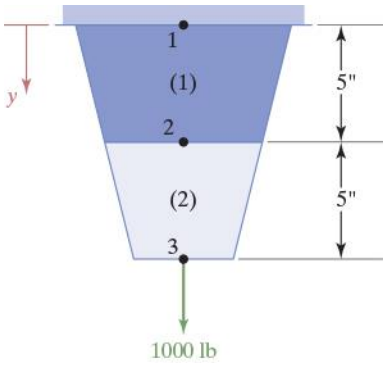
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Chapter 1

1.1

(a) Using two elements



$$A(y) = 0.25 - 0.0125y$$

$$A_1 = 0.25\ \text{in}^2 \qquad A_2 = 0.1875\ \text{in}^2$$

$$A_3 = 0.125\ \text{in}^2$$

$$K_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

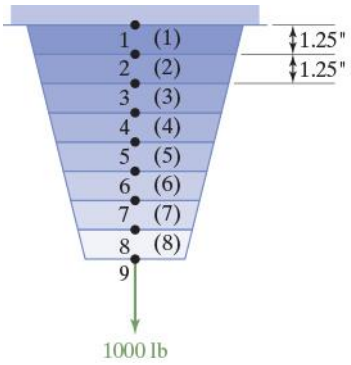
$$K_1 = \frac{(0.25 + 0.1875)(10.4 \times 10^6)}{2(5)} = 455{,}000\ \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{(0.1875 + 0.125)\,(10.4 \times 10^6)}{2(5)} = 325{,}000\ \frac{\text{lb}}{\text{in}}$$

$$10^3 \begin{bmatrix} \overset{1}{\cancel{455}} & \overset{0}{-\cancel{455}} & \\ \underset{0}{-\cancel{455}} & 455+325 & -325 \\ & -325 & 325 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$$u_1 = 0, \quad \underline{\underline{u_2 = 0.002197\ \text{in}}} \quad \underline{\underline{u_3 = 0.005274\ \text{in}}} \quad \Leftarrow$$

(b)



$$A_1 = 0.25\ \text{in}^2$$

$$A_2 = 0.234375\ \text{in}^2$$

$$A_3 = 0.21875\ \text{in}^2$$

$$A_4 = 0.203125\ \text{in}^2$$

$$A_5 = 0.1875\ \text{in}^2$$

$$A_6 = 0.171875\ \text{in}^2$$

$$A_7 = 0.15625\ \text{in}^2$$

$$A_8 = 0.140625\ \text{in}^2$$

$$A_9 = 0.125\ \text{in}^2$$

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1.1 Cont.

$$K_1 = 2015000\,\frac{\text{lb}}{\text{in}} \qquad K_2 = 1885000\,\frac{\text{lb}}{\text{in}} \qquad K_3 = 1755000\,\frac{\text{lb}}{\text{in}}$$

$$K_4 = 1625000\,\frac{\text{lb}}{\text{in}} \qquad K_5 = 1495000\,\frac{\text{lb}}{\text{in}} \qquad K_6 = 1365000\,\frac{\text{lb}}{\text{in}}$$

$$K_7 = 1235000\,\frac{\text{lb}}{\text{in}} \qquad K_8 = 1105000\,\frac{\text{lb}}{\text{in}}$$

1 ~~2015000~~	0 ~~−2015000~~							
~~−2015000~~ 0	2015000 +1885000	−1885000						
	−1885000	1885000 +1755000	−1755000					
		−1755000	1755000 +1625000	−1625000				
			−1625000	1625000 +1495000	−1495000			
				−1495000	1495000 +1365000	−1365000		
					−1365000	1365000 +1235000	−1235000	
						−1235000	1235000 +1105000	−1105000
							−1105000	1105000

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$u_1 = 0,\ u_2 = 0.00049628,\ u_3 = 0.0010268,\ u_4 = 0.0015965,\ u_5 = 0.0022119,$
$u_6 = 0.0028808,\ u_7 = 0.0036134,\ u_8 = 0.0044232,\ u_9 = 0.0053281\,\text{in}.$ ⇦

y (in)	exact defl.	Two-element	eight element
0	0	0	0
1.25	0.00049645		0.00049628
2.5	0.0010272		0.0010268
3.75	0.0015972		0.0015965
5.0	0.0022129	0.002197	0.0022119
6.25	0.0028822		0.0028808
7.5	0.0036154		0.0036134
8.75	0.0044259		0.0044232
10.0	0.0053319	0.005274	0.0053281

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1.2

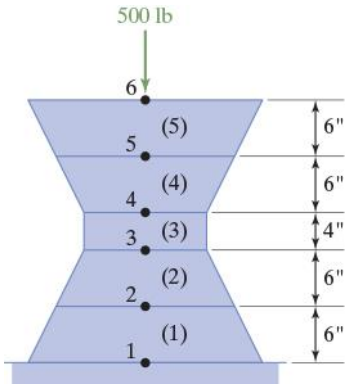
$$K = \frac{A_{\text{avg.}} E}{\ell}$$

$$K_1 = K_5 = \frac{\left(\frac{10.5+12}{2}\right)(3)(3.27\times 10^6)}{6}$$

$$K_1 = K_5 = 18{,}393{,}750 \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_4 = \frac{\left(\frac{10.5+9}{2}\right)(3)(3.27\times 10^6)}{6} = 15{,}941{,}250 \frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{(9)(3)(3.27\times 10^6)}{4} = 22{,}072{,}500 \frac{\text{lb}}{\text{in}}$$



$$\begin{bmatrix}
\overset{1}{\cancel{18393750}} & \overset{0}{\cancel{-18393750}} & & & & \\
\cancel{-18393750} & 18393750 & -15941250 & & & \\
0 & +15941250 & & & & \\
 & -15941250 & 15941250 & -22072500 & & \\
 & & +22072500 & & & \\
 & & -22072500 & 22072500 & -15941250 & \\
 & & & +15941250 & & \\
 & & & -15941250 & 15941250 & -18393750 \\
 & & & & +18393750 & \\
 & & & & -18393750 & 18393750
\end{bmatrix}
\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}
= \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -500 \end{Bmatrix}$$

$$\{u\} = \begin{Bmatrix} 0 \\ -2.71831\times 10^{-5} \\ -5.85483\times 10^{-5} \\ -8.12009\times 10^{-5} \\ -1.12566\times 10^{-4} \\ -1.39749\times 10^{-4} \end{Bmatrix} \text{in} \qquad \Leftarrow$$

$$\sigma^{(3)} = \frac{(-8.12009\times 10^{-5} + 5.85483)\times 10^{-5} \times (3.27\times 10^6)}{4}$$

$$\sigma^{(3)} = \underline{\underline{18.5 \text{ Psi C}}}$$

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1.2 Cont.

as a check:

$$\sigma^{(3)} = \frac{F}{A} = \frac{500}{(9)(3)} = 18.5 \text{ Psi}$$

$$\sigma = \frac{(u_{i+1} - u_i)}{\ell} E$$

$$\sigma^{(1)} = \frac{(-2.71831 \times 10^{-5} - 0)(3.27 \times 10^{6})}{6} = \underline{\underline{14.8 \text{ Psi C}}}$$

$$\sigma^{(5)} = \frac{(-1.39749 + 1.12566) \times 10^{-4} \times (3.27 \times 10^{6})}{6} = \underline{\underline{14.8 \text{ Psi C}}}$$

as a check:

$$\sigma^{(1)} = \sigma^{(5)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5 + 12}{2}\right)(3)} = 14.8 \text{ Psi C}$$

$$\sigma^{(2)} = \frac{\left(-5.85483 + 2.71831\right) \times 10^{-5} \times (3.27 \times 10^{6})}{6} = \underline{\underline{17.1 \text{ Psi C}}}$$

$$\sigma^{(4)} = \frac{(-1.12566 \times 10^{-4} + 8.12009 \times 10^{-5})(3.27 \times 10^{6})}{6} = \underline{\underline{17.1 \text{ Psi C}}}$$

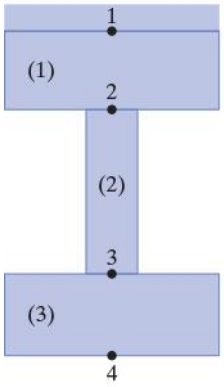
as a check:

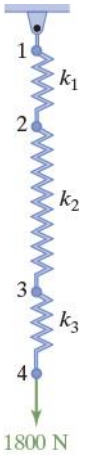
$$\sigma^{(2)} = \sigma^{(4)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5 + 9}{2}\right)(3)} = 17.1 \text{ Psi}$$

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1.3





$$K = \frac{AE}{\ell}$$

$$K_1 = K_3 = \frac{(0.08)(0.006)(68.9\times10^9)}{0.025}$$

$$K_1 = K_3 = 1.32288\times10^9 \ \frac{\text{N}}{\text{m}}$$

$$K_2 = \frac{(0.02)(0.006)(68.9\times10^9)}{0.1}$$

$$K_2 = 0.08268\times10^9 \ \frac{\text{N}}{\text{m}}$$

$$10^9\begin{bmatrix} \overset{1}{\cancel{1.32288}} & \overset{0}{\cancel{-1.32288}} & & \\ \overset{0}{\cancel{-1.32288}} & 1.32288+0.08268 & -0.08268 & \\ & -0.08268 & 0.08268+1.32288 & -1.32288 \\ & & -1.32288 & 1.32288 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1800 \end{Bmatrix}$$

$$\begin{bmatrix} 1.40556 & -0.08268 & 0 \\ -0.08268 & 1.40556 & -1.32288 \\ 0 & -1.32288 & 1.32288 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1800\times10^{-9} \end{Bmatrix}$$

$$\underline{\underline{u_2 = 1.360668\times10^{-6} \text{ m}}} \quad \underline{\underline{u_3 = 2.313135\times10^{-5} \text{ m}}} \quad \underline{\underline{u_4 = 2.449202\times10^{-5} \text{ m}}} \Leftarrow$$

$$\sigma^{(1)} = E\frac{u_2-u_1}{\ell} = (68.9\times10^9)\frac{(1.360668\times10^{-6}-0)}{0.025} = 3750000 = \underline{\underline{3.75 \text{ MPa}}}$$

as a check: $\sigma^{(1)} = \dfrac{F}{A} = \dfrac{1800 \text{ N}}{(0.08)(0.006)} = \underline{\underline{3.75 \text{ MPa}}}$

$$\sigma^{(2)} = E\frac{u_3-u_2}{\ell} = (68.9\times10^9)\frac{(2.313135\times10^{-5}-1.360668\times10^{-6})}{0.1} = \underline{\underline{15 \text{ MPa}}} \Leftarrow$$

as a check: $\sigma^{(2)} = \dfrac{1800 \text{ N}}{(0.02)(0.006)} = \underline{\underline{15 \text{ MPa}}}$

$$\sigma^{(3)} = E\frac{u_4-u_3}{\ell} = (68.9\times10^9)\frac{(2.449202-2.313135)\times10^{-5}}{0.025} = \underline{\underline{3.75 \text{ MPa}}}$$

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1.4

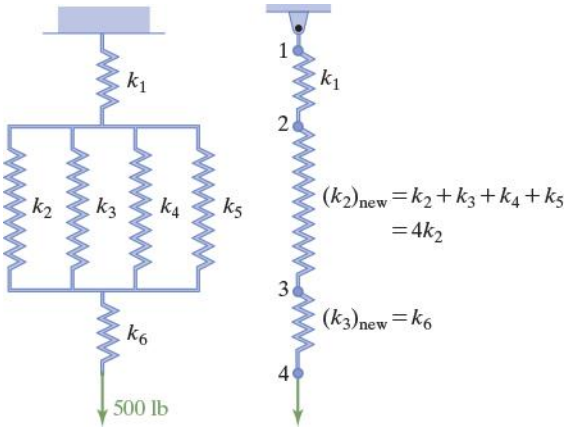
$$K_1 = K_6 = \frac{(4)(0.125)(28\times10^6)}{2}$$

$$K_1 = K_6 = 7\times10^6 \ \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_3 = K_4 = K_5 = \frac{(0.625)(0.125)(28\times10^6)}{8}$$

$$K_2 = 273438 \ \frac{\text{lb}}{\text{in}}$$

$$(K_2)_{\text{new}} = (4)(273438) = 1093750 \ \frac{\text{lb}}{\text{in}}$$



$$\begin{bmatrix} \overset{1}{\cancel{7\times10^6}} & \overset{0}{\cancel{-7\times10^6}} & & \\ \overset{0}{\cancel{-7\times10^6}} & 7\times10^6 + 1093750 & -1093750 & \\ & -1093750 & 1093750 + 7\times10^6 & -7\times10^6 \\ & & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$\begin{bmatrix} 8093750 & -1093750 & 0 \\ -1093750 & 8093750 & -7\times10^6 \\ 0 & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$u_2 = \underline{\underline{7.142857\times10^{-5} \text{ in}}} \quad u_3 = \underline{\underline{5.285714\times10^{-4} \text{ in}}} \quad u_4 = \underline{\underline{6\times10^{-4} \text{ in}}} \quad \Leftarrow$$

$$\sigma^{(1)} = E\left(\frac{u_2 - u_1}{\ell}\right) = 28\times10^6\left(\frac{7.142857\times10^{-5} \text{ in}}{2}\right) = \underline{\underline{1000 \ \frac{\text{lb}}{\text{in}^2}}}$$

as a check: $\sigma^{(1)} = \dfrac{F}{A} = \dfrac{500}{(4)(0.125)} = 1000 \ \dfrac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = E\left(\frac{u_3 - u_2}{\ell}\right) = 28\times10^6\left(\frac{5.285714\times10^{-4} - 7.142857\times10^{-5}}{8}\right) = \underline{\underline{1600 \ \frac{\text{lb}}{\text{in}^2}}} \quad \Leftarrow$$

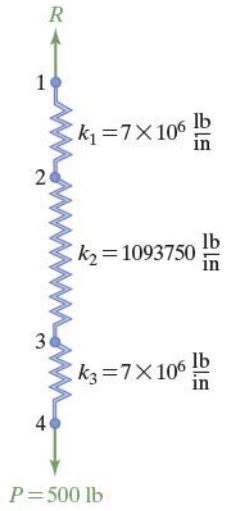
as a check: $\sigma^{(2)} = \dfrac{500}{(2.5)(0.125)} = 1600 \ \dfrac{\text{lb}}{\text{in}^2}$

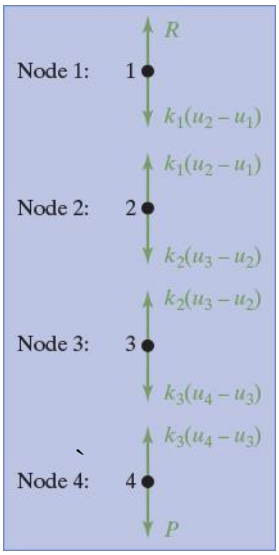
$$\sigma^{(3)} = E\left(\frac{u_4 - u_3}{\ell}\right) = 28\times10^6\left(\frac{6\times10^{-4} - 5.285714\times10^{-4}}{2}\right) = \underline{\underline{1000 \ \frac{\text{lb}}{\text{in}^2}}}$$

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1.5





$$R - K_1(u_2 - u_1) = 0$$

$$K_1(u_2 - u_1) - K_2(u_3 - u_2) = 0$$

$$K_2(u_3 - u_2) - K_3(u_4 - u_3) = 0$$

$$K_3(u_4 - u_3) - P = 0$$

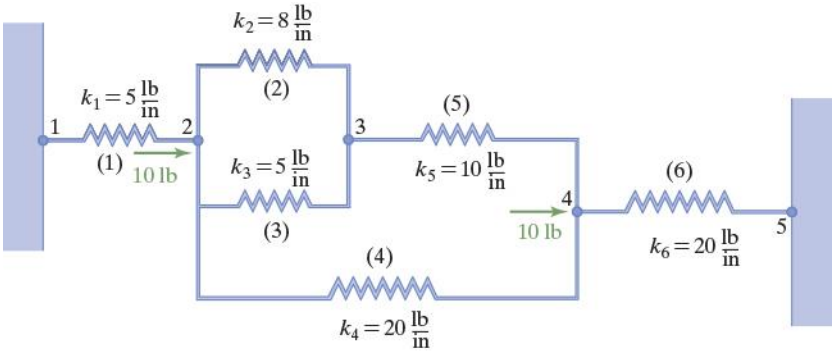
In matrix form:

$$\begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1 + K_2 & -K_2 & \\ & -K_2 & K_2 + K_3 & -K_3 \\ & & K_3 & K_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -R \\ 0 \\ 0 \\ P \end{Bmatrix}$$

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1.6



Size of the global matrix: 5 × 5

$$[K]^{(1)} = \begin{matrix} \\ ① \\ ② \end{matrix}\begin{bmatrix} ① & ② \\ 5 & -5 \\ -5 & 5 \end{bmatrix}\begin{matrix} \\ ① \\ ② \end{matrix} \qquad [K]^{(2)} = \begin{matrix} \\ ② \\ ③ \end{matrix}\begin{bmatrix} ② & ③ \\ 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{matrix} \\ ② \\ ③ \end{matrix}$$

$$[K]^{(3)} = \begin{matrix} \\ ② \\ ③ \end{matrix}\begin{bmatrix} ② & ③ \\ 5 & -5 \\ -5 & 5 \end{bmatrix}\begin{matrix} \\ ② \\ ③ \end{matrix} \qquad [K]^{(4)} = \begin{matrix} \\ ② \\ ④ \end{matrix}\begin{bmatrix} ② & ④ \\ 20 & -20 \\ -20 & 20 \end{bmatrix}\begin{matrix} \\ ② \\ ④ \end{matrix}$$

$$[K]^{(5)} = \begin{matrix} \\ ③ \\ ④ \end{matrix}\begin{bmatrix} ③ & ④ \\ 10 & -10 \\ -10 & 10 \end{bmatrix}\begin{matrix} \\ ③ \\ ④ \end{matrix} \qquad [K]^{(6)} = \begin{matrix} \\ ④ \\ ⑤ \end{matrix}\begin{bmatrix} ④ & ⑤ \\ 20 & -20 \\ -20 & 20 \end{bmatrix}\begin{matrix} \\ ④ \\ ⑤ \end{matrix}$$

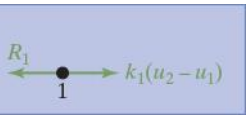
$$[K]^{(6)} = \begin{bmatrix} 5 & -5 & & \\ -5 & 5+8+5+20 & -8-5 & -20 \\ & -8-5 & 8+5+10 & -10 \\ & -20 & -10 & 20+10+20 & -20 \\ & & & -20 & 20 \end{bmatrix}$$

applying B.Cs and loads; $u_1 = u_5 = 0 \quad F_2 = 10\ \text{lb}, \quad F_4 = 10\ \text{lb}$

$$\begin{bmatrix} 38 & -13 & -20 \\ -13 & 23 & -10 \\ -20 & -10 & 50 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 10 \end{Bmatrix}$$

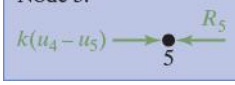
$$u_2 = \underline{\underline{0.962\ \text{in}}} \quad u_3 = \underline{\underline{0.874\ \text{in}}} \quad u_4 = \underline{\underline{0.760\ \text{in}}} \qquad \Leftarrow$$

Node 1:



$$R_1 = 5(0.962 - 0) = \underline{\underline{4.8\ \text{lb}}} \qquad \Leftarrow$$

Node 5:



$$R_5 = 20(0.760 - 0) = \underline{\underline{15.2\ \text{lb}}}$$

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1.6 Cont.

$K(u_4 - u_5)$

note also as a check, $R_1 + R_5 = \sum F_{\text{external}}$ 4.8 + 15.2 = 10 +10

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1.7

$$150\begin{bmatrix}
\frac{1}{150}\cancel{5.88} & \overset{0}{\cancel{5.88}} & & & & & & \\
-5.88 & 5.88+2.27 & -2.27 & & & & & \\
 & -2.27 & 2.27+10 & -10 & & & & \\
 & & -10 & 10+0.581 & -0.581 & & & \\
 & & & -0.581 & 0.581+0.781 & -0.781 & & \\
 & & & & -0.781 & 0.781+2.22 & -2.22 & \\
 & & & & & -2.22 & 2.22+1.47 & -1.47 \\
 & & & & & & \cancel{-1.47}\ 0 & \cancel{1.47}\ \frac{1}{150}
\end{bmatrix}\begin{Bmatrix}T_1\\T_2\\T_3\\T_4\\T_5\\T_6\\T_7\\T_8\end{Bmatrix}=\begin{Bmatrix}10\\0\\0\\0\\0\\0\\0\\68\end{Bmatrix}$$

$$\{T\}=\begin{Bmatrix}10\\12.04\\17.31\\18.51\\39.12\\54.46\\59.85\\68\end{Bmatrix}^{\circ}\text{F} \quad \Leftarrow$$

$$q=UA\Delta T$$

$$q=(1.47)(150)(68-59.85)=\underline{\underline{1800\ \frac{\text{Btu}}{\text{hr}}}} \quad \Leftarrow$$

or as another example:

$$q=(0.781)(150)(54.46-39.12)=1800\ \frac{\text{Btu}}{\text{hr}}$$

or

$$q=\frac{1}{\sum\text{Resistance}}A(T_{\text{in}}-T_{\text{out}})=\frac{(150)(68-10)}{0.17+0.44+0.1+1.72+1.28+0.45+0.68}=1800\ \frac{\text{Btu}}{\text{hr}}$$

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1.8

$$150\begin{bmatrix} \frac{1}{150} & 0 & & & & & \\ \cancel{5.88} & \cancel{-5.88} & & & & & \\ -5.88 & 5.88+1.23 & -1.23 & & & & \\ & -1.23 & 1.23+0.76 & -0.76 & & & \\ & & -0.76 & 0.76+0.053 & -0.053 & & \\ & & & -0.053 & 0.053+2.22 & -2.22 & \\ & & & & -2.22 & 2.22+1.47 & -1.47 \\ & & & & & \cancel{-1.47}\ 0 & \cancel{1.47}\ \frac{1}{150} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 20.37 \\ 22.12 \\ 24.95 \\ 65.57 \\ 66.54 \\ 68 \end{Bmatrix} °\text{F} \quad \Leftarrow$$

$q = UA\Delta T$

as an example:

$$q = (1.47)(150)(68-66.54) = \underline{\underline{320\ \frac{\text{Btu}}{\text{hr}}}}$$

or

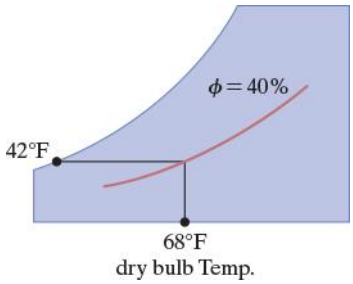
$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{(150)(68-20)}{0.17+0.81+1.32+19+0.45+0.68} = 320\ \frac{\text{Btu}}{\text{hr}}$$

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1.9

with the help of a psychometric chart, using a dry bulb Temp. of $68\,°\text{F}$ and $\phi = 40\%$, we identify condensation temperature to be $42\,°\text{F}$. Thus condensation will occur between surfaces 4 and 5.



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1.10

$$1000\begin{bmatrix} \frac{1}{1000} & 0 & & & \\ \cancel{1.47} & \cancel{-1.47} & & & \\ -1.47 & 1.47+0.053 & -0.053 & & \\ & -0.053 & 0.053+2.22 & -2.22 & \\ & & -2.22 & 2.22+1.47 & -1.47 \\ & & & \cancel{-1.47} & \cancel{1.47} \\ & & & 0 & \frac{1}{1000} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 15 \\ 16.81 \\ 66.99 \\ 68.19 \\ 70 \end{Bmatrix} ^\circ\text{F} \quad \Leftarrow$$

$q = UA\Delta T$

as an example:

$$q = (1.47)(1000)(70 - 68.19) = \underline{\underline{2660 \frac{\text{Btu}}{\text{hr}}}} \quad \Leftarrow$$

or

$$q = (1.47)(1000)(16.81 - 15) = 2660 \frac{\text{Btu}}{\text{hr}}$$

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1.11

$$22.5\begin{bmatrix} \frac{1}{22.5} & 0 & & \\ \cancel{5.88} & \cancel{-5.88} & & \\ -5.88 & 5.88+2.56 & -2.56 & \\ & -2.56 & 2.56+1.47 & -1.47 \\ & & \cancel{-1.47} & \cancel{1.47} \\ & & 0 & \frac{1}{22.5} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 26.85 \\ 42.59 \\ 70 \end{Bmatrix} °\text{F}$$

$$q = U_1 A \Delta T = (5.88)(22.5)(26.85 - 20) = \underline{\underline{910 \frac{\text{Btu}}{\text{hr}}}}$$

or as another example: ⇦

$$q = U_2 A \Delta T = (2.56)(22.5)(42.59 - 26.85) = \underline{\underline{910 \frac{\text{Btu}}{\text{hr}}}}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A(T_{\text{in}} - T_{\text{out}}) = \frac{1}{(0.17 + 0.39 + 0.68)}(22.5)(70 - 20)$$

$$\underline{\underline{q = 910 \frac{\text{Btu}}{\text{hr}}}} \quad ⇦$$

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1.12

$$(A_1)_c = (A_5)_c = \left(\frac{12+10.5}{2}\right)(3) - (3)\frac{\pi}{4}(0.5)^2 = 33.161\,\text{in}^2$$

$$(A_2)_c = (A_4)_c = \frac{(10.5+9)}{2}(3) - 3\frac{\pi}{4}(0.5)^2 = 28.661\,\text{in}^2$$

$$(A_3)_c = (9)(3) - 3\frac{\pi}{4}(0.5)^2 = 26.411\,\text{in}^2$$

$$(K_1)_c = (K_5)_c = \frac{(33.161)(3.27\times10^6)}{6} = 18{,}072{,}745\,\frac{\text{lb}}{\text{in}}$$

$$(K_2)_c = (K_4)_c = \frac{(28.661)(3.27\times10^6)}{6} = 15{,}620{,}245\,\frac{\text{lb}}{\text{in}}$$

$$(K_3)_c = \frac{(26.411)(3.27\times10^6)}{4} = 21{,}590{,}992\,\frac{\text{lb}}{\text{in}}$$

$$(K_1)_s = (K_2)_s = (K_4)_s = (K_5)_s = \frac{(3)\left(\frac{\pi}{4}\right)(0.5)^2(29\times10^6)}{6} = 2{,}847{,}068\,\frac{\text{lb}}{\text{in}}$$

$$(K_3)_s = \frac{(3)\left(\frac{\pi}{4}\right)(0.5)^2(29\times10^6)}{4} = 4{,}270{,}602\,\frac{\text{lb}}{\text{in}}$$

The Combined stiffnesses:

$$K_1 = K_5 = 18{,}072{,}745 + 2{,}847{,}068 = 20{,}919{,}813$$

$$K_2 = K_4 = 15{,}620{,}245 + 2{,}847{,}068 = 18{,}467{,}313$$

$$K_3 = 21{,}590{,}992 + 4{,}270{,}602 = 25{,}861{,}594$$

$$\begin{bmatrix}
\overset{1}{\cancel{20{,}919{,}813}} & \overset{0}{\cancel{-20{,}919{,}813}} & & & & \\
\cancel{-20{,}919{,}813} & 20{,}919{,}813 & -18{,}467{,}313 & & & \\
\overset{0}{} & +18{,}467{,}313 & & & & \\
 & -18{,}467{,}313 & 18{,}467{,}313 & -25{,}861{,}594 & & \\
 & & +25{,}861{,}594 & & & \\
 & & -25{,}861{,}594 & 25{,}861{,}594 & -18{,}467{,}313 & \\
 & & & +18{,}467{,}313 & & \\
 & & & -18{,}467{,}313 & 18{,}467{,}313 & -20{,}919{,}813 \\
 & & & & +20{,}919{,}813 & \\
 & & & & -20{,}919{,}813 & 20{,}919{,}813
\end{bmatrix}
\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} =
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1000 \end{Bmatrix}$$

$$u_1 = 0 \qquad u_2 = -4.78016\times10^{-5}\ \text{in} \qquad u_3 = -1.01951\times10^{-4}\ \text{in}$$

$$\underline{\underline{u_4 = -1.40618\times10^{-4}}} \qquad \underline{\underline{u_5 = -1.94768\times10^{-4}\ \text{in}}} \qquad \underline{\underline{u_6 = -2.42570\times10^{-4}\ \text{in}}} \quad \Leftarrow$$

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1.12 Cont.

$$\sigma^{(1)}_{\text{Concrete}} = E_c \frac{(u_2 - u_1)}{\ell} = \frac{(3.27\times10^6)(-4.78016\times10^{-5})}{6} = \underline{\underline{26 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma^{(11}_{\text{Steel}} = E_s \frac{(u_2 - u_1)}{\ell} = \underline{\underline{231 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

as a check:

$$\sigma^{(1)}_{\text{Concrete}}(A_1)_c + \sigma^{(1)}_s(A_1)_s = (26)(33.161) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = \underline{\underline{1000 \text{ lb}}}$$

$$\sigma^{(2)}_c = E_c\left(\frac{u_3 - u_2}{\ell}\right) = \frac{(3.27\times10^6)(-1.01951\times10^{-4} + 4.78016\times10^{-5})}{6} = \underline{\underline{30 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma^{(2)}_s = 231 \frac{\text{lb}}{\text{in}^2} \text{ C}$$

Check: $\sigma^{(2)}_c(A_2)_c + \sigma^{(2)}_s(A_2)_s = (30)(28.661) + (231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000 \text{ lb}$

$$\sigma^{(3)}_c = E_c \frac{(u_4 - u_3)}{\ell} = \frac{(3.27\times10^6)(-1.40618\times10^{-4} + 1.01951\times10^{-4})}{4} = \underline{\underline{32 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma^{(3)}_s = E_s \frac{(u_4 - u_3)}{\ell} = \frac{(29\times10^6)(-1.40618\times10^{-4} + 1.01951\times10^{-4})}{4} = \underline{\underline{280 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

Check: $\sigma^{(3)}_c(A_3)_c + \sigma^{(3)}_s(A_3)_s = (32)(26.411) + (280)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000 \text{ lb}$

$$\sigma^{(4)}_c = E_c \frac{(u_5 - u_4)}{\ell} = \frac{(3.27\times10^6)(-1.94768\times10^{-4} + 1.40618\times10^{-4})}{6} = \underline{\underline{30 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma^{(4)}_s = 231 \frac{\text{lb}}{\text{in}^2} \text{ C}$$

$$\sigma^{(5)}_c = E_c \frac{(u_6 - u_5)}{\ell} = \frac{(3.27\times10^6)(-2.42570 + 1.94768)\times10^{-4}}{6} = \underline{\underline{26 \frac{\text{lb}}{\text{in}^2} \text{ C}}}$$

$$\sigma^{(5)}_s = 231 \frac{\text{lb}}{\text{in}^2} \text{ C}$$

note $\sigma^{(1)}_c = \sigma^{(5)}_c$ and $\sigma^{(2)}_c = \sigma^{(4)}_c$ as expected.

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1.13

$$\Lambda^{(e)} = \frac{A_{\text{avg}}\, E}{2\ell}\left(u_{i+1} - u_i\right)^2$$

$$\Lambda_{\text{total}} = \Lambda^{(1)} + \Lambda^{(2)} + \Lambda^{(3)} + \Lambda^{(4)} + \Lambda^{(5)}$$

$$\Lambda^{(1)} = \Lambda^{(5)} = \frac{(A_1)_c E_c}{2\ell}(u_{i+1} - u_i)^2 + \frac{(A_1)_s E_s}{2\ell}(u_{i+1} - u_i)^2$$

$$= \frac{(A_1)_c E_c + (A_1)_s E_s}{2\ell}(u_2 - u_1)^2$$

$$\Lambda^{(1)} = \left[\frac{(33.161)(3.27\times10^6) + 3\left(\frac{\pi}{4}\right)(0.5)^2(29\times10^6)}{(2)(6)}\right](-4.78016\times10^{-5})^2 = 0.024\ \text{lb}\cdot\text{in}$$

or

$$\Lambda^{(5)} = \left[\frac{(33.161)(3.27\times10^6) + 3\left(\frac{\pi}{4}\right)(0.5)^2(29\times10^6)}{(2)(6)}\right] = [(-2.4257 + 1.94768)\times10^{-4}]^2 = \overset{\surd}{0.024}$$

$$\Lambda^{(2)} = \frac{(A_2)_c E_c + (A_2)_s E_s}{2\ell}(u_3 - u_2)$$

$$= \left[\frac{(28.661)(3.27\times10^6) + 3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(6)}\right][(-1.01951\times10^{-4} + 4.78016\times10^{-5})]^2 = 0.027$$

or

$$\Lambda^{(4)} = \left[\frac{(28.661)(3.27\times10^6) + 3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(6)}\right] = [(-1.94768 + 1.40618)\times10^{-4}]^2 = \overset{\surd}{0.027}$$

$$\Lambda^{(3)} = \frac{(A_3)_c E_c + (A_3)_s E_s}{2\ell}(u_4 - u_3)$$

$$\Lambda^{(3)} = \left[\frac{(26.411)(3.27\times10^6) + 3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(4)}\right][(-1.40618 + 1.01951\times10^{-4}]^2$$

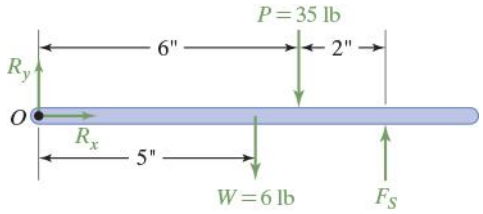
$$= 0.019\ \text{lb}\cdot\text{in}$$

$$\Lambda_{\text{total}} = (2)(0.024) + 0.019 + 2(0.027) = \underline{\underline{0.121\ \text{lb}\cdot\text{in}}} \quad \Leftarrow$$

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1.14



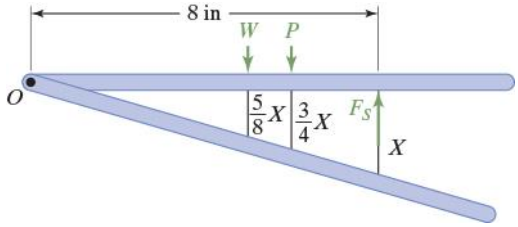
(a) $\oplus \sum M_0 = 0$

$(-6 \text{ lb})(5 \text{ in}) + (35 \text{ lb})(6 \text{ in}) + F_s(8 \text{ in}) = 0$

$F_s = 30 \text{ lb}$

$F_s = KX \quad 30 \text{ lb} = 60\ X \quad \underline{\underline{X = 0.5 \text{ in}}}$ ⇦

(b)



$$\frac{\partial \pi}{\partial u_i} = \frac{\partial}{\partial u_i}\sum_{i=1}^{n}\Lambda^{(e)} - \frac{\partial}{\partial u_i}\sum_{i=1}^{m}F_i u_i = 0$$

$$\frac{\partial}{\partial X}\left(\frac{1}{2}KX^2\right) - \frac{\partial}{\partial X}\left(35\left(\frac{3}{4}X\right)\right) - (6)\left(\frac{5}{8}X\right) = 0$$

$$KX - (35)\left(\frac{3}{4}\right) - (6)\left(\frac{5}{8}\right) = 0$$

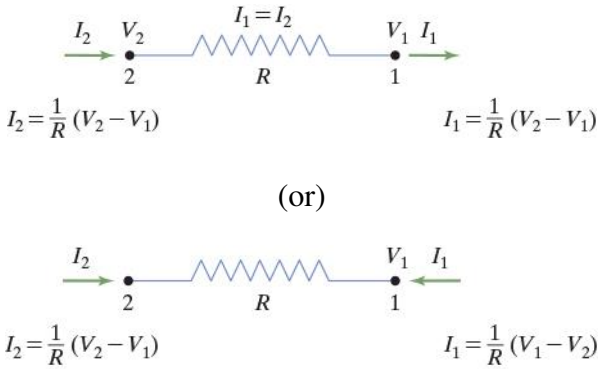
$$X = \frac{(35)\left(\frac{3}{4}\right) + (6)\left(\frac{5}{8}\right)}{60}$$

$\underline{\underline{X = 0.5 \text{ in}}}$ ⇦

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1.15



Because of the fact that charge is conserved in a circuit (Kirchhoff's current law), at any time, the algebraic sum of the currents entering any node must be zero. Thus we can write

$$I_1 = \frac{1}{R}(V_1 - V_2)$$

$$I_2 = \frac{1}{R}(V_2 - V_1)$$

Note $I_1 + I_2 = 0$

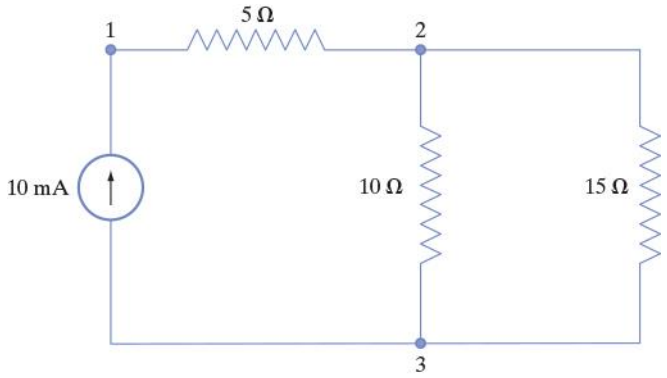
In matrix form:

$$\frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

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1.16



$$[K]^{(1)} = \frac{1}{5}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(2)} = \frac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(3)} = \frac{1}{15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}; \text{ Apply } V_3 = 0 \text{ as a boundary condition}$$

$$\begin{bmatrix} \frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{10}+\frac{1}{15} & -\frac{1}{10}-\frac{1}{15} \\ 0 & \cancel{-\frac{1}{10}-\frac{1}{15}} & \cancel{\frac{1}{10}+\frac{1}{15}} \\ & 0 & 1 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0.01 \\ 0 \\ 0 \end{Bmatrix}$$

$$V_1 = 0.11 \qquad V_2 = 0.06$$

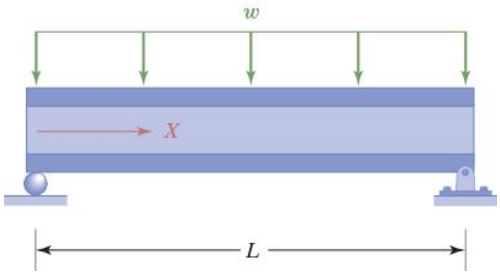
$$\underline{\underline{V_1 - V_2 = 0.05 \text{ Volts}}} \quad \Leftarrow$$

$$\underline{\underline{V_2 - V_3 = 0.06 \text{ Volts}}} \quad \Leftarrow$$

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1.17



$$\frac{d^2Y}{dX^2} = \frac{M(X)}{EI} = \frac{wX(L-X)}{2EI}$$

$$\frac{dY}{dX} = \frac{1}{2EI} = \left(\frac{wX^2L}{2} - \frac{wX^3}{3}\right) + C_1$$

$$Y = \frac{1}{2EI}\left(\frac{wX^3L}{6} - \frac{wX^4}{12}\right) + C_1X + C_2$$

Applying Boundary Conditions:

$Y = 0 \quad @ \quad X = 0 \quad C_2 = 0$

$$Y = 0 \quad @ \quad X = L \quad C_1 = -\frac{wL^3}{24EI}$$

$$\underline{\underline{Y_{\text{exact}} = \frac{-wX}{24EI}(X^3 - 2LX^2 + L^3)}} \quad \Leftarrow$$

(a) Note that the assumed Solution satisfies the boundary conditions. $Y = C_1\left[\left(\frac{X}{L}\right)^2 - \left(\frac{X}{L}\right)\right]$

$$\frac{dY}{dX} = C_1\left[\frac{2X}{L^2} - \frac{1}{L}\right]$$

$$\frac{d^2Y}{dX^2} = C_1\left(\frac{2}{L^2}\right)$$

$$\frac{2C_1}{L^2} - \frac{wX(L-X)}{2EI} = \mathrm{R}$$

We may force the error function to equal zero at $X = \frac{L}{2}$

$$\frac{2C_1}{L^2} - \frac{w\frac{L}{2}\left(L - \frac{L}{2}\right)}{2EI} = 0 \rightarrow C_1 = \frac{wL^4}{16EI}$$

$$\underline{\underline{Y = \frac{wL^4}{16EI}\left[\left(\frac{X}{L}\right)^2 - \left(\frac{X}{L}\right)\right]}} \quad \Leftarrow$$

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1.17 Cont.

(b) $\int_0^L R\,dX = 0$

$$\int_0^L \left[\frac{2c_1}{L^2} - \frac{wX(L-X)}{2EI}\right] dX = 0$$

$$\frac{2c_1}{L^2}L - \frac{w}{2EI}\left(\frac{L^3}{2} - \frac{L^3}{3}\right) = 0 \rightarrow c_1 = \frac{wL^4}{24EI}$$

$$\underline{\underline{Y = \frac{wL^4}{24EI}\left(\left(\frac{X}{L}\right)^2 - \left(\frac{X}{L}\right)\right)}} \quad \Leftarrow$$

For $W24 \times 104$ Beam $I = 3100\,\text{in}^4$; $E = 29 \times 10^6\ \frac{\text{lb}}{\text{in}^2}$

Exact

$$Y_{\max} = \frac{-5wL^4}{384EI} = \frac{-5\left(5000\ \frac{\text{lb}}{\text{ft}} \times \frac{1\,\text{ft}}{12\,\text{in}}\right)\left(20\,\text{ft} \times \frac{12\,\text{in}}{\text{ft}}\right)^4}{(384)\left(29 \times 10^6\ \frac{\text{lb}}{\text{in}^2}\right)\left(3100\,\text{in}^4\right)} = \underline{\underline{-0.20\,\text{in}}}$$

Collocation

$$Y_{\max} = \frac{-wL^4}{64EI} = \frac{-\left(5000\ \frac{\text{lb}}{\text{ft}} \times \frac{1\,\text{ft}}{12\,\text{in}}\right)\left(20\,\text{ft} \times \frac{12\,\text{in}}{\text{ft}}\right)^4}{(64)\left(29 \times 10^6\ \frac{\text{lb}}{\text{in}^2}\right)(3100\,\text{in}^4)} = \underline{\underline{-0.24\,\text{in}}} \quad \Leftarrow$$

Subdomain

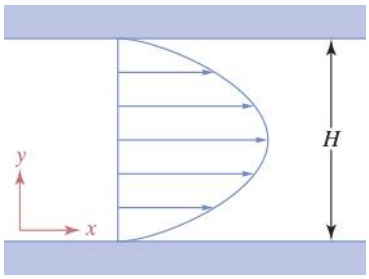
$$Y_{\max} = \frac{-wL^4}{96EI} = \frac{-\left(5000\ \frac{\text{lb}}{\text{ft}} \times \frac{1\,\text{ft}}{12\,\text{in}}\right)\left(20\,\text{ft} \times \frac{12\,\text{in}}{\text{ft}}\right)^4}{(96)\left(29 \times 10^6\ \frac{\text{lb}}{\text{in}^2}\right)\left(3100\,\text{in}^4\right)} = \underline{\underline{-0.16\,\text{in}}}$$

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1.18 See Section 1.7

1.19



$$\mu\frac{d^2u}{dy^2}=\frac{dP}{dX}$$

$$\frac{du}{dy}=\frac{1}{\mu}\frac{dP}{dX}y+c_1$$

$$u(y)=\frac{1}{\mu}\frac{dP}{dX}\frac{y^2}{2}+c_1y+c_2$$

applying B.Cs $u(0)=0$ and $u(H)=0$

$$0=0+0+c_2 \quad \text{and} \quad 0=\frac{1}{\mu}\frac{dP}{dX}\frac{H^2}{2}+c_1H$$

$$c_2=0 \qquad \text{and} \qquad c_1=-\frac{H}{2\mu}\frac{dP}{dX}$$

$$\underline{\underline{u(y)=-\frac{1}{2\mu}\frac{dP}{dX}(Hy-y^2)}}$$

note because Pressure drops in the direction of flow, $\frac{dP}{dX}<0$ in velocity equation.

(a) $\mu\frac{d^2u}{dy^2}=\frac{dP}{dX}$ $\qquad u_{\text{assumed}}=c_1\sin\left(\frac{\pi y}{H}\right)$

note the assumed solution Satisfies the boundary conditions.

$$\frac{du}{dy}=\frac{d}{dy}\left(c_1\sin\left(\frac{\pi y}{H}\right)\right)=c_1\frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)$$

$$\frac{d^2u}{dy^2}=\frac{d}{dy}\left(c_1\frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)\right)=-c_1\left(\frac{\pi}{H}\right)^2\sin\left(\frac{\pi y}{H}\right)$$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2\sin\left(\frac{\pi y}{H}\right)\right]-\frac{dP}{dX}=\text{R}$$

We may force the error function to equal zero at $y=\frac{H}{2}$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2\right]\sin\left(\frac{\pi}{H}\frac{H}{2}\right)-\frac{dP}{dX}=0$$

$$c_1=-\frac{H^2}{\mu\pi^2}\frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y)=-\frac{H^2}{\mu\pi^2}\frac{dP}{dX}\left(\sin\left(\frac{\pi y}{H}\right)\right)}} \qquad \Leftarrow$$

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1.19 Cont.

(b) $\int_0^H R\,dy = 0$

$$\int_0^H \left[\mu \left[-c_1 \left(\frac{\pi}{H} \right)^2 \sin \frac{\pi y}{H} \right] - \frac{dP}{dX} \right] dy = 0$$

$$\mu c_1 \left(\frac{\mu}{H} \right)^2 \left[\frac{1}{\frac{\pi}{H}} \cos \frac{\pi y}{H} \right]_0^H - \frac{dP}{dX} H = 0$$

$$\mu c_1 \frac{\pi}{H} (-2) = H \frac{dP}{dX}$$

$$c_1 = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} \left[\sin \left(\frac{\pi y}{H} \right) \right]}} \quad \Leftarrow$$

For Comparison, let's use:

$$\mu = 0.02 \frac{\text{N} \cdot \text{S}}{\text{m}^2}; \quad H = 0.01 \text{ mm} = 1 \times 10^{-5} \text{ m}; \quad \frac{dP}{dX} = -1 \times 10^8 \frac{\text{Pa}}{\text{m}}$$

evaluating max vel. @ $y = \frac{H}{2}$

Exact

$$u_{\max} = -\frac{1}{2\mu} \frac{dP}{dX} \left(H \left(\frac{H}{2} \right) - \left(\frac{H}{2} \right)^2 \right) = -\frac{1}{2\mu} \frac{dP}{dX} \frac{H^2}{4} = -\frac{(-1 \times 10^8)(1 \times 10^{-5})^2}{(8)(0.02)} = \underline{\underline{0.06 \frac{\text{m}}{\text{s}}}}$$

Collocation method

$$u_{\max} = -\frac{H^2}{\mu\pi^2} \frac{dP}{dX} \left(\sin \left(\pi \frac{\frac{H}{2}}{H} \right) \right) = -\frac{H^2}{\mu\pi^2} \frac{dP}{dX} = -\frac{(1 \times 10^{-5})^2 (-1 \times 10^8)}{(0.02)\pi^2} = \underline{\underline{0.05 \frac{\text{m}}{\text{s}}}}$$

Subdomain method

$$u_{\max} = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} \left(\sin \left(\pi \frac{\frac{H}{2}}{H} \right) \right) = -\frac{H^2}{2\mu\pi} \frac{dP}{dX} = -\frac{(1 \times 10^{-5})^2 (-1 \times 10^8)}{(2)(0.02)\pi} = \underline{\underline{0.08 \frac{\text{m}}{\text{s}}}}$$

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1.20

(a) *Galerkin method*

$$\int_0^H \sin\left(\frac{\pi y}{H}\right)\left[\mu\left(-C_1\left(\frac{\pi}{H}\right)^2 \sin\frac{\pi y}{H}\right)-\frac{dP}{dX}\right]dy=0$$

$$-\mu C_1\left(\frac{\pi}{H}\right)^2\int_0^H \sin^2\left(\frac{\pi y}{H}\right)dy=\frac{dP}{dX}\int_0^H \sin\left(\frac{\pi y}{H}\right)dy$$

$$-\mu C_1\left(\frac{\pi}{H}\right)^2\left[\frac{1}{2}y-\frac{1}{4\left(\frac{\pi}{H}\right)}\sin\left(\frac{2\pi}{H}y\right)\right]_0^H=\frac{dP}{dX}\left[-\frac{1}{\frac{\pi}{H}}\cos\left(\frac{\pi}{H}y\right)\right]_0^H$$

$$-\mu C_1\left(\frac{\pi}{H}\right)^2\left(\frac{1}{2}H\right)=-\frac{H}{\pi}\frac{dP}{dX}(-2)$$

$$C_1=\frac{-4H^2}{\mu\pi^3}\frac{dP}{dX} \quad \text{then,} \quad \underline{\underline{u(y)=-\frac{4H^2}{\mu\pi^3}\frac{dP}{dX}\sin\left(\frac{\pi y}{H}\right)}} \quad \Leftarrow$$

(b) *least-squares method*

$$\int_0^H R\frac{\partial R}{\partial C_1}dy=\int_0^H \overbrace{\sin\left(\frac{\pi y}{H}\right)\left[\mu\left(-C_1\left(\frac{\pi}{H}\right)^2\right)\sin\frac{\pi y}{H}-\frac{dP}{dX}\right]}^{R}\overbrace{\left[-\mu\left(\frac{\pi}{H}\right)^2\sin\frac{\pi y}{H}\right]}^{\frac{\partial R}{\partial c_1}}dy=0$$

results in:

$$C_1=-\frac{4H^2}{\mu\pi^3}\frac{dP}{dX} \quad \text{and} \quad \underline{\underline{u(y)=-\frac{4H^2}{\mu\pi^3}\frac{dP}{dX}\sin\left(\frac{\pi y}{H}\right)}} \quad \Leftarrow$$

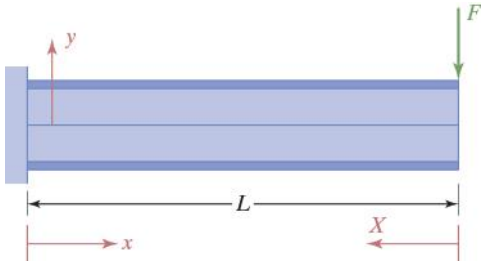
using values from Problem 1.19 for comparison

$$u_{\max}=\frac{-4H^2}{\mu\pi^3}\frac{dP}{dX}=\frac{-4(1\times10^{-5})^2(-1\times10^8)}{(0.02)\pi^3}=\underline{\underline{0.06\,\frac{\text{m}}{\text{s}}}} \quad \Leftarrow$$

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1.21



$$\frac{d^2y}{dX^2} = -\frac{PX}{EI}$$

$$\frac{dy}{dX} = \frac{-p}{EI}\frac{X^2}{2} + c_1$$

$$y = -\frac{p}{EI}\frac{X^3}{6} + c_1X + c_2$$

B.Cs ① $y(L) = 0$, ② $\left.\frac{dy}{dX}\right|_{X=L} = 0$

② $\quad 0 = \frac{-P}{EI}\frac{L^2}{2} + c_1 \qquad c_1 = \frac{P}{EI}\frac{L^2}{2}$

① $\quad 0 = -P\frac{L^3}{6} + \frac{P}{EI}\frac{L^2}{2}L + C_2 \qquad C_2 = \frac{P}{EI}\left(-\frac{1}{3}L^3\right)$

$$\underline{\underline{y_{\text{exact}} = \frac{P}{6EI}(-X^3 + 3L^2X - 2L^3)}} \quad \Leftarrow$$

In terms of x: Substitute for $X = L - x$ and simplify

$$y_{\text{exact}} = \frac{P}{6EI}(-(L-x)^3 + 3L^2(L-x) - 2L^3)$$

$$\underline{\underline{y_{\text{exact}} = \frac{P}{6EI}\left(x^3 - 3L^2x\right)}} \quad \Leftarrow$$

$$EI\frac{d^2y}{dx^2} + P(L-x) = 0$$

B.Cs $y(0) = 0$ and $\left.\frac{dy}{dx}\right|_{x=0} = 0$

Let us assume: $y_{\text{assumed}} = c_1x^2 + c_2x^3$

note, the assumed solution satisfies the boundary conditions.

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1.21 Cont.

$$\frac{dy}{dx} = 2c_1 x + 3c_2 x^2$$

$$\frac{d^2 y}{dx^2} = 2c_1 + 6c_2 x$$

then Residual R becomes:

$$R = EI(2c_1 + 6c_2 x) + P(L - x)$$

Subdomain method:

$$\int_0^{\frac{L}{2}} R\,dx = 0 \qquad \int_0^{\frac{L}{2}} [EI(2c_1 + 6c_2 x) + P(L - x)]dx = 0 \qquad (1)$$

$$\int_{\frac{L}{2}}^{L} R\,dx = 0 \qquad \int_{\frac{L}{2}}^{L} [EI(2c_1 + 6c_2 x) + P(L - x)]dx = 0 \qquad (2)$$

Integrating (1):

$$EI\left(2c_1\left(\frac{L}{2}\right) + 6c_2\frac{\left(\frac{L}{2}\right)^2}{2}\right) + P\left(L\left(\frac{L}{2}\right) - \frac{\left(\frac{L}{2}\right)^2}{2}\right) = 0$$

Simplifying:

$$2c_1 + \frac{3}{2}c_2 L = -\frac{3PL}{4EI} \qquad (3)$$

Integrating (2) and Simplifying:

$$c_1 + \frac{9}{4}c_2 L = -\frac{PL}{8EI} \qquad (4)$$

Solving (3) and (4) Simultaneously:

$$\underline{\underline{c_1 = -\frac{PL}{2EI}}} \quad \text{and} \quad \underline{\underline{c_2 = \frac{P}{6EI}}} \qquad \Leftarrow$$

$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

note, because the assumed solution has the same functional form as the exact solution, the coefficients are exact.

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1.21 Cont.

Galerkin method:

$$\int_0^L x^2 R\,dx = 0 \qquad \int_0^L x^2[EI(2c_1 + 6c_2) + P(L-x)]\,dx = 0 \tag{1}$$

$$\int_0^L x^3 R\,dx = 0 \qquad \int_0^L x^3[EI(2c_1 + 6c_2) + P(L-x)]\,dx = 0 \tag{2}$$

Integrating and Simplifying (1) and (2), we get:

$$\begin{cases} \frac{2}{3}c_1 + \frac{3}{2}c_2 L = -\frac{PL}{12EI} & (3) \\ \frac{1}{2}c_1 + \frac{6}{5}c_2 L = -\frac{PL}{20EI} & (4) \end{cases}$$

Solving (3) and (4) Simultaneously

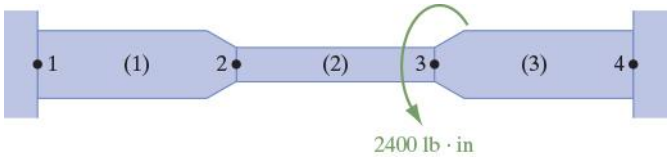
$$\underline{\underline{c_1 = -\frac{PL}{2EI}}} \quad \text{and} \quad \underline{\underline{c_2 = \frac{P}{6EI}}} \qquad \Leftarrow$$

$$y = \frac{P}{6EI}(x^3 - 3Lx^2)$$

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1.22



$$J_1 = J_3 = \frac{1}{2}\pi r^4$$

$$J_1 = J_3 = \frac{1}{2}\pi\left(\frac{1.5}{2}\right)^4 = 0.497 \text{ in}^4$$

$$J_2 = \frac{1}{2}\pi\left(\frac{1}{2}\right)^4 = 0.0982 \text{ in}^4$$

$$[K]^{(1)} = [K]^{(3)} = \frac{JG}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{(0.497)(9.8\times10^6)}{(2)(12)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 202942 & -202942 \\ -202942 & 202942 \end{bmatrix}$$

$$[K]^{(2)} = \frac{(0.0982)(11.2\times10^6)}{(1.5)(12)}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 61102 & -61102 \\ -61102 & 61102 \end{bmatrix}$$

$$\begin{bmatrix} \overset{1}{\cancel{202942}} & \overset{0}{\cancel{-202942}} & & \\ \underset{0}{\cancel{-202942}} & 202942+61102 & -61102 & \\ & -61102 & 61102+202942 & \underset{0}{\cancel{-202942}} \\ & & \underset{0}{\cancel{-202942}} & \underset{1}{\cancel{202942}} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 2400 \\ 0 \end{Bmatrix}$$

$$\theta_1 = 0 \quad \underline{\underline{\theta_2 = 0.002222 \text{ rad}}} \quad \underline{\underline{\theta_3 = 0.009604 \text{ rad}}} \quad \theta_4 = 0$$

$$R_4 = K_3(\theta_3 - 0) = 202942 \times 0.009604 = \underline{\underline{1949 \text{ lb}\cdot\text{in}}}$$

$$R_1 = K_1(\theta_2 - 0) = 202942 \times 0.002222 = \underline{\underline{451 \text{ lb}\cdot\text{in}}}$$

note they add up to 2400 lb·in

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1.23

$$\begin{bmatrix} 264044 & -61102 \\ -61102 & 264044 \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 1200 \\ 1200 \end{Bmatrix}$$

$$\underline{\underline{\theta_2 = \theta_3 = 0.005913 \text{ rad}}} \quad \Longleftarrow$$

$$\underline{R_1} = \underline{\underline{R_4}} = K_1(\theta_2 - 0) = K_3(\theta_3 - 0) = 202942 \times 0.005913 = \underline{\underline{1200 \text{ lb} \cdot \text{in}}}$$

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1.24

$A_1 = 0.5\ \text{in}^2$

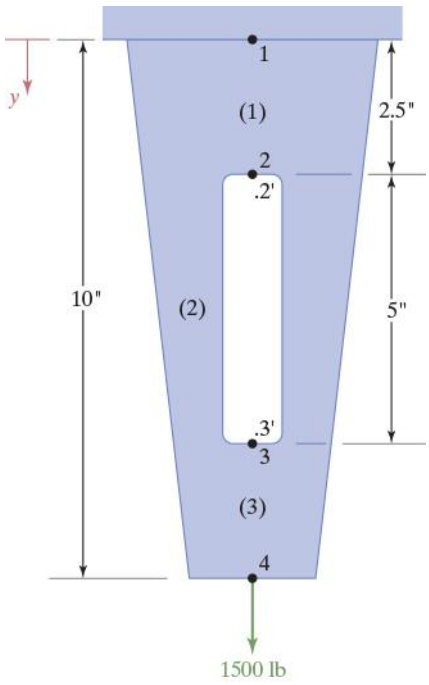
$A_2 = 0.4375\ \text{in}^2$ $\qquad A_{2'} = 0.4375.\ (0.5)(0.125)$

$= 0.375\ \text{in}^2$

$A_3 = 0.3125\ \text{in}^2$ $\qquad A_{3'} = 0.3125 - (0.5)(0.125)$

$= 0.25\ \text{in}^2$

$A_4 = 0.25\ \text{in}^2$



$$K = \frac{A_{\text{avg}} E}{\ell}$$

$$K_1 = \frac{\left(\frac{0.5 + 0.4375}{2}\right)(10.6 \times 10^6)}{2.5} = 1{,}987{,}500\ \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{\left(\frac{0.375 + 0.25}{2}\right)(10.6 \times 10^6)}{5} = 662{,}500\ \frac{\text{lb}}{\text{in}}$$

$$K_3 = \frac{\left(\frac{0.3125 + 0.25}{2}\right)(10.6 \times 10^6)}{2.5} = 1{,}192{,}500\ \frac{\text{lb}}{\text{in}}$$

without the whole:

$A(y) = (4 - 0.2y)(0.125)$

$= 0.5 - 0.025y$

$$\begin{bmatrix} \overset{1}{\cancel{1987500}} & \overset{0}{\cancel{-1{,}987{,}500}} & & \\ \cancel{-1{,}987{,}500} & 1{,}987{,}500 + 662500 & -662500 & \\ 0 & & & \\ & -662500 & 662500 + 1192500 & -1192500 \\ & & -1192500 & 1192500 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1500 \end{Bmatrix}$$

$u_1 = 0 \quad \underline{\underline{u_2 = 7.5472 \times 10^{-4}\ \text{in}}} \quad \underline{\underline{u_3 = 0.003020\ \text{in}}} \quad \underline{\underline{u_4 = 0.004277\ \text{in}}}$

$$\sigma^{(1)} = (10.6 \times 10^6)\left(\frac{7.5472 \times 10^{-4}}{2.5}\right) = 3200\ \frac{\text{lb}}{\text{in}^2}$$

as a check: $\sigma^{(1)} = \dfrac{1500}{\frac{0.5 + 0.4375}{2}} = 3200\ \dfrac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = (10.6 \times 10^6)\left(\frac{0.00302 - 7.5472 \times 10^{-4}}{5}\right) = 4800\ \frac{\text{lb}}{\text{in}^2}$$

check: $\sigma^{(2)} = \dfrac{1500}{\left(\frac{0.375 + 0.25}{2}\right)} = 4800\ \dfrac{\text{lb}}{\text{in}^2}$

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1.24 Cont.

$$\sigma^{(3)} = (10.6 \times 10^6)\left(\frac{0.004277 - 0.00302}{2.5}\right) = 5300 \frac{\text{lb}}{\text{in}^2}$$

check: $\sigma^{(3)} = \dfrac{1500}{\dfrac{0.3125 + 0.25}{2}} = 5300 \text{ Psi}$

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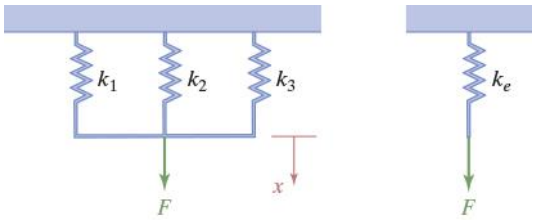
1.25

Springs in Parallel

$$F = F_1 + F_2 + F_3$$

$$k_e x = k_1 x + k_2 x + k_3 x \quad \Leftarrow$$

$$\underline{\underline{k_e = k_1 + k_2 + k_3}}$$



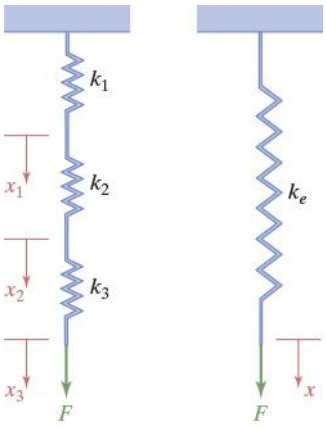
Springs in Series

$$x = x_1 + x_2 + x_3$$

$$\frac{F}{k_e} = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

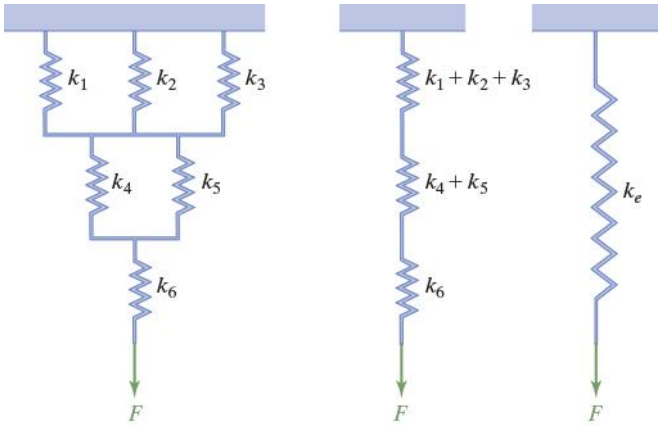
$$\underline{\underline{k_e = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}}}} \quad \Leftarrow$$



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1.26

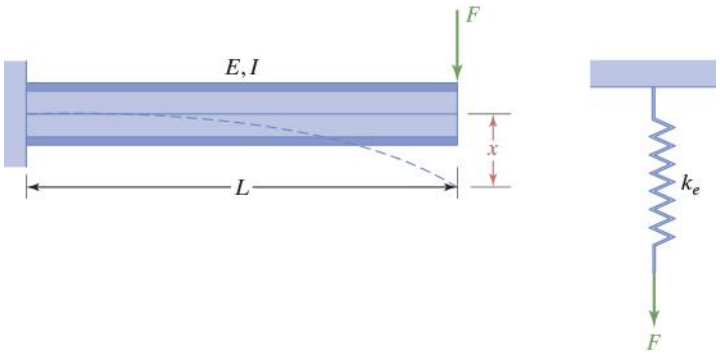


$$k_e = \frac{1}{\frac{1}{k_1 + k_2 + k_3} + \frac{1}{k_4 + k_5} + \frac{1}{k_6}} \quad \Leftarrow$$

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1.27



From Table 4.1

$$X = \frac{FL^3}{3EI}$$

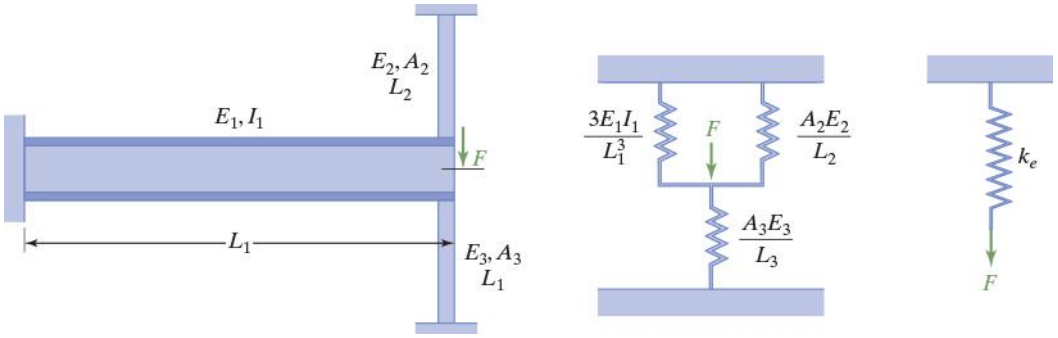
$$F = \frac{3EI}{L^3}x$$

$$\underline{\underline{K_e = \frac{3EI}{L^3}}} \quad \Leftarrow$$

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1.28



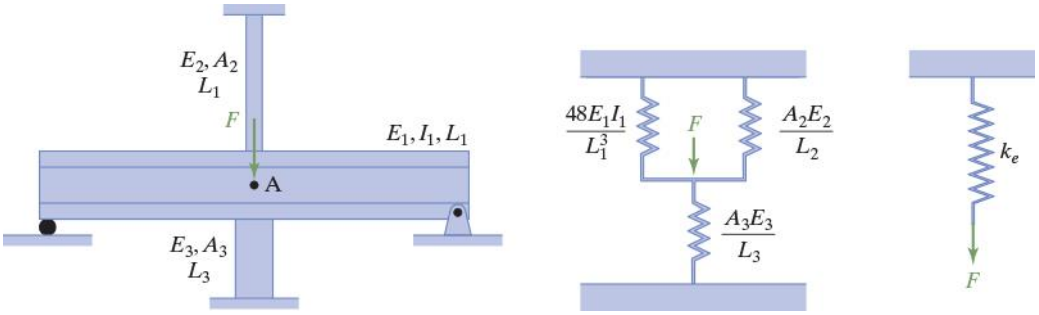
$$K_e = \frac{3E_1I_1}{L_1^3} + \frac{A_2E_2}{L_2} + \frac{A_3E_3}{L_3} \quad \Leftarrow$$

Note, deflection of each spring is the same, therefore, springs are in Parallel.

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1.29



From Table 4.1, the K_e for the beam is:

$$(K_e)_{\text{beam}} = \frac{48E_1I_1}{L_1^3}$$

$$K_e = \frac{48E_1I_1}{L_1^3} + \frac{A_2E_2}{L_2} + \frac{A_3E_3}{L_3}$$

Note, deflection of each spring is the same, therefore, springs are in parallel.

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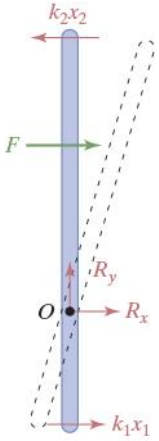
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1.30

$$\circlearrowleft + \sum M_0 = 0$$

$$-FL + (K_2 x_2)(2L) + (K_1 x_1)(L) = 0$$

Note $x_2 = 2x_1$ and simplify



$$(2K_2 x_1)(2L) + K_1 x_1 L = FL$$

$$\underline{\underline{x_1 = \frac{F}{4K_2 + K_1}}} \quad \Leftarrow$$

$$\pi = \sum_{e=1}^{n} \Lambda^{(e)} - \sum F_i u_i$$

$$\pi = \frac{1}{2} K_2 x_2^2 + \frac{1}{2} K_1 x_1^2 - F x_1 = \frac{1}{2} K_2 (2x_1)^2 + \frac{1}{2} K_1 x_1^2 - F x_1$$

$$\pi = 2K_2 x_1^2 + \frac{1}{2} K_1 x_1^2 - F x_1$$

$$\frac{\partial \pi}{\partial x_1} = 4K_2 x_1 + K_1 x_1 - F = 0$$

$$\underline{\underline{x_1 = \frac{F}{4K_2 + K_1}}} \quad \Leftarrow$$

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