Problem Solving Approach to Mathematics for Elementary School Teachers 13th edition Billstein Solutions Manual

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INSTRUCTOR'S GUIDE TO MATHEMATICS ACTIVITIES FOR ELEMENTARY SCHOOL TEACHERS

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to accompany

A PROBLEM SOLVING APPROACH TO MATHEMATICS FOR ELEMENTARY SCHOOL TEACHERS

THIRTEENTH EDITION

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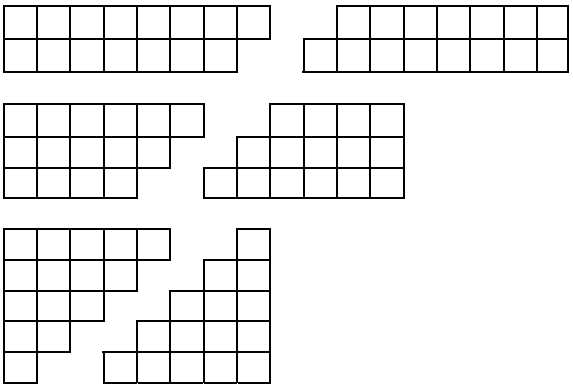
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Chapter 1
An Introduction to Problem Solving

Activity 1

Box 1: UNDERSTAND THE PROBLEM

1. Answers will vary.
2. Sample Responses: Shapes are congruent if they have exactly the same shape and size.
   Shapes are congruent if they can be made to match exactly by placing one on top of the other.
3. a. 

   b. 3 ways
4. Answers will vary.
5. Answers will vary.

Box 2: DEVISE A PLAN

1. Squares could be moved around to try to construct the staircases or they could be sketched on graph paper.
2. Simplify the problem, look for a pattern, and make a table
3. Answers will vary.

(Continued on next page.)

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Activity 1 Continued

Box 4: LOOK BACK

1. To form two congruent staircases, the number of squares must be even and you must be able to form a rectangle in which both dimensions are greater than one and one dimension is odd and the other is even using the squares.

3. a. 3 ways

   b. Since the two staircases must fit together to form a rectangle with area 120, find the dimensions of all rectangles with integral dimensions and an area of 120. The possible dimensions are 1×120, 2×60, 3×40, 4×30, 5×24, 6×20, 8×15, 10×12.

   To form a staircase, both dimensions must be greater than 1, and one dimension must be even and the other odd. This leaves 3 possibilities, a 3×40 rectangle, a 5×24 rectangle, and an 8×15 rectangle.

   The 3×40 rectangle can be divided into two 3-step staircases. The bottom step is 21 squares long, the middle step is 20 steps long, and the top step is 19 squares long.

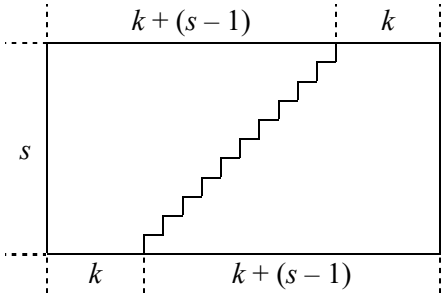
   The 5×24 rectangle can be divided into two 5-step staircases. The bottom step is 14 squares long, the next step is 13 squares long, the middle step is 12 steps long, the fourth step is 11 squares long and the top step is 10 squares long.

   The 8×15 rectangle can be divided into two 8-step staircases. The steps are 11, 10, 9, 8, 7, 6, 5, and 4 steps long.

4. To determine whether n squares can be used to construct two congruent staircases, find the factor pairs of n. There is a pair of staircases for each factor pair in which both factors are greater than 1 and one factor is odd and the other even.

   (Since the two staircases must be congruent, each must contain the same number of squares. So if n squares can be arranged into two congruent staircases, n must be even.

   Suppose a staircase has s steps and that the top step contains k squares. There are $s - 1$ steps below the top step and each step down contains one more square than the step above it, so the bottom step contains $s - 1$ more squares than the top step. That is, the bottom step contains $k + (s - 1)$ squares. When two of the staircases are put together to form a rectangle, the height of the rectangle is s and the length is $k + k + (s - 1) = 2k + (s - 1)$.

   If s is odd, $s - 1$ is even and $2k + (s - 1)$ is even since the sum of two even numbers is even.

   If s is even, $s - 1$ is odd and $2k + (s - 1)$ is odd since the sum of an odd and an even number is odd.

   So if n squares can be arranged into two congruent staircases, you must be able to make a rectangle with the squares in which one dimension is odd and the other dimension is even.

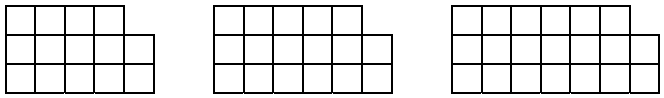
   Suppose one dimension of a rectangle is odd and the other is even. Orient the rectangle so the height is odd and the length is even. Starting at the center of the middle row of squares, form steps going up to the right and steps going down to the left. Since the number of rows above the middle row and the number of rows below it are equal, this process will result in two congruent staircases.)

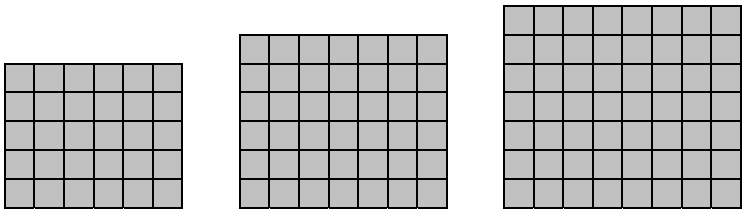
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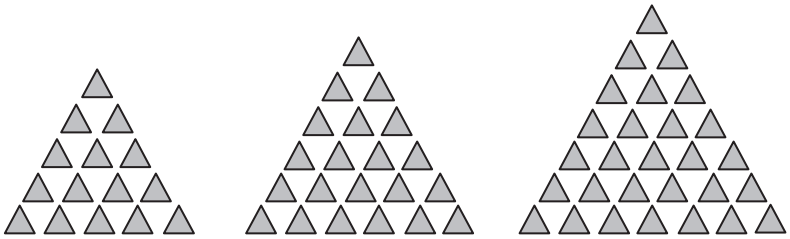
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Activity 2

Box 1

1. 

2. 

3. 

4. 6, 3, 4, 5, 6, 7

5. 14, 17, 20, 23, 26, 29

6. 25, 18, 11, 4, –3, –10

7. 15, 21, 28, 36, 45, 55

8. 28, 39, 52, 67, 84, 103

9. 32, 64, 128, 256, 512, 1024

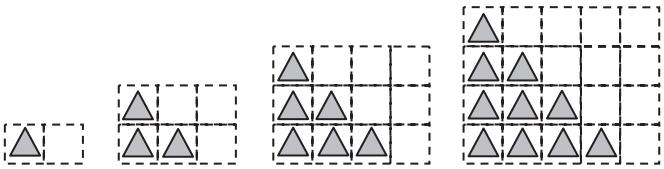
10. 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$

11. 34, 55, 89, 144, 233, 377

12. Each term of the sequence in Exercise 5 is the number of squares in the corresponding term of the sequence in Exercise 1. Similarly, each term of the sequence in Exercise 7 is the number of triangles in the corresponding term of the sequence in Exercise 3.

13. a. The number of triangles in each term of the sequence in Exercise 3 is half the number of squares in the corresponding term of the sequence in Exercise 2.

    b. Sample Response:



(Continued on next page.)

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Activity 2 Continued

Box 2

1. 7, 12, 17, 22, 27, 32, 37, 42, 47
2. 2, 5, 8, 11, 14, 17, 20, 23, 26

   Each term of the sequence in Exercise 1 on page 5 is a rectangular array 3 squares tall and *n* squares long (where *n* is the term number) with the top right-hand square removed. Thus the number of squares used to form each term is 1 less than 3 times *n*.

3. 44, 41, 38, 35, 32, 29, 26, 23, 20
4. 7, 13, 23, 37, 55, 77, 103, 133, 167
5. 2, 6, 12, 20, 30, 42, 56, 72, 90

   Each term of the sequence in Exercise 2 on page 5 is a rectangular array of squares. The height of the rectangle is equal to the term number and its length is 1 more than the term number. Since the area of the rectangle is the length times the width, the number of squares in the array is *the term number* times *the term number plus 1*. That is, *the term number* times *the next term number*.

6. In Exercises 1–3, the difference between successive terms is a constant. In Exercise 1 the difference is always 5, in Exercise 2 it is always 3, and in Exercise 3 it is –3. In each case, the difference is the constant by which the term number was multiplied.

   The differences between successive terms in Exercises 4 and 5 are not constant. Note that, in these cases, the rule for generating the sequence does not involve multiplying the term number by a constant.

7. Answers will vary. One possibility is to look for a pattern in the differences between successive terms and to continue that pattern to find the next five terms in the sequence. For example, in Exercise 5 the differences are the sequence of even numbers 4, 6, 8, 10, 12, 14, 16, 18… Thus, the next five terms would be 90 + **20** = 110, 110 + **22** = 132, 132 + **24** = 156, 156 + **26** = 182, and 182 + **28** = 210.

8. 2, 4, 8, 16, 30, **52**, **84**,**128**, 186, **260**

9. The first four terms of the sequences are the same, but from there on they are different. So no matter how many of the initial terms of a sequence you may know, there may be more than one way to extend the sequence.

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Activity 3

2.

Number of People	2	4	6	8	10	$2N$
Number of Pairs	1	2	3	4	5	N
Min Number of Moves	3	8	15	24	35	$N^2 + 2N$
Sequence of Moves	RLR	RLLRRL LR	RLLRRR LLLRRR LLR	RLLRRR LLLLRR RRLLLL RRRLLR	RLLRRR LLLLRR RRRLLL LLRRRR RLLLLR RRLLR	1R, 2L, 3R, ... , $(N-1)$L, NR, NL, NR, $(N-1)$L, ... , 3R, 2L, 1R if N is odd 1R, 2L, 3R, ... , $(N-1)$R, NL, NR, NL, $(N-1)$R, ... , 3R, 2L, 1R if N is even

4. 12 people

5. 255 moves

6.

Number of People	1	3	5	7	9	N
Min Number of Moves	1	5	11	19	29	$(N^2 + 4N - 1)/4$
Sequence of Moves	R	RLRRL	RLLRRR LLRRL	RLLRRR LLLRRR RLLLRR L	RLLRRR LLLLRR RRRLLL LRRRRL LLRRL	

The Legend of the Tower of Brahma

1.

Number of Disks	1	2	3	4	5	n
Number of Moves	1	3	7	15	31	$2^n - 1$

2. a. $2^{10} - 1 = 17$ min 3 sec

   b. $2^{30} - 1 \approx 34$ years

   c. $2^{50} - 1 \approx 36$ million years

   d. $2^{64} - 1 \approx 585$ billion years

Activity 4

2. a. 1 Subtracting 1 leaves 4 on the display.
   If Player A subtracts 1, that leaves 3 on the display and B wins by subtracting 3.
   If A subtracts 2, that leaves 2 on the display and B wins by subtracting 2.
   If A subtracts 3, it leaves 1 on the display and B wins by subtracting 1.

   b. To be certain to win, you want to be the first player. At the end of your turn you want to leave 16, 12, 8, or 4 on the display.

3. a. In this game you want to be the second player. At the end of your turn you want to leave 20, 15, 10, or 5 on the display.

   b. In this game you want to be the second player. At the end of your turn you want to leave 40, 30, 20, or 10 on the display.

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Activity 5

Box 1

1. The results for Steps 1-5 will vary depending on the number selected by each person.
2. Sample Response: 10 because that was the answer I got for Persons 1 through 5.
3. Step 1: n
   Step 2: $3n$
   Step 3: $3n + 30$
   Step 4: $(3n + 30) \div 3 = n + 10$
   Step 5: $n + 10 - n$
   The result is 10.

Box 3

1. Answers will vary.
2. The display on the calculator shows the month and day of a person's birthday. For example, 1021 for October 21.
3. Step 1 m
   Step 2: $5m$
   Step 3: $5m + 20$
   Step 4: $20m + 80$
   Step 5: $20m + 73$
   Step 6: $100m + 365$
   Step 7: $100m + 365 + d$
   Step 8: $100m + \text{d}$
   In Step 8, multiplying the month, m, by 100 moves the month to the hundreds place in the final answer. If m is a single digit number, that digit will be in the hundreds place of the answer. If m is a two-digit number, its tens digit will be in the thousands place of the answer and its units digit will be in the hundreds place. When the day, d, is added, the digits of d will appear in the tens and units places of the final answer.

Activity 6

Box 1

1. 153 (or 1 if it isn't a multi-digit number)
2. Sample Response: Since $7^3 = 343$, from the last two clues I knew the digits in the number had to be less than 7. From the second clue, I knew the digits could only be 1, 3, or 5.
3. 25 cards or 85 cards

Box 2

1. No. If there were green gumballs in the jar, it would be labeled correctly, but no jar has a correct label.
2. Red
3. No. If so, the jar labeled GREEN would have GREEN gumballs, but all labels are incorrect.
4. The correct label for the jar labeled Red - Green is Red, the correct label for the jar labeled Red is Green, and the correct label for the jar labeled Green is Red - Green.

(Continued on next page.)

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Activity 6 Continued

Box 3

State	Alabama	Alaska	Oklahoma	Minnesota
Flower	Camellia	Forget Me Not	Mistletoe	Lady's Slipper
Bird	Yellowhammer	Willow ptarmigan	Flycatcher	Loon

Clues b, d, and c are the keys to solving the problem.
From b, we know that Alaska and forget me not go together.
From d, we know that Alabama and yellowhammer go together.
From c, either camellia or lady's slipper goes with Minnesota. If camellia and Minnesota go together, then the state bird can't be the willow ptarmigan (clue e) or the loon (clue c) so it must be the flycatcher. But this would contradict clue a, so Minnesota, lady's slipper, and loon must go together.

Box 4

Two answers are possible. Freddie or Susie can be in either First or Fourth place.

First	Second	Third	Fourth
Yellow	Purple	Green	Red
Freddie or Susie	Liz	Joe	Susie or Freddie

Activity 7

Box 1

Term Number	1	2	3	4	5	6	7	8
Term	4	11	18	25	32	39	46	53
Difference		7	7	7	7	7	7	7

Constant difference = 7

Term Number		*Constant Difference*			*What Was Done?*		*To Get*	
1	′□	7	→	7	–3	=	4	First Term
2	′□	7	→	14	–3	=	11	Second Term
3	′□	7	→	21	–3	=	18	Third Term
10	′□	7	→	70	–3	=	67	Tenth Term
50	′□	7	→	350	–3	=	347	Fiftieth Term

Each term is 3 less than 7 times the term number,

Rule: nth term = $7n - 3$

Box 2

					Rule	**25th Term**	**100th Term**
1.	25,	29,	33,	37	$4T + 5$	105	405
2.	30,	37,	44,	51	$7T - 5$	170	695
3.	5,	7,	9,	11	$2T - 5$	45	195
4.	90,	88,	86,	84	$-2T + 100$	50	–100

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5. 65, 62, 59, 56 $-3T + 80$ 5 –220

Activity 8

Box 1

1.

Beginning of month	Number of newborn pairs	Number of 1-month-old pairs	Number of pairs 2 months old or older	Total number of pairs
1	1	0	0	1
2	0	1	0	1
3	1	0	1	2
4	1	1	1	3
5	2	1	2	5
6	3	2	3	8
7	5	3	5	13

2. a. The number of 1-month old pairs in month N ($N \geq 2$) is equal to the number of newborn pairs in month $N - 1$.

   b. The number of pairs 2 months old or older in month N ($N \geq 2$) is equal to the sum of the number of 1-month old pairs and the number of pairs 2-months old or older in month $N - 1$.

   c. From the second month on, the number of newborn pairs each month is equal to the number of pairs 2 months old or older.

Box 2: CREATING A SPREADSHEET

2. a. 1 was added to the 1 in cell A2, so a 2 appears in cell A3.

   b. The formula "= A3 + 1" is copied into cell A4, the formula "= A4 + 1" is copied into cell A5, and so on. Thus for each cell A4 through A14, 1 is added to the number in the preceding cell. The result is that the numbers 3 through 13 appear in cells A4 through A14 respectively.

   c. To find the number of pairs after one year, you must extend the table to the beginning of the 13th month.

3. a. The content of cell B2 was copied into cell C3, so a 1 appears in cell C3.

4. - 6.

Month	Newborn	1 Month Old	2 Months Old	Total Pairs
1	1	0	0	1
2	0	1	0	1
3	1	0	1	2
4	1	1	1	3
5	2	1	2	5
6	3	2	3	8
7	5	3	5	13
8	8	5	8	21
9	13	8	13	34
10	21	13	21	55
11	34	21	34	89
12	55	34	55	144
13	89	55	89	233

7. 233 pairs

8. From the third term on, each term equals the sum of the two preceding terms.

(Continued on next page.)

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Activity 8 Continued

Box 3: LOOKING FOR PATTERNS

Fibonacci #s	Sums	Fib. # – 1	Squares	Sum of Squares	Products	Quotients
1	1	0	1	1	1	1
1	2	0	1	2	2	2
2	4	1	4	6	6	1.5
3	7	2	9	15	15	1.666666667
5	12	4	25	40	40	1.6
8	20	7	64	104	104	1.625
13	33	12	169	273	273	1.615384615
21	54	20	441	714	714	1.619047619
34	88	33	1156	1870	1870	1.617647059
55	143	54	3025	4895	4895	1.618181818
89	232	88	7921	12816	12816	1.617977528
144	376	143	20736	33552	33552	1.618055556
233	609	232	54289	87841	87841	1.618025751
377	986	376	142129	229970	229970	1.618037135
610	1596	609	372100	602070	602070	1.618032787
987	2583	986	974169	1576239	1576239	1.618034448
1597	4180	1596	2550409	4126648	4126648	1.618033813
2584	6764	2583	6677056	10803704	10803704	1.618034056
4181	10945	4180	17480761	28284465	28284465	1.618033963
6765	17710	6764	45765225	74049690	0	0

1. c. The first sum is 1 less than the third Fibonacci number, the second sum is 1 less than the fourth Fibonacci number, and so on—the nth sum is 1 less than the $n + 2$ Fibonacci number.

2. c. The numbers in the two columns are the same.

   d. The sum of the squares of the first n Fibonacci numbers equals the product of the nth Fibonacci number and the $n + 1$ Fibonacci number.

3. The quotients approach 1.61803.

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Activity 9

1. Answers will vary.
2. See table. The number of layers doubles.
3.

Folds	0	1	2	3	4	5	6	7	8
No. Layers (Std. Form)	1	2	4	8	16	32	64	128	256
No. Layers (Fact. Form)	1	2	$2 \cdot 2$	$2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
No. Layers (Exp. Form)	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8
Approximate Height (cm)	0.008	0.016	0.031	0.063	0.125	0.25	0. 5	1.0	2

4. Answers will vary. The paper can be folded 5-6 times.
5. 1 fold, 2^1; 0 folds 2^0
6. See table.
7. After 14 folds, the stack is 128 cm tall and after 15 folds it is 256 cm tall, so depending on the student's height, their answer will be 14 or 15 folds.
8. b. The height of the stack after 30 folds would be about 8.4 km. This is higher than the Willis Tower (0.442 km) and less than the altitude of an orbiting satellite (altitudes range from about 150 km to 1000 km at the point nearest to Earth) and the distance to the moon (about 385,000 km).

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INSTRUCTOR'S SOLUTIONS MANUAL

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Northern Arizona University

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Northern Arizona University

to accompany

A PROBLEM SOLVING APPROACH TO MATHEMATICS FOR ELEMENTARY SCHOOL TEACHERS

THIRTEENTH EDITION

Rick Billstein
University of Montana

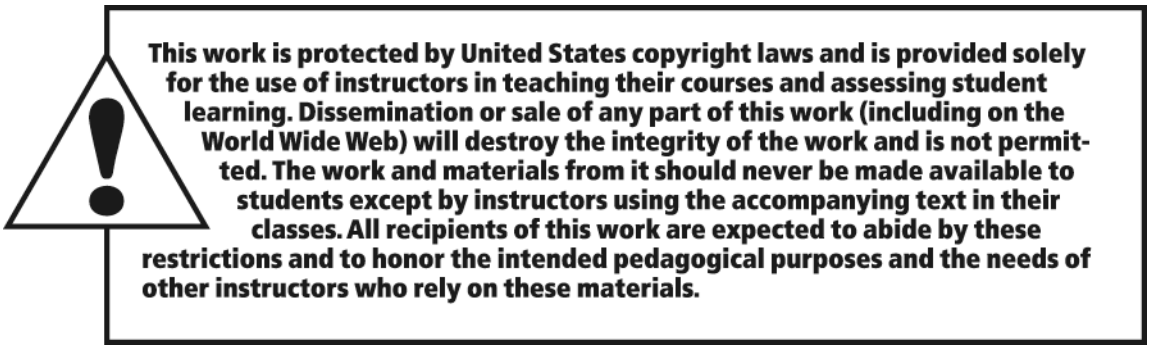
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CHAPTER 1

AN INTRODUCTION TO PROBLEM SOLVING

Assessment 1-1A: Mathematics and Problem Solving

1. (a) List the numbers:

$$\begin{array}{rcrcccrcr} 1 & + & 2 & + & \cdots & + & 98 & + & 99 \\ 99 & + & 98 & + & \cdots & + & 2 & + & 1 \\ \hline 100 & + & 100 & + & \cdots & + & 100 & + & 100 \end{array}$$

There are 99 sums of 100. Thus the total can be found by computing $\frac{99 \cdot 100}{2} = \mathbf{4950}$. (Another way of looking at this problem is to realize there are $\frac{99}{2} = 49.5$ pairs of sums, each of 100; thus $49.5 \cdot 100 = 4950$.)

(b) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{1001-1}{2} + 1 = 501$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 1 & + & 3 & + & \cdots & + & 999 & + & 1001 \\ 1001 & + & 999 & + & \cdots & + & 3 & + & 1 \\ \hline 1002 & + & 1002 & + & \cdots & + & 1002 & + & 1002 \end{array}$$

There are 501 sums of 1002. Thus the total can be found by computing $\frac{501 \cdot 1002}{2} = \mathbf{251,001}$.

(c) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{300-3}{3} + 1 = 100$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 3 & + & 6 & + & \cdots & + & 297 & + & 300 \\ 300 & + & 297 & + & \cdots & + & 6 & + & 3 \\ \hline 303 & + & 303 & + & \cdots & + & 303 & + & 303 \end{array}$$

There are 100 sums of 303. Thus the total can be found by computing $\frac{100 \cdot 303}{2} = \mathbf{15,150}$.

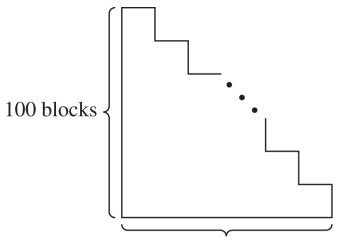
(d) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{400-4}{4} + 1 = 100$ terms.

List the numbers:

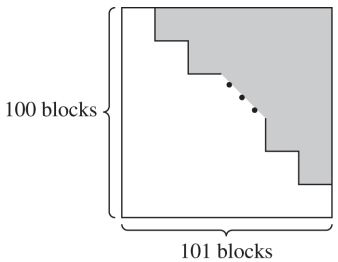
$$\begin{array}{rcrcccrcr} 4 & + & 8 & + & \cdots & + & 396 & + & 400 \\ 400 & + & 396 & + & \cdots & + & 8 & + & 4 \\ \hline 404 & + & 404 & + & \cdots & + & 404 & + & 404 \end{array}$$

There are 100 sums of 404. Thus the total can be found by computing $\frac{100 \cdot 404}{2} = \mathbf{20,200}$.

2. (a)



(b)



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When the stack in (a) and a stack of the same size is placed differently next to the original stack in (a), a rectangle containing 100 (101) blocks is created. Since each block is represented twice, the desired sum is $100(101)/2 = \mathbf{5050}$.

While the above represents a specific example, the same thinking can be used for any natural number n to arrive at a formula $n(n+1)/2$.

3. There are $\frac{147-36}{1} + 1 = 112$ terms.

List the numbers:

$$\begin{array}{ccccccccc} 36 & + & 37 & + & \cdots & + & 146 & + & 147 \\ 147 & + & 146 & + & \cdots & + & 37 & + & 36 \\ \hline 183 & + & 183 & + & \cdots & + & 183 & + & 183 \end{array}$$

There are 112 sums of 183. Thus the total can be found by computing $\frac{112 \cdot 183}{2} = \mathbf{10,248}$.

4. **(a)** Make a table as follows; there are 9 rows so there are **9 different ways**.

6-cookie packages	2-cookie packages	single-cookie packages
1	2	0
1	1	2
1	0	4
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10

(b) Make a table as follows; there are 12 rows so there are **12 different ways**.

6-cookie packages	2-cookie packages	single-cookie packages
2	0	0
1	3	0
1	2	2
1	1	4
1	0	6
0	6	0
0	5	2
0	4	4
0	3	6
0	2	8
0	1	10
0	0	12

5. If each layer of boxes has 7 more than the previous layer we can add powers of 7:

$7^0 = 1$ (red box)

$7^1 = 7$ (blue boxes)

$7^2 = 49$ (black boxes)

$7^3 = 343$ (yellow boxes)

$7^4 = 2401$ (gold boxes)

$1 + 7 + 49 + 343 + 2401 =$ **2801 boxes** altogether.

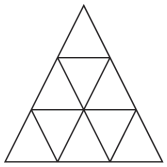
6. Using strategies from Poyla's problem solving list identify subgoals (solve simpler problems) and make diagrams to solve the original problem.



1 triangle; name this the "unit" triangle.



This triangle is made of 4 unit triangles. Counting the large triangle there are 5 triangles

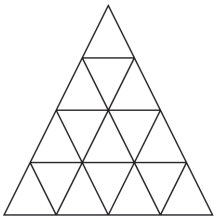


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Unit triangles	4 unit triangles	9 unit triangles
9	3	1

13 total triangles



Unit triangles	4 unit triangles	9 unit triangles	16 unit triangles
16	7	3	1

There are 27 triangles in the original figure.

7. Observe that $E = (1+1)+(3+1)+\cdots+(97+1) = O+49$. Thus, E is 49 more than O.

Alternative strategy:

$$O + E = 1+2+3+4+5+6+\cdots+97+98$$
$$= \frac{98(99)}{2} = 49(99)$$
$$E = 2(1+2+3+4+\cdots+49) = 2\left(\frac{49(50)}{2}\right) = 49(50)$$
$$O = O + E - E = 49(99) - 49(50) = 49(49).$$

So O is 49 less than E.

8. Bubba is last; Cory must be between Alababa and Dandy; Dandy is faster than Cory. Listing from fastest to slowest, the finishing order is then **Dandy, Cory, Alababa, and Bubba**.

9. Make a table.

$20 bills	$10 bills	$5 bills
2	1	0
2	0	2
1	3	0
1	2	2
1	1	4
1	0	6
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10

There are twelve rows so there are **twelve different ways**.

10. The diagonal from the left, top corner to the right, bottom corner sums to $17 + 22 + 27 = 66$.

The first row sums to $17 + a + 7 = 24 + a$. So $a = 66 - 24 = \mathbf{42}$. The last column sums to $7 + b + 27 = 34 + b$. So $b = 66 - 34 = \mathbf{32}$. The first column sums to $17 + 12 + c = 29 + c$. So $c = 66 - 29 = \mathbf{37}$. The second column sums to $42 + 22 + d = 64 + d$. So $d = 66 - 64 = \mathbf{2}$.

11. Debbie and Amy began reading on the same day, since 72 pages for Debbie ÷ 9 pages per day = 8 days. Thus Amy is on 6 pages per day × 8 days = **page 48**.

12. The last three digits must sum to 20, so the second to last digit must be $20 - (7 + 4) = 9$. Since the sum of the 11th , 12th, and 13th digits is also 20, the 11th digit is $20 - (7 + 9) = 4$.

A		7								4	7	9	4

We can continue in this fashion until we find that A is 9, or we can observe the repeating pattern from back to front, 4, 9, 7, 4, 9, 7, … and discover that **A is 9**.

13. Choose the box labeled Oranges and Apples (Box B). Retrieve a fruit from Box B. Since Box B is mislabeled, Box B should be labeled as having the fruit you retrieved. For example, if you retrieved an apple, then Box B should be labeled Apples. Since Box A is mislabeled, the Oranges and Apples label should be placed on Box A. These leave only one possibility for Box C; it should be labeled Oranges. If an orange was retrieved from Box B, then Box C would be labeled Oranges and Apples and Box A should be labeled Apples.

14. The electrician made $1315 for 4 days at $50 per hour. She spent $15 per day on gasoline so 4 • $15 = $60 on gasoline. The total is then $1315 + $60 = $ 1375. At $50 per hour, she worked $\frac{1375}{50}$ = **27.5 hours**.

15. Working backward: Top – 6 rungs – 7 rungs + 5 rungs – 3 rungs = top – 11 rungs, which is located at the middle. From the middle rung travel up 11 to the top or down 11 to the bottom. Along with the starting rung, then, there are 11 + 11 + 1 = **23 rungs.**

16. There are several different ways to solve this. One is to use a variable. So, let a be equal to the number of apple pies that are baked. This means the number of cherry pies that are baked is

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represented by $4 - a$. So, using 9 slices for an apple pie and 7 slices for a cherry pie, we get:

$9a + 7(4 - a) = 34$

$9a + 28 - 7a = 34$

$2a = 6$

$a = 3$

So the number of **apple pies is 3**. Therefore, the number of **cherry pies is 1**.

Since the number of pies is small, another solution strategy is to make a table of all possible cases:

Apple Pies	Cherry Pies	Apple slices	Cherry slices	Total slices
0	4	0	28	28
1	3	9	21	30
2	2	18	14	32
3	**1**	**27**	**7**	**34**
4	0	36	0	36

From the table, we can see there are 34 slices when there are three apple pies and one cherry pie.

17. **Al made $50**. Examine it step by step. First, Al spent $100 on the CD player. At this point, he is down $100. Once he sold it for $125, he is now $25 ahead. When he later bought it back for $150, he was down $125; but after selling it again for $175, he is now ahead $50.

18. Since the bat is $49 more than the ball, and the total spent is $50, we can use the guess and check method to solve. The table below represents possible guesses:

Cost of bat	Cost of ball	sum
$49.00	$0.00	$49.00
$49.25	$0.25	$49.50
$49.50	**$0.50**	**$50.00**

Another method would be to use a variable. Let the cost of the ball (in dollars) be the variable b. The cost of a bat in dollars, therefore, would be $b + 49$. Together the two costs must add up to 50, so:

$b + (b + 49) = 50$

$2b + 49 = 50$

$2b = 1$

$b = ½$. In terms of money, $b = \$0.50$, **50 cents**. Since the ball costs 50 cents, **the bat must cost $49.50**, and the sum of the price of bat and ball does equal $50.

Assessment 1-1B

1. **(a)** List the numbers:

$$\begin{array}{rcrcrcrcr} 1 & + & 2 & + & \cdots & + & 48 & + & 49 \\ 49 & + & 48 & + & \cdots & + & 2 & + & 1 \\ \hline 50 & + & 50 & + & \cdots & + & 50 & + & 50 \end{array}$$

There are 49 sums of 50. Thus the total can be found by computing $\frac{49 \cdot 50}{2} = \mathbf{1225}$. (Another way of looking at this problem is to realize there are $\frac{49}{2} = 24.5$ pairs of sums, each of 50; thus $24.5 \cdot 50 = 1225$.)

(b) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{2009 - 1}{2} + 1 = 1005$ terms.

List the numbers:

$$\begin{array}{rcrcrcrcr} 1 & + & 3 & + & \cdots & + & 2007 & + & 2009 \\ 2009 & + & 2007 & + & \cdots & + & 3 & + & 1 \\ \hline 2010 & + & 2010 & + & \cdots & + & 2010 & + & 2010 \end{array}$$

There are 1005 sums of 2010. Thus the total can be found by computing $\frac{1005 \cdot 2010}{2} = \mathbf{1,010,025}$.

(c) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{600 - 6}{6} + 1 = 100$ terms.

List the numbers:

$$\begin{array}{rcrcrcrcr} 6 & + & 12 & + & \cdots & + & 594 & + & 600 \\ 600 & + & 594 & + & \cdots & + & 12 & + & 6 \\ \hline 606 & + & 606 & + & \cdots & + & 606 & + & 606 \end{array}$$

There are 100 sums of 606. Thus the total can be found by computing $\frac{606 \cdot 100}{2} = \mathbf{30,300}$.

(d) The number of terms in any sequence of numbers may be found by subtracting the first term from the last (or the last from the first if the first is greater than the last), dividing the result by the common

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difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{1000-5}{5}+1=200$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 1000 & + & 995 & + & \cdots & + & 10 & + & 5 \\ 5 & + & 10 & + & \cdots & + & 995 & + & 1000 \\ \hline 1005 & + & 1005 & + & \cdots & + & 1005 & + & 1005 \end{array}$$

There are 200 sums of 1005. Thus the total can be found by computing $\frac{1005\cdot 200}{2}=$ **100,500**.

2. (a) The diagram illustrates how the numbers can be paired to form 50 sums of 101. The sum of the first 100 natural numbers is $50(101) =$ **5050**.

(b) A diagram similar to the one in 2a would illustrate how the numbers can be paired to form 100 sums of 202. Because there are an odd number of terms, the middle term, 101, is left unpaired. So, the sum of the first 201 natural numbers is 100 • 202 + 101 = **20,301**.

3. There are $\frac{203-58}{1}+1=146$ terms. terms

List the numbers:

$$\begin{array}{rcrcccrcr} 58 & + & 59 & + & \cdots & + & 202 & + & 203 \\ 203 & + & 202 & + & \cdots & + & 59 & + & 58 \\ \hline 261 & + & 261 & + & \cdots & + & 261 & + & 261 \end{array}$$

There are 146 sums of 261. Thus the total can be found by computing $\frac{146\cdot 261}{2}=$ **19,053**.

4. There are many answers to this problem. A systematic list is a good approach. Using only two numbers and addition, 5 rows give 5 different ways.

One number	Eleven minus the number
1	10
2	9
3	8
4	7
5	6

We can view 6 + 5 as a different way than 5 + 6 and continue in this manner to find 10 different ways. Or, we can find 7 more ways using three numbers as follows.

1	1	9
2	1	8
3	1	7
4	1	6
5	1	5
1	2	8

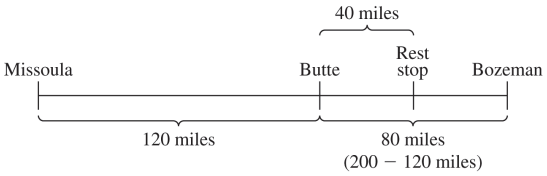
5. Since there are two different color socks in the drawer, drawing only two socks does not guarantee finding a matching pair; you could get one sock of each color. However, once you draw a third sock, you are guaranteed to have a matching pair, since the third sock must match one or the other of the previous two socks.

6. There are 13 squares of one unit each; 4 squares of four units each; and one square 9 units; for a total of 18 squares.

7.
$$\begin{aligned} P &= 1+3+5+7+\ldots+99 \\ Q &= \quad 5+7+\ldots+99+101 \\ Q-P &= (5+\ldots+99+101)- \\ &\quad (1+3+5+\ldots+99) \\ &= (101)-(1+3) \\ &= 97 \end{aligned}$$

Q is larger than P by 97.

8. A diagram will help.



The next step is 120 miles + 40 miles = **160 miles** from Missoula.

9. (a) Marc must have five pennies to make an even $1.00. The minimum number of coins would have as many quarters as possible, or three quarters. The remaining $20¢$ must consist of at least one dime and one nickel; the only possibility is one dime and two nickels. The minimum is 5 pennies, 2 nickels, 1 dime, and 3 quarters, or **11 coins**.

(b) The maximum number of coins is achieved by having as many pennies as possible. It is a requirement to have one quarter, one dime, and one nickel $= 40¢$, so there may then be 60 pennies for a total of **63 coins**.

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10. Adding all the numbers gives 99. This means that each row, diagonal, and column must add to $99 \div 3 = 33$. Write 33 as a sum of the numbers in all possible ways:

$19 + 11 + 3$
$19 + 9 + 5$
$17 + 13 + 3$
$17 + 11 + 5$
$17 + 9 + 7$
$15 + 13 + 5$
$15 + 11 + 7$
$13 + 11 + 9$

Summarizing the pattern:

Number	Nr. sums with number
3	2
5	3
7	2
9	3
11	4
13	3
15	2
17	3
19	2

Thus 11 must be in the center of the square and 5, 9, 13, and 17 must be in the corners. One solution would be:

17	7	9
3	11	19
13	15	5

11. Answers may vary; two solutions might be to:

(a) Put four marbles on each tray of the balance scale. Take the heavier four and weigh two on each tray. Take the heavier two and weigh one on each tray; the heavier marble will be evident on this third weighing.

(b) This alternative shows the heavier marble can be found more efficiently, two steps rather than three. Put three marbles in each tray of the balance scale.

(i) If the two trays are the same weight, the heavier marble is one of the remaining two. Weigh them to find the heavier.

(ii) If one side is heavier, take two of the three marbles and weigh them. If they are the same weight, the remaining marble is the heavier. If not, the heavier will be evident on this second weighing.

12. **(a)** There are:

1 partridge $\times$ 12 days $=$ 12 gifts;
2 doves $\times$ 11 days $=$ 22 gifts;
3 hens $\times$ 10 days $=$ 30 gifts;
4 birds $\times$ 9 days $=$ 36 gifts;
5 rings $\times$ 8 days $=$ 40 gifts;
6 geese $\times$ 7 days $=$ 42 gifts;
7 swans $\times$ 6 days $=$ 42 gifts;
8 maids $\times$ 5 days $=$ 40 gifts;

9 ladies $\times$ 4 days $=$ 36 gifts;
10 lords $\times$ 3 days $=$ 30 gifts;
11 pipers $\times$ 2 days $=$ 22 gifts; and
12 drummers $\times$ 1 day $=$ 12 gifts.

So the **gifts given the most by your true love was 42 geese and 42 swans**.

(b) $12 + 22 + \cdots + 22 + 12 =$ **364 gifts total**.

13. **(a)** There must be 1 or 3 quarters for an amount ending in 5. Then dimes can add to \$1.15 plus 4 pennies to realize \$1.19. Thus:

Quarters	Dimes	Pennies	Total
3	**4**	**4**	**\$1.19**
1	**9**	**4**	**\$1.19**

and in neither case can change for \$1.00 be made.

(b) Two or zero quarters would allow an amount ending in 0. Then more combinations of dimes or pennies could add to \$1.00.

14. If the price of 15 sandwiches equals the price of 20 salads, each sandwich will buy $\frac{20}{15} = \frac{4}{3}$ salads. Thus 3 sandwiches $= 3\left(\frac{4}{3}\right) =$ **4 salads**.

15. Use a variable and a table

12 AM	5 AM	9 AM	12 PM
T	T − 15	2(T − 15)	2(T − 15) + 10

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$$\begin{aligned} \text{So,}\quad 2(T-15)+10 &= 32\\ 2T-30+10 &= 32\\ 2T-20 &= 32\\ 2T &= 52\\ T &= \textbf{26 degrees}. \end{aligned}$$

16. One way to solve this problem is to write an equation using a variable. For example, let p equal the number of puzzles Seth bought. So the number of trucks Seth bought would be $5 - p$. Since each puzzle cost \$9 and each truck cost \$5, we get

$$9p + 5(5-p) = 33$$
$$9p + 25 - 5p = 33$$
$$4p = 8$$
$$p = 2$$

So Seth bought **2 puzzles**. Since he bought 5 gifts all together, he also bought **3 trucks.**

Another way to solve it is to guess and check, making a table to keep track of results

Puzzles	Trucks	Cost of puzzles	Cost of Trucks	Total cost
0	5	\$0	\$25	\$25
1	4	\$9	\$20	\$29
2	**3**	**\$18**	**\$15**	**\$33**
3	2	\$27	\$10	\$37

The table shows that by buying **2 puzzles and 3 trucks**, Seth will spend \$33.

17. The first line tells us that hamburgers equal \$10. Using that information, we can use the second line to figure out 4 hot dogs cost \$8, so each hot dog costs \$2. Going to the third line, since two hot dogs cost \$4, each drink must cost \$1. So on the fourth line, one drink and one hamburger cost \$11. **The value of the question mark is \$11**.

18. This is difficult to visualize, so the strategy of examining a simpler case and looking for a pattern might be the best method to solve this problem. It would also help if you are able to make a model of the shapes and act out the situation.

Imagine a $3 \times 3 \times 3$ large cube. It would be made up of a total of 27 small cubes. Taking off one layer all around would involve taking off the top layer, the bottom layer, the front, the back, the left side, and the right side. Doing this one step at a time: taking off the top layer takes away 9 cubes. The same if you take off the bottom layer, so you've removed 18 cubes so far. Taking off the front and the back would remove 3 more cubes each (the other 6 cubes on those faces were removed when the top and bottom were removed). Finally, taking off the left and right sides removes one more cube each. In total, 26 cubes are removed, leaving only 1 cube.

Now, imagine a $4 \times 4 \times 4$ cube. There would be a total of 64 small cubes making it. Removing the top and bottom layers takes a total of 32 cubes away; removing the front and the back would take away 16 more cubes; finally, removing the left and right sides takes away 8 more cubes, leaving only 8 cubes.

So, when starting with a $3 \times 3 \times 3$ large cube, we are left with 1 small cube $(1 \times 1 \times 1 = 1)$; when starting with a $4 \times 4 \times 4$ large cube, we are left with 8 small cubes $(2 \times 2 \times 2 = 8)$. Therefore, the pattern seems to be, when given large cube of dimension $n \times n \times n$, to find the number of small cubes that make it after removing one layer of small cubes all around the larger cube, you would take $(n-2) \times (n-2) \times (n-2)$. So for a $10 \times 10 \times 10$ large cube, the number of small cubes left would be $8 \times 8 \times 8 =$ **512 small cubes.**

Assessment 1-2A: Explorations with Patterns

1. (a) Each figure in the sequence adds one box each to the top and bottom rows. The next would be:



(b) Each figure in the sequence adds one upright and one inverted triangle. The next would be:



(c) Each figure in the sequence adds one box to the base and one row to the overall triangle. The next would be:



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2. **(a)** Terms that continue a pattern are **17, 21, 25**, , … . This is an **arithmetic** sequence because each successive term is obtained from the previous term by addition of 4.

(b) Terms that continue a pattern are **220, 270, 320**, … . This is **arithmetic** because each successive term is obtained from the previous term by addition of 50.

(c) Terms that continue a pattern are **27, 81, 243**, … . This is **geometric** because each successive term is obtained from the previous term by multiplying by 3.

(d) Terms that continue a pattern are $\mathbf{10^9, 10^{11}, 10^{13}}$, … . This is **geometric** because each successive term is obtained from the previous term by multiplying by 10^2.

(e) Terms that continue a pattern are $\mathbf{193+10\times 2^{30}, 193+11\times 2^{30}, 193+12\times 2^{30}}$, … . This is **arithmetic** because each successive term is obtained from the previous term by addition of 2^{30}.

3. In these problems, let a_n represent the nth term in a sequence, a_1 represent the first term, d represent the common difference between terms in an arithmetic sequence, and r represent the common ratio between terms in a geometric sequence. In an arithmetic sequence, $a_n = a_1 + (n-1)d$; in a geometric sequence $a_n = a_1 r^{n-1}$. Thus:

(a) Arithmetic sequence: $a_1 = 1$ and $d = 4$:

(i) $a_{100} = 1 + (100-1)\cdot 4$
$= 1 + 99 \cdot 4 = \mathbf{397}$.

(ii) $a_n = 1 + (n-1)\cdot 4$
$= 1 + 4n - 4 = \mathbf{4n - 3}$.

(b) Arithmetic sequence: $a_1 = 70$ and $d = 50$:

(i) $a_{100} = 70 + (100-1)\cdot 50$
$= 70 + 99 \cdot 50 = \mathbf{5020}$.

(ii) $a_n = 70 + (n-1)\cdot 50$
$= 70 + 50n - 50$ or $\mathbf{50n + 20}$.

(c) Geometric sequence: $a_1 = 1$ and $r = 3$:

(i) $a_{100} = 1 \cdot 3^{100-1} = \mathbf{3^{99}}$.

(ii) $a_n = 1 \cdot 3^{n-1} = \mathbf{3^{n-1}}$.

(d) Geometric sequence:
$a_1 = 10$ and $r = 10^2$:

(i) $a_{100} = 10 \cdot (10^2)^{(100-1)} = 10 \cdot (10^2)^{99}$
$= 10 \cdot 10^{198} = \mathbf{10^{199}}$.

(ii) $a_n = 10 \cdot (10^2)^{(n-1)}$
$= 10 \cdot 10^{(2n-2)} = \mathbf{10^{2n-1}}$.

(e) Arithmetic sequence:
$a_1 = 193 + 7 \cdot 2^{30}$ and $d = 2^{30}$:

(i)
$a_{100} = 193 + 7 \cdot 2^{30} + (100-1)\cdot 2^{30}$
$= 193 + 7 \cdot 2^{30} + 99 \cdot 2^{30}$
$= \mathbf{193 + 106 \times 2^{30}}$.

(ii) $a_n = 193 + 7 \cdot 2^{30} + (n-1)\cdot 2^{30}$
$= \mathbf{193 + (n + 6) \times 2^{30}}$.

4. **2, 7, 12**, … . Each term is the 5th number on a clock face (clockwise) from, the preceding term.

5. **(a)** Make a table.

Number of term	Term
1	$1 \cdot 1 \cdot 1 = 1$
2	$2 \cdot 2 \cdot 2 = 8$
3	$3 \cdot 3 \cdot 3 = 27$
4	$4 \cdot 4 \cdot 4 = 64$
5	$5 \cdot 5 \cdot 5 = 125$
6	$6 \cdot 6 \cdot 6 = 216$
7	$7 \cdot 7 \cdot 7 = 343$
8	$8 \cdot 8 \cdot 8 = 512$
9	$9 \cdot 9 \cdot 9 = 729$
10	$10 \cdot 10 \cdot 10 = 1000$
11	$11 \cdot 11 \cdot 11 = 1331$

The 11^{th} term **1331** is the least 4-digit number greater than 1000.

(b) The 9^{th} term **729** is the greatest 3-digit number in this pattern.

(c) $10^4 = 10{,}000$; The greatest number less than 10^4 is $21 \cdot 21 \cdot 21 = \mathbf{9261}$.

(d) The cell A14 corresponds to the 14th term, which is $14 \cdot 14 \cdot 14 = \mathbf{2744}$.

6. **(a)** The number of matchstick squares in each windmill form an arithmetic sequence with $a_1 = 5$ and $d = 4$. The number of matchstick squares required to build the

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10th windmill is thus $5 + (10 - 1) \cdot 4 = 5 + 9 \cdot 4 =$ **41 squares**.

(b) The nth windmill would require $5 + (n - 1) \cdot 4 = 5 + 4n - 4 =$ **$4n + 1$ squares**.

(c) There are 16 matchsticks in the original windmill. Each additional windmill adds 12 matchsticks.

This is an arithmetic sequence with $a_1 = 16$ and $d = 12$, so $a_n = 16 + (n - 1) \cdot 12 =$ **$12n + 4$ matchsticks**.

7. (a) Each cube adds four squares to the preceding figure; or 6, 10, 14, … . This is an arithmetic sequence with $a_1 = 6$ and $d = 4$. Thus $a_{15} = 6 + (15 - 1) \cdot 4 =$ **62 squares** to be painted in the 10th figure.

(b) This is an arithmetic sequence with $a_1 = 6$ and $d = 6$. The nth term is thus: $a_n = 6 + (n - 1) \cdot 4 =$ **$4n + 2$**.

8. Since the first year begins with 700 students, after the first year there would be 760, after the second there would be 820, … , and after the twelfth year the number of students would be the 13^{th} term in the sequence.

This then is an arithmetic sequence with $a_1 = 700$ and $d = 60$, so the 13^{th} term (current enrollment + twelve more years) is: $700 + (13 - 1) \cdot 60 =$ **1420 students.**

9. Using the general expression for the nth term of an arithmetic sequence with $a_1 = 24{,}000$ and $a_9 = 31{,}680$ yields:

$$31680 = 24000 + (9 - 1)d$$
$$31680 = 24000 + 8d \Rightarrow d = 960,$$

the amount by which Juan's income increased each year.

To find the year in which his income was \$45120:

$$45120 = 24000 + (n - 1) \cdot 960$$
$$45120 = 23040 + 960n$$
$$\Rightarrow n = 23.$$

Juan's income was \$45,120 in his **23rd year**.

10. The number that fits into the last triangle is **8**. The numbers inside the triangle are found by multiplying the number at the top of the triangle by the number at the bottom left of the triangle; then subtracting from that the number at the bottom right of the triangle. So, $2 \times 5 - 2 =$ **8**.

11. (a) To build an up-down-up staircase with 3 steps up and 3 steps down, each current step will have a block placed on it; plus a block will be added to each end. In total, there will be five more blocks, for a total of 9 blocks used. To build an up-down-up staircase with 4 steps up and 4 steps down, each step from the previous iteration will have a block placed on it (5 blocks) plus a block at each end (2 blocks), adding seven total blocks, bringing the total to 9 + 7 = 16 blocks. The table below illustrates the pattern:

Steps up/down	Blocks added	Total blocks
1	0	1
2	3	4
3	5	9
4	7	16

Therefore, to build an up-down-up staircase with 5 steps up and 5 steps down, 9 blocks need to be added to the previous 16 blocks to arrive at a total of **25 blocks**.

(b) Based on the table and answer above, the total blocks are always the square of the number of steps up and down. So, if the number of steps up and down are n, then the total number of blocks will be $\boldsymbol{n^2}$.

12. (a) Using the general expression for the nth term of an arithmetic sequence with $a_1 = 51$, $a_n = 251$, and $d = 1$ yields:

$$251 = 51 + (n - 1) \cdot 1 \Rightarrow$$
$$251 = 50 + n \Rightarrow n = 201.$$

There are **201 terms** in the sequence.

(b) Using the general expression for the nth term of a geometric sequence with $a_1 = 1$, $a_n = 2^{60}$, and $r = 2$ yields:

$$2^{60} = 1(2)^{n-1} = 2^{n-1}$$
$$\Rightarrow n - 1 = 60 \Rightarrow n = 61.$$

There are **61 terms** in the sequence.

(c) Using the general expression for the nth term of an arithmetic sequence with $a_1 = 10$, $a_n = 2000$, and $d = 10$ yields:

$$2000 = 10 + (n - 1) \cdot 10 \Rightarrow$$
$$2000 = 10n \Rightarrow n = 200.$$

There are **200 terms** in the sequence.

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(d) Using the general expression for the n^{th} term of a geometric sequence with $a_1 = 1$, $a_n = 1024$, and $r = 2$ yields:

$$1024 = 1(2)^{n-1} \Rightarrow$$
$$2^{10} = 2^{n-1} \Rightarrow n - 1 = 10$$
$$\Rightarrow n = 11.$$

There are **11 terms** in the sequence.

13. (a) First term: $(1)^2 + 2 = \mathbf{3}$;
Second term: $(2)^2 + 2 = \mathbf{6}$;
Third term: $(3)^2 + 2 = \mathbf{11}$;
Fourth term: $(4)^2 + 2 = \mathbf{18}$; and
Fifth term: $(5)^2 + 2 = \mathbf{27}$.

(b) First term: $5(1) + 1 = \mathbf{6}$;
Second term: $5(2) + 1 = \mathbf{11}$;
Third term: $5(3) + 1 = \mathbf{16}$;
Fourth term: $5(4) + 1 = \mathbf{21}$; and
Fifth term: $5(5) + 1 = \mathbf{26}$.

(c) First term: $10^{(1)} - 1 = \mathbf{9}$;
Second term: $10^{(2)} - 1 = \mathbf{99}$;
Third term: $10^{(3)} - 1 = \mathbf{999}$;
Fourth term: $10^{(4)} - 1 = \mathbf{9999}$; and
Fifth term: $10^{(5)} - 1 = \mathbf{99999}$.

(d) First term: $3(1) - 2 = \mathbf{1;}$
Second term: $3(2) - 2 = \mathbf{4;}$
Third term: $3(3) - 2 = \mathbf{7;}$
Fourth term: $3(4) - 2 = \mathbf{10;}$ and
Fifth term: $3(5) - 2 = \mathbf{13.}$

14. Answers may vary; examples are:

(a) If $n = 5$, then $\frac{5+5}{5} = 2 \neq 5 + 1 = 6$.

(b) If $n = 2$, then $(2 + 4)^2 = 6^2 = 36$ does not equal $2^2 + 4^2 = 20$.

15. (a) There are 1, 5, 11, 19, 29 tiles in the five figures. Each figure adds 2n tiles to the preceding figure, thus a_6 the 6th term has $29 + 12 = \mathbf{41\ tiles}$.

(b) $n^2 = 1, 4, 9, 16, 25, \ldots$. Adding $(n - 1)$ to n^2 yields $1, 5, 11, 19, 29, \ldots$, which is the proper sequence. Thus the n*th* term has $\boldsymbol{n^2 + n - 1}$.

(c) If $n^2 + (n - 1) = 1259$;

Then $n^2 + n - 1260 = 0$. This implies $(n - 35)(n + 36) = 0$, so $n = 35$.

There are 1259 tiles in the **35th figure**.

16. The nth term of the arithmetic sequence is $200 + n(200)$. The sequence can also be generated by adding 200 to the previous term. The n^{th} term of the geometric sequence is 2^n. The sequence can also be generated by multiplying the previous term by 2. Make a table.

Number of the term	Arithmetic term	Geometric term
7	1600	128
8	1800	256
9	2000	512
10	2200	1024
11	2400	2048
12	2600	4096

With the **12th term**, the geometric sequence is greater.

17. (a) Start with one piece of paper. Cutting it into five pieces gives us 5. Taking each of the pieces and cutting it into five pieces again gives $5 \cdot 5 = 25$ pieces. Continuing this process gives a geometric sequence: 1, 5, 25, 125, … . After the 5th cut there are $\mathbf{5^5 = 3125}$ pieces of paper.

(b) The number of pieces after the nth cut would be $\mathbf{5}^n$.

18. (a) For an arithmetic sequence there is a common difference between the terms. Between 39 and 69 there are three differences so we can find the common difference by subtracting 39 from 69 and dividing the answer by three: $69 - 39 = 30$ and $30 \div 3 = 10$. The common difference is 10 and we can find the missing terms: 39 – 10 = **29** and 39 + 10 = **49** and 49 + 10 = **59**.

(b) For an arithmetic sequence there is a common difference between the terms. Between 200 and 800 there are three differences so we can find the common difference by subtracting 200 from 800 and dividing the answer by three: $800 - 200 = 600$ and $600 \div 3 = 200$. The

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common difference is 200 and we can find the missing terms: 200 – 200 = **0** and 200 + 200 = **400** and 400 + 200 = **600**.

(c) For a geometric sequence there is a common ration between the terms. Between 5^4 and 5^{10} there are three common ratios used so we can find the common ratio by dividing 5^{10} by 5^4 and then taking the cube root:

$5^{10} \div 5^4 = 5^6$ and $(5^6)^{\left(\frac{1}{3}\right)} = 5^2$. The common ratio is 52 and we can find the missing terms:

$5^4 \div 5^2 = \mathbf{5^2}, 5^4 \cdot 5^2 = \mathbf{5^6}, 5^6 \cdot 5^2 = \mathbf{5^8}$.

19. (a) Let's call the missing terms a, b, c, d, e and f, then the sequence becomes:

$a, b, 1, 1, c, d, e, f$

$b + 1 = 1 \rightarrow b = 0$

$a + b = 1 \rightarrow a + 0 = 1 \rightarrow a = 1$

$1 + 1 = c \rightarrow c = 2$

$1 + c = d \rightarrow 1 + 2 = d \rightarrow d = 3$

$c + d = e \rightarrow 2 + 3 = e \rightarrow e = 5$

$d + e = f \rightarrow 3 + 5 = f \rightarrow = 8.$

The missing terms are **1, 0, 2, 3, 5**, and **8**.

(b) Let's call the missing terms a, b, c, and d, then the sequence becomes:

$a, b, c, 10, 13, d, 36, 59$

$c + 10 = 13 \rightarrow c = 3$

$b + c = 10 \rightarrow b + 3 = 10 \rightarrow b = 7$

$a + b = c \rightarrow a + 7 = 3 \rightarrow = {}^{-}4$

$10 + 13 = d \rightarrow d = 23$

The missing terms are **-4, 7, 3**, and **23**.

(c) If a Fibonacci-type sequence is a sequence in which the first two terms are arbitrary and in which every term starting from the third is the sum of the previous two terms, then we can add 0 and 2 to get the third term and continue the pattern:

$0 + 2 = 2$

$2 + 2 = 4$

$2 + 4 = 6$

$4 + 6 = 10$

$6 + 10 = 16$

$10 + 16 = 26$

The missing terms are **2, 4, 6, 10, 16**, and **26**.

20. (a)

Year 1 $80 + .05(80) = 84$

Year 2 $84 + .05(84) = 88.2$

Year 3 $88.2 + .05(88.2) = 92.61$

Year 4 $92.61 + .05(92.61) = 97.2405$

Year 5 $97.2405 + .05(97.2405) = 102.102525$

$\approx$ **$102.10**.

(b) This is a geometric sequence with $a_1 = 80$ and $\boldsymbol{r} = 1.05$, so the price after $\boldsymbol{n}$ years is $\mathbf{80 \cdot 1.05^n}$.

Assessment 1-2B

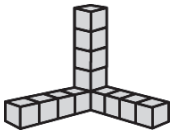
1. (a) In a clockwise direction, the shaded area moves to a new position separated from the original by one open space, then two open spaces, then by three, etc. The separation in each successive step increases by one unit; next would be:



(b) Each figure in the sequence adds one row of boxes to the base. Next would be:



(c) Each figure in the sequence adds one box to the top and each leg of the figure. Next would be:



2. (a) Terms that continue a pattern are **18, 22, 26**, … . This is an **arithmetic** sequence because each successive term is obtained from the previous term by addition of 4.

(b) Terms that continue a pattern are **39, 52, 65**, … . This is an **arithmetic** sequence because each successive term is obtained from the previous term by addition of 13.

(c) Terms that continue a pattern are $\mathbf{4^4}$, $\mathbf{4^5}$, $\mathbf{4^6}$, … . This is a **geometric** sequence because each successive term is obtained from the previous term by multiplying by 4.

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(d) Terms that continue a pattern are $\mathbf{2^{14}, 2^{18}, 2^{22}}$, … . This is a **geometric** sequence because each successive term is obtained from the previous term by multiplying by 24.

(e) Terms that continue a pattern are $\mathbf{100+10\times 2^{50}, 100+12\times 2^{50}, 100+14\times 2^{50}}$,.... This is an **arithmetic** sequence because each successive term is obtained from the previous term by adding by $2\bullet 2^{50}$.

3. In these problems, a_n represents the nth term in a sequence, a_1 represents the first term, d represent the common difference between terms in an arithmetic sequence, and r represents the common ratio between terms in a geometric sequence.

In an arithmetic sequence, $a_n = a_1 + (n-1)d$; in a geometric sequence, $a_n = a_1 r^{n-1}$. Thus:

(a) Arithmetic sequence: $a_1 = 2$ and $d = 4$.

(i) $a_{100} = 2 + (100-1)\cdot 4 = \mathbf{398}$.

(ii) $a_n = 2 + (n-1)\cdot 4$
$= 2 + 4n - 4 = \mathbf{4n - 2}$.

(b) Arithmetic sequence: $a_1 = 0$ and $d = 13$.

(i) $a_{100} = 0 + (100-1)\cdot 13 = \mathbf{1287}$.

(ii) $a_n = 0 + (n-1)\cdot 13$
$= \mathbf{13n - 13}$.

(c) Geometric sequence: $a_1 = 4$ and $r = 4$.

(i) $a_{100} = 4\cdot 4^{99} = \mathbf{4^{100}}$.

(ii) $a_n = 4\cdot 4^{n-1} = \mathbf{4^n}$.

(d) Geometric sequence: $a_1 = 2^2$ and $r = 2^4$.

(i)
$a_{100} = 2^2\cdot(2^4)^{99} = 2^2\cdot 2^{396} = \mathbf{2^{398}}$.

(ii) $a_n = 2^2\cdot(2^4)^{(n-1)}$
$= 2^2\cdot 2^{4n-4} = \mathbf{2^{4n-2}}$.

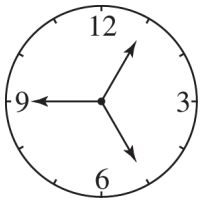
(e) Arithmetic sequence:
$a_1 = 100 + 4\cdot 2^{50}$ and $d = 2\cdot 2^{50} = 2^{51}$:

(i)
$a_{100} = 100 + 4\cdot 2^{50} + (100-1)\cdot 2^{51}$
$= 100 + 2\cdot 2^{51} + 99\cdot 2^{51}$
$= 100 + 101\cdot 2^{51}$
$= 100 + 101\cdot 2\cdot 2^{50}$
$= \mathbf{100 + 202\times 2^{50}}$.

(ii) $a_n = 100 + 4\cdot 2^{50} + (n-1)\cdot 2^{51}$
$= 100 + 2\cdot 2^{51} + (n-1)\cdot 2^{51}$
$= 100 + (n+1)\cdot 2^{51}$
$= 100 + (n+1)\cdot 2\cdot 2^{50}$
$= \mathbf{100 + 2(n+1)2^{50}}$.

4. The hands must move 8 hours to move from 1 to 9 on the clock face. To move from 9 to 5, the hand must move 8 hours also. To move from 5 to 1, the hand must move another 8 hours. If we add 8 hours to 1 o'clock, we will land on the 9.

This pattern will continue, so the next three terms are **9, 5, 1**.



5. **(a)** Answers may vary: two possible answers are:

(i)
The sum of the first *n* odd numbers is n^2; e.g., $1 + 3 + 5 + 7 = 4^2$.

(ii) **Square the average of the first and last terms;** e.g.,

$$1 + 3 + 5 + 7 = \left(\frac{1+7}{2}\right)^2 = 4^2.$$

(b) There are $\frac{35-1}{2}+1=18$ terms in this sequence.

(i) $1 + 3 + 5 + 7 + \cdots + 35 = 18^2$
$= \mathbf{324}$.

(ii) $\left(\frac{1+35}{2}\right)^2 = 18^2 = \mathbf{324}$.

6. **(a)** Note that 5 toothpicks are added to form each succeeding hexagon. This is an arithmetic sequence $a_1 = 6$ and $d = 5$,

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so $a_{10} = 6 + (10 - 1) \cdot 5 =$
$6 + 9 \cdot 5 =$ **51 toothpicks**.

(b) n hexagons would require
$6 + (n - 1) \cdot 5 = 6 + 5n - 5 =$ **$5n + 1$ toothpicks**.

7. (a) Looking at the second figure, there are $3 + 1 = 4$ triangles. In the third figure, there are $5 + 3 + 1 = 9$ triangles. The fourth figure would then have $7 + 5 + 3 + 1 = 16$ triangles. An alternative to simply adding $7, 5, 3,$ and 1 together is to note that $7 + 1 = 8$ and $5 + 3 = 8$. There are $\frac{4}{2} = 2$ of these sums, and $2 \cdot 8 = 16$. Then the 100th figure would have $100 + 99 = 199$ triangles in the base, $99 + 98 = 197$ triangles in the second row, and so on until the 100th row where there would be 1 triangle. $199 + 1 = 200$; $197 + 3 = 200$; etc. and so the sum of each pair is 200 and there are $\frac{100}{2} = 50$ of these pairs. $50 \cdot 200 = 10{,}000,$ or **10,000 triangles** in the 100th figure.

(b) The number of triangles in the nth figure is $\frac{n}{2}$(number of triangles in base + 1). The number of triangles in the base is $n + (n - 1)$, or $2n - 1$. $(2n - 1) + 1 = 2n$. Then $\frac{n}{2}(2n) = n^2$, or **n^2 triangles** in the nth figure.

8. This is a geometric sequence with $a_1 = \dfrac{15{,}360}{2}$ and $r = \frac{1}{2}$. The nth term of a geometric sequence is $a_n = a_1 r^{n-1}$; thus the 10th term would be $15360\left(\frac{1}{2}\right)^{10} =$ **15 liters**.

Note the progression of terms in the following table:

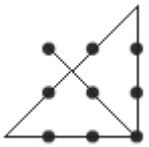
After Day	Amount of Water Remaining
1	$15{,}360 \cdot \frac{1}{2} = 7680$ liters
2	$7680 \cdot \frac{1}{2} = 3840$ liters
⋮	⋮
9	$60 \cdot \frac{1}{2} = 30$ liters
10	$30 \cdot \frac{1}{2} = 15$ liters

9. This is an arithmetic sequence with $a_1 = 8\frac{1}{6}$ (i.e., 8 a.m. plus 10 minutes, or $\frac{10}{60}$ of an hour) and $d = \frac{5}{6}$ (or $\frac{50}{60}$ of an hour). Thus

$a_8 = 8\frac{1}{6} + (8 - 1) \cdot \frac{5}{6} = 14,$ or **2:00 p.m.**

(14 is 2:00 p.m. on a 24-hour clock.)

10. Answers will be a rotation of the following figure:



11. (a) In the first drawing, there are 6 toothpicks; in the second drawing, there are 10, and the third drawing has 14 toothpicks. This it is an arithmetic sequence with $a_1 = 6$, $d = 4$. So, for the tenth figure, we

$$a_{10} = 6 + (10 - 1) \cdot 4$$

would have $a_{10} = 6 + 9 \cdot 4$

$$a_{10} = \mathbf{42.}$$

(b) The n^{th} term for this arithmetic sequence is

$a_n = 6 + (n - 1) \cdot 4$

$a_n = 6 + 4n - 4$

$a_n =$ **4n + 2.**

(c) For a total of 102 toothpicks, to find the figure, we have

$102 = 4n + 2$

$100 = 4n$

$\mathbf{25} = n.$

12. (a) The n^{th} term for this geometric sequence is 3^{n-1}. Thus $3^{99} = 3^{n-1}$.

So $99 = n - 1$, and $n = 100$.

There are **100 terms** in the sequence.

(b) The n^{th} term for this arithmetic sequence is $9 + (n - 1) \cdot 4$. Thus $353 = 9 + (n - 1) \cdot 4$. Solving for n, $n = 87$. There are **87 terms** in the sequence.

(c) The n^{th} term for this arithmetic sequence is $38 + (n - 1) \cdot 1$. Thus $238 = 38 + (n - 1) \cdot 1$. Solving for n, $n = 201$. There are **201 terms** in the sequence.

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13. **(a)** First term: $5(1) - 1 = \mathbf{4}$
Second term: $5(2) - 1 = \mathbf{9}$
Third term: $5(3) - 1 = \mathbf{14}$
Fourth term: $5(4) - 1 = \mathbf{19}$
Fifth term: $5(5) - 1 = \mathbf{24}$

(b) First term: $6(1) - 2 = \mathbf{4}$
Second term: $6(2) - 2 = \mathbf{10}$
Third term: $6(3) - 2 = \mathbf{16}$
Fourth term: $6(4) - 2 = \mathbf{22}$
Fifth term: $6(5) - 2 = \mathbf{28}$

(c) First term: $5 \cdot 1 + 1 = \mathbf{6}$
Second term: $5 \cdot 2 + 1 = \mathbf{11}$
Third term: $5 \cdot 3 + 1 = \mathbf{16}$
Fourth term: $5 \cdot 4 + 1 = \mathbf{21}$
Fifth term: $5 \cdot 5 + 1 = \mathbf{26}$

(d) First term: $1^2 - 1 = \mathbf{0}$
Second term: $2^2 - 1 = \mathbf{3}$
Third term: $3^2 - 1 = \mathbf{8}$
Fourth term: $4^2 - 1 = \mathbf{15}$
Fifth term: $5^2 - 1 = \mathbf{24}$

14. Answers may vary; examples are:

(a) If $n = 6$, then $\frac{3+6}{3} = 3 \neq 6$.

(b) If $n = 4$, then
$(4 - 2)^2 = 4 \neq 4^2 - 2^2 = 12$.

15. **(a)** The first figure has 2 tiles, the second has 5 tiles, the third has 8 tiles, … . This is an arithmetic sequence where the ***n*th** term is $2 + (n - 1) \cdot 3$.

Thus the 7^{th} term has $2 + (7 - 1) \cdot 3 = \mathbf{20}$ **tiles.**

(b) The nth term is $2 + (n - 1) \cdot 3 = 2 + 3n - 3 = \mathbf{3n - 1}$.

(c) The question can be written as: Is there an n such that $3n - 1 = 449$. Since $3n - 1 = 449 \Rightarrow 3n = 450 \Rightarrow n = 150$, the answer is **yes, the 150th figure**.

16. The n^{th} term of the arithmetic sequence is $-100 + n(300)$. The sequence can also be generated by adding 300 to the previous term. The n^{th} term of the geometric sequence is 3^{n-1}. The sequence can also be generated by multiplying the previous term by 3. Make a table

Number of the term	Arithmetic term	Geometric term
7	2000	729
8	2300	2187
9	2600	6561

With the **9th term**, the geometric sequence is greater.

17. Use a table of Fibonacci numbers to find the pattern, F_n is the nth Fibonacci number:

Generation	Male	Female	Number in Generation	Total
1	1	0	1	1
2	0	1	1	2
3	1	1	2	4
4	1	2	3	7
5	2	3	5	12
6	3	5	8	20
⋮	⋮	⋮	⋮	⋮
n	F_{n-2}	F_{n-1}	F_n	$F_{n+2}-1$

The sum of the first n Fibonacci numbers is $F_{n+2} - 1$. $F_{12} = 144$, so there are **143 bees** in all 10 generations.

18. **(a)** For an arithmetic sequence there is a common difference between the terms. Between 49 and 64 there are three differences so we can find the common difference by subtracting 49 from 64 and dividing the answer by three: $64 - 49 = 15$ and $15 \div 3 = 5$. The common difference is 5 and we can find the missing terms: $49 - 5 =$ **44** and $49 + 5 =$ **54** and $54 + 5 =$ **59**.

(b) For a geometric sequence there is a common ratio between the terms. Between 1 and 625 there are four common ratios used so we can find the common ratio by dividing 625 by 1 and then taking the fourth root: $625 \div 1 = 625$ and $625^{\left(\frac{1}{4}\right)} = 5$. The common ratio is 5 and we can find the missing terms: $1 \cdot 5 = \mathbf{5}, 5 \cdot 5 = \mathbf{25}, 25 \cdot 5 = \mathbf{125}$.

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(c) For a geometric sequence there is a common ratio between the terms. Between 310 and 319 there are three common ratios used so we can find the common ratio by dividing 319 by 310 and then taking the cube root: $3^{19} \div 3^{10} = 3^9$ and $(3^9)^{\left(\frac{1}{3}\right)} = 3^3$. The common ratio is 33 and we can find the missing terms:
$3^{10} \div 3^3 = \mathbf{3^7}, 3^{10} \cdot 3^3 = \mathbf{3^{13}}, 3^{13} \cdot 3^3 = \mathbf{3^{16}}$.

(d) For an arithmetic sequence there is a common difference between the terms. Between $\boldsymbol{a}$ and $5\boldsymbol{a}$ there are four differences so we can find the common difference by subtracting $\boldsymbol{a}$ from $5\boldsymbol{a}$ and dividing the answer by four:
$5a - a = 4a$ and $4a \div 4 = a$. The common difference is $\boldsymbol{a}$ and we can find the missing terms: $\boldsymbol{a} + \boldsymbol{a} = \mathbf{2}\boldsymbol{a}$, $2\boldsymbol{a} + \boldsymbol{a} = \mathbf{3}\boldsymbol{a}$, $3\boldsymbol{a} + \boldsymbol{a} = \mathbf{4}\boldsymbol{a}$.

19. (a) Let's call the missing terms x and y, then the sequence becomes 1, x, y, 7, 11 and if it is a Fibonacci-type sequence then:

$1 + x = y$

$x + y = 7$

$y + 7 = 11 \rightarrow y = 11 - 7 = 4$

and $x + y = 7 \rightarrow x + 4 = 7 \rightarrow x = 3$.

The missing terms are 3 and 4.

(b) Let's call the missing terms x, y. and z, then the sequence becomes x, 2, y, 4, z and if it is a Fibonacci-type sequence then:

$x + 2 = y$

$2 + y = 4 \rightarrow y = 2$

$y + 4 = z \rightarrow 2 + 4 = z \rightarrow z = 6$

and $x + 2 = 2 \rightarrow x = 0$.

The missing terms are 0, 2, and 6.

(c) Let's call the missing terms x, y. and z, then the sequence becomes x, y, 3, 4, z and if it is a Fibonacci-type sequence then:

$x + y = 3$

$y + 3 = 4 \rightarrow y = 4 - 3 = 1$

$3 + 4 = 7$

and $x + y = 3 \rightarrow x + 1 = 3 \rightarrow x = 2$.

The missing terms are 2, 1, and 7.

Mathematical Connections 1-2: Review Problems

15. Order the teams from 1 to 10, and consider a simpler problem of counting how many games are played if each team plays each other once. The first team plays nine teams. The second team also plays nine teams, but one of these games has already been counted. The third team also plays 9 teams, but two of these games were counted in the previous two summands. Continuing in this manner, the total is 10 + 9 + 8 + … + 3 + 2 + 1 = 9(10) / 2 = 45 games. Double this amount to obtain **90 games must be played** for each team to play each other twice.

16. **7 ways**. Make a table:

Quarters	Dimes	Nickels
1	1	1
1	0	3
0	4	0
0	3	2
0	2	4
0	1	6
0	0	8

17. If the problem is interpreted to stated that at least one 12-person tent is used, **then there are 10 ways**. This can be seen by the table below, which illustrates the ways 2-,3-,5-, and 6-person tents can be combined accommodate 14 people.

6-Person	5-Person	3-Person	2-Person
2	0	0	1
1	1	1	0
1	0	2	1
1	0	0	4
0	2	0	2
0	1	3	0
0	1	1	3
0	0	4	1
0	0	2	4
0	0	0	7

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Chapter 1 Review

1. Make a plan. Every 7 days (every week) the day will change from Sunday to Sunday. 365 days per year $\div 7$ days per week ≈ 52 weeks per year $+\frac{1}{7}$ weeks per year. Thus the day of the week will change from Sunday to Sunday 52 times and then change from Sunday to Monday. **July 4 will be a Monday**.

2. $\$5.90 \div 2 = \$2.95$ more on one of the items. That is $\$20 + \$2.95 =$ **$22.95 for the more expensive item** and $\$20 - \$2.95 = \$17.05$ for the less expensive item. Check that both items add up to \$40: $\$22.95 + \$17.05 = \$40$.

3. **(a)** **15, 21, 28. Neither**. The successive differences of terms increases by one; e.g., $10 + 5, 15 + 6, \ldots$.

(b) **32, 27, 22. Arithmetic** Subtract 5 from each term to obtain the subsequent term.

(c) **400, 200, 100. Geometric** Each term is half the previous term.

(d) **21, 34, 55. Neither** Each term is the sum of the previous two terms—this is the Fibonacci sequence.

(e) **17, 20, 23. Arithmetic** Add 3 to each term to obtain the subsequent term.

(f) **256, 1024, 4096. Geometric** Multiply each term by 4 to obtain the subsequent term.

(g) **16, 20, 24. Arithmetic** Add 4 to each term to obtain the subsequent term.

(h) **125, 216, 343. Neither** Each term is the 3rd power of the counting numbers $= 13, 23, 33, \ldots$.

4. **(a)** The successive differences are 3. Each term is 3 more than the previous term. This suggests that it is an arithmetic sequence of the form $3n + ?$. Since the first term is 5, $3(1) + ? = 5$. The n^{th} term would be $\boldsymbol{3n + 2}$.

(b) Each term given is 3 times the previous term. This suggests that the sequence is geometric. $\boldsymbol{n^{th}}$ term will be $\boldsymbol{3^n}$.

(c) The only number that changes in successive terms is the exponent. For the first term, the exponent is 2; for the second term, the exponent is 3; for the third term, the exponent is 4; and so on. So, for the n^{th} term, the exponent will be $n + 1$. Therefore, the n^{th} term will be $\boldsymbol{2^{n+1} - 1}$.

5. **(a)** $3(1) - 2 = \mathbf{1}$;
$3(2) - 2 = \mathbf{4}$;
$3(3) - 2 = \mathbf{7}$;
$3(4) - 2 = \mathbf{10}$; and
$3(5) - 2 = \mathbf{13}$.

(b) $1^2 + 1 = \mathbf{2}$;
$2^2 + 2 = \mathbf{6}$;
$3^2 + 3 = \mathbf{12}$;
$4^2 + 4 = \mathbf{20}$; and
$5^2 + 5 = \mathbf{30}$.

(c) $4(1) - 1 = \mathbf{3}$;
$4(2) - 1 = \mathbf{7}$;
$4(3) - 1 = \mathbf{11}$;
$4(4) - 1 = \mathbf{15}$; and
$4(5) - 1 = \mathbf{19}$.

6. **(a)** $a_1 = 2, d = 2, a_n = 200$.
So $200 = 2 + (n - 1) \cdot 2 \Rightarrow n = 100$.
Sum is $\frac{100(2+200)}{2} = \mathbf{10{,}100}$.

(b) $a_1 = 51,\ d = 1, a_n = 151$.
So $151 = 51 + (n - 1) \cdot 1 \Rightarrow n = 101$.
Sum is $\frac{101 \cdot (51+151)}{2} = \mathbf{10{,}201}$.

7. **(a)** Answers will vary; for example 5 and 3 are odd numbers, but $5 + 3 = 8$, which is not odd.

(b) 15 is odd; and it does not end in a 1 or a 3.

(c) The sum of any two even numbers is always even. An even number is one divisible by 2, so any even number can be represented by $2 + 2 + 2 + \cdots$.
Regardless of how many twos are added, the result is always a multiple of 2, or an even number.

8. All rows, columns, and diagonals must add to 34; i.e., the sum of the digits in row 1. Complete rows or columns with one number missing, then two, etc. to work through the square:

16	3	2	13
5	10	**11**	**8**
9	**6**	7	12
4	**15**	14	**1**

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9. The ten middle tables will hold two each and the two end tables will hold three each, totaling **26 people.**

10. **(a)** $\square + 2^{60} = 2^{61}$

$\square = 2^{61} - 2^{60}$

$\square = 2(2^{60}) - 2^{60}$

$\square = 2^{60}(2-1)$

$\square = \mathbf{2^{60}}$

(b) $\square^2 = 625$

$\sqrt{\square^2} = \sqrt{625}$

$\square = \mathbf{25}$

11. $100 \div 5 = 20$ plus $1 = \mathbf{21}$ **posts**. 1 must be added because both end posts must be counted.

12. 1 mile $=$ 5280 feet.

$5280 \div 6$ feet $=$ 880 turns per mile.

880×50000 miles $=$ **44,000,000 turns**.

13. There are 9 students between 7 and 17 (8 through 16). There must be 9 between them in both directions, since they are direct opposites. $9 + 9 + 2 =$ **20 students**.

14. Let l be a large box, m be a medium box, and s be a small box:

$3l + (3l \times 2m$ each$) + [(3 \times 2)m \times 5s$ each$]$

$3l + 6m + 30s =$ **39 total boxes**.

15. Extend the pattern of doubling the number of ants each day. This is a geometric sequence with $a_1 = 1500$, $a_n = 100{,}000$, and $r = 2$.

$100{,}000 = 1500 \cdot 2^{n-1} \Rightarrow$

$66\frac{2}{3} = 2^{n-1}$.

Since $2^{7-1} < 66\frac{2}{3}$ and $2^{8-1} > 66\frac{2}{3}$, the ant farm will fill sometime **between the 7th and 8th day**.

16. The best strategy would be one of guessing and checking:

(i) Ten 3's + two 5's = 40… close but too low.

(ii) Nine 3's + three 5's = 42… still too low.

(iii) Eight 3's + Four 5's = 44.

They must have answered **four 5-point questions**.

17. Let $\ell =$ length of the longest piece,

$m =$ length of the middle-sized piece, and

$s =$ lenth of the shortest piece.

Then $\ell = 3m$ and $s = m - 10$.

So $\ell + m + s = 90 \Rightarrow$

$3m + m + (m - 10) = 90 \Rightarrow$

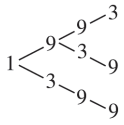
$5m = 100$.

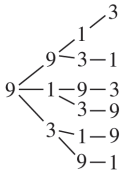
Thus $m =$ **20 cm**;

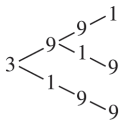
$\ell = 3m =$ **60 cm**; and

$s = m - 10 =$ **10 cm**.

18. Make a diagram that demonstrates all the ways four-digit numbers can be formed from left (thousands place) to right (ones place). **12 four-digit numbers** can be formed.







19. Answer may vary. Fill the 4-cup container with water and pour the water into the 7-cup container. Fill the 4-cup container again and pour water into the 7-cup container until it is full. Four minus three (1) cups of water will remain in the 4-cup container. Empty the 7-cup container and pour the contents of the 4-cup container into the 7- cup container. The 7-cup container now holds 1 cup of water. Refill the 4-cup container and pour it into the 7-cup container. The 7-cup container now contains exactly 5 cups of water.

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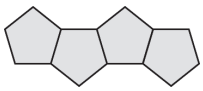
20. A possible pattern is to increase each rectangle by one row of dots and one column of dots to obtain the next term in the sequence. Make a table.

Number of the term	Row of dots	Column of dots	Term (row × column)
1	1	2	2
2	2	3	6
3	3	4	12
4	4	5	20
5	5	6	30
6	6	7	42
7	7	8	56
⋮			
100	100	101	10100
⋮			
n	n	$n+1$	$n(n+1)$

We also observe that the number of the term corresponds to the number of rows in the arrays and that the number of columns in the array is the number of the term plus one. Thus, the next three terms are **30, 42, and 56**. **The 100th term is 10,100 and the n^{th} term is $n(n+1)$**.

21. A possible pattern is that each successive figure is constructed by adjoining another pentagon to the previous figure

(a)



(b) Observe that the perimeter of the first figure is 5 and that when a new pentagon is adjoined 4 new sides are added and one side (where the new pentagon is adjoined) is lost. Make a table.

Number of terms	1-unit sides (perimeter)
1	**5**
2	$5-1+4=$ **8**
3	$8-1+4=$ **11**
4	$11-1+4=$ **14**

(c) and **(d)** Looking at the terms in the sequence and noting that the difference of terms is three, we suspect that the sequence is arithmetic and conjecture that the n^{th} term is $3n+2$. However, we need to be sure. Looking at the 4th term (part a) we observe that the pentagons on the end Contribute 4 sides to the perimeter and the "middle" pentagons contribute 3 sides. Thus, in the n^{th} figure there will be 2 "end" pentagons that contribute 4 1-unit sides and $n-2$ "middle" pentagons that contribute $3(n-2)$ 1 unit sides. The total will be $3(n-2)+2(4)=$ **$3n+2$ units**. Thus the 100th term is $3(100)+2=$ **302**.

22. **(a)** The circled terms will constitute an arithmetic sequence because the common difference will be twice the difference in the original series.

(b) The new sequence will be a geometric sequence because the ratio will be the square of the ratio of the original series.

23. When $n=1$, then $n^2-n=1^2-1=0$, so the first term, $a_1=0$. When $n=2$, then $n^2-n=2^2-2$ or 2. Thus $a_1+a_2=2$; hence $a_2=2$. For $n=3$, $n^2-n=3^2-3=6$. Substituting for a_1 and a_2, we get $a_3=6-2=4$. For $n=4$, $4^2-4=12$. Substituting for a_1, a_2, and a_3, we get $0+2+4+a_4=12$. Hence $\boldsymbol{a_4=6}$.

24. **(a)** Let's call the missing terms a, and b then the sequence becomes:

$13, a, b, 27$

$13+a=b \rightarrow a=b-13$

$a+b=27 \rightarrow b-13+b=27$

$\rightarrow 2b=40$

$\rightarrow b=20$

$a=b-13 \rightarrow a=7$

So **7 and 20** are the missing terms.

(b) Let's call the missing terms a, and b then the sequence becomes:

$137, a, b, 163$

$137+a=b \rightarrow a=b-137$

$a+b=163 \rightarrow b-137+b=163$

$\rightarrow 2b=300$

$\rightarrow b=150$

$a=b-137 \rightarrow a=13$

So **13 and 150** are the missing terms.

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(c) Let's call the missing terms x, and y, then the sequence becomes:

$$b, x, y, a$$

$$b + x = y \rightarrow x = y - b$$

$$x + y = a \rightarrow y - b + y = a$$

$$\rightarrow 2y = a + b$$

$$\rightarrow y = \frac{a+b}{2}$$

$$x = \frac{a+b}{2} - b \rightarrow x = \frac{a+b}{2} - \frac{2b}{2}$$

$$\rightarrow x = \frac{a-b}{2}$$

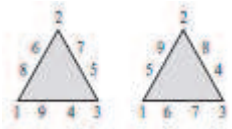
So the missing terms are $\boldsymbol{\frac{a-b}{2}}$ **and** $\boldsymbol{\frac{a+b}{2}}$.

25. The first line tells us that since three cylinders =15, each cylinder must be worth 5. Using that information, we can use the second line to figure out that a circle must be worth 4. That information can be used on the third line to determine that the cup-like shape must be equal to 1. So, putting it all together in the fourth line, we have 4 + 5 + 1 = **10**.

26. The pattern involves taking two squares that are diagonal, summing their numbers, and putting that sum in the square that is below one of the diagonal squares and to the left of the other diagonal square. For example, the top square is 4; the square diagonal to it (to the right) is 2; their sum is 6, which is the number that appears in the square below the 4 and to the left of the 2. You can also see it where 9 + 10 = 19, and 1 + 6 =7. So using this pattern, the question mark must be **equal to 3.**

27. Since 1, 2, and 3 are already on the triangle, the numbers 4-9 will be used to fill in the six question marks. On the left side, 1+2 = 3, so the two question marks on that side must sum to 14. Similarly on the bottom, the two question marks must sum to 13, and on the right side, the two question marks must sum to 12. So on the right side with the available numbers, only two possibilities, 5+7, or 4+8, sum to 12.

If the two numbers on the right side are 5 and 7, the bottom two numbers (which must sum to 13) are 4 and 9, and the two numbers on the left side must be 6 and 8. If the two numbers on the right side are 4 and 8, the two bottom numbers must be 6 and 7, leaving 5 and 9 to be the numbers on the right. Below are the two solutions presented on the triangle:



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CHAPTER 2

INTRODUCTION TO LOGIC AND SETS

Assessment 2-1A: Reasoning and Logic: An Introduction

1. (a) **False statement**. A statement is a sentence that is either true or false, but not both.

(b) **False statement.** Los Angeles is a city, not a state.

(c) **Not a statement.** Questions are not statements.

(d) **True statement.**

2. (a) **There exists at least one** natural number n such that $n + 8 = 11$.

(b) **There exists at least one** natural number n such that $n^2 = 4$.

(c) For **all** natural numbers n, $n + 3 = 3 + n$.

(d) For **all** natural numbers n, $5n + 4n = 9n$.

3. (a) For **all** natural numbers n, $n + 8 = 11$.

(b) For **all** natural numbers n, $n^2 = 4$.

(c) There is **no** natural number x such that $x + 3 = 3 + x$.

(d) There is **no** natural number x such that $5x + 4x = 9x$.

4. (a) The book **does not have** 500 pages.

(b) $3 \cdot 5 \neq 15$.

(c) **Some** dogs **do not have** four legs.

(d) **No** rectangles are squares.

(e) **All** rectangles are squares.

(f) **Some** dogs have fleas.

5. (a) If $n = 4$, or $n = 5$, then $n < 6$ *and* $n > 3$, so the statement is **true**, since it can be shown to work for some natural numbers n.

(b) All natural numbers are greater than zero; so, since the condition is that $n > 0$ *or* $n < 5$, the statement is **true**.

6. (a)

p	$\sim p$	$\sim(\sim p)$
T	**F**	**T**
F	**T**	**F**

(b)

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	**F**	**T**	**F**
F	**T**	**T**	**F**

(c) **Yes**. The truth table entries are the same.

(d) **No**. The truth table entries are not the same.

7. (a)

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$	
T	**T**	**T**	**F**	**T**	
T	**F**	**F**	**F**	**F**	
F	**T**	**T**	**T**	**T**	
F	**F**	**T**	**T**	**T**	

(b) $p \rightarrow q \equiv \sim p \vee q$. Answers will vary. Here are two possible examples:1. Let p be "the Bobcats win" and q be "the Bobcats make the playoffs". Then Column 3 would read "If the Bobcats win, then the Bobcats make the playoffs". Column 5 would read "The Bobcats lose or the Bobcats make the playoffs."2. Let p be "it is summer vacation" and q be "I am at home". Then Column 3 would read "If it is summer vacation, then I am at home". Column 5 would read "It is not summer vacation or I am at home."

(c) In this problem, p is the statement "2 + 3 = 5" and q is the statement "4 + 6 = 10". That would make the statement in this problem be in the form of $\sim p \vee q$. So, it will be logically equivalent to a statement in the form of $p \rightarrow q$ or "if 2 + 3 = 5, then 4 + 6 = 10."

8. (a) $\mathbf{q} \wedge \mathbf{r}$. Both q and r are true.

(b) $\mathbf{r} \vee \sim \mathbf{q}$. r is true or q is not true.

(c) $\sim(\mathbf{q} \wedge \mathbf{r})$. q and r are not both true.

(d) $\sim \mathbf{q}$. q is not true.

9. (a) **False**. The statement is a conjunction. In order for a conjunction to be true, both p and q must be true; otherwise, the conjunction is false. The two parts of the conjunction could be stated as such: p is the statement $2 + 3 = 5$ and q is the

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