} \cdots E_1 H_j H_{j-1} \cdots H_1 C$$

and hence A is row equivalent to C.

(b) If A and B are nonsingular $n \times n$ matrices, then A and B are row equivalent to I. Since A is row equivalent to I and I is row equivalent to B, it follows from part (a) that A is row equivalent to B.

25. If U is any row echelon form of A, then A can be reduced to U using row operations, so A is row equivalent to U. If B is row equivalent to A, then it follows from the result in Exercise 24(a) that B is row equivalent to U.

26. If B is row equivalent to A, then there exist elementary matrices $E_1, E_2, \ldots, E_k$ such that

$$B = E_k E_{k-1} \cdots E_1 A$$

Let $M = E_k E_{k-1} \cdots E_1$. The matrix M is nonsingular since each of the E_i's is nonsingular.

Conversely, suppose there exists a nonsingular matrix M such that $B = MA$. Since M is nonsingular, it is row equivalent to I. Thus, there exist elementary matrices $E_1, E_2, \ldots, E_k$ such that

$$M = E_k E_{k-1} \cdots E_1 I$$

It follows that

$$B = MA = E_k E_{k-1} \cdots E_1 A$$

Therefore, B is row equivalent to A.

27. If A is nonsingular, then A is row equivalent to I. If B is row equivalent to A, then using the result from Exercise 24(a), we can conclude that B is row equivalent to I. Therefore, B must be nonsingular. So it is not possible for B to be singular and also be row equivalent to a nonsingular matrix.

28. (a) The system $V\mathbf{c} = \mathbf{y}$ is given by

$$\begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & & & & \\ 1 & x_{n+1} & x_{n+1}^2 & \cdots & x_{n+1}^n \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n+1} \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n+1} \end{pmatrix}$$

Comparing the ith row of each side, we have

$$c_1 + c_2 x_i + \cdots + c_{n+1} x_i^n = y_i$$

Thus

$$p(x_i) = y_i \qquad i = 1, 2, \ldots, n+1$$

(b) If $x_1, x_2, \ldots, x_{n+1}$ are distinct and $V\mathbf{c} = \mathbf{0}$, then we can apply part (a) with $\mathbf{y} = \mathbf{0}$. Thus if $p(x) = c_1 + c_2 x + \cdots + c_{n+1} x^n$, then

$$p(x_i) = 0 \qquad i = 1, 2, \ldots, n+1$$

The polynomial $p(x)$ has $n+1$ roots. Since the degree of $p(x)$ is less than $n+1$, $p(x)$ must be the zero polynomial. Hence

$$c_1 = c_2 = \cdots = c_{n+1} = 0$$

Since the system $V\mathbf{c} = \mathbf{0}$ has only the trivial solution, the matrix V must be nonsingular.

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29. True. If A is row equivalent to I, then A is nonsingular, so if $AB = AC$, then we can multiply both sides of this equation by A^{-1}.

$$\begin{aligned} A^{-1}AB &= A^{-1}AC \\ B &= C \end{aligned}$$

30. True. If E and F are elementary matrices, then they are both nonsingular and the product of two nonsingular matrices is a nonsingular matrix. Indeed, $G^{-1} = F^{-1}E^{-1}$.

31. True. If $\mathbf{a} + \mathbf{a}_2 = \mathbf{a}_3 + 2\mathbf{a}_4$, then

$$\mathbf{a} + \mathbf{a}_2 - \mathbf{a}_3 - 2\mathbf{a}_4 = \mathbf{0}$$

If we let $\mathbf{x} = (1, 1, -1, -2)^T$, then $\mathbf{x}$ is a solution to $A\mathbf{x} = \mathbf{0}$. Since $\mathbf{x} \neq \mathbf{0}$ the matrix A must be singular.

32. False. Let I be the 2×2 identity matrix and let $A = I$, $B = -I$, and

$$C = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

Since B and C are nonsingular, they are both row equivalent to A; however,

$$B + C = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

is singular, so it cannot be row equivalent to A.

6 PARTITIONED MATRICES

2. $B = A^TA = \begin{pmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_n^T \end{pmatrix} (\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n) = \begin{pmatrix} \mathbf{a}_1^T\mathbf{a}_1 & \mathbf{a}_1^T\mathbf{a}_2 & \cdots & \mathbf{a}_1^T\mathbf{a}_n \\ \mathbf{a}_2^T\mathbf{a}_1 & \mathbf{a}_2^T\mathbf{a}_2 & \cdots & \mathbf{a}_2^T\mathbf{a}_n \\ \vdots & & & \\ \mathbf{a}_n^T\mathbf{a}_1 & \mathbf{a}_n^T\mathbf{a}_2 & \cdots & \mathbf{a}_n^T\mathbf{a}_n \end{pmatrix}$

5. (a) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 4 & -2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix} (1 \;\; 2 \;\; 3) = \begin{pmatrix} 6 & 0 & 1 \\ 11 & -1 & 4 \end{pmatrix}$

(c) Let

$$A_{11} = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix} \qquad A_{12} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A_{21} = (0 \quad 0) \qquad A_{22} = (1 \quad 0)$$

The block multiplication is performed as follows:

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} A_{11}^T & A_{21}^T \\ A_{12}^T & A_{22}^T \end{pmatrix} = \begin{pmatrix} A_{11}A_{11}^T + A_{12}A_{12}^T & A_{11}A_{21}^T + A_{12}A_{22}^T \\ A_{21}A_{11}^T + A_{22}A_{12}^T & A_{21}A_{21}^T + A_{22}A_{22}^T \end{pmatrix}$$

$$= \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & 0 \end{array}\right)$$

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6. (a)

$$\begin{aligned} XY^T &= \mathbf{x}_1\mathbf{y}_1^T + \mathbf{x}_2\mathbf{y}_2^T + \mathbf{x}_3\mathbf{y}_3^T \\ &= \begin{pmatrix} 2 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix} \begin{pmatrix} 4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} + \begin{pmatrix} 20 & 5 \\ 12 & 3 \end{pmatrix} \end{aligned}$$

(b) Since $\mathbf{y}_i\mathbf{x}_i^T = (\mathbf{x}_i\mathbf{y}_i^T)^T$ for $j = 1, 2, 3$, the outer product expansion of YX^T is just the transpose of the outer product expansion of XY^T. Thus

$$\begin{aligned} YX^T &= \mathbf{y}_1\mathbf{x}_1^T + \mathbf{y}_2\mathbf{x}_2^T + \mathbf{y}_3\mathbf{x}_3^T \\ &= \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} + \begin{pmatrix} 20 & 12 \\ 5 & 3 \end{pmatrix} \end{aligned}$$

7. It is possible to perform both block multiplications. To see this, suppose A_{11} is a $k \times r$ matrix, A_{12} is a $k \times (n-r)$ matrix, A_{21} is an $(m-k) \times r$ matrix and A_{22} is $(m-k) \times (n-r)$. It is possible to perform the block multiplication of AA^T since the matrix multiplications $A_{11}A_{11}^T$, $A_{11}A_{21}^T$, $A_{12}A_{12}^T$, $A_{12}A_{22}^T$, $A_{21}A_{11}^T$, $A_{21}A_{21}^T$, $A_{22}A_{12}^T$, $A_{22}A_{22}^T$ are all possible. It is possible to perform the block multiplication of A^TA since the matrix multiplications $A_{11}^TA_{11}$, $A_{11}^TA_{12}$, $A_{21}^TA_{21}$, $A_{21}^TA_{11}$, $A_{12}^TA_{12}$, $A_{22}^TA_{21}$, $A_{22}^TA_{22}$ are all possible.

8. $AX = A(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_r) = (A\mathbf{x}_1, A\mathbf{x}_2, \ldots, A\mathbf{x}_r)$
$B = (\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_r)$
$AX = B$ if and only if the column vectors of AX and B are equal

$$A\mathbf{x}_j = \mathbf{b}_j \qquad j = 1, \ldots, r$$

9. (a) Since D is a diagonal matrix, its jth column will have d_{jj} in the jth row and the other entries will all be 0. Thus $\mathbf{d}_j = d_{jj}\mathbf{e}_j$ for $j = 1, \ldots, n$.

(b)

$$\begin{aligned} AD &= A(d_{11}\mathbf{e}_1,\ d_{22}\mathbf{e}_2, \ldots, d_{nn}\mathbf{e}_n) \\ &= (d_{11}A\mathbf{e}_1,\ d_{22}A\mathbf{e}_2, \ldots, d_{nn}A\mathbf{e}_n) \\ &= (d_{11}\mathbf{a}_1,\ d_{22}\mathbf{a}_2, \ldots, d_{nn}\mathbf{a}_n) \end{aligned}$$

10. (a)

$$U\Sigma = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} \Sigma_1 \\ O \end{pmatrix} = U_1\Sigma_1 + U_2O = U_1\Sigma_1$$

(b) If we let $X = U\Sigma$, then

$$X = U_1\Sigma_1 = (\sigma_1\mathbf{u}_1, \sigma_2\mathbf{u}_2, \ldots, \sigma_n\mathbf{u}_n)$$

and it follows that

$$A = U\Sigma V^T = XV^T = \sigma_1\mathbf{u}_1\mathbf{v}_1^T + \sigma_2\mathbf{u}_2\mathbf{v}_2^T + \cdots + \sigma_n\mathbf{u}_n\mathbf{v}_n^T$$

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11.

$$\begin{pmatrix} A_{11}^{-1} & C \\ O & A_{22}^{-1} \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} \\ O & A_{22} \end{pmatrix} = \begin{pmatrix} I & A_{11}^{-1}A_{12} + CA_{22} \\ O & I \end{pmatrix}$$

If

$$A_{11}^{-1}A_{12} + CA_{22} = O$$

then

$$C = -A_{11}^{-1}A_{12}A_{22}^{-1}$$

Let

$$B = \begin{pmatrix} A_{11}^{-1} & -A_{11}^{-1}A_{12}A_{22}^{-1} \\ O & A_{22}^{-1} \end{pmatrix}$$

Since $AB = BA = I$, it follows that $B = A^{-1}$.

12. Let $\mathbf{0}$ denote the zero vector in $\mathbb{R}^n$. If A is singular, then there exists a vector $\mathbf{x}_1 \neq \mathbf{0}$ such that $A\mathbf{x}_1 = \mathbf{0}$. If we set

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{0} \end{pmatrix}$$

then

$$M\mathbf{x} = \begin{pmatrix} A & O \\ O & B \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} A\mathbf{x}_1 + O\mathbf{0} \\ O\mathbf{x}_1 + B\mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

By Theorem 1.5.2, M must be singular. Similarly, if B is singular, then there exists a vector $\mathbf{x}_2 \neq \mathbf{0}$ such that $B\mathbf{x}_2 = \mathbf{0}$. So if we set

$$\mathbf{x} = \begin{pmatrix} \mathbf{0} \\ \mathbf{x}_2 \end{pmatrix}$$

then $\mathbf{x}$ is a nonzero vector and $M\mathbf{x}$ is equal to the zero vector.

15.

$$A^{-1} = \begin{pmatrix} O & I \\ I & -B \end{pmatrix}, \quad A^2 = \begin{pmatrix} I & B \\ B & I \end{pmatrix}, \quad A^3 = \begin{pmatrix} B & I \\ I & 2B \end{pmatrix}$$

and hence

$$A^{-1} + A^2 + A^3 = \begin{pmatrix} I + B & 2I + B \\ 2I + B & I + B \end{pmatrix}$$

16. The block form of S^{-1} is given by

$$S^{-1} = \begin{pmatrix} I & -A \\ O & I \end{pmatrix}$$

It follows that

$$S^{-1}MS = \begin{pmatrix} I & -A \\ O & I \end{pmatrix} \begin{pmatrix} AB & O \\ B & O \end{pmatrix} \begin{pmatrix} I & A \\ O & I \end{pmatrix}$$

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$$= \begin{pmatrix} I & -A \\ O & I \end{pmatrix} \begin{pmatrix} AB & ABA \\ B & BA \end{pmatrix}$$

$$= \begin{pmatrix} O & O \\ B & BA \end{pmatrix}$$

17. The block multiplication of the two factors yields

$$\begin{pmatrix} I & O \\ B & I \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} \\ O & C \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ BA_{11} & BA_{12}+C \end{pmatrix}$$

If we equate this matrix with the block form of A and solve for B and C, we get

$$B = A_{21}A_{11}^{-1} \qquad \text{and} \qquad C = A_{22} - A_{21}A_{11}^{-1}A_{12}$$

To check that this works note that

$$\begin{aligned} BA_{11} &= A_{21}A_{11}^{-1}A_{11} = A_{21} \\ BA_{12}+C &= A_{21}A_{11}^{-1}A_{12} + A_{22} - A_{21}A_{11}^{-1}A_{12} = A_{22} \end{aligned}$$

and hence

$$\begin{pmatrix} I & O \\ B & I \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} \\ O & C \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = A$$

18. In order for the block multiplication to work, we must have

$$XB = S \qquad \text{and} \qquad YM = T$$

Since both B and M are nonsingular, we can satisfy these conditions by choosing $X = SB^{-1}$ and $Y = TM^{-1}$.

19. (a)

$$BC = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} (c) = \begin{pmatrix} b_1 c \\ b_2 c \\ \vdots \\ b_n c \end{pmatrix} = c\mathbf{b}$$

(b)

$$\begin{aligned} A\mathbf{x} &= (\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n) \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \\ &= \mathbf{a}_1(x_1) + \mathbf{a}_2(x_2) + \cdots + \mathbf{a}_n(x_n) \end{aligned}$$

(c) It follows from parts (a) and (b) that

$$\begin{aligned} A\mathbf{x} &= \mathbf{a}_1(x_1) + \mathbf{a}_2(x_2) + \cdots + \mathbf{a}_n(x_n) \\ &= x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n \end{aligned}$$

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20. If $A\mathbf{x} = \mathbf{0}$ for all $\mathbf{x} \in \mathbb{R}^n$, then

$$\mathbf{a}_j = A\mathbf{e}_j = \mathbf{0} \quad \text{for} \quad j = 1, \ldots, n$$

and hence A must be the zero matrix.

21. If

$$B\mathbf{x} = C\mathbf{x} \quad \text{for all} \quad \mathbf{x} \in \mathbb{R}^n$$

then

$$(B - C)\mathbf{x} = \mathbf{0} \quad \text{for all} \quad \mathbf{x} \in \mathbb{R}^n$$

It follows from Exercise 20 that

$$\begin{aligned} B - C &= O \\ B &= C \end{aligned}$$

22. (a)

$$\begin{pmatrix} A^{-1} & \mathbf{0} \\ -\mathbf{c}^T A^{-1} & 1 \end{pmatrix} \begin{pmatrix} A & \mathbf{a} \\ \mathbf{c}^T & \beta \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ x_{n+1} \end{pmatrix} = \begin{pmatrix} A^{-1} & \mathbf{0} \\ -\mathbf{c}^T A^{-1} & 1 \end{pmatrix} \begin{pmatrix} \mathbf{b} \\ b_{n+1} \end{pmatrix}$$

$$\begin{pmatrix} I & A^{-1}\mathbf{a} \\ \mathbf{0}^T & -\mathbf{c}^T A^{-1}\mathbf{a} + \beta \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ x_{n+1} \end{pmatrix} = \begin{pmatrix} A^{-1}\mathbf{b} \\ -\mathbf{c}^T A^{-1}\mathbf{b} + b_{n+1} \end{pmatrix}$$

(b) If

$$\mathbf{y} = A^{-1}\mathbf{a} \qquad \text{and} \qquad \mathbf{z} = A^{-1}\mathbf{b}$$

then

$$(-\mathbf{c}^T\mathbf{y} + \beta)x_{n+1} = -\mathbf{c}^T\mathbf{z} + b_{n+1}$$

$$x_{n+1} = \frac{-\mathbf{c}^T\mathbf{z} + b_{n+1}}{-\mathbf{c}^T\mathbf{y} + \beta} \quad (\beta - \mathbf{c}^T\mathbf{y} \neq 0)$$

and

$$\mathbf{x} + x_{n+1}A^{-1}\mathbf{a} = A^{-1}\mathbf{b}$$

$$\mathbf{x} = A^{-1}\mathbf{b} - x_{n+1}A^{-1}\mathbf{a} = \mathbf{z} - x_{n+1}\mathbf{y}$$

MATLAB EXERCISES

1. In parts (a), (b), (c) it should turn out that $A1 = A4$ and $A2 = A3$. In part (d) $A1 = A3$ and $A2 = A4$. Exact equality will not occur in parts (c) and (d) because of roundoff error.

2. The solution $\mathbf{x}$ obtained using the \ operation will be more accurate and yield the smaller residual vector. The computation of $\mathbf{x}$ is also more efficient since the solution is computed using Gaussian elimination with partial pivoting and this involves less arithmetic than computing the inverse matrix and multiplying it times $\mathbf{b}$.

3. (a) Since $A\mathbf{x} = \mathbf{0}$ and $\mathbf{x} \neq \mathbf{0}$, it follows from Theorem 1.5.2 that A is singular.

(b) The columns of B are all multiples of $\mathbf{x}$. Indeed,

$$B = (\mathbf{x}, 2\mathbf{x}, 3\mathbf{x}, 4\mathbf{x}, 5\mathbf{x}, 6\mathbf{x})$$

and hence

$$AB = (A\mathbf{x}, 2A\mathbf{x}, 3A\mathbf{x}, 4A\mathbf{x}, 5A\mathbf{x}, 6A\mathbf{x}) = O$$

(c) If $D = B + C$, then

$$AD = AB + AC = O + AC = AC$$

4. By construction, B is upper triangular whose diagonal entries are all equal to 1. Thus B is row equivalent to I and hence B is nonsingular. If one changes B by setting $b_{10,1} = -1/256$ and computes $B\mathbf{x}$, the result is the zero vector. Since $\mathbf{x} \neq \mathbf{0}$, the matrix B must be singular.

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5. (a) Since A is nonsingular, its reduced row echelon form is I. If $E_1, \ldots, E_k$ are elementary matrices such that $E_k \cdots E_1 A = I$, then these same matrices can be used to transform $(A \;\; \mathbf{b})$ to its reduced row echelon form U. It follows then that

$$U = E_k \cdots E_1 (A \;\; \mathbf{b}) = A^{-1}(A \;\; \mathbf{b}) = (I \;\; A^{-1}\mathbf{b})$$

Thus, the last column of U should be equal to the solution $\mathbf{x}$ of the system $A\mathbf{x} = \mathbf{b}$.

(b) After the third column of A is changed, the new matrix A is now singular. Examining the last row of the reduced row echelon form of the augmented matrix $(A \; \mathbf{b})$, we see that the system is inconsistent.

(c) The system $A\mathbf{x} = \mathbf{c}$ is consistent since $\mathbf{y}$ is a solution. There is a free variable x_3, so the system will have infinitely many solutions.

(f) The vector $\mathbf{v}$ is a solution since

$$A\mathbf{v} = A(\mathbf{w} + 3\mathbf{z}) = A\mathbf{w} + 3A\mathbf{z} = \mathbf{c}$$

For this solution, the free variable $x_3 = v_3 = 3$. To determine the general solution just set $\mathbf{x} = \mathbf{w} + t\mathbf{z}$. This will give the solution corresponding to $x_3 = t$ for any real number t.

6. (c) There will be no walks of even length from V_i to V_j whenever $i + j$ is odd.

(d) There will be no walks of length k from V_i to V_j whenever $i + j + k$ is odd.

(e) The conjecture is still valid for the graph containing the additional edges.

(f) If the edge $\{V_6, V_8\}$ is included, then the conjecture is no longer valid. There is now a walk of length 1 from V_6 to V_8 and $i + j + k = 6 + 8 + 1$ is odd.

8. The change in part (b) should not have a significant effect on the survival potential for the turtles. The change in part (c) will effect the $(2, 2)$ and $(3, 2)$ of the Leslie matrix. The new values for these entries will be $l_{22} = 0.9540$ and $l_{32} = 0.0101$. With these values, the Leslie population model should predict that the survival period will double but the turtles will still eventually die out.

9. (b) $\mathbf{x1} = \mathbf{c} - V\mathbf{x2}$.

10. (b)

$$A^{2k} = \begin{pmatrix} I & kB \\ kB & I \end{pmatrix}$$

This can be proved using mathematical induction. In the case $k = 1$

$$A^2 = \begin{pmatrix} O & I \\ I & B \end{pmatrix} \begin{pmatrix} O & I \\ I & B \end{pmatrix} = \begin{pmatrix} I & B \\ B & I \end{pmatrix}$$

If the result holds for $k = m$

$$A^{2m} = \begin{pmatrix} I & mB \\ mB & I \end{pmatrix}$$

then

$$\begin{aligned} A^{2m+2} &= A^2 A^{2m} \\ &= \begin{pmatrix} I & B \\ B & I \end{pmatrix} \begin{pmatrix} I & mB \\ mB & I \end{pmatrix} \end{aligned}$$

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$$= \begin{pmatrix} I & (m+1)B \\ (m+1)B & I \end{pmatrix}$$

It follows by mathematical induction that the result holds for all positive integers k.

(b)

$$A^{2k+1} = AA^{2k} = \begin{pmatrix} O & I \\ I & B \end{pmatrix} \begin{pmatrix} I & kB \\ kB & I \end{pmatrix} = \begin{pmatrix} kB & I \\ I & (k+1)B \end{pmatrix}$$

11. (a) By construction, the entries of A were rounded to the nearest integer. The matrix $B = A^TA$ must also have integer entries and it is symmetric since

$$B^T = (A^TA)^T = A^T(A^T)^T = A^TA = B$$

(b)

$$LDL^T = \begin{pmatrix} I & O \\ E & I \end{pmatrix} \begin{pmatrix} B_{11} & O \\ O & F \end{pmatrix} \begin{pmatrix} I & E^T \\ O & I \end{pmatrix}$$

$$= \begin{pmatrix} B_{11} & B_{11}E^T \\ EB_{11} & EB_{11}E^T + F \end{pmatrix}$$

where

$$E = B_{21}B_{11}^{-1} \quad \text{and} \quad F = B_{22} - B_{21}B_{11}^{-1}B_{12}$$

It follows that

$$\begin{aligned} B_{11}E^T &= B_{11}(B_{11}^{-1})^T B_{21}^T = B_{11}B_{11}^{-1}B_{12} = B_{12} \\ EB_{11} &= B_{21}B_{11}^{-1}B_{11} = B_{21} \end{aligned}$$

$$\begin{aligned} EB_{11}E^T + F &= B_{21}E^T + B_{22} - B_{21}B_{11}^{-1}B_{12} \\ &= B_{21}B_{11}^{-1}B_{12} + B_{22} - B_{21}B_{11}^{-1}B_{12} \\ &= B_{22} \end{aligned}$$

Therefore

$$LDL^T = B$$

CHAPTER TEST A

1. The statement is false. If the row echelon form has free variables and the linear system is consistent, then there will be infinitely many solutions. However, it is possible to have an inconsistent system whose coefficient matrix will reduce to an echelon form with free variables. For example, if

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

then A involves one free variable, but the system $A\mathbf{x} = \mathbf{b}$ is inconsistent.

2. The statement is true since the zero vector will always be a solution.

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3. The statement is true. A matrix A is nonsingular if and only if it is row equivalent to the I (the identity matrix). A will be row equivalent to I if and only if its reduced row echelon form is I.

4. The statement is true. If A is nonsingular, then A is row equivalent to I. So there exist elementary matrices $E_1, E_2, \ldots, E_k$, such that

$$A = E_k E_{k-1} \cdots E_1 I = E_k E_{k-1} \cdots E_1$$

5. The statement is false. For example, if $A = I$ and $B = -I$, the matrices A and B are both nonsingular, but $A + B = O$ is singular.

6. The statement is false. For example, if A is any matrix of the form

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

Then $A = A^{-1}$.

7. The statement is false.

$$(A - B)^2 = A^2 - BA - AB + B^2 \neq A^2 - 2AB + B^2$$

since in general $BA \neq AB$. For example, if

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

then

$$(A - B)^2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

however,

$$A^2 - 2AB + B^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix}$$

8. The statement is false. If A is nonsingular and $AB = AC$, then we can multiply both sides of the equation by A^{-1} and conclude that $B = C$. However, if A is singular, then it is possible to have $AB = AC$ and $B \neq C$. For example, if

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$$

$$AC = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$$

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9. The statement is false. In general, AB and BA are usually not equal, so it is possible for $AB = O$ and BA to be a nonzero matrix. For example, if

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad BA = \begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}$$

10. The statement is true. If $\mathbf{x} = (1, 2, -1)^T$, then $\mathbf{x} \neq \mathbf{0}$ and $A\mathbf{x} = \mathbf{0}$, so A must be singular.

11. The statement is true. If $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_3$ and $\mathbf{x} = (1, 0, 1)^T$, then

$$A\mathbf{x} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = 1\mathbf{a}_1 + 0\mathbf{a}_2 + 1\mathbf{a}_3 = \mathbf{b}$$

So $\mathbf{x}$ is a solution to $A\mathbf{x} = \mathbf{b}$.

12. The statement is true. If $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$, then $\mathbf{x} = (1, 1, 1)^T$ is a solution to $A\mathbf{x} = \mathbf{b}$, since

$$A\mathbf{x} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3 = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 = \mathbf{b}$$

If $\mathbf{a}_2 = \mathbf{a}_3$, then we can also express $\mathbf{b}$ as a linear combination

$$\mathbf{b} = \mathbf{a}_1 + 0\mathbf{a}_2 + 2\mathbf{a}_3$$

Thus $\mathbf{y} = (1, 0, 2)^T$ is also a solution to the system. However, if there is more than one solution, then the reduced row echelon form of A must involve a free variable. A consistent system with a free variable must have infinitely many solutions.

13. The statement is true. An elementary matrix E of type I or type II is symmetric. So in either case we have $E^T = E$ is elementary. If E is an elementary matrix of type III formed from the identity matrix by adding a nonzero multiple c of row k to row j, then E^T will be the elementary matrix of type III formed from the identity matrix by adding c times row j to row k.

14. The statement is false. An elementary matrix is a matrix that is constructed by performing exactly one elementary row operation on the identity matrix. The product of two elementary matrices will be a matrix formed by performing *two* elementary row operations on the identity matrix. For example,

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

are elementary matrices, however;

$$E_1E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

is not an elementary matrix.

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15. The statement is true. The row vectors of A are $x_1\mathbf{y}^T, x_2\mathbf{y}^T, \ldots, x_n\mathbf{y}^T$. Note, all of the row vectors are multiples of $\mathbf{y}^T$. Since $\mathbf{x}$ and $\mathbf{y}$ are nonzero vectors, at least one of these row vectors must be nonzero. However, if any nonzero row is picked as a pivot row, then since all of the other rows are multiples of the pivot row, they will all be eliminated in the first step of the reduction process. The resulting row echelon form will have exactly one nonzero row.

CHAPTER TEST B

1.

$$\left(\begin{array}{rrrr|r} 1 & -1 & 3 & 2 & 1 \\ -1 & 1 & -2 & 1 & -2 \\ 2 & -2 & 7 & 7 & 1 \end{array}\right) \rightarrow \left(\begin{array}{rrrr|r} 1 & -1 & 3 & 2 & 1 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 3 & -1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{rrrr|r} 1 & -1 & 0 & -7 & 4 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

The free variables are x_2 and x_4. If we set $x_2 = a$ and $x_4 = b$, then

$$x_1 = 4 + a + 7b \qquad \text{and} \qquad x_3 = -1 - 3b$$

and hence the solution set consists of all vectors of the form

$$\mathbf{x} = \begin{pmatrix} 4 + a + 7b \\ a \\ -1 - 3b \\ b \end{pmatrix}$$

2. (a) A linear equation in 3 unknowns corresponds to a plane in 3-space.

(b) Given 2 equations in 3 unknowns, each equation corresponds to a plane. If one equation is a multiple of the other, then the equations represent the same plane and any point on the that plane will be a solution to the system. If the two planes are distinct, then they are either parallel or they intersect in a line. If they are parallel they do not intersect, so the system will have no solutions. If they intersect in a line, then there will be infinitely many solutions.

(c) A homogeneous linear system is always consistent since it has the trivial solution $\mathbf{x} = \mathbf{0}$. It follows from part (b) then that a homogeneous system of 2 equations in 3 unknowns must have infinitely many solutions. Geometrically the 2 equations represent planes that both pass through the origin, so if the planes are distinct they must intersect in a line.

3. (a) If the system is consistent and there are two distinct solutions, then there must be a free variable and hence there must be infinitely many solutions. In fact, all vectors of the form $\mathbf{x} = \mathbf{x}_1 + c(\mathbf{x}_1 - \mathbf{x}_2)$ will be solutions since

$$A\mathbf{x} = A\mathbf{x}_1 + c(A\mathbf{x}_1 - A\mathbf{x}_2) = \mathbf{b} + c(\mathbf{b} - \mathbf{b}) = \mathbf{b}$$

(b) If we set $\mathbf{z} = \mathbf{x}_1 - \mathbf{x}_2$, then $\mathbf{z} \neq \mathbf{0}$ and $A\mathbf{z} = \mathbf{0}$. Therefore, it follows from Theorem 1.5.2 that A must be singular.

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4. (a) The system will be consistent if and only if the vector $\mathbf{b} = (3, 1)^T$ can be written as a linear combination of the column vectors of A. Linear combinations of the column vectors of A are vectors of the form

$$c_1 \begin{pmatrix} \alpha \\ 2\alpha \end{pmatrix} + c_2 \begin{pmatrix} \beta \\ 2\beta \end{pmatrix} = (c_1\alpha + c_2\beta) \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Since $\mathbf{b}$ is not a multiple of $(1, 2)^T$ the system must be inconsistent.

(b) To obtain a consistent system, choose $\mathbf{b}$ to be a multiple of $(1, 2)^T$. If this is done the second row of the augmented matrix will zero out in the elimination process and you will end up with one equation in 2 unknowns. The reduced system will have infinitely many solutions.

5. (a) To transform A to B, you need to interchange the second and third rows of A. The elementary matrix that does this is

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(b) To transform A to C using a column operation, you need to subtract twice the second column of A from the first column. The elementary matrix that does this is

$$F = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

6. If $\mathbf{b} = 3\mathbf{a}_1 + \mathbf{a}_2 + 4\mathbf{a}_3$, then $\mathbf{b}$ is a linear combination of the column vectors of A and it follows from the consistency theorem that the system $A\mathbf{x} = \mathbf{b}$ is consistent. In fact, $\mathbf{x} = (3, 1, 4)^T$ is a solution to the system.

7. If $\mathbf{a}_1 - 3\mathbf{a}_2 + 2\mathbf{a}_3 = \mathbf{0}$, then $\mathbf{x} = (1, -3, 2)^T$ is a solution to $A\mathbf{x} = \mathbf{0}$. It follows from Theorem 1.5.2 that A must be singular.

8. If

$$A = \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}$$

then

$$A\mathbf{x} = \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = B\mathbf{x}$$

9. In general, the product of two symmetric matrices is not necessarily symmetric. For example, if

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix}$$

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then A and B are both symmetric but their product

$$AB = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ 4 & 10 \end{pmatrix}$$

is not symmetric.

10. If E and F are elementary matrices, then they are both nonsingular and their inverses are elementary matrices of the same type. If $C = EF$, then C is a product of nonsingular matrices, so C is nonsingular and $C^{-1} = F^{-1}E^{-1}$.

11.

$$A^{-1} = \begin{pmatrix} I & O & O \\ O & I & O \\ O & -B & I \end{pmatrix}$$

12. (a) The column partition of A and the row partition of B must match up, so k must be equal to 5. There is really no restriction on r, it can be any integer in the range $1 \leq r \leq 9$. In fact, $r = 10$ will work when B has block structure

$$\begin{pmatrix} B_{11} \\ B_{21} \end{pmatrix}$$

(b) The (2,2) block of the product is given by $A_{21}B_{12} + A_{22}B_{22}$

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Chapter 2

Determinants

1 THE DETERMINANT OF A MATRIX

1. (c) $\det(A) = -3$

7. Given that $a_{11} = 0$ and $a_{21} \neq 0$, let us interchange the first two rows of A and also multiply the third row through by $-a_{21}$. We end up with the matrix

$$\begin{pmatrix} a_{21} & a_{22} & a_{23} \\ 0 & a_{12} & a_{13} \\ -a_{21}a_{31} & -a_{21}a_{32} & -a_{21}a_{33} \end{pmatrix}$$

Now if we add a_{31} times the first row to the third, we obtain the matrix

$$\begin{pmatrix} a_{21} & a_{22} & a_{23} \\ 0 & a_{12} & a_{13} \\ 0 & a_{31}a_{22} - a_{21}a_{32} & a_{31}a_{23} - a_{21}a_{33} \end{pmatrix}$$

This matrix will be row equivalent to I if and only if

$$\begin{vmatrix} a_{12} & a_{13} \\ a_{31}a_{22} - a_{21}a_{32} & a_{31}a_{23} - a_{21}a_{33} \end{vmatrix} \neq 0$$

Thus the original matrix A will be row equivalent to I if and only if

$$a_{12}a_{31}a_{23} - a_{12}a_{21}a_{33} - a_{13}a_{31}a_{22} + a_{13}a_{21}a_{32} \neq 0$$

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