# *Calculus Early Transcendentals 11th edition Anton Solutions Manual*

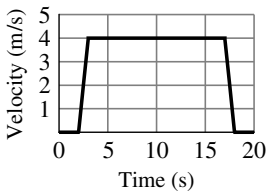
SKU: 9781118883822-SOLUTIONS

# The Derivative

## Exercise Set 2.1

**1.** **(a)** $m_{\tan} = (50-10)/(15-5) = 40/10 = 4$ m/s.

**(b)**




**2.** At $t = 4$ s, $m_{\tan} \approx (90-0)/(10-2) = 90/8 = 11.25$ m/s. At $t = 8$ s, $m_{\tan} \approx (140-0)/(10-4) = 140/6 \approx 23.33$ m/s.

**3.** **(a)** $(10-10)/(3-0) = 0$ cm/s.

**(b)** $t = 0$, $t = 2$, $t = 4.2$, and $t = 8$ (horizontal tangent line).

**(c)** maximum: $t = 1$ (slope $> 0$), minimum: $t = 3$ (slope $< 0$).

**(d)** $(3-18)/(4-2) = -7.5$ cm/s (slope of estimated tangent line to curve at $t = 3$).

**4.** **(a)** decreasing (slope of tangent line decreases with increasing time)

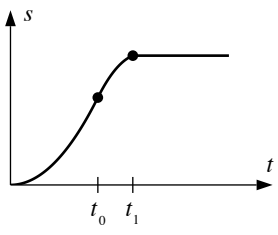
**(b)** increasing (slope of tangent line increases with increasing time)

**(c)** increasing (slope of tangent line increases with increasing time)

**(d)** decreasing (slope of tangent line decreases with increasing time)

**5.** It is a straight line with slope equal to the velocity.

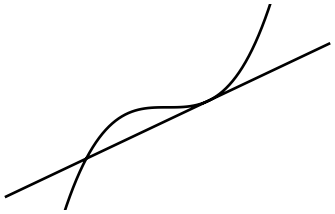
**6.** The velocity increases from time 0 to time $t_0$, so the slope of the curve increases during that time. From time $t_0$ to time $t_1$, the velocity, and the slope, decrease. At time $t_1$, the velocity, and hence the slope, instantaneously drop to zero, so there is a sharp bend in the curve at that point.



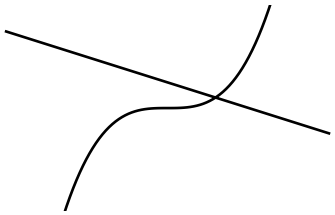



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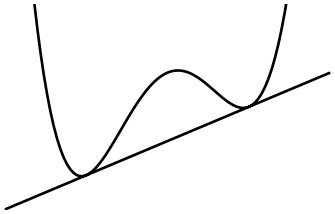
**7.**



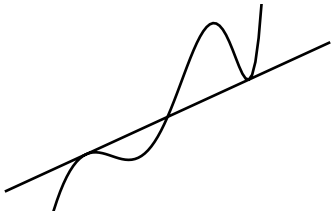
**8.**



**9.**



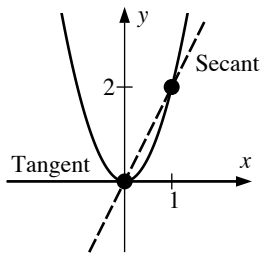
**10.**



**11.** **(a)** $m_{\text{sec}} = \dfrac{f(1)-f(0)}{1-0} = \dfrac{2}{1} = 2$

**(b)** $m_{\text{tan}} = \lim\limits_{x_1 \to 0} \dfrac{f(x_1)-f(0)}{x_1-0} = \lim\limits_{x_1 \to 0} \dfrac{2x_1^2-0}{x_1-0} = \lim\limits_{x_1 \to 0} 2x_1 = 0$

**(c)** $m_{\text{tan}} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \lim\limits_{x_1 \to x_0} \dfrac{2x_1^2-2x_0^2}{x_1-x_0} = \lim\limits_{x_1 \to x_0} (2x_1+2x_0) = 4x_0$




**(d)** The tangent line is the $x$-axis.

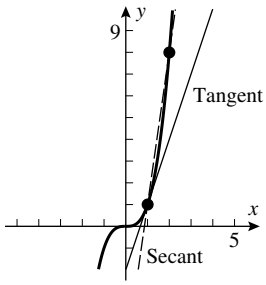
**12.** **(a)** $m_{\text{sec}} = \dfrac{f(2)-f(1)}{2-1} = \dfrac{2^3-1^3}{1} = 7$

**(b)** $m_{\text{tan}} = \lim\limits_{x_1 \to 1} \dfrac{f(x_1)-f(1)}{x_1-1} = \lim\limits_{x_1 \to 1} \dfrac{x_1^3-1}{x_1-1} = \lim\limits_{x_1 \to 1} \dfrac{(x_1-1)(x_1^2+x_1+1)}{x_1-1} = \lim\limits_{x_1 \to 1} (x_1^2+x_1+1) = 3$

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**(c)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{x_1^3 - x_0^3}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} (x_1^2 + x_1 x_0 + x_0^2) = 3x_0^2$

**(d)**

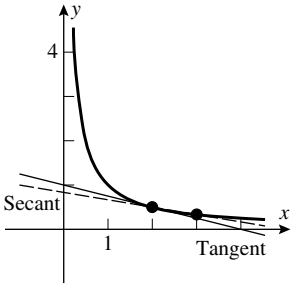



**13. (a)** $m_{\text{sec}} = \dfrac{f(3) - f(2)}{3 - 2} = \dfrac{1/3 - 1/2}{1} = -\dfrac{1}{6}$

**(b)** $m_{\tan} = \lim\limits_{x_1 \to 2} \dfrac{f(x_1) - f(2)}{x_1 - 2} = \lim\limits_{x_1 \to 2} \dfrac{1/x_1 - 1/2}{x_1 - 2} = \lim\limits_{x_1 \to 2} \dfrac{2 - x_1}{2x_1(x_1 - 2)} = \lim\limits_{x_1 \to 2} \dfrac{-1}{2x_1} = -\dfrac{1}{4}$

**(c)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{1/x_1 - 1/x_0}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{x_0 - x_1}{x_0 x_1 (x_1 - x_0)} = \lim\limits_{x_1 \to x_0} \dfrac{-1}{x_0 x_1} = -\dfrac{1}{x_0^2}$

**(d)**

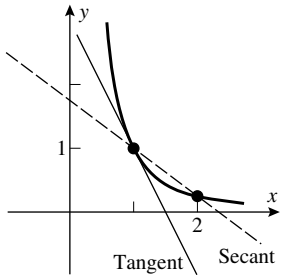



**14. (a)** $m_{\text{sec}} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{1/4 - 1}{1} = -\dfrac{3}{4}$

**(b)** $m_{\tan} = \lim\limits_{x_1 \to 1} \dfrac{f(x_1) - f(1)}{x_1 - 1} = \lim\limits_{x_1 \to 1} \dfrac{1/x_1^2 - 1}{x_1 - 1} = \lim\limits_{x_1 \to 1} \dfrac{1 - x_1^2}{x_1^2(x_1 - 1)} = \lim\limits_{x_1 \to 1} \dfrac{-(x_1 + 1)}{x_1^2} = -2$

**(c)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{1/x_1^2 - 1/x_0^2}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{x_0^2 - x_1^2}{x_0^2 x_1^2 (x_1 - x_0)} = \lim\limits_{x_1 \to x_0} \dfrac{-(x_1 + x_0)}{x_0^2 x_1^2} = -\dfrac{2}{x_0^3}$

**(d)**




**15. (a)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 - 1) - (x_0^2 - 1)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 - x_0^2)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} (x_1 + x_0) = 2x_0$

**(b)** $m_{\tan} = 2(-1) = -2$

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**16.** **(a)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 + 3x_1 + 2) - (x_0^2 + 3x_0 + 2)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{(x_1^2 - x_0^2) + 3(x_1 - x_0)}{x_1 - x_0} =$
$= \lim\limits_{x_1 \to x_0} (x_1 + x_0 + 3) = 2x_0 + 3$

**(b)** $m_{\tan} = 2(2) + 3 = 7$

**17.** **(a)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{(x_1 + \sqrt{x_1}\,) - (x_0 + \sqrt{x_0}\,)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \left(1 + \dfrac{1}{\sqrt{x_1} + \sqrt{x_0}}\right) = 1 + \dfrac{1}{2\sqrt{x_0}}$

**(b)** $m_{\tan} = 1 + \dfrac{1}{2\sqrt{1}} = \dfrac{3}{2}$

**18.** **(a)** $m_{\tan} = \lim\limits_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{1/\sqrt{x_1} - 1/\sqrt{x_0}}{x_1 - x_0} = \lim\limits_{x_1 \to x_0} \dfrac{\sqrt{x_0} - \sqrt{x_1}}{\sqrt{x_0}\,\sqrt{x_1}\,(x_1 - x_0)} =$
$= \lim\limits_{x_1 \to x_0} \dfrac{-1}{\sqrt{x_0}\,\sqrt{x_1}\,(\sqrt{x_1} + \sqrt{x_0}\,)} = -\dfrac{1}{2x_0^{3/2}}$

**(b)** $m_{\tan} = -\dfrac{1}{2(4)^{3/2}} = -\dfrac{1}{16}$

**19.** True. Let $x = 1 + h$.

**20.** False. A secant line meets the curve in at least two places, but a tangent line might meet it only once.

**21.** False. Velocity represents the rate at which position changes.

**22.** True. The units of the rate of change are obtained by dividing the units of $f(x)$ (inches) by the units of $x$ (tons).

**23.** **(a)** 72°F at about 4:30 P.M. **(b)** About $(67 - 43)/6 = 4$°F/h.

**(c)** Decreasing most rapidly at about 9 P.M.; rate of change of temperature is about $-7$°F/h (slope of estimated tangent line to curve at 9 P.M.).

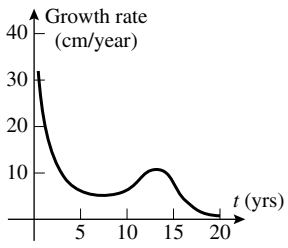
**24.** For $V = 10$ the slope of the tangent line is about $(0 - 5)/(20 - 0) = -0.25$ atm/L, for $V = 25$ the slope is about $(1 - 2)/(25 - 0) = -0.04$ atm/L.

**25.** **(a)** During the first year after birth.

**(b)** About 6 cm/year (slope of estimated tangent line at age 5).

**(c)** The growth rate is greatest at about age 14; about 10 cm/year.

**(d)**




**26.** **(a)** The object falls until $s = 0$. This happens when $1250 - 16t^2 = 0$, so $t = \sqrt{1250/16} = \sqrt{78.125} > \sqrt{25} = 5$; hence the object is still falling at $t = 5$ sec.

**(b)** $\dfrac{f(6) - f(5)}{6 - 5} = \dfrac{674 - 850}{1} = -176$. The average velocity is $-176$ ft/s.

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**(c)** $v_{\text{inst}} = \lim\limits_{h\to 0} \dfrac{f(5+h)-f(5)}{h} = \lim\limits_{h\to 0} \dfrac{[1250-16(5+h)^2]-850}{h} = \lim\limits_{h\to 0} \dfrac{-160h-16h^2}{h} = \lim\limits_{h\to 0}(-160-16h) = -160$ ft/s.

**27.** **(a)** $0.3\cdot 40^3 = 19{,}200$ ft **(b)** $v_{\text{ave}} = 19{,}200/40 = 480$ ft/s

**(c)** Solve $s = 0.3t^3 = 1000; t \approx 14.938$ so $v_{\text{ave}} \approx 1000/14.938 \approx 66.943$ ft/s.

**(d)** $v_{\text{inst}} = \lim\limits_{h\to 0} \dfrac{0.3(40+h)^3 - 0.3\cdot 40^3}{h} = \lim\limits_{h\to 0} \dfrac{0.3(4800h+120h^2+h^3)}{h} = \lim\limits_{h\to 0} 0.3(4800+120h+h^2) = 1440$ ft/s

**28.** **(a)** $v_{\text{ave}} = \dfrac{4.5(12)^2 - 4.5(0)^2}{12-0} = 54$ ft/s

**(b)** $v_{\text{inst}} = \lim\limits_{t_1\to 6} \dfrac{4.5t_1^2 - 4.5(6)^2}{t_1-6} = \lim\limits_{t_1\to 6} \dfrac{4.5(t_1^2-36)}{t_1-6} = \lim\limits_{t_1\to 6} \dfrac{4.5(t_1+6)(t_1-6)}{t_1-6} = \lim\limits_{t_1\to 6} 4.5(t_1+6) = 54$ ft/s

**29.** **(a)** $v_{\text{ave}} = \dfrac{6(4)^4 - 6(2)^4}{4-2} = 720$ ft/min

**(b)** $v_{\text{inst}} = \lim\limits_{t_1\to 2} \dfrac{6t_1^4 - 6(2)^4}{t_1-2} = \lim\limits_{t_1\to 2} \dfrac{6(t_1^4-16)}{t_1-2} = \lim\limits_{t_1\to 2} \dfrac{6(t_1^2+4)(t_1^2-4)}{t_1-2} = \lim\limits_{t_1\to 2} 6(t_1^2+4)(t_1+2) = 192$ ft/min

**30.** See the discussion before Definition 2.1.1.

**31.** The instantaneous velocity at $t=1$ equals the limit as $h\to 0$ of the average velocity during the interval between $t=1$ and $t=1+h$.

## Exercise Set 2.2

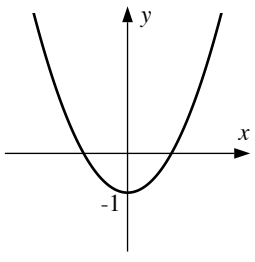
**1.** $f'(1) = 2.5$, $f'(3) = 0$, $f'(5) = -2.5$, $f'(6) = -1$.

**2.** $f'(4) < f'(0) < f'(2) < 0 < f'(-3)$.

**3.** **(a)** $f'(a)$ is the slope of the tangent line. **(b)** $f'(2) = m = 3$ **(c)** The same, $f'(2) = 3$.

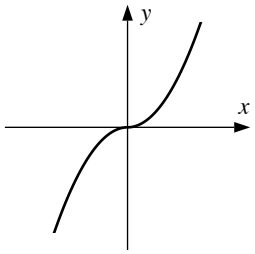
**4.** $f'(1) = \dfrac{2-(-1)}{1-(-1)} = \dfrac{3}{2}$

**5.**




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**6.**




**7.** $y-(-1)=5(x-3),\ y=5x-16$

**8.** $y-3=-4(x+2),\ y=-4x-5$

**9.** $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}\dfrac{2(x+h)^2-2x^2}{h}=\lim\limits_{h\to0}\dfrac{4xh+2h^2}{h}=4x$; $f'(1)=4$ so the tangent line is given by $y-2=4(x-1)$, $y=4x-2$.

**10.** $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}\dfrac{1/(x+h)^2-1/x^2}{h}=\lim\limits_{h\to0}\dfrac{x^2-(x+h)^2}{hx^2(x+h)^2}=\lim\limits_{h\to0}\dfrac{-2xh-h^2}{hx^2(x+h)^2}=\lim\limits_{h\to0}\dfrac{-2x-h}{x^2(x+h)^2}=-\dfrac{2}{x^3}$; $f'(-1)=2$ so the tangent line is given by $y-1=2(x+1)$, $y=2x+3$.

**11.** $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}\dfrac{(x+h)^3-x^3}{h}=\lim\limits_{h\to0}(3x^2+3xh+h^2)=3x^2$; $f'(0)=0$ so the tangent line is given by $y-0=0(x-0)$, $y=0$.

**12.** $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}\dfrac{[2(x+h)^3+1]-[2x^3+1]}{h}=\lim\limits_{h\to0}(6x^2+6xh+2h^2)=6x^2$; $f(-1)=2(-1)^3+1=-1$ and $f'(-1)=6$ so the tangent line is given by $y+1=6(x+1)$, $y=6x+5$.

**13.** $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}\dfrac{\sqrt{x+1+h}-\sqrt{x+1}}{h}=\lim\limits_{h\to0}\dfrac{\sqrt{x+1+h}-\sqrt{x+1}}{h}\dfrac{\sqrt{x+1+h}+\sqrt{x+1}}{\sqrt{x+1+h}+\sqrt{x+1}}=\lim\limits_{h\to0}\dfrac{h}{h(\sqrt{x+1+h}+\sqrt{x+1})}=\dfrac{1}{2\sqrt{x+1}}$; $f(8)=\sqrt{8+1}=3$ and $f'(8)=\dfrac{1}{6}$ so the tangent line is given by $y-3=\dfrac{1}{6}(x-8)$, $y=\dfrac{1}{6}x+\dfrac{5}{3}$.

**14.** $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}=\lim\limits_{h\to0}\dfrac{\sqrt{2x+2h+1}-\sqrt{2x+1}}{h}\dfrac{\sqrt{2x+2h+1}+\sqrt{2x+1}}{\sqrt{2x+2h+1}+\sqrt{2x+1}}=$

$=\lim\limits_{h\to0}\dfrac{2h}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})}=\lim\limits_{h\to0}\dfrac{2}{\sqrt{2x+2h+1}+\sqrt{2x+1}}=\dfrac{1}{\sqrt{2x+1}}$; $f(4)=\sqrt{2\cdot4+1}=\sqrt{9}=3$ and $f'(4)=1/3$ so the tangent line is given by $y-3=\dfrac{1}{3}(x-4)$, $y=\dfrac{1}{3}x+\dfrac{5}{3}$.

**15.** $f'(x)=\lim\limits_{\Delta x\to0}\dfrac{\dfrac{1}{x+\Delta x}-\dfrac{1}{x}}{\Delta x}=\lim\limits_{\Delta x\to0}\dfrac{\dfrac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}=\lim\limits_{\Delta x\to0}\dfrac{-\Delta x}{x\Delta x(x+\Delta x)}=\lim\limits_{\Delta x\to0}-\dfrac{1}{x(x+\Delta x)}=-\dfrac{1}{x^2}$.

**16.** $f'(x)=\lim\limits_{\Delta x\to0}\dfrac{\dfrac{1}{(x+\Delta x)+1}-\dfrac{1}{x+1}}{\Delta x}=\lim\limits_{\Delta x\to0}\dfrac{\dfrac{(x+1)-(x+\Delta x+1)}{(x+1)(x+\Delta x+1)}}{\Delta x}=\lim\limits_{\Delta x\to0}\dfrac{x+1-x-\Delta x-1}{\Delta x(x+1)(x+\Delta x+1)}=$

$=\lim_{\Delta x\to0}\frac{-\Delta x}{\Delta x(x+1)(x+\Delta x+1)}=\lim\limits_{\Delta x\to0}\dfrac{-1}{(x+1)(x+\Delta x+1)}=-\dfrac{1}{(x+1)^2}$.

**17.** $f'(x)=\lim\limits_{\Delta x\to0}\dfrac{(x+\Delta x)^2-(x+\Delta x)-(x^2-x)}{\Delta x}=\lim\limits_{\Delta x\to0}\dfrac{2x\Delta x+(\Delta x)^2-\Delta x}{\Delta x}=\lim\limits_{\Delta x\to0}(2x-1+\Delta x)=2x-1$.

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**18.** $f'(x) = \lim\limits_{\Delta x\to 0} \dfrac{(x+\Delta x)^4 - x^4}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4}{\Delta x} =$

$= \lim\limits_{\Delta x\to 0} (4x^3 + 6x^2\Delta x + 4x(\Delta x)^2 + (\Delta x)^3) = 4x^3.$

**19.** $f'(x) = \lim\limits_{\Delta x\to 0} \dfrac{\dfrac{1}{\sqrt{x+\Delta x}} - \dfrac{1}{\sqrt{x}}}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{\sqrt{x} - \sqrt{x+\Delta x}}{\Delta x\sqrt{x}\sqrt{x+\Delta x}} = \lim\limits_{\Delta x\to 0} \dfrac{x - (x+\Delta x)}{\Delta x\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})} =$

$= \lim\limits_{\Delta x\to 0} \dfrac{-1}{\sqrt{x}\sqrt{x+\Delta x}(\sqrt{x} + \sqrt{x+\Delta x})} = -\dfrac{1}{2x^{3/2}}.$

**20.** $f'(x) = \lim\limits_{\Delta x\to 0} \dfrac{\dfrac{1}{\sqrt{x+\Delta x-1}} - \dfrac{1}{\sqrt{x-1}}}{\Delta x} = \lim\limits_{\Delta x\to 0} \dfrac{\sqrt{x-1} - \sqrt{x+\Delta x-1}}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}} \dfrac{\sqrt{x-1} + \sqrt{x+\Delta x-1}}{\sqrt{x-1} + \sqrt{x+\Delta x-1}} =$

$= \lim\limits_{\Delta x\to 0} \dfrac{-\Delta x}{\Delta x\sqrt{x-1}\sqrt{x+\Delta x-1}(\sqrt{x-1} + \sqrt{x+\Delta x-1})} = \lim\limits_{\Delta x\to 0} \dfrac{-1}{\sqrt{x-1}\sqrt{x+\Delta x-1}(\sqrt{x-1} + \sqrt{x+\Delta x-1})} =$

$-\dfrac{1}{2(x-1)^{3/2}}.$

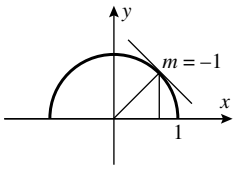
**21.** $f'(t) = \lim\limits_{h\to 0} \dfrac{f(t+h) - f(t)}{h} = \lim\limits_{h\to 0} \dfrac{[4(t+h)^2 + (t+h)] - [4t^2 + t]}{h} = \lim\limits_{h\to 0} \dfrac{4t^2 + 8th + 4h^2 + t + h - 4t^2 - t}{h} =$

$\lim\limits_{h\to 0} \dfrac{8th + 4h^2 + h}{h} = \lim\limits_{h\to 0} (8t + 4h + 1) = 8t + 1.$

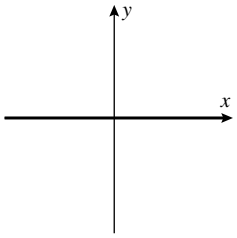
**22.** $\dfrac{dV}{dr} = \lim\limits_{h\to 0} \dfrac{\frac{4}{3}\pi(r+h)^3 - \frac{4}{3}\pi r^3}{h} = \lim\limits_{h\to 0} \dfrac{\frac{4}{3}\pi(r^3 + 3r^2h + 3rh^2 + h^3 - r^3)}{h} = \lim\limits_{h\to 0} \dfrac{4}{3}\pi(3r^2 + 3rh + h^2) = 4\pi r^2.$

**23.** **(a)** D **(b)** F **(c)** B **(d)** C **(e)** A **(f)** E

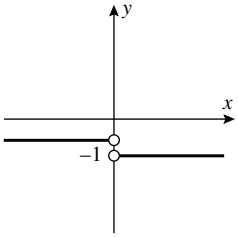
**24.** $f'(\sqrt{2}/2)$ is the slope of the tangent line to the unit circle at $(\sqrt{2}/2, \sqrt{2}/2)$. This line is perpendicular to the line $y = x$, so its slope is -1.



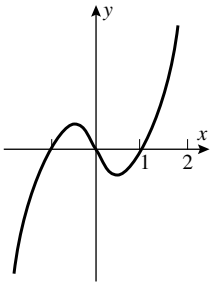

**25.** **(a)**



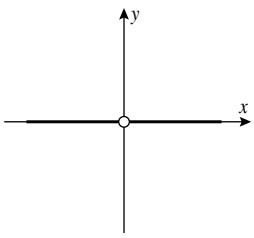

**(b)**



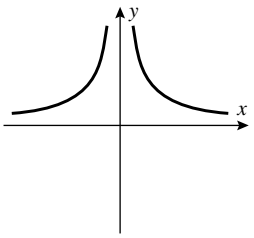

**(c)**



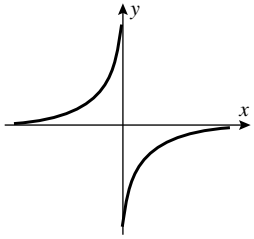

**26.** **(a)**




**(b)**




**(c)**




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**27.** False. If the tangent line is horizontal then $f'(a) = 0$.

**28.** True. $f'(-2)$ equals the slope of the tangent line.

**29.** False. E.g. $|x|$ is continuous but not differentiable at $x = 0$.

**30.** True. See Theorem 2.2.3.

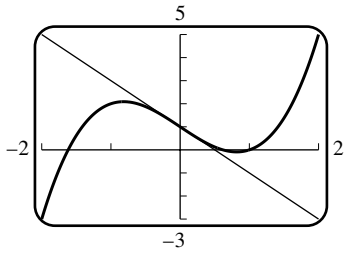
**31.** **(a)** $f(x) = \sqrt{x}$ and $a = 1$ **(b)** $f(x) = x^2$ and $a = 3$

**32.** **(a)** $f(x) = \cos x$ and $a = \pi$ **(b)** $f(x) = x^7$ and $a = 1$

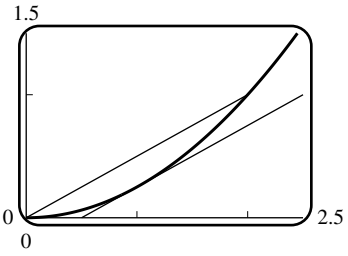
**33.** $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{(1-(x+h)^2)-(1-x^2)}{h} = \lim\limits_{h\to 0} \dfrac{-2xh-h^2}{h} = \lim\limits_{h\to 0}(-2x-h) = -2x$, and $\left.\dfrac{dy}{dx}\right|_{x=1} = -2$.

**34.** $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{\dfrac{x+2+h}{x+h} - \dfrac{x+2}{x}}{h} = \lim\limits_{h\to 0} \dfrac{x(x+2+h)-(x+2)(x+h)}{hx(x+h)} = \lim\limits_{h\to 0} \dfrac{-2}{x(x+h)} = \dfrac{-2}{x^2}$, and $\left.\dfrac{dy}{dx}\right|_{x=-2} = -\dfrac{1}{2}$.

**35.** $y = -2x + 1$




**36.**




**37.** **(b)**

| $w$ | 1.5 | 1.1 | 1.01 | 1.001 | 1.0001 | 1.00001 |
|---|---|---|---|---|---|---|
| $\dfrac{f(w)-f(1)}{w-1}$ | 1.6569 | 1.4355 | 1.3911 | 1.3868 | 1.3863 | 1.3863 |

| $w$ | 0.5 | 0.9 | 0.99 | 0.999 | 0.9999 | 0.99999 |
|---|---|---|---|---|---|---|
| $\dfrac{f(w)-f(1)}{w-1}$ | 1.1716 | 1.3393 | 1.3815 | 1.3858 | 1.3863 | 1.3863 |

**38.** **(b)**

| $w$ | $\frac{\pi}{4}+0.5$ | $\frac{\pi}{4}+0.1$ | $\frac{\pi}{4}+0.01$ | $\frac{\pi}{4}+0.001$ | $\frac{\pi}{4}+0.0001$ | $\frac{\pi}{4}+0.00001$ |
|---|---|---|---|---|---|---|
| $\dfrac{f(w)-f(\pi/4)}{w-\pi/4}$ | 0.50489 | 0.67060 | 0.70356 | 0.70675 | 0.70707 | 0.70710 |

| $w$ | $\frac{\pi}{4}-0.5$ | $\frac{\pi}{4}-0.1$ | $\frac{\pi}{4}-0.01$ | $\frac{\pi}{4}-0.001$ | $\frac{\pi}{4}-0.0001$ | $\frac{\pi}{4}-0.00001$ |
|---|---|---|---|---|---|---|
| $\dfrac{f(w)-f(\pi/4)}{w-\pi/4}$ | 0.85114 | 0.74126 | 0.71063 | 0.70746 | 0.70714 | 0.70711 |

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**39.** **(a)** $\dfrac{f(3)-f(1)}{3-1}=\dfrac{2.2-2.12}{2}=0.04;\ \dfrac{f(2)-f(1)}{2-1}=\dfrac{2.34-2.12}{1}=0.22;\ \dfrac{f(2)-f(0)}{2-0}=\dfrac{2.34-0.58}{2}=0.88.$

**(b)** The tangent line at $x=1$ appears to have slope about 0.8, so $\dfrac{f(2)-f(0)}{2-0}$ gives the best approximation and $\dfrac{f(3)-f(1)}{3-1}$ gives the worst.

**40.** **(a)** $f'(0.5)\approx\dfrac{f(1)-f(0)}{1-0}=\dfrac{2.12-0.58}{1}=1.54.$

**(b)** $f'(2.5)\approx\dfrac{f(3)-f(2)}{3-2}=\dfrac{2.2-2.34}{1}=-0.14.$

**41.** **(a)** dollars/ft

**(b)** $f'(x)$ is roughly the price per additional foot.

**(c)** If each additional foot costs extra money (this is to be expected) then $f'(x)$ remains positive.

**(d)** From the approximation $1000=f'(300)\approx\dfrac{f(301)-f(300)}{301-300}$ we see that $f(301)\approx f(300)+1000$, so the extra foot will cost around \$1000.

**42.** **(a)** $\dfrac{\text{gallons}}{\text{dollars/gallon}}=\text{gallons}^2/\text{dollar}$

**(b)** The increase in the amount of paint that would be sold for one extra dollar per gallon.

**(c)** It should be negative since an increase in the price of paint would decrease the amount of paint sold.

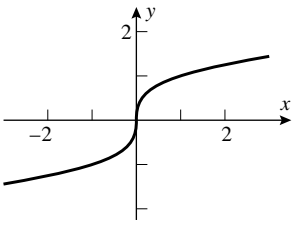
**(d)** From $-100=f'(10)\approx\dfrac{f(11)-f(10)}{11-10}$ we see that $f(11)\approx f(10)-100$, so an increase of one dollar per gallon would decrease the amount of paint sold by around 100 gallons.

**43.** **(a)** $F\approx 200$ lb, $dF/d\theta\approx 50$ **(b)** $\mu=(dF/d\theta)/F\approx 50/200=0.25$

**44.** The derivative at time $t=100$ of the velocity with respect to time is equal to the slope of the tangent line, which is approximately $m\approx\dfrac{12500-0}{140-40}=125$ ft/s$^2$. Thus the mass is approximately $M(100)\approx\dfrac{T}{dv/dt}=\dfrac{7680982\text{ lb}}{125\text{ ft/s}^2}\approx$ 61000 slugs.

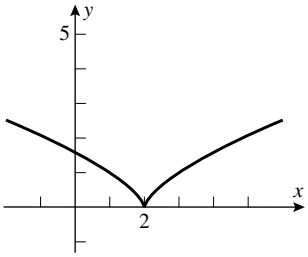
**45.** **(a)** $T\approx 115°$F, $dT/dt\approx -3.35°$F/min **(b)** $k=(dT/dt)/(T-T_0)\approx(-3.35)/(115-75)=-0.084$

**46.** **(a)** $\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\sqrt[3]{x}=0=f(0)$, so $f$ is continuous at $x=0$. $\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{\sqrt[3]{h}-0}{h}=\lim\limits_{h\to 0}\dfrac{1}{h^{2/3}}=+\infty$, so $f'(0)$ does not exist.

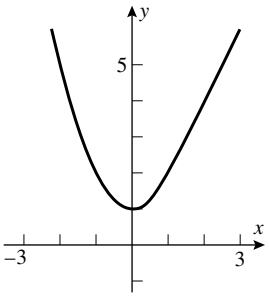



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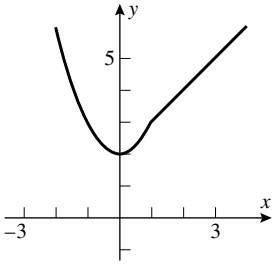
**(b)** $\lim_{x\to 2} f(x) = \lim_{x\to 2}(x-2)^{2/3} = 0 = f(2)$ so $f$ is continuous at $x = 2$. $\lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \dfrac{h^{2/3}-0}{h} = \lim_{h\to 0} \dfrac{1}{h^{1/3}}$ which does not exist so $f'(2)$ does not exist.



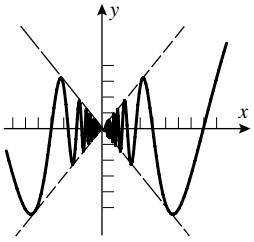

**47.** $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$, so $f$ is continuous at $x = 1$. $\lim_{h\to 0^-} \dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \dfrac{[(1+h)^2+1]-2}{h} = \lim_{h\to 0^-}(2+h) = 2$; $\lim_{h\to 0^+} \dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \dfrac{2(1+h)-2}{h} = \lim_{h\to 0^+} 2 = 2$, so $f'(1) = 2$.




**48.** $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$ so $f$ is continuous at $x = 1$. $\lim_{h\to 0^-} \dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \dfrac{[(1+h)^2+2]-3}{h} = \lim_{h\to 0^-}(2+h) = 2$; $\lim_{h\to 0^+} \dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \dfrac{[(1+h)+2]-3}{h} = \lim_{h\to 0^+} 1 = 1$, so $f'(1)$ does not exist.

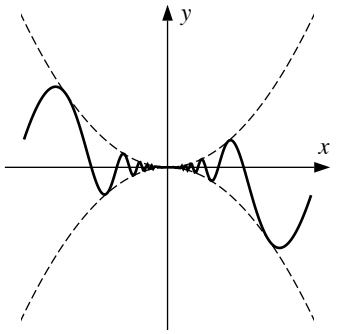



**49.** Since $-|x| \le x\sin(1/x) \le |x|$ it follows by the Squeezing Theorem (Theorem 1.6.4) that $\lim_{x\to 0} x\sin(1/x) = 0$. The derivative cannot exist: consider $\dfrac{f(x)-f(0)}{x} = \sin(1/x)$. This function oscillates between $-1$ and $+1$ and does not tend to any number as $x$ tends to zero.




**50.** For continuity, compare with $\pm x^2$ to establish that the limit is zero. The difference quotient is $x\sin(1/x)$ and (see Exercise 49) this has a limit of zero at the origin.

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**51.** Let $\epsilon = |f'(x_0)/2|$. Then there exists $\delta > 0$ such that if $0 < |x - x_0| < \delta$, then $\left|\dfrac{f(x) - f(x_0)}{x - x_0} - f'(x_0)\right| < \epsilon$. Since $f'(x_0) > 0$ and $\epsilon = f'(x_0)/2$ it follows that $\dfrac{f(x) - f(x_0)}{x - x_0} > \epsilon > 0$. If $x = x_1 < x_0$ then $f(x_1) < f(x_0)$ and if $x = x_2 > x_0$ then $f(x_2) > f(x_0)$.

**52.** $g'(x_1) = \lim\limits_{h\to 0} \dfrac{g(x_1 + h) - g(x_1)}{h} = \lim\limits_{h\to 0} \dfrac{f(m(x_1 + h) + b) - f(mx_1 + b)}{h} = m \lim\limits_{h\to 0} \dfrac{f(x_0 + mh) - f(x_0)}{mh} = mf'(x_0)$.

**53.** **(a)** Let $\epsilon = |m|/2$. Since $m \neq 0$, $\epsilon > 0$. Since $f(0) = f'(0) = 0$ we know there exists $\delta > 0$ such that $\left|\dfrac{f(0 + h) - f(0)}{h}\right| < \epsilon$ whenever $0 < |h| < \delta$. It follows that $|f(h)| < \frac{1}{2}|hm|$ for $0 < |h| < \delta$. Replace $h$ with $x$ to get the result.

**(b)** For $0 < |x| < \delta, |f(x)| < \frac{1}{2}|mx|$. Moreover $|mx| = |mx - f(x) + f(x)| \leq |f(x) - mx| + |f(x)|$, which yields $|f(x) - mx| \geq |mx| - |f(x)| > \frac{1}{2}|mx| > |f(x)|$, i.e. $|f(x) - mx| > |f(x)|$.

**(c)** If any straight line $y = mx + b$ is to approximate the curve $y = f(x)$ for small values of $x$, then $b = 0$ since $f(0) = 0$. The inequality $|f(x) - mx| > |f(x)|$ can also be interpreted as $|f(x) - mx| > |f(x) - 0|$, i.e. the line $y = 0$ is a better approximation than is $y = mx$.

**54.** Let $g(x) = f(x) - [f(x_0) + f'(x_0)(x - x_0)]$ and $h(x) = f(x) - [f(x_0) + m(x - x_0)]$; note that $h(x) - g(x) = (f'(x_0) - m)(x - x_0)$. If $m \neq f'(x_0)$ then there exists $\delta > 0$ such that if $0 < |x - x_0| < \delta$ then $\left|\dfrac{f(x) - f(x_0)}{x - x_0} - f'(x_0)\right| < \dfrac{1}{2}|f'(x_0) - m|$. Multiplying by $|x - x_0|$ gives $|g(x)| < \dfrac{1}{2}|h(x) - g(x)|$. Hence $2|g(x)| < |h(x) + (-g(x))| \leq |h(x)| + |g(x)|$, so $|g(x)| < |h(x)|$. In words, $f(x)$ is closer to $f(x_0) + f'(x_0)(x - x_0)$ than it is to $f(x_0) + m(x - x_0)$. So the tangent line gives a better approximation to $f(x)$ than any other line through $(x_0, f(x_0))$. Clearly any line not passing through that point gives an even worse approximation for $x$ near $x_0$, so the tangent line gives the best linear approximation.

**55.** See discussion around Definition 2.2.2.

**56.** See Theorem 2.2.3.

## Exercise Set 2.3

**1.** $28x^6$, by Theorems 2.3.2 and 2.3.4.

**2.** $-36x^{11}$, by Theorems 2.3.2 and 2.3.4.

**3.** $24x^7 + 2$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

**4.** $2x^3$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

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**5.** 0, by Theorem 2.3.1.

**6.** $\sqrt{2}$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

**7.** $-\dfrac{1}{3}(7x^6+2)$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

**8.** $\dfrac{2}{5}x$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

**9.** $-3x^{-4}-7x^{-8}$, by Theorems 2.3.3 and 2.3.5.

**10.** $\dfrac{1}{2\sqrt{x}}-\dfrac{1}{x^2}$, by Theorems 2.3.3 and 2.3.5.

**11.** $24x^{-9}+1/\sqrt{x}$, by Theorems 2.3.3, 2.3.4, and 2.3.5.

**12.** $-42x^{-7}-\dfrac{5}{2\sqrt{x}}$, by Theorems 2.3.3, 2.3.4, and 2.3.5.

**13.** $f'(x)=ex^{e-1}-\sqrt{10}\,x^{-1-\sqrt{10}}$, by Theorems 2.3.3 and 2.3.5.

**14.** $f'(x)=-\dfrac{2}{3}x^{-4/3}$, by Theorems 2.3.3 and 2.3.4.

**15.** $(3x^2+1)^2=9x^4+6x^2+1$, so $f'(x)=36x^3+12x$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

**16.** $3ax^2+2bx+c$, by Theorems 2.3.1, 2.3.2, 2.3.4, and 2.3.5.

**17.** $y'=10x-3$, $y'(1)=7$.

**18.** $y'=\dfrac{1}{2\sqrt{x}}-\dfrac{2}{x^2}$, $y'(1)=-3/2$.

**19.** $2t-1$, by Theorems 2.3.2 and 2.3.5.

**20.** $\dfrac{1}{3}-\dfrac{1}{3t^2}$, by Theorems 2.3.3, 2.3.4, and 2.3.5.

**21.** $dy/dx=1+2x+3x^2+4x^3+5x^4$, $dy/dx|_{x=1}=15$.

**22.** $\dfrac{dy}{dx}=\dfrac{-3}{x^4}-\dfrac{2}{x^3}-\dfrac{1}{x^2}+1+2x+3x^2$, $\left.\dfrac{dy}{dx}\right|_{x=1}=0$.

**23.** $y=(1-x^2)(1+x^2)(1+x^4)=(1-x^4)(1+x^4)=1-x^8$, $\dfrac{dy}{dx}=-8x^7$, $dy/dx|_{x=1}=-8$.

**24.** $dy/dx=24x^{23}+24x^{11}+24x^7+24x^5$, $dy/dx|_{x=1}=96$.

**25.** $f'(1)\approx\dfrac{f(1.01)-f(1)}{0.01}=\dfrac{-0.999699-(-1)}{0.01}=0.0301$, and by differentiation, $f'(1)=3(1)^2-3=0$.

**26.** $f'(1)\approx\dfrac{f(1.01)-f(1)}{0.01}\approx\dfrac{0.980296-1}{0.01}\approx-1.9704$, and by differentiation, $f'(1)=-2/1^3=-2$.

**27.** The estimate will depend on your graphing utility and on how far you zoom in. Since $f'(x)=1-\dfrac{1}{x^2}$, the exact value is $f'(1)=0$.

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**28.** The estimate will depend on your graphing utility and on how far you zoom in. Since $f'(x) = \dfrac{1}{2\sqrt{x}} + 2$, the exact value is $f'(1) = 5/2$.

**29.** $32t$, by Theorems 2.3.2 and 2.3.4.

**30.** $2\pi$, by Theorems 2.3.2 and 2.3.4.

**31.** $3\pi r^2$, by Theorems 2.3.2 and 2.3.4.

**32.** $-2\alpha^{-2} + 1$, by Theorems 2.3.2, 2.3.4, and 2.3.5.

**33.** True. By Theorems 2.3.4 and 2.3.5, $\dfrac{d}{dx}[f(x) - 8g(x)] = f'(x) - 8g'(x)$; substitute $x = 2$ to get the result.

**34.** True. $\dfrac{d}{dx}[ax^3 + bx^2 + cx + d] = 3ax^2 + 2bx + c$.

**35.** False. $\dfrac{d}{dx}[4f(x) + x^3]\Big|_{x=2} = (4f'(x) + 3x^2)\big|_{x=2} = 4f'(2) + 3 \cdot 2^2 = 32$

**36.** False. $f(x) = x^6 - x^3$ so $f'(x) = 6x^5 - 3x^2$ and $f''(x) = 30x^4 - 6x$, which is not equal to $2x(4x^3 - 1) = 8x^4 - 2x$.

**37.** **(a)** $\dfrac{dV}{dr} = 4\pi r^2$ **(b)** $\dfrac{dV}{dr}\Big|_{r=5} = 4\pi(5)^2 = 100\pi$

**38.** $\dfrac{d}{d\lambda}\left[\dfrac{\lambda\lambda_0 + \lambda^6}{2 - \lambda_0}\right] = \dfrac{1}{2 - \lambda_0}\dfrac{d}{d\lambda}(\lambda\lambda_0 + \lambda^6) = \dfrac{1}{2 - \lambda_0}(\lambda_0 + 6\lambda^5) = \dfrac{\lambda_0 + 6\lambda^5}{2 - \lambda_0}$.

**39.** $y - 2 = 5(x + 3)$, $y = 5x + 17$.

**40.** $y + 2 = -(x - 2)$, $y = -x$.

**41.** **(a)** $dy/dx = 21x^2 - 10x + 1$, $d^2y/dx^2 = 42x - 10$ **(b)** $dy/dx = 24x - 2$, $d^2y/dx^2 = 24$

**(c)** $dy/dx = -1/x^2$, $d^2y/dx^2 = 2/x^3$ **(d)** $dy/dx = 175x^4 - 48x^2 - 3$, $d^2y/dx^2 = 700x^3 - 96x$

**42.** **(a)** $y' = 28x^6 - 15x^2 + 2$, $y'' = 168x^5 - 30x$ **(b)** $y' = 3$, $y'' = 0$

**(c)** $y' = \dfrac{2}{5x^2}$, $y'' = -\dfrac{4}{5x^3}$ **(d)** $y' = 8x^3 + 9x^2 - 10$, $y'' = 24x^2 + 18x$

**43.** **(a)** $y' = -5x^{-6} + 5x^4$, $y'' = 30x^{-7} + 20x^3$, $y''' = -210x^{-8} + 60x^2$

**(b)** $y = x^{-1}$, $y' = -x^{-2}$, $y'' = 2x^{-3}$, $y''' = -6x^{-4}$

**(c)** $y' = 3ax^2 + b$, $y'' = 6ax$, $y''' = 6a$

**44.** **(a)** $dy/dx = 10x - 4$, $d^2y/dx^2 = 10$, $d^3y/dx^3 = 0$

**(b)** $dy/dx = -6x^{-3} - 4x^{-2} + 1$, $d^2y/dx^2 = 18x^{-4} + 8x^{-3}$, $d^3y/dx^3 = -72x^{-5} - 24x^{-4}$

**(c)** $dy/dx = 4ax^3 + 2bx$, $d^2y/dx^2 = 12ax^2 + 2b$, $d^3y/dx^3 = 24ax$

**45.** **(a)** $f'(x) = 6x$, $f''(x) = 6$, $f'''(x) = 0$, $f'''(2) = 0$

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**(b)** $\dfrac{dy}{dx} = 30x^4 - 8x$, $\dfrac{d^2y}{dx^2} = 120x^3 - 8$, $\left.\dfrac{d^2y}{dx^2}\right|_{x=1} = 112$

**(c)** $\dfrac{d}{dx}\left[x^{-3}\right] = -3x^{-4}$, $\dfrac{d^2}{dx^2}\left[x^{-3}\right] = 12x^{-5}$, $\dfrac{d^3}{dx^3}\left[x^{-3}\right] = -60x^{-6}$, $\dfrac{d^4}{dx^4}\left[x^{-3}\right] = 360x^{-7}$, $\left.\dfrac{d^4}{dx^4}\left[x^{-3}\right]\right|_{x=1} = 360$

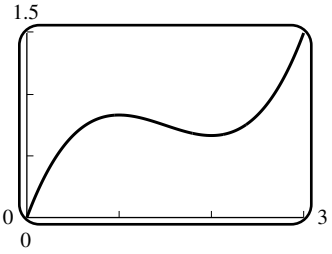
**46.** **(a)** $y' = 16x^3 + 6x^2$, $y'' = 48x^2 + 12x$, $y''' = 96x + 12$, $y'''(0) = 12$

**(b)** $y = 6x^{-4}$, $\dfrac{dy}{dx} = -24x^{-5}$, $\dfrac{d^2y}{dx^2} = 120x^{-6}$, $\dfrac{d^3y}{dx^3} = -720x^{-7}$, $\dfrac{d^4y}{dx^4} = 5040x^{-8}$, $\left.\dfrac{d^4y}{dx^4}\right|_{x=1} = 5040$

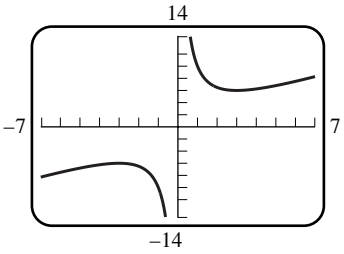
**47.** $y' = 3x^2 + 3$, $y'' = 6x$, and $y''' = 6$ so $y''' + xy'' - 2y' = 6 + x(6x) - 2(3x^2 + 3) = 6 + 6x^2 - 6x^2 - 6 = 0$.

**48.** $y = x^{-1}$, $y' = -x^{-2}$, $y'' = 2x^{-3}$ so $x^3y'' + x^2y' - xy = x^3(2x^{-3}) + x^2(-x^{-2}) - x(x^{-1}) = 2 - 1 - 1 = 0$.

**49.** The graph has a horizontal tangent at points where $\dfrac{dy}{dx} = 0$, but $\dfrac{dy}{dx} = x^2 - 3x + 2 = (x-1)(x-2) = 0$ if $x = 1, 2$. The corresponding values of $y$ are $5/6$ and $2/3$ so the tangent line is horizontal at $(1, 5/6)$ and $(2, 2/3)$.




**50.** Find where $f'(x) = 0$ : $f'(x) = 1 - 9/x^2 = 0$, $x^2 = 9$, $x = \pm 3$. The tangent line is horizontal at $(3, 6)$ and $(-3, -6)$.




**51.** The $y$-intercept is $-2$ so the point $(0, -2)$ is on the graph; $-2 = a(0)^2 + b(0) + c$, $c = -2$. The $x$-intercept is 1 so the point (1,0) is on the graph; $0 = a + b - 2$. The slope is $dy/dx = 2ax + b$; at $x = 0$ the slope is $b$ so $b = -1$, thus $a = 3$. The function is $y = 3x^2 - x - 2$.

**52.** Let $P(x_0, y_0)$ be the point where $y = x^2 + k$ is tangent to $y = 2x$. The slope of the curve is $\dfrac{dy}{dx} = 2x$ and the slope of the line is 2 thus at $P$, $2x_0 = 2$ so $x_0 = 1$. But $P$ is on the line, so $y_0 = 2x_0 = 2$. Because $P$ is also on the curve we get $y_0 = x_0^2 + k$ so $k = y_0 - x_0^2 = 2 - (1)^2 = 1$.

**53.** The points $(-1, 1)$ and $(2, 4)$ are on the secant line so its slope is $(4 - 1)/(2 + 1) = 1$. The slope of the tangent line to $y = x^2$ is $y' = 2x$ so $2x = 1$, $x = 1/2$.

**54.** The points $(1, 1)$ and $(4, 2)$ are on the secant line so its slope is $1/3$. The slope of the tangent line to $y = \sqrt{x}$ is $y' = 1/(2\sqrt{x})$ so $1/(2\sqrt{x}) = 1/3$, $2\sqrt{x} = 3$, $x = 9/4$.

**55.** $y' = -2x$, so at any point $(x_0, y_0)$ on $y = 1 - x^2$ the tangent line is $y - y_0 = -2x_0(x - x_0)$, or $y = -2x_0x + x_0^2 + 1$. The point $(2, 0)$ is to be on the line, so $0 = -4x_0 + x_0^2 + 1$, $x_0^2 - 4x_0 + 1 = 0$. Use the quadratic formula to get $x_0 = \dfrac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$. The points are $(2 + \sqrt{3}, -6 - 4\sqrt{3})$ and $(2 - \sqrt{3}, -6 + 4\sqrt{3})$.

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**56.** Let $P_1(x_1, ax_1^2)$ and $P_2(x_2, ax_2^2)$ be the points of tangency. $y' = 2ax$ so the tangent lines at $P_1$ and $P_2$ are $y - ax_1^2 = 2ax_1(x - x_1)$ and $y - ax_2^2 = 2ax_2(x - x_2)$. Solve for $x$ to get $x = \frac{1}{2}(x_1 + x_2)$ which is the $x$-coordinate of a point on the vertical line halfway between $P_1$ and $P_2$.

**57.** $y' = 3ax^2 + b$; the tangent line at $x = x_0$ is $y - y_0 = (3ax_0^2 + b)(x - x_0)$ where $y_0 = ax_0^3 + bx_0$. Solve with $y = ax^3 + bx$ to get

$$\begin{aligned}(ax^3 + bx) - (ax_0^3 + bx_0) &= (3ax_0^2 + b)(x - x_0)\\ ax^3 + bx - ax_0^3 - bx_0 &= 3ax_0^2x - 3ax_0^3 + bx - bx_0\\ x^3 - 3x_0^2x + 2x_0^3 &= 0\\ (x - x_0)(x^2 + xx_0 - 2x_0^2) &= 0\\ (x - x_0)^2(x + 2x_0) &= 0, \text{ so } x = -2x_0.\end{aligned}$$

**58.** Let $(x_0, y_0)$ be the point of tangency. Note that $y_0 = 1/x_0$. Since $y' = -1/x^2$, the tangent line has the equation $y - y_0 = (-1/x_0^2)(x - x_0)$, or $y - \dfrac{1}{x_0} = -\dfrac{1}{x_0^2}x + \dfrac{1}{x_0}$ or $y = -\dfrac{1}{x_0^2}x + \dfrac{2}{x_0}$, with intercepts at $\left(0, \dfrac{2}{x_0}\right) = (0, 2y_0)$ and $(2x_0, 0)$. The distance from the $y$-intercept to the point of tangency is $\sqrt{(x_0 - 0)^2 + (y_0 - 2y_0)^2}$, and the distance from the $x$-intercept to the point of tangency is $\sqrt{(x_0 - 2x_0)^2 + (y_0 - 0)^2}$ so that they are equal (and equal the distance $\sqrt{x_0^2 + y_0^2}$ from the point of tangency to the origin).

**59.** $y' = -\dfrac{1}{x^2}$; the tangent line at $x = x_0$ is $y - y_0 = -\dfrac{1}{x_0^2}(x - x_0)$, or $y = -\dfrac{x}{x_0^2} + \dfrac{2}{x_0}$. The tangent line crosses the $x$-axis at $2x_0$, the $y$-axis at $2/x_0$, so that the area of the triangle is $\dfrac{1}{2}(2/x_0)(2x_0) = 2$.

**60.** $f'(x) = 3ax^2 + 2bx + c$; there is a horizontal tangent where $f'(x) = 0$. Use the quadratic formula on $3ax^2 + 2bx + c = 0$ to get $x = (-b \pm \sqrt{b^2 - 3ac})/(3a)$ which gives two real solutions, one real solution, or none if

**(a)** $b^2 - 3ac > 0$ **(b)** $b^2 - 3ac = 0$ **(c)** $b^2 - 3ac < 0$

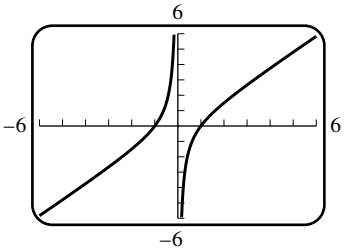
**61.** $F = GmMr^{-2}$, $\dfrac{dF}{dr} = -2GmMr^{-3} = -\dfrac{2GmM}{r^3}$

**62.** $dR/dT = 0.04124 - 3.558 \times 10^{-5}T$ which decreases as $T$ increases from 0 to 700. When $T = 0$, $dR/dT = 0.04124\,\Omega/°\text{C}$; when $T = 700$, $dR/dT = 0.01633\,\Omega/°\text{C}$. The resistance is most sensitive to temperature changes at $T = 0°\text{C}$, least sensitive at $T = 700°\text{C}$.

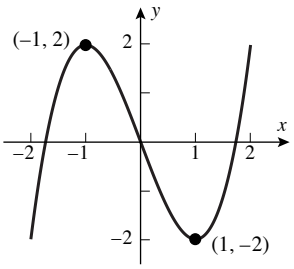
**63.** Since $dT/dx = (1/2)0.453x^{-1/2} = 0.2265/\sqrt{x}$, we have $dT/dx|_{x=9} = 0.2265/3 = 0.0755$ s/m.

**64.** Since $dr/dt = (2/3)1.93t^{-1/3} = 3.86/3\sqrt[3]{t}$, we have $dr/dt|_{t=0.602} = 3.86/3\sqrt[3]{0.602} \approx 1.52$ ($10^5$ km/day).

**65.** $f'(x) = 1 + 1/x^2 > 0$ for all $x \neq 0$

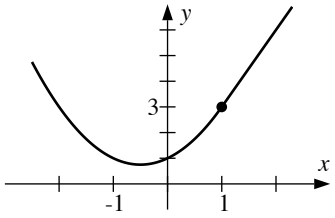



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**66.** $f'(x) = 3x^2 - 3 = 0$ when $x = \pm 1$; $f'(x) > 0$ for $-\infty < x < -1$ and $1 < x < +\infty$

**67.** $f$ is continuous at 1 because $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = f(1)$; also $\lim\limits_{x\to 1^-} f'(x) = \lim\limits_{x\to 1^-} (2x+1) = 3$ and $\lim\limits_{x\to 1^+} f'(x) = \lim\limits_{x\to 1^+} 3 = 3$ so $f$ is differentiable at 1, and the derivative equals 3.




**68.** $f$ is not continuous at $x = 9$ because $\lim\limits_{x\to 9^-} f(x) = -63$ and $\lim\limits_{x\to 9^+} f(x) = 3$. $f$ cannot be differentiable at $x = 9$, for if it were, then $f$ would also be continuous, which it is not.

**69.** $f$ is continuous at 1 because $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = f(1)$. Also, $\lim\limits_{x->1^-} \dfrac{f(x)-f(1)}{x-1}$ equals the derivative of $x^2$ at $x = 1$, namely $2x|_{x=1} = 2$, while $\lim\limits_{x->1^+} \dfrac{f(x)-f(1)}{x-1}$ equals the derivative of $\sqrt{x}$ at $x = 1$, namely $\left.\dfrac{1}{2\sqrt{x}}\right|_{x=1} = \dfrac{1}{2}$. Since these are not equal, $f$ is not differentiable at $x = 1$.

**70.** $f$ is continuous at 1/2 because $\lim\limits_{x\to 1/2^-} f(x) = \lim\limits_{x\to 1/2^+} f(x) = f(1/2)$; also $\lim\limits_{x\to 1/2^-} f'(x) = \lim\limits_{x\to 1/2^-} 3x^2 = 3/4$ and $\lim\limits_{x\to 1/2^+} f'(x) = \lim\limits_{x\to 1/2^+} 3x/2 = 3/4$ so $f'(1/2) = 3/4$, and $f$ is differentiable at $x = 1/2$.

**71.** **(a)** $f(x) = 3x - 2$ if $x \geq 2/3$, $f(x) = -3x + 2$ if $x < 2/3$ so $f$ is differentiable everywhere except perhaps at $2/3$. $f$ is continuous at $2/3$, also $\lim\limits_{x\to 2/3^-} f'(x) = \lim\limits_{x\to 2/3^-} (-3) = -3$ and $\lim\limits_{x\to 2/3^+} f'(x) = \lim\limits_{x\to 2/3^+} (3) = 3$ so $f$ is not differentiable at $x = 2/3$.

**(b)** $f(x) = x^2 - 4$ if $|x| \geq 2$, $f(x) = -x^2 + 4$ if $|x| < 2$ so $f$ is differentiable everywhere except perhaps at $\pm 2$. $f$ is continuous at $-2$ and 2, also $\lim\limits_{x\to 2^-} f'(x) = \lim\limits_{x\to 2^-} (-2x) = -4$ and $\lim\limits_{x\to 2^+} f'(x) = \lim\limits_{x\to 2^+} (2x) = 4$ so $f$ is not differentiable at $x = 2$. Similarly, $f$ is not differentiable at $x = -2$.

**72.** **(a)** $f'(x) = -(1)x^{-2}$, $f''(x) = (2\cdot 1)x^{-3}$, $f'''(x) = -(3\cdot 2\cdot 1)x^{-4}$; $f^{(n)}(x) = (-1)^n \dfrac{n(n-1)(n-2)\cdots 1}{x^{n+1}}$

**(b)** $f'(x) = -2x^{-3}$, $f''(x) = (3\cdot 2)x^{-4}$, $f'''(x) = -(4\cdot 3\cdot 2)x^{-5}$; $f^{(n)}(x) = (-1)^n \dfrac{(n+1)(n)(n-1)\cdots 2}{x^{n+2}}$

**73.** **(a)**

$$\frac{d^2}{dx^2}[cf(x)] = \frac{d}{dx}\left[\frac{d}{dx}[cf(x)]\right] = \frac{d}{dx}\left[c\frac{d}{dx}[f(x)]\right] = c\frac{d}{dx}\left[\frac{d}{dx}[f(x)]\right] = c\frac{d^2}{dx^2}[f(x)]$$

$$\frac{d^2}{dx^2}[f(x)+g(x)] = \frac{d}{dx}\left[\frac{d}{dx}[f(x)+g(x)]\right] = \frac{d}{dx}\left[\frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]\right] = \frac{d^2}{dx^2}[f(x)] + \frac{d^2}{dx^2}[g(x)]$$

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**(b)** Yes, by repeated application of the procedure illustrated in part **(a)**.

**74.** $\lim_{w\to 2}\dfrac{f'(w)-f'(2)}{w-2}=f''(2)$; $f'(x)=8x^7-2$, $f''(x)=56x^6$, so $f''(2)=56(2^6)=3584$.

**75.** **(a)** $f'(x)=nx^{n-1}$, $f''(x)=n(n-1)x^{n-2}$, $f'''(x)=n(n-1)(n-2)x^{n-3}$, …, $f^{(n)}(x)=n(n-1)(n-2)\cdots 1$

**(b)** From part **(a)**, $f^{(k)}(x)=k(k-1)(k-2)\cdots 1$ so $f^{(k+1)}(x)=0$ thus $f^{(n)}(x)=0$ if $n>k$.

**(c)** From parts **(a)** and **(b)**, $f^{(n)}(x)=a_n n(n-1)(n-2)\cdots 1$.

**76.** **(a)** If a function is differentiable at a point then it is continuous at that point, thus $f'$ is continuous on $(a,b)$ and consequently so is $f$.

**(b)** $f$ and all its derivatives up to $f^{(n-1)}(x)$ are continuous on $(a,b)$.

**77.** Let $g(x)=x^n, f(x)=(mx+b)^n$. Use Exercise 52 in Section 2.2, but with $f$ and $g$ permuted. If $x_0=mx_1+b$ then Exercise 52 says that $f$ is differentiable at $x_1$ and $f'(x_1)=mg'(x_0)$. Since $g'(x_0)=nx_0^{n-1}$, the result follows.

**78.** $f(x)=4x^2+12x+9$ so $f'(x)=8x+12=2\cdot 2(2x+3)$, as predicted by Exercise 75.

**79.** $f(x)=27x^3-27x^2+9x-1$ so $f'(x)=81x^2-54x+9=3\cdot 3(3x-1)^2$, as predicted by Exercise 75.

**80.** $f(x)=(x-1)^{-1}$ so $f'(x)=(-1)\cdot 1(x-1)^{-2}=-1/(x-1)^2$.

**81.** $f(x)=3(2x+1)^{-2}$ so $f'(x)=3(-2)2(2x+1)^{-3}=-12/(2x+1)^3$.

**82.** $f(x)=\dfrac{x+1-1}{x+1}=1-(x+1)^{-1}$, and $f'(x)=-(-1)(x+1)^{-2}=1/(x+1)^2$.

**83.** $f(x)=\dfrac{2x^2+4x+2+1}{(x+1)^2}=2+(x+1)^{-2}$, so $f'(x)=-2(x+1)^{-3}=-2/(x+1)^3$.

**84.** **(a)** If $n=0$ then $f(x)=x^0=1$ so $f'(x)=0$ by Theorem 2.3.1. This equals $0x^{0-1}$, so the Extended Power Rule holds in this case.

**(b)** $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{1/(x+h)^m-1/x^m}{h}=\lim_{h\to 0}\dfrac{x^m-(x+h)^m}{hx^m(x+h)^m}=$

$$=\lim_{h\to 0}\frac{(x+h)^m-x^m}{h}\cdot\lim_{h\to 0}\left(-\frac{1}{x^m(x+h)^m}\right)=\frac{d}{dx}\left(x^m\right)\cdot\left(-\frac{1}{x^{2m}}\right)=mx^{m-1}\cdot\left(-\frac{1}{x^{2m}}\right)=-mx^{-m-1}=nx^{n-1}.$$

## Exercise Set 2.4

**1.** **(a)** $f(x)=2x^2+x-1$, $f'(x)=4x+1$ **(b)** $f'(x)=(x+1)\cdot(2)+(2x-1)\cdot(1)=4x+1$

**2.** **(a)** $f(x)=3x^4+5x^2-2$, $f'(x)=12x^3+10x$ **(b)** $f'(x)=(3x^2-1)\cdot(2x)+(x^2+2)\cdot(6x)=12x^3+10x$

**3.** **(a)** $f(x)=x^4-1$, $f'(x)=4x^3$ **(b)** $f'(x)=(x^2+1)\cdot(2x)+(x^2-1)\cdot(2x)=4x^3$

**4.** **(a)** $f(x)=x^3+1$, $f'(x)=3x^2$ **(b)** $f'(x)=(x+1)(2x-1)+(x^2-x+1)\cdot(1)=3x^2$

**5.** $f'(x)=(3x^2+6)\dfrac{d}{dx}\left(2x-\dfrac{1}{4}\right)+\left(2x-\dfrac{1}{4}\right)\dfrac{d}{dx}(3x^2+6)=(3x^2+6)(2)+\left(2x-\dfrac{1}{4}\right)(6x)=18x^2-\dfrac{3}{2}x+12$

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**6.** $f'(x) = (2-x-3x^3)\dfrac{d}{dx}(7+x^5) + (7+x^5)\dfrac{d}{dx}(2-x-3x^3) = (2-x-3x^3)(5x^4) + (7+x^5)(-1-9x^2) = -24x^7 - 6x^5 + 10x^4 - 63x^2 - 7$

**7.** $f'(x) = (x^3+7x^2-8)\dfrac{d}{dx}(2x^{-3}+x^{-4}) + (2x^{-3}+x^{-4})\dfrac{d}{dx}(x^3+7x^2-8) = (x^3+7x^2-8)(-6x^{-4}-4x^{-5}) + +(2x^{-3}+x^{-4})(3x^2+14x) = -15x^{-2} - 14x^{-3} + 48x^{-4} + 32x^{-5}$

**8.** $f'(x) = (x^{-1}+x^{-2})\dfrac{d}{dx}(3x^3+27) + (3x^3+27)\dfrac{d}{dx}(x^{-1}+x^{-2}) = (x^{-1}+x^{-2})(9x^2) + (3x^3+27)(-x^{-2}-2x^{-3}) = 3 + 6x - 27x^{-2} - 54x^{-3}$

**9.** $f'(x) = 1\cdot(x^2+2x+4) + (x-2)\cdot(2x+2) = 3x^2$

**10.** $f'(x) = (2x+1)(x^2-x) + (x^2+x)(2x-1) = 4x^3 - 2x$

**11.** $f'(x) = \dfrac{(x^2+1)\frac{d}{dx}(3x+4) - (3x+4)\frac{d}{dx}(x^2+1)}{(x^2+1)^2} = \dfrac{(x^2+1)\cdot 3 - (3x+4)\cdot 2x}{(x^2+1)^2} = \dfrac{-3x^2-8x+3}{(x^2+1)^2}$

**12.** $f'(x) = \dfrac{(x^4+x+1)\frac{d}{dx}(x-2) - (x-2)\frac{d}{dx}(x^4+x+1)}{(x^4+x+1)^2} = \dfrac{(x^4+x+1)\cdot 1 - (x-2)\cdot(4x^3+1)}{(x^4+x+1)^2} = \dfrac{-3x^4+8x^3+3}{(x^4+x+1)^2}$

**13.** $f'(x) = \dfrac{(3x-4)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(3x-4)}{(3x-4)^2} = \dfrac{(3x-4)\cdot 2x - x^2\cdot 3}{(3x-4)^2} = \dfrac{3x^2-8x}{(3x-4)^2}$

**14.** $f'(x) = \dfrac{(3x-4)\frac{d}{dx}(2x^2+5) - (2x^2+5)\frac{d}{dx}(3x-4)}{(3x-4)^2} = \dfrac{(3x-4)\cdot 4x - (2x^2+5)\cdot 3}{(3x-4)^2} = \dfrac{6x^2-16x-15}{(3x-4)^2}$

**15.** $f(x) = \dfrac{2x^{3/2}+x-2x^{1/2}-1}{x+3}$, so

$$f'(x) = \frac{(x+3)\frac{d}{dx}(2x^{3/2}+x-2x^{1/2}-1) - (2x^{3/2}+x-2x^{1/2}-1)\frac{d}{dx}(x+3)}{(x+3)^2} =$$
$$= \frac{(x+3)\cdot(3x^{1/2}+1-x^{-1/2}) - (2x^{3/2}+x-2x^{1/2}-1)\cdot 1}{(x+3)^2} = \frac{x^{3/2}+10x^{1/2}+4-3x^{-1/2}}{(x+3)^2}$$

**16.** $f(x) = \dfrac{-2x^{3/2}-x+4x^{1/2}+2}{x^2+3x}$, so

$$f'(x) = \frac{(x^2+3x)\frac{d}{dx}(-2x^{3/2}-x+4x^{1/2}+2) - (-2x^{3/2}-x+4x^{1/2}+2)\frac{d}{dx}(x^2+3x)}{(x^2+3x)^2} =$$
$$= \frac{(x^2+3x)\cdot(-3x^{1/2}-1+2x^{-1/2}) - (-2x^{3/2}-x+4x^{1/2}+2)\cdot(2x+3)}{(x^2+3x)^2} =$$
$$= \frac{x^{5/2}+x^2-9x^{3/2}-4x-6x^{1/2}-6}{(x^2+3x)^2}$$

**17.** This could be computed by two applications of the product rule, but it's simpler to expand $f(x)$: $f(x) = 14x + 21 + 7x^{-1} + 2x^{-2} + 3x^{-3} + x^{-4}$, so $f'(x) = 14 - 7x^{-2} - 4x^{-3} - 9x^{-4} - 4x^{-5}$.

**18.** This could be computed by two applications of the product rule, but it's simpler to expand $f(x)$: $f(x) = -6x^7 - 4x^6 + 16x^5 - 3x^{-2} - 2x^{-3} + 8x^{-4}$, so $f'(x) = -42x^6 - 24x^5 + 80x^4 + 6x^{-3} + 6x^{-4} - 32x^{-5}$.

**19.** In general, $\dfrac{d}{dx}\left[g(x)^2\right] = 2g(x)g'(x)$ and $\dfrac{d}{dx}\left[g(x)^3\right] = \dfrac{d}{dx}\left[g(x)^2 g(x)\right] = g(x)^2 g'(x) + g(x)\dfrac{d}{dx}\left[g(x)^2\right] = g(x)^2 g'(x) + g(x)\cdot 2g(x)g'(x) = 3g(x)^2 g'(x)$.

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Letting $g(x) = x^7 + 2x - 3$, we have $f'(x) = 3(x^7 + 2x - 3)^2(7x^6 + 2)$.

**20.** In general, $\dfrac{d}{dx}\left[g(x)^2\right] = 2g(x)g'(x)$, so $\dfrac{d}{dx}\left[g(x)^4\right] = \dfrac{d}{dx}\left[\left(g(x)^2\right)^2\right] = 2g(x)^2 \cdot \dfrac{d}{dx}\left[g(x)^2\right] = 2g(x)^2 \cdot 2g(x)g'(x) = 4g(x)^3 g'(x)$

Letting $g(x) = x^2 + 1$, we have $f'(x) = 4(x^2 + 1)^3 \cdot 2x = 8x(x^2 + 1)^3$.

**21.** $\dfrac{dy}{dx} = \dfrac{(x+3)\cdot 2 - (2x-1)\cdot 1}{(x+3)^2} = \dfrac{7}{(x+3)^2}$, so $\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{7}{16}$.

**22.** $\dfrac{dy}{dx} = \dfrac{(x^2-5)\cdot 4 - (4x+1)\cdot(2x)}{(x^2-5)^2} = \dfrac{-4x^2 - 2x - 20}{(x^2-5)^2}$, so $\left.\dfrac{dy}{dx}\right|_{x=1} = -\dfrac{26}{16} = -\dfrac{13}{8}$.

**23.** $\dfrac{dy}{dx} = \left(\dfrac{3x+2}{x}\right)\dfrac{d}{dx}\left(x^{-5}+1\right) + \left(x^{-5}+1\right)\dfrac{d}{dx}\left(\dfrac{3x+2}{x}\right) = \left(\dfrac{3x+2}{x}\right)\left(-5x^{-6}\right) + \left(x^{-5}+1\right)\left[\dfrac{x(3) - (3x+2)(1)}{x^2}\right] = \left(\dfrac{3x+2}{x}\right)\left(-5x^{-6}\right) + \left(x^{-5}+1\right)\left(-\dfrac{2}{x^2}\right)$; so $\left.\dfrac{dy}{dx}\right|_{x=1} = 5(-5) + 2(-2) = -29$.

**24.** $\dfrac{dy}{dx} = (2x^7 - x^2)\dfrac{d}{dx}\left(\dfrac{x-1}{x+1}\right) + \left(\dfrac{x-1}{x+1}\right)\dfrac{d}{dx}(2x^7 - x^2) = (2x^7 - x^2)\left[\dfrac{(x+1)(1) - (x-1)(1)}{(x+1)^2}\right] +$

$+ \left(\dfrac{x-1}{x+1}\right)(14x^6 - 2x) = (2x^7 - x^2)\cdot\dfrac{2}{(x+1)^2} + \left(\dfrac{x-1}{x+1}\right)(14x^6 - 2x)$; so $\left.\dfrac{dy}{dx}\right|_{x=1} = (2-1)\dfrac{2}{4} + 0(14-2) = \dfrac{1}{2}$.

**25.** $f'(x) = \dfrac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$, so $f'(1) = 0$.

**26.** $f'(x) = \dfrac{(x^2+1)\cdot 2x - (x^2-1)\cdot 2x}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$, so $f'(1) = 1$.

**27.** **(a)** $g'(x) = \sqrt{x}f'(x) + \dfrac{1}{2\sqrt{x}}f(x)$, $g'(4) = (2)(-5) + \dfrac{1}{4}(3) = -37/4$.

**(b)** $g'(x) = \dfrac{xf'(x) - f(x)}{x^2}$, $g'(4) = \dfrac{(4)(-5) - 3}{16} = -23/16$.

**28.** **(a)** $g'(x) = 6x - 5f'(x)$, $g'(3) = 6(3) - 5(4) = -2$.

**(b)** $g'(x) = \dfrac{2f(x) - (2x+1)f'(x)}{f^2(x)}$, $g'(3) = \dfrac{2(-2) - 7(4)}{(-2)^2} = -8$.

**29.** **(a)** $F'(x) = 5f'(x) + 2g'(x), F'(2) = 5(4) + 2(-5) = 10$.

**(b)** $F'(x) = f'(x) - 3g'(x), F'(2) = 4 - 3(-5) = 19$.

**(c)** $F'(x) = f(x)g'(x) + g(x)f'(x), F'(2) = (-1)(-5) + (1)(4) = 9$.

**(d)** $F'(x) = [g(x)f'(x) - f(x)g'(x)]/g^2(x), F'(2) = [(1)(4) - (-1)(-5)]/(1)^2 = -1$.

**30.** **(a)** $F'(x) = 6f'(x) - 5g'(x), F'(\pi) = 6(-1) - 5(2) = -16$.

**(b)** $F'(x) = f(x) + g(x) + x(f'(x) + g'(x))$, $F'(\pi) = 10 - 3 + \pi(-1+2) = 7 + \pi$.

**(c)** $F'(x) = 2f(x)g'(x) + 2f'(x)g(x) = 2(20) + 2(3) = 46$.

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**(d)** $F'(x) = \dfrac{(4+g(x))f'(x) - f(x)g'(x)}{(4+g(x))^2} = \dfrac{(4-3)(-1) - 10(2)}{(4-3)^2} = -21.$

**31.** $\dfrac{dy}{dx} = \dfrac{2x(x+2) - (x^2-1)}{(x+2)^2}$, $\dfrac{dy}{dx} = 0$ if $x^2 + 4x + 1 = 0$. By the quadratic formula, $x = \dfrac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$. The tangent line is horizontal at $x = -2 \pm \sqrt{3}$.

**32.** $\dfrac{dy}{dx} = \dfrac{2x(x-1) - (x^2+1)}{(x-1)^2} = \dfrac{x^2 - 2x - 1}{(x-1)^2}$. The tangent line is horizontal when it has slope 0, i.e. $x^2 - 2x - 1 = 0$ which, by the quadratic formula, has solutions $x = \dfrac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$, the tangent line is horizontal when $x = 1 \pm \sqrt{2}$.

**33.** The tangent line is parallel to the line $y = x$ when it has slope 1. $\dfrac{dy}{dx} = \dfrac{2x(x+1) - (x^2+1)}{(x+1)^2} = \dfrac{x^2 + 2x - 1}{(x+1)^2} = 1$ if $x^2 + 2x - 1 = (x+1)^2$, which reduces to $-1 = +1$, impossible. Thus the tangent line is never parallel to the line $y = x$.

**34.** The tangent line is perpendicular to the line $y = x$ when the tangent line has slope $-1$. $y = \dfrac{x+2+1}{x+2} = 1 + \dfrac{1}{x+2}$, hence $\dfrac{dy}{dx} = -\dfrac{1}{(x+2)^2} = -1$ when $(x+2)^2 = 1$, $x^2 + 4x + 3 = 0$, $(x+1)(x+3) = 0$, $x = -1, -3$. Thus the tangent line is perpendicular to the line $y = x$ at the points $(-1, 2), (-3, 0)$.

**35.** Fix $x_0$. The slope of the tangent line to the curve $y = \dfrac{1}{x+4}$ at the point $(x_0, 1/(x_0+4))$ is given by $\dfrac{dy}{dx} = \dfrac{-1}{(x+4)^2}\Big|_{x=x_0} = \dfrac{-1}{(x_0+4)^2}$. The tangent line to the curve at $(x_0, y_0)$ thus has the equation $y - y_0 = \dfrac{-(x-x_0)}{(x_0+4)^2}$, and this line passes through the origin if its constant term $y_0 - x_0\dfrac{-1}{(x_0+4)^2}$ is zero. Then $\dfrac{1}{x_0+4} = \dfrac{-x_0}{(x_0+4)^2}$, so $x_0 + 4 = -x_0$, $x_0 = -2$.

**36.** $y = \dfrac{2x+5}{x+2} = \dfrac{2x+4+1}{x+2} = 2 + \dfrac{1}{x+2}$, and hence $\dfrac{dy}{dx} = \dfrac{-1}{(x+2)^2}$, thus the tangent line at the point $(x_0, y_0)$ is given by $y - y_0 = \dfrac{-1}{(x_0+2)^2}(x - x_0)$ , where $y_0 = 2 + \dfrac{1}{x_0+2}$. If this line is to pass through $(0, 2)$, then $2 - y_0 = \dfrac{-1}{(x_0+2)^2}(-x_0)$, $\dfrac{-1}{x_0+2} = \dfrac{x_0}{(x_0+2)^2}$, $-x_0 - 2 = x_0$, so $x_0 = -1$.

**37.** **(a)** Their tangent lines at the intersection point must be perpendicular.

**(b)** They intersect when $\dfrac{1}{x} = \dfrac{1}{2-x}$, $x = 2 - x$, $x = 1$, $y = 1$. The first curve has derivative $y = -\dfrac{1}{x^2}$, so the slope when $x = 1$ is $-1$. Second curve has derivative $y = \dfrac{1}{(2-x)^2}$ so the slope when $x = 1$ is 1. Since the two slopes are negative reciprocals of each other, the tangent lines are perpendicular at the point $(1, 1)$.

**38.** The curves intersect when $a/(x-1) = x^2 - 2x + 1$, or $(x-1)^3 = a, x = 1 + a^{1/3}$. They are perpendicular when their slopes are negative reciprocals of each other, i.e. $\dfrac{-a}{(x-1)^2}(2x-2) = -1$, which has the solution $x = 2a + 1$. Solve $x = 1 + a^{1/3} = 2a + 1, 2a^{2/3} = 1, a = 2^{-3/2}$. Thus the curves intersect and are perpendicular at the point $(2a+1, 1/2)$ provided $a = 2^{-3/2}$.

**39.** $F'(x) = xf'(x) + f(x)$, $F''(x) = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$.

**40.** **(a)** $F'''(x) = xf'''(x) + 3f''(x)$.

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**(b)** Assume that $F^{(n)}(x) = xf^{(n)}(x) + nf^{(n-1)}(x)$ for some $n$ (for instance $n = 3$, as in part **(a)**). Then $F^{(n+1)}(x) = xf^{(n+1)}(x) + (1+n)f^{(n)}(x) = xf^{(n+1)}(x) + (n+1)f^{(n)}(x)$, which is an inductive proof.

**41.** $R'(p) = p \cdot f'(p) + f(p) \cdot 1 = f(p) + pf'(p)$, so $R'(120) = 9000 + 120 \cdot (-60) = 1800$. Increasing the price by a small amount $\Delta p$ dollars would increase the revenue by about $1800\Delta p$ dollars.

**42.** $R'(p) = p \cdot f'(p) + f(p) \cdot 1 = f(p) + pf'(p)$, so $R'(120) = 9000 + 120 \cdot (-80) = -600$. Increasing the price by a small amount $\Delta p$ dollars would decrease the revenue by about $600\Delta p$ dollars.

**43.** $f(x) = \dfrac{1}{x^n}$ so $f'(x) = \dfrac{x^n \cdot (0) - 1 \cdot (nx^{n-1})}{x^{2n}} = -\dfrac{n}{x^{n+1}} = -nx^{-n-1}$.

## Exercise Set 2.5

**1.** $f'(x) = -4\sin x + 2\cos x$

**2.** $f'(x) = \dfrac{-10}{x^3} + \cos x$

**3.** $f'(x) = 4x^2 \sin x - 8x \cos x$

**4.** $f'(x) = 4 \sin x \cos x$

**5.** $f'(x) = \dfrac{\sin x(5 + \sin x) - \cos x(5 - \cos x)}{(5 + \sin x)^2} = \dfrac{1 + 5(\sin x - \cos x)}{(5 + \sin x)^2}$

**6.** $f'(x) = \dfrac{(x^2 + \sin x)\cos x - \sin x(2x + \cos x)}{(x^2 + \sin x)^2} = \dfrac{x^2 \cos x - 2x \sin x}{(x^2 + \sin x)^2}$

**7.** $f'(x) = \sec x \tan x - \sqrt{2}\sec^2 x$

**8.** $f'(x) = (x^2 + 1)\sec x \tan x + (\sec x)(2x) = (x^2 + 1)\sec x \tan x + 2x \sec x$

**9.** $f'(x) = -4 \csc x \cot x + \csc^2 x$

**10.** $f'(x) = -\sin x - \csc x + x \csc x \cot x$

**11.** $f'(x) = \sec x(\sec^2 x) + (\tan x)(\sec x \tan x) = \sec^3 x + \sec x \tan^2 x$

**12.** $f'(x) = (\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x) = -\csc^3 x - \csc x \cot^2 x$

**13.** $f'(x) = \dfrac{(1 + \csc x)(-\csc^2 x) - \cot x(0 - \csc x \cot x)}{(1 + \csc x)^2} = \dfrac{\csc x(-\csc x - \csc^2 x + \cot^2 x)}{(1 + \csc x)^2}$, but $1 + \cot^2 x = \csc^2 x$ (identity), thus $\cot^2 x - \csc^2 x = -1$, so $f'(x) = \dfrac{\csc x(-\csc x - 1)}{(1 + \csc x)^2} = -\dfrac{\csc x}{1 + \csc x}$.

**14.** $f'(x) = \dfrac{(1 + \tan x)(\sec x \tan x) - (\sec x)(\sec^2 x)}{(1 + \tan x)^2} = \dfrac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1 + \tan x)^2} =$

$= \dfrac{\sec x(\tan x + \tan^2 x - \sec^2 x)}{(1 + \tan x)^2} = \dfrac{\sec x(\tan x - 1)}{(1 + \tan x)^2}$

**15.** $f(x) = \sin^2 x + \cos^2 x = 1$ (identity), so $f'(x) = 0$.

**16.** $f'(x) = 2 \sec x \tan x \sec x - 2 \tan x \sec^2 x = \dfrac{2 \sin x}{\cos^3 x} - 2\dfrac{\sin x}{\cos^3 x} = 0$; also, $f(x) = \sec^2 x - \tan^2 x = 1$ (identity), so $f'(x) = 0$.

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**17.** $f(x) = \dfrac{\tan x}{1 + x\tan x}$ (because $\sin x \sec x = (\sin x)(1/\cos x) = \tan x$), so

$f'(x) = \dfrac{(1 + x\tan x)(\sec^2 x) - \tan x[x(\sec^2 x) + (\tan x)(1)]}{(1 + x\tan x)^2} = \dfrac{\sec^2 x - \tan^2 x}{(1 + x\tan x)^2} = \dfrac{1}{(1 + x\tan x)^2}$ (because $\sec^2 x - \tan^2 x = 1$).

**18.** $f(x) = \dfrac{(x^2 + 1)\cot x}{3 - \cot x}$ (because $\cos x \csc x = (\cos x)(1/\sin x) = \cot x$), so

$f'(x) = \dfrac{(3 - \cot x)[2x\cot x - (x^2 + 1)\csc^2 x] - (x^2 + 1)\cot x \csc^2 x}{(3 - \cot x)^2} = \dfrac{6x\cot x - 2x\cot^2 x - 3(x^2 + 1)\csc^2 x}{(3 - \cot x)^2}$.

**19.** $dy/dx = -x\sin x + \cos x$, $d^2y/dx^2 = -x\cos x - \sin x - \sin x = -x\cos x - 2\sin x$

**20.** $dy/dx = -\csc x \cot x$, $d^2y/dx^2 = -[(\csc x)(-\csc^2 x) + (\cot x)(-\csc x \cot x)] = \csc^3 x + \csc x \cot^2 x$

**21.** $dy/dx = x(\cos x) + (\sin x)(1) - 3(-\sin x) = x\cos x + 4\sin x$,
$d^2y/dx^2 = x(-\sin x) + (\cos x)(1) + 4\cos x = -x\sin x + 5\cos x$

**22.** $dy/dx = x^2(-\sin x) + (\cos x)(2x) + 4\cos x = -x^2\sin x + 2x\cos x + 4\cos x$,
$d^2y/dx^2 = -[x^2(\cos x) + (\sin x)(2x)] + 2[x(-\sin x) + \cos x] - 4\sin x = (2 - x^2)\cos x - 4(x + 1)\sin x$

**23.** $dy/dx = (\sin x)(-\sin x) + (\cos x)(\cos x) = \cos^2 x - \sin^2 x$,
$d^2y/dx^2 = (\cos x)(-\sin x) + (\cos x)(-\sin x) - [(\sin x)(\cos x) + (\sin x)(\cos x)] = -4\sin x \cos x$

**24.** $dy/dx = \sec^2 x$, $d^2y/dx^2 = 2\sec^2 x \tan x$

**25.** Let $f(x) = \tan x$, then $f'(x) = \sec^2 x$.

**(a)** $f(0) = 0$ and $f'(0) = 1$, so $y - 0 = (1)(x - 0)$, $y = x$.

**(b)** $f\left(\dfrac{\pi}{4}\right) = 1$ and $f'\left(\dfrac{\pi}{4}\right) = 2$, so $y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$, $y = 2x - \dfrac{\pi}{2} + 1$.

**(c)** $f\left(-\dfrac{\pi}{4}\right) = -1$ and $f'\left(-\dfrac{\pi}{4}\right) = 2$, so $y + 1 = 2\left(x + \dfrac{\pi}{4}\right)$, $y = 2x + \dfrac{\pi}{2} - 1$.

**26.** Let $f(x) = \sin x$, then $f'(x) = \cos x$.

**(a)** $f(0) = 0$ and $f'(0) = 1$, so $y - 0 = (1)(x - 0)$, $y = x$.

**(b)** $f(\pi) = 0$ and $f'(\pi) = -1$, so $y - 0 = (-1)(x - \pi)$, $y = -x + \pi$.

**(c)** $f\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$ and $f'\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$, so $y - \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\left(x - \dfrac{\pi}{4}\right)$, $y = \dfrac{1}{\sqrt{2}}x - \dfrac{\pi}{4\sqrt{2}} + \dfrac{1}{\sqrt{2}}$.

**27.** **(a)** If $y = x\sin x$ then $y' = \sin x + x\cos x$ and $y'' = 2\cos x - x\sin x$ so $y'' + y = 2\cos x$.

**(b)** Differentiate the result of part **(a)** twice more to get $y^{(4)} + y'' = -2\cos x$.

**28.** **(a)** If $y = \cos x$ then $y' = -\sin x$ and $y'' = -\cos x$, so $y'' + y = (-\cos x) + (\cos x) = 0$; if $y = \sin x$ then $y' = \cos x$ and $y'' = -\sin x$ so $y'' + y = (-\sin x) + (\sin x) = 0$.

**(b)** $y' = A\cos x - B\sin x$, $y'' = -A\sin x - B\cos x$, so $y'' + y = (-A\sin x - B\cos x) + (A\sin x + B\cos x) = 0$.

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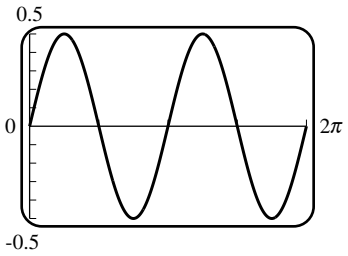
**29.** **(a)** $f'(x) = \cos x = 0$ at $x = \pm\pi/2, \pm 3\pi/2$.

**(b)** $f'(x) = 1 - \sin x = 0$ at $x = -3\pi/2, \pi/2$.

**(c)** $f'(x) = \sec^2 x \geq 1$ always, so no horizontal tangent line.

**(d)** $f'(x) = \sec x \tan x = 0$ when $\sin x = 0$, $x = \pm 2\pi, \pm\pi, 0$.

**30.** **(a)**




**(b)** $y = \sin x \cos x = (1/2)\sin 2x$ and $y' = \cos 2x$. So $y' = 0$ when $2x = (2n+1)\pi/2$ for $n = 0, 1, 2, 3$ or $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

**31.** $x = 10\sin\theta$, $dx/d\theta = 10\cos\theta$; if $\theta = 60°$, then $dx/d\theta = 10(1/2) = 5$ ft/rad $= \pi/36$ ft/deg $\approx 0.087$ ft/deg.

**32.** $s = 3800\csc\theta, ds/d\theta = -3800\csc\theta\cot\theta$; if $\theta = 30°$, then $ds/d\theta = -3800(2)(\sqrt{3}) = -7600\sqrt{3}$ ft/rad $= -380\sqrt{3}\pi/9$ ft/deg $\approx -230$ ft/deg.

**33.** $D = 50\tan\theta, dD/d\theta = 50\sec^2\theta$; if $\theta = 45°$, then $dD/d\theta = 50(\sqrt{2})^2 = 100$ m/rad $= 5\pi/9$ m/deg $\approx 1.75$ m/deg.

**34.** **(a)** From the right triangle shown, $\sin\theta = r/(r+h)$ so $r + h = r\csc\theta$, $h = r(\csc\theta - 1)$.

**(b)** $dh/d\theta = -r\csc\theta\cot\theta$; if $\theta = 30°$, then $dh/d\theta = -6378(2)(\sqrt{3}) \approx -22{,}094$ km/rad $\approx -386$ km/deg.

**35.** False. $g'(x) = f(x)\cos x + f'(x)\sin x$

**36.** True, if $f(x)$ is continuous at $x = 0$, then $g'(0) = \lim\limits_{h\to 0}\dfrac{g(h) - g(0)}{h} = \lim\limits_{h\to 0}\dfrac{f(h)\sin h}{h} = \lim\limits_{h\to 0} f(h) \cdot \lim\limits_{h\to 0}\dfrac{\sin h}{h} = f(0)\cdot 1 = f(0)$.

**37.** True. $f(x) = \dfrac{\sin x}{\cos x} = \tan x$, so $f'(x) = \sec^2 x$.

**38.** False. $g'(x) = f(x)\cdot\dfrac{d}{dx}(\sec x) + f'(x)\sec x = f(x)\sec x\tan x + f'(x)\sec x$, so $g'(0) = f(0)\sec 0\tan 0 + f'(0)\sec 0 = 8\cdot 1\cdot 0 + (-2)\cdot 1 = -2$. The second equality given in the problem is wrong: $\lim\limits_{h\to 0}\dfrac{f(h)\sec h - f(0)}{h} = -2$ but $\lim\limits_{h\to 0}\dfrac{8(\sec h - 1)}{h} = 0$.

**39.** $\dfrac{d^4}{dx^4}\sin x = \sin x$, so $\dfrac{d^{4k}}{dx^{4k}}\sin x = \sin x$; $\dfrac{d^{87}}{dx^{87}}\sin x = \dfrac{d^3}{dx^3}\dfrac{d^{4\cdot 21}}{dx^{4\cdot 21}}\sin x = \dfrac{d^3}{dx^3}\sin x = -\cos x$.

**40.** $\dfrac{d^{100}}{dx^{100}}\cos x = \dfrac{d^{4k}}{dx^{4k}}\cos x = \cos x$.

**41.** $f'(x) = -\sin x$, $f''(x) = -\cos x$, $f'''(x) = \sin x$, and $f^{(4)}(x) = \cos x$ with higher order derivatives repeating this pattern, so $f^{(n)}(x) = \sin x$ for $n = 3, 7, 11, \ldots$

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**42.** $f(x) = \sin x, f'(x) = \cos x, f''(x) = -\sin x, f'''(x) = -\cos x, f^{(4)}(x) = \sin x$, and the right-hand sides continue with a period of 4, so that $f^{(n)}(x) = \sin x$ when $n = 4k$ for some $k$.

**43.** **(a)** all $x$ **(b)** all $x$ **(c)** $x \neq \pi/2 + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

**(d)** $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(e)** $x \neq \pi/2 + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(f)** $x \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

**(g)** $x \neq (2n+1)\pi$, $n = 0, \pm 1, \pm 2, \ldots$ **(h)** $x \neq n\pi/2$, $n = 0, \pm 1, \pm 2, \ldots$ **(i)** all $x$

**44.** **(a)** $\dfrac{d}{dx}[\cos x] = \lim\limits_{h\to 0} \dfrac{\cos(x+h) - \cos x}{h} = \lim\limits_{h\to 0} \dfrac{\cos x \cos h - \sin x \sin h - \cos x}{h} =$

$= \lim\limits_{h\to 0} \left[\cos x \left(\dfrac{\cos h - 1}{h}\right) - \sin x \left(\dfrac{\sin h}{h}\right)\right] = (\cos x)(0) - (\sin x)(1) = -\sin x.$

**(b)** $\dfrac{d}{dx}[\cot x] = \dfrac{d}{dx}\left[\dfrac{\cos x}{\sin x}\right] = \dfrac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x} = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \dfrac{-1}{\sin^2 x} = -\csc^2 x.$

**(c)** $\dfrac{d}{dx}[\sec x] = \dfrac{d}{dx}\left[\dfrac{1}{\cos x}\right] = \dfrac{0 \cdot \cos x - (1)(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \sec x \tan x.$

**(d)** $\dfrac{d}{dx}[\csc x] = \dfrac{d}{dx}\left[\dfrac{1}{\sin x}\right] = \dfrac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x} = -\dfrac{\cos x}{\sin^2 x} = -\csc x \cot x.$

**45.** $\dfrac{d}{dx}\sin x = \lim\limits_{w\to x} \dfrac{\sin w - \sin x}{w - x} = \lim\limits_{w\to x} \dfrac{2\sin\frac{w-x}{2}\cos\frac{w+x}{2}}{w - x} = \lim\limits_{w\to x} \dfrac{\sin\frac{w-x}{2}}{\frac{w-x}{2}} \cos\dfrac{w+x}{2} = 1 \cdot \cos x = \cos x.$

**46.** $\dfrac{d}{dx}[\cos x] = \lim\limits_{w\to x} \dfrac{\cos w - \cos x}{w - x} = \lim\limits_{w\to x} \dfrac{-2\sin(\frac{w-x}{2})\sin(\frac{w+x}{2})}{w - x} = -\lim\limits_{w\to x} \sin\left(\dfrac{w+x}{2}\right) \lim\limits_{w\to x} \dfrac{\sin(\frac{w-x}{2})}{\frac{w-x}{2}} = -\sin x.$

**47.** **(a)** $\lim\limits_{h\to 0} \dfrac{\tan h}{h} = \lim\limits_{h\to 0} \dfrac{\left(\frac{\sin h}{\cos h}\right)}{h} = \lim\limits_{h\to 0} \dfrac{\left(\frac{\sin h}{h}\right)}{\cos h} = \dfrac{1}{1} = 1.$

**(b)** $\dfrac{d}{dx}[\tan x] = \lim\limits_{h\to 0} \dfrac{\tan(x+h) - \tan x}{h} = \lim\limits_{h\to 0} \dfrac{\frac{\tan x + \tan h}{1 - \tan x \tan h} - \tan x}{h} = \lim\limits_{h\to 0} \dfrac{\tan x + \tan h - \tan x + \tan^2 x \tan h}{h(1 - \tan x \tan h)} =$

$\lim\limits_{h\to 0} \dfrac{\tan h(1 + \tan^2 x)}{h(1 - \tan x \tan h)} = \lim\limits_{h\to 0} \dfrac{\tan h \sec^2 x}{h(1 - \tan x \tan h)} = \sec^2 x \lim\limits_{h\to 0} \dfrac{\frac{\tan h}{h}}{1 - \tan x \tan h} = \sec^2 x \dfrac{\lim\limits_{h\to 0} \frac{\tan h}{h}}{\lim\limits_{h\to 0}(1 - \tan x \tan h)} = \sec^2 x.$

**48.** $\lim\limits_{x\to 0} \dfrac{\tan(x+y) - \tan y}{x} = \lim\limits_{h\to 0} \dfrac{\tan(y+h) - \tan y}{h} = \dfrac{d}{dy}(\tan y) = \sec^2 y.$

**49.** By Exercises 49 and 50 of Section 1.6, we have $\lim\limits_{h\to 0} \dfrac{\sin h}{h} = \dfrac{\pi}{180}$ and $\lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} = 0$. Therefore:

**(a)** $\dfrac{d}{dx}[\sin x] = \lim\limits_{h\to 0} \dfrac{\sin(x+h) - \sin x}{h} = \sin x \lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} + \cos x \lim\limits_{h\to 0} \dfrac{\sin h}{h} = (\sin x)(0) + (\cos x)(\pi/180) =$

$\dfrac{\pi}{180}\cos x.$

**(b)** $\dfrac{d}{dx}[\cos x] = \lim\limits_{h\to 0} \dfrac{\cos(x+h) - \cos x}{h} = \lim\limits_{h\to 0} \dfrac{\cos x \cos h - \sin x \sin h - \cos x}{h} = \cos x \lim\limits_{h\to 0} \dfrac{\cos h - 1}{h} - \sin x \lim\limits_{h\to 0} \dfrac{\sin h}{h} =$

$0 \cdot \cos x - \dfrac{\pi}{180} \cdot \sin x = -\dfrac{\pi}{180}\sin x.$

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**50.** If $f$ is periodic, then so is $f'$. Proof: Suppose $f(x+p) = f(x)$ for all $x$. Then $f'(x+p) = \lim_{h\to 0} \dfrac{f(x+p+h)-f(x+p)}{h} = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = f'(x)$. However, $f'$ may be periodic even if $f$ is not. For example, $f(x) = x + \sin x$ is not periodic, but $f'(x) = 1 + \cos x$ has period $2\pi$.

## Exercise Set 2.6

**1.** $(f \circ g)'(x) = f'(g(x))g'(x)$, so $(f \circ g)'(0) = f'(g(0))g'(0) = f'(0)(3) = (2)(3) = 6$.

**2.** $(f \circ g)'(2) = f'(g(2))g'(2) = 5(-3) = -15$.

**3.** **(a)** $(f \circ g)(x) = f(g(x)) = (2x-3)^5$ and $(f \circ g)'(x) = f'(g(x))g'(x) = 5(2x-3)^4(2) = 10(2x-3)^4$.

**(b)** $(g \circ f)(x) = g(f(x)) = 2x^5 - 3$ and $(g \circ f)'(x) = g'(f(x))f'(x) = 2(5x^4) = 10x^4$.

**4.** **(a)** $(f \circ g)(x) = 5\sqrt{4+\cos x}$ and $(f \circ g)'(x) = f'(g(x))g'(x) = \dfrac{5}{2\sqrt{4+\cos x}}(-\sin x)$.

**(b)** $(g \circ f)(x) = 4 + \cos(5\sqrt{x})$ and $(g \circ f)'(x) = g'(f(x))f'(x) = -\sin(5\sqrt{x})\dfrac{5}{2\sqrt{x}}$.

**5.** **(a)** $F'(x) = f'(g(x))g'(x)$, $F'(3) = f'(g(3))g'(3) = -1(7) = -7$.

**(b)** $G'(x) = g'(f(x))f'(x)$, $G'(3) = g'(f(3))f'(3) = 4(-2) = -8$.

**6.** **(a)** $F'(x) = f'(g(x))g'(x)$, $F'(-1) = f'(g(-1))g'(-1) = f'(2)(-3) = (4)(-3) = -12$.

**(b)** $G'(x) = g'(f(x))f'(x), G'(-1) = g'(f(-1))f'(-1) = -5(3) = -15$.

**7.** $f'(x) = 37(x^3+2x)^{36}\dfrac{d}{dx}(x^3+2x) = 37(x^3+2x)^{36}(3x^2+2)$.

**8.** $f'(x) = 6(3x^2+2x-1)^5\dfrac{d}{dx}(3x^2+2x-1) = 6(3x^2+2x-1)^5(6x+2) = 12(3x^2+2x-1)^5(3x+1)$.

**9.** $f'(x) = -2\left(x^3 - \dfrac{7}{x}\right)^{-3}\dfrac{d}{dx}\left(x^3 - \dfrac{7}{x}\right) = -2\left(x^3 - \dfrac{7}{x}\right)^{-3}\left(3x^2 + \dfrac{7}{x^2}\right)$.

**10.** $f(x) = (x^5 - x + 1)^{-9}$, $f'(x) = -9(x^5-x+1)^{-10}\dfrac{d}{dx}(x^5-x+1) = -9(x^5-x+1)^{-10}(5x^4-1) = \dfrac{-9(5x^4-1)}{(x^5-x+1)^{10}}$.

**11.** $f(x) = 4(3x^2-2x+1)^{-3}$, $f'(x) = -12(3x^2-2x+1)^{-4}\dfrac{d}{dx}(3x^2-2x+1) = -12(3x^2-2x+1)^{-4}(6x-2) = \dfrac{24(1-3x)}{(3x^2-2x+1)^4}$.

**12.** $f'(x) = \dfrac{1}{2\sqrt{x^3-2x+5}}\dfrac{d}{dx}(x^3-2x+5) = \dfrac{3x^2-2}{2\sqrt{x^3-2x+5}}$.

**13.** $f'(x) = \dfrac{1}{2\sqrt{4+\sqrt{3x}}}\dfrac{d}{dx}(4+\sqrt{3x}) = \dfrac{\sqrt{3}}{4\sqrt{x}\sqrt{4+\sqrt{3x}}}$.

**14.** $f'(x) = \dfrac{1}{3}\left(12+\sqrt{x}\right)^{-2/3}\cdot\dfrac{1}{2}\dfrac{1}{\sqrt{x}} = \dfrac{1}{6(12+\sqrt{x})^{2/3}\sqrt{x}}$.

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**15.** $f'(x) = \cos(1/x^2)\dfrac{d}{dx}(1/x^2) = -\dfrac{2}{x^3}\cos(1/x^2).$

**16.** $f'(x) = \left(\sec^2\sqrt{x}\right)\dfrac{d}{dx}\sqrt{x} = \left(\sec^2\sqrt{x}\right)\dfrac{1}{2\sqrt{x}}.$

**17.** $f'(x) = 20\cos^4 x\dfrac{d}{dx}(\cos x) = 20\cos^4 x(-\sin x) = -20\cos^4 x\sin x.$

**18.** $f'(x) = 4 + 20(\sin^3 x)\dfrac{d}{dx}(\sin x) = 4 + 20\sin^3 x\cos x.$

**19.** $f'(x) = 2\cos(3\sqrt{x})\dfrac{d}{dx}[\cos(3\sqrt{x})] = -2\cos(3\sqrt{x})\sin(3\sqrt{x})\dfrac{d}{dx}(3\sqrt{x}) = -\dfrac{3\cos(3\sqrt{x})\sin(3\sqrt{x})}{\sqrt{x}}.$

**20.** $f'(x) = 4\tan^3(x^3)\dfrac{d}{dx}[\tan(x^3)] = 4\tan^3(x^3)\sec^2(x^3)\dfrac{d}{dx}(x^3) = 12x^2\tan^3(x^3)\sec^2(x^3).$

**21.** $f'(x) = 4\sec(x^7)\dfrac{d}{dx}[\sec(x^7)] = 4\sec(x^7)\sec(x^7)\tan(x^7)\dfrac{d}{dx}(x^7) = 28x^6\sec^2(x^7)\tan(x^7).$

**22.** $f'(x) = 3\cos^2\left(\dfrac{x}{x+1}\right)\dfrac{d}{dx}\cos\left(\dfrac{x}{x+1}\right) = 3\cos^2\left(\dfrac{x}{x+1}\right)\left[-\sin\left(\dfrac{x}{x+1}\right)\right]\dfrac{(x+1)(1)-x(1)}{(x+1)^2} =$

$= -\dfrac{3}{(x+1)^2}\cos^2\left(\dfrac{x}{x+1}\right)\sin\left(\dfrac{x}{x+1}\right).$

**23.** $f'(x) = \dfrac{1}{2\sqrt{\cos(5x)}}\dfrac{d}{dx}[\cos(5x)] = -\dfrac{5\sin(5x)}{2\sqrt{\cos(5x)}}.$

**24.** $f'(x) = \dfrac{1}{2\sqrt{3x-\sin^2(4x)}}\dfrac{d}{dx}[3x-\sin^2(4x)] = \dfrac{3-8\sin(4x)\cos(4x)}{2\sqrt{3x-\sin^2(4x)}}.$

**25.** $f'(x) = -3\left[x+\csc(x^3+3)\right]^{-4}\dfrac{d}{dx}\left[x+\csc(x^3+3)\right] =$

$= -3\left[x+\csc(x^3+3)\right]^{-4}\left[1-\csc(x^3+3)\cot(x^3+3)\dfrac{d}{dx}(x^3+3)\right] =$

$= -3\left[x+\csc(x^3+3)\right]^{-4}\left[1-3x^2\csc(x^3+3)\cot(x^3+3)\right].$

**26.** $f'(x) = -4\left[x^4-\sec(4x^2-2)\right]^{-5}\dfrac{d}{dx}\left[x^4-\sec(4x^2-2)\right] =$

$= -4\left[x^4-\sec(4x^2-2)\right]^{-5}\left[4x^3-\sec(4x^2-2)\tan(4x^2-2)\dfrac{d}{dx}(4x^2-2)\right] =$

$= -16x\left[x^4-\sec(4x^2-2)\right]^{-5}\left[x^2-2\ \sec(4x^2-2)\tan(4x^2-2)\right].$

**27.** $\dfrac{dy}{dx} = x^3(2\sin 5x)\dfrac{d}{dx}(\sin 5x) + 3x^2\sin^2 5x = 10x^3\sin 5x\cos 5x + 3x^2\sin^2 5x.$

**28.** $\dfrac{dy}{dx} = \sqrt{x}\left[3\tan^2(\sqrt{x})\sec^2(\sqrt{x})\dfrac{1}{2\sqrt{x}}\right] + \dfrac{1}{2\sqrt{x}}\tan^3(\sqrt{x}) = \dfrac{3}{2}\tan^2(\sqrt{x})\sec^2(\sqrt{x}) + \dfrac{1}{2\sqrt{x}}\tan^3(\sqrt{x}).$

**29.** $\dfrac{dy}{dx} = x^5\sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\left(\dfrac{1}{x}\right) + \sec\left(\dfrac{1}{x}\right)(5x^4) = x^5\sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right)\left(-\dfrac{1}{x^2}\right) + 5x^4\sec\left(\dfrac{1}{x}\right) =$

$= -x^3\sec\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right) + 5x^4\sec\left(\dfrac{1}{x}\right).$

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**30.** $\dfrac{dy}{dx} = \dfrac{\sec(3x+1)\cos x - 3\sin x \sec(3x+1)\tan(3x+1)}{\sec^2(3x+1)} = \cos x\cos(3x+1) - 3\sin x\sin(3x+1).$

**31.** $\dfrac{dy}{dx} = -\sin(\cos x)\dfrac{d}{dx}(\cos x) = -\sin(\cos x)(-\sin x) = \sin(\cos x)\sin x.$

**32.** $\dfrac{dy}{dx} = \cos(\tan 3x)\dfrac{d}{dx}(\tan 3x) = 3\sec^2 3x\cos(\tan 3x).$

**33.** $\dfrac{dy}{dx} = 3\cos^2(\sin 2x)\dfrac{d}{dx}[\cos(\sin 2x)] = 3\cos^2(\sin 2x)[-\sin(\sin 2x)]\dfrac{d}{dx}(\sin 2x) = -6\cos^2(\sin 2x)\sin(\sin 2x)\cos 2x.$

**34.** $\dfrac{dy}{dx} = \dfrac{(1-\cot x^2)(-2x\csc x^2\cot x^2) - (1+\csc x^2)(2x\csc^2 x^2)}{(1-\cot x^2)^2} = -2x\csc x^2\ \dfrac{1+\cot x^2+\csc x^2}{(1-\cot x^2)^2}$, since $\csc^2 x^2 = 1+\cot^2 x^2$.

**35.** $\dfrac{dy}{dx} = (5x+8)^7\dfrac{d}{dx}(1-\sqrt{x})^6 + (1-\sqrt{x})^6\dfrac{d}{dx}(5x+8)^7 = 6(5x+8)^7(1-\sqrt{x})^5\dfrac{-1}{2\sqrt{x}} + 7\cdot 5(1-\sqrt{x})^6(5x+8)^6 = \dfrac{-3}{\sqrt{x}}(5x+8)^7(1-\sqrt{x})^5 + 35(1-\sqrt{x})^6(5x+8)^6.$

**36.** $\dfrac{dy}{dx} = (x^2+x)^5\dfrac{d}{dx}\sin^8 x + (\sin^8 x)\dfrac{d}{dx}(x^2+x)^5 = 8(x^2+x)^5\sin^7 x\cos x + 5(\sin^8 x)(x^2+x)^4(2x+1).$

**37.** $\dfrac{dy}{dx} = 3\left[\dfrac{x-5}{2x+1}\right]^2\dfrac{d}{dx}\left[\dfrac{x-5}{2x+1}\right] = 3\left[\dfrac{x-5}{2x+1}\right]^2\cdot\dfrac{11}{(2x+1)^2} = \dfrac{33(x-5)^2}{(2x+1)^4}.$

**38.** $\dfrac{dy}{dx} = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{d}{dx}\left(\dfrac{1+x^2}{1-x^2}\right) = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{(1-x^2)(2x)-(1+x^2)(-2x)}{(1-x^2)^2} = 17\left(\dfrac{1+x^2}{1-x^2}\right)^{16}\dfrac{4x}{(1-x^2)^2} = \dfrac{68x(1+x^2)^{16}}{(1-x^2)^{18}}.$

**39.** $\dfrac{dy}{dx} = \dfrac{(4x^2-1)^8(3)(2x+3)^2(2) - (2x+3)^3(8)(4x^2-1)^7(8x)}{(4x^2-1)^{16}} = \dfrac{2(2x+3)^2(4x^2-1)^7[3(4x^2-1)-32x(2x+3)]}{(4x^2-1)^{16}} = -\dfrac{2(2x+3)^2(52x^2+96x+3)}{(4x^2-1)^9}.$

**40.** $\dfrac{dy}{dx} = 12[1+\sin^3(x^5)]^{11}\dfrac{d}{dx}[1+\sin^3(x^5)] = 12[1+\sin^3(x^5)]^{11}3\sin^2(x^5)\dfrac{d}{dx}\sin(x^5) =$
$= 180x^4[1+\sin^3(x^5)]^{11}\sin^2(x^5)\cos(x^5).$

**41.** $\dfrac{dy}{dx} = 5\left[x\sin 2x + \tan^4(x^7)\right]^4\dfrac{d}{dx}\left[x\sin 2x\tan^4(x^7)\right] =$
$= 5\left[x\sin 2x + \tan^4(x^7)\right]^4\left[x\cos 2x\dfrac{d}{dx}(2x) + \sin 2x + 4\tan^3(x^7)\dfrac{d}{dx}\tan(x^7)\right] =$
$= 5\left[x\sin 2x + \tan^4(x^7)\right]^4\left[2x\cos 2x + \sin 2x + 28x^6\tan^3(x^7)\sec^2(x^7)\right].$

**42.** $\dfrac{dy}{dx} = 4\tan^3\left(2 + \dfrac{(7-x)\sqrt{3x^2+5}}{x^3+\sin x}\right)\sec^2\left(2 + \dfrac{(7-x)\sqrt{3x^2+5}}{x^3+\sin x}\right)$
$\times\left(-\dfrac{\sqrt{3x^2+5}}{x^3+\sin x} + 3\dfrac{(7-x)x}{\sqrt{3x^2+5}\,(x^3+\sin x)} - \dfrac{(7-x)\sqrt{3x^2+5}\,(3x^2+\cos x)}{(x^3+\sin x)^2}\right)$

**43.** $\dfrac{dy}{dx} = \cos 3x - 3x\sin 3x$; if $x=\pi$ then $\dfrac{dy}{dx} = -1$ and $y = -\pi$, so the equation of the tangent line is $y+\pi = -(x-\pi)$, or $y = -x$.

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**44.** $\dfrac{dy}{dx} = 3x^2\cos(1+x^3)$; if $x=-3$ then $y=-\sin 26$, $\dfrac{dy}{dx} = 27\cos 26$, so the equation of the tangent line is $y+\sin 26 = 27(\cos 26)(x+3)$, or $y = 27(\cos 26)x + 81\cos 26 - \sin 26$.

**45.** $\dfrac{dy}{dx} = -3\sec^3(\pi/2-x)\tan(\pi/2-x)$; if $x=-\pi/2$ then $\dfrac{dy}{dx} = 0, y=-1$, so the equation of the tangent line is $y+1=0$, or $y=-1$

**46.** $\dfrac{dy}{dx} = 3\left(x-\dfrac{1}{x}\right)^2\left(1+\dfrac{1}{x^2}\right)$; if $x=2$ then $y=\dfrac{27}{8}$, $\dfrac{dy}{dx} = 3\dfrac{9}{4}\dfrac{5}{4} = \dfrac{135}{16}$, so the equation of the tangent line is $y-27/8 = (135/16)(x-2)$, or $y = \dfrac{135}{16}x - \dfrac{27}{2}$.

**47.** $\dfrac{dy}{dx} = \sec^2(4x^2)\dfrac{d}{dx}(4x^2) = 8x\sec^2(4x^2)$, $\left.\dfrac{dy}{dx}\right|_{x=\sqrt{\pi}} = 8\sqrt{\pi}\sec^2(4\pi) = 8\sqrt{\pi}$. When $x=\sqrt{\pi}$, $y=\tan(4\pi)=0$, so the equation of the tangent line is $y = 8\sqrt{\pi}(x-\sqrt{\pi}) = 8\sqrt{\pi}x - 8\pi$.

**48.** $\dfrac{dy}{dx} = 12\cot^3 x\dfrac{d}{dx}\cot x = -12\cot^3 x\csc^2 x$, $\left.\dfrac{dy}{dx}\right|_{x=\pi/4} = -24$. When $x=\pi/4, y=3$, so the equation of the tangent line is $y-3=-24(x-\pi/4)$, or $y=-24x+3+6\pi$.

**49.** $\dfrac{dy}{dx} = 2x\sqrt{5-x^2} + \dfrac{x^2}{2\sqrt{5-x^2}}(-2x)$, $\left.\dfrac{dy}{dx}\right|_{x=1} = 4 - 1/2 = 7/2$. When $x=1, y=2$, so the equation of the tangent line is $y-2=(7/2)(x-1)$, or $y=\dfrac{7}{2}x - \dfrac{3}{2}$.

**50.** $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}} - \dfrac{x}{2}(1-x^2)^{3/2}(-2x)$, $\left.\dfrac{dy}{dx}\right|_{x=0} = 1$. When $x=0, y=0$, so the equation of the tangent line is $y=x$.

**51.** $\dfrac{dy}{dx} = x(-\sin(5x))\dfrac{d}{dx}(5x) + \cos(5x) - 2\sin x\dfrac{d}{dx}(\sin x) = -5x\sin(5x) + \cos(5x) - 2\sin x\cos x =$
$= -5x\sin(5x) + \cos(5x) - \sin(2x)$,
$\dfrac{d^2y}{dx^2} = -5x\cos(5x)\dfrac{d}{dx}(5x) - 5\sin(5x) - \sin(5x)\dfrac{d}{dx}(5x) - \cos(2x)\dfrac{d}{dx}(2x) = -25x\cos(5x) - 10\sin(5x) - 2\cos(2x)$.

**52.** $\dfrac{dy}{dx} = \cos(3x^2)\dfrac{d}{dx}(3x^2) = 6x\cos(3x^2)$, $\dfrac{d^2y}{dx^2} = 6x(-\sin(3x^2))\dfrac{d}{dx}(3x^2) + 6\cos(3x^2) = -36x^2\sin(3x^2) + 6\cos(3x^2)$.

**53.** $\dfrac{dy}{dx} = \dfrac{(1-x)+(1+x)}{(1-x)^2} = \dfrac{2}{(1-x)^2} = 2(1-x)^{-2}$ and $\dfrac{d^2y}{dx^2} = -2(2)(-1)(1-x)^{-3} = 4(1-x)^{-3}$.

**54.** $\dfrac{dy}{dx} = x\sec^2\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\left(\dfrac{1}{x}\right) + \tan\left(\dfrac{1}{x}\right) = -\dfrac{1}{x}\sec^2\left(\dfrac{1}{x}\right) + \tan\left(\dfrac{1}{x}\right)$,
$\dfrac{d^2y}{dx^2} = -\dfrac{2}{x}\sec\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\sec\left(\dfrac{1}{x}\right) + \dfrac{1}{x^2}\sec^2\left(\dfrac{1}{x}\right) + \sec^2\left(\dfrac{1}{x}\right)\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = \dfrac{2}{x^3}\sec^2\left(\dfrac{1}{x}\right)\tan\left(\dfrac{1}{x}\right)$.

**55.** $y = \cot^3(\pi-\theta) = -\cot^3\theta$ so $dy/dx = 3\cot^2\theta\csc^2\theta$.

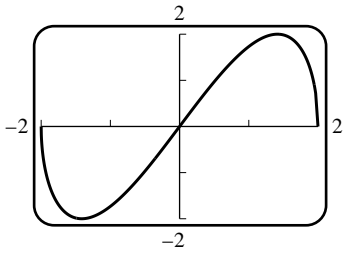
**56.** $6\left(\dfrac{au+b}{cu+d}\right)^5\dfrac{ad-bc}{(cu+d)^2}$.

**57.** $\dfrac{d}{d\omega}[a\cos^2\pi\omega + b\sin^2\pi\omega] = -2\pi a\cos\pi\omega\sin\pi\omega + 2\pi b\sin\pi\omega\cos\pi\omega = \pi(b-a)(2\sin\pi\omega\cos\pi\omega) = \pi(b-a)\sin 2\pi\omega$.

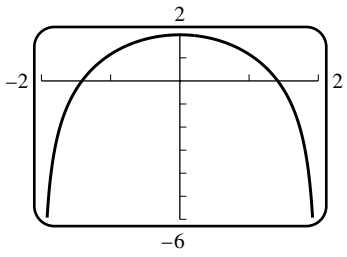
**58.** $2\csc^2(\pi/3-y)\cot(\pi/3-y)$.

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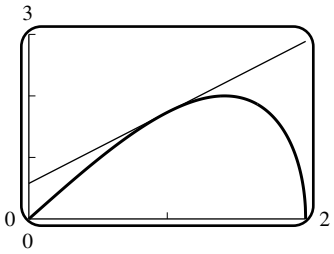
**59. (a)**



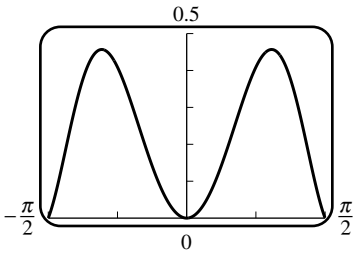

**(c)** $f'(x) = x\dfrac{-x}{\sqrt{4-x^2}} + \sqrt{4-x^2} = \dfrac{4-2x^2}{\sqrt{4-x^2}}$.



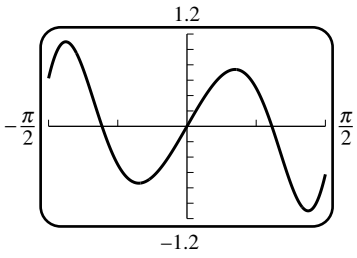

**(d)** $f(1) = \sqrt{3}$ and $f'(1) = \dfrac{2}{\sqrt{3}}$ so the tangent line has the equation $y - \sqrt{3} = \dfrac{2}{\sqrt{3}}(x-1)$.




**60. (a)**

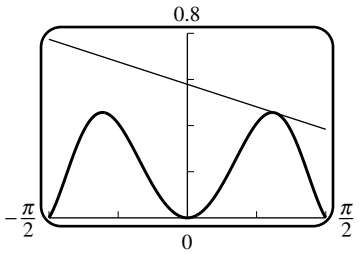



**(c)** $f'(x) = 2x\cos(x^2)\cos x - \sin x \sin(x^2)$.




**(d)** $f(1) = \sin 1 \cos 1$ and $f'(1) = 2\cos^2 1 - \sin^2 1$, so the tangent line has the equation $y - \sin 1 \cos 1 = (2\cos^2 1 - \sin^2 1)(x-1)$.

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**61.** False. $\dfrac{d}{dx}[\sqrt{y}\,] = \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx} = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

**62.** False. $dy/dx = f'(u)g'(x) = f'(g(x))\,g'(x)$.

**63.** False. $dy/dx = -\sin[g(x)]\,g'(x)$.

**64.** True. Let $u = 3x^3$ and $v = \sin u$, so $y = v^3$. Then $\dfrac{dy}{dx} = \dfrac{dy}{dv}\dfrac{dv}{du}\dfrac{du}{dx} = 3v^2 \cdot (\cos u) \cdot 9x^2 = 3\sin^2(3x^3) \cdot \cos(3x^3) \cdot 9x^2 = 27x^2 \sin^2(3x^3)\,\cos(3x^3)$.

**65.** **(a)** $dy/dt = -A\omega \sin \omega t, d^2y/dt^2 = -A\omega^2 \cos \omega t = -\omega^2 y$

**(b)** One complete oscillation occurs when $\omega t$ increases over an interval of length $2\pi$, or if $t$ increases over an interval of length $2\pi/\omega$.

**(c)** $f = 1/T$

**(d)** Amplitude $= 0.6$ cm, $T = 2\pi/15$ s/oscillation, $f = 15/(2\pi)$ oscillations/s.

**66.** $dy/dt = 3A\cos 3t, d^2y/dt^2 = -9A\sin 3t$, so $-9A\sin 3t + 2A\sin 3t = 4\sin 3t$, $-7A\sin 3t = 4\sin 3t, -7A = 4$, and $A = -4/7$

**67.** By the chain rule, $\dfrac{d}{dx}\left[\sqrt{x + f(x)}\right] = \dfrac{1 + f'(x)}{2\sqrt{x + f(x)}}$. From the graph, $f(x) = \dfrac{4}{3}x + 5$ for $x < 0$, so $f(-1) = \dfrac{11}{3}$, $f'(-1) = \dfrac{4}{3}$, and $\dfrac{d}{dx}\left[\sqrt{x + f(x)}\right]\Big|_{x=-1} = \dfrac{7/3}{2\sqrt{8/3}} = \dfrac{7\sqrt{6}}{24}$.

**68.** $2\sin(\pi/6) = 1$, so we can assume $f(x) = -\frac{5}{2}x + 5$. Thus for sufficiently small values of $|x - \pi/6|$ we have $\dfrac{d}{dx}[f(2\sin x)]\Big|_{x=\pi/6} = f'(2\sin x)\dfrac{d}{dx}2\sin x\Big|_{x=\pi/6} = -\dfrac{5}{2}2\cos x\Big|_{x=\pi/6} = -\dfrac{5}{2}2\dfrac{\sqrt{3}}{2} = -\dfrac{5}{2}\sqrt{3}$.

**69.** **(a)** $p \approx 10$ lb/in$^2$, $dp/dh \approx -2$ lb/in$^2$/mi. **(b)** $\dfrac{dp}{dt} = \dfrac{dp}{dh}\dfrac{dh}{dt} \approx (-2)(0.3) = -0.6$ lb/in$^2$/s.

**70.** **(a)** $F = \dfrac{45}{\cos\theta + 0.3\sin\theta}, \dfrac{dF}{d\theta} = -\dfrac{45(-\sin\theta + 0.3\cos\theta)}{(\cos\theta + 0.3\sin\theta)^2}$; if $\theta = 30°$, then $dF/d\theta \approx 10.5$ lb/rad $\approx 0.18$ lb/deg.

**(b)** $\dfrac{dF}{dt} = \dfrac{dF}{d\theta}\dfrac{d\theta}{dt} \approx (0.18)(-0.5) = -0.09$ lb/s.

**71.** With $u = \sin x$, $\dfrac{d}{dx}(|\sin x|) = \dfrac{d}{dx}(|u|) = \dfrac{d}{du}(|u|)\dfrac{du}{dx} = \dfrac{d}{du}(|u|)\cos x = \begin{cases} \cos x, & u > 0 \\ -\cos x, & u < 0 \end{cases} = \begin{cases} \cos x, & \sin x > 0 \\ -\cos x, & \sin x < 0 \end{cases}$
$= \begin{cases} \cos x, & 0 < x < \pi \\ -\cos x, & -\pi < x < 0 \end{cases}$

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**72.** $\dfrac{d}{dx}(\cos x) = \dfrac{d}{dx}[\sin(\pi/2 - x)] = -\cos(\pi/2 - x) = -\sin x.$

**73.** **(a)** For $x \neq 0, |f(x)| \leq |x|$, and $\lim\limits_{x\to 0} |x| = 0$, so by the Squeezing Theorem, $\lim\limits_{x\to 0} f(x) = 0$.

**(b)** If $f'(0)$ were to exist, then the limit (as $x$ approaches 0) $\dfrac{f(x) - f(0)}{x - 0} = \sin(1/x)$ would have to exist, but it doesn't.

**(c)** For $x \neq 0$, $f'(x) = x\left(\cos\dfrac{1}{x}\right)\left(-\dfrac{1}{x^2}\right) + \sin\dfrac{1}{x} = -\dfrac{1}{x}\cos\dfrac{1}{x} + \sin\dfrac{1}{x}$.

**(d)** If $x = \dfrac{1}{2\pi n}$ for an integer $n \neq 0$, then $f'(x) = -2\pi n\cos(2\pi n) + \sin(2\pi n) = -2\pi n$. This approaches $+\infty$ as $n \to -\infty$, so there are points $x$ arbitrarily close to 0 where $f'(x)$ becomes arbitrarily large. Hence $\lim\limits_{x\to 0} f'(x)$ does not exist.

**74.** **(a)** $-x^2 \leq x^2\sin(1/x) \leq x^2$, so by the Squeezing Theorem $\lim\limits_{x\to 0} f(x) = 0$.

**(b)** $f'(0) = \lim\limits_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x\to 0} x\sin(1/x) = 0$ by Exercise 73, part **(a)**.

**(c)** For $x \neq 0$, $f'(x) = 2x\sin(1/x) + x^2\cos(1/x)(-1/x^2) = 2x\sin(1/x) - \cos(1/x)$.

**(d)** If $f'(x)$ were continuous at $x = 0$ then so would $\cos(1/x) = 2x\sin(1/x) - f'(x)$ be, since $2x\sin(1/x)$ is continuous there. But $\cos(1/x)$ oscillates at $x = 0$.

**75.** **(a)** $g'(x) = 3[f(x)]^2 f'(x)$, $g'(2) = 3[f(2)]^2 f'(2) = 3(1)^2(7) = 21$.

**(b)** $h'(x) = f'(x^3)(3x^2)$, $h'(2) = f'(8)(12) = (-3)(12) = -36$.

**76.** $F'(x) = f'(g(x))g'(x) = \sqrt{3(x^2-1)+4}\cdot 2x = 2x\sqrt{3x^2+1}$.

**77.** $F'(x) = f'(g(x))g'(x) = f'(\sqrt{3x-1})\dfrac{3}{2\sqrt{3x-1}} = \dfrac{\sqrt{3x-1}}{(3x-1)+1}\dfrac{3}{2\sqrt{3x-1}} = \dfrac{1}{2x}$.

**78.** $\dfrac{d}{dx}[f(x^2)] = f'(x^2)(2x)$, thus $f'(x^2)(2x) = x^2$ so $f'(x^2) = x/2$ if $x \neq 0$.

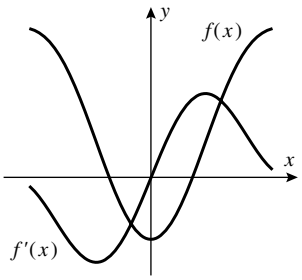
**79.** $\dfrac{d}{dx}[f(3x)] = f'(3x)\dfrac{d}{dx}(3x) = 3f'(3x) = 6x$, so $f'(3x) = 2x$. Let $u = 3x$ to get $f'(u) = \dfrac{2}{3}u$; $\dfrac{d}{dx}[f(x)] = f'(x) = \dfrac{2}{3}x$.

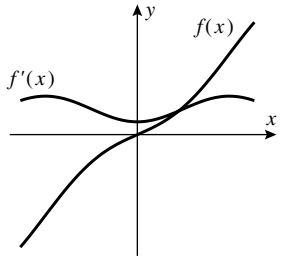
**80.** **(a)** If $f(-x) = f(x)$, then $\dfrac{d}{dx}[f(-x)] = \dfrac{d}{dx}[f(x)]$, $f'(-x)(-1) = f'(x)$, $f'(-x) = -f'(x)$ so $f'$ is odd.

**(b)** If $f(-x) = -f(x)$, then $\dfrac{d}{dx}[f(-x)] = -\dfrac{d}{dx}[f(x)]$, $f'(-x)(-1) = -f'(x)$, $f'(-x) = f'(x)$ so $f'$ is even.

**81.** For an even function, the graph is symmetric about the $y$-axis; the slope of the tangent line at $(a, f(a))$ is the negative of the slope of the tangent line at $(-a, f(-a))$. For an odd function, the graph is symmetric about the origin; the slope of the tangent line at $(a, f(a))$ is the same as the slope of the tangent line at $(-a, f(-a))$.

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**82.** $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dv}\dfrac{dv}{dw}\dfrac{dw}{dx}$.

**83.** $\dfrac{d}{dx}[f(g(h(x)))] = \dfrac{d}{dx}[f(g(u))],\ u = h(x),\ \dfrac{d}{du}[f(g(u))]\dfrac{du}{dx} = f'(g(u))g'(u)\dfrac{du}{dx} = f'(g(h(x)))g'(h(x))h'(x)$.

**84.** $g'(x) = f'\left(\dfrac{\pi}{2} - x\right) \cdot \dfrac{d}{dx}\left(\dfrac{\pi}{2} - x\right) = -f'\left(\dfrac{\pi}{2} - x\right)$, so $g'$ is the negative of the co-function of $f'$.

The derivatives of $\sin x$, $\tan x$, and $\sec x$ are $\cos x$, $\sec^2 x$, and $\sec x \tan x$, respectively. The negatives of the co-functions of these are $-\sin x$, $-\csc^2 x$, and $-\csc x \cot x$, which are the derivatives of $\cos x$, $\cot x$, and $\csc x$, respectively.

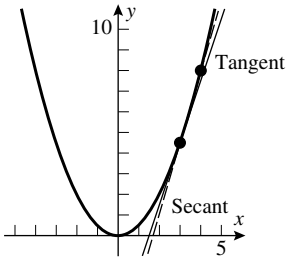
## Chapter 2 Review Exercises

**2. (a)** $m_{\text{sec}} = \dfrac{f(4) - f(3)}{4 - 3} = \dfrac{(4)^2/2 - (3)^2/2}{1} = \dfrac{7}{2}$

**(b)** $m_{\tan} = \lim\limits_{w \to 3} \dfrac{f(w) - f(3)}{w - 3} = \lim\limits_{w \to 3} \dfrac{w^2/2 - 9/2}{w - 3} = \lim\limits_{w \to 3} \dfrac{w^2 - 9}{2(w - 3)} = \lim\limits_{w \to 3} \dfrac{(w + 3)(w - 3)}{2(w - 3)} = \lim\limits_{w \to 3} \dfrac{w + 3}{2} = 3.$

**(c)** $m_{\tan} = \lim\limits_{w \to x} \dfrac{f(w) - f(x)}{w - x} = \lim\limits_{w \to x} \dfrac{w^2/2 - x^2/2}{w - x} = \lim\limits_{w \to x} \dfrac{w^2 - x^2}{2(w - x)} = \lim\limits_{w \to x} \dfrac{w + x}{2} = x.$

**(d)**




**3. (a)** $m_{\tan} = \lim\limits_{w \to x} \dfrac{f(w) - f(x)}{w - x} = \lim\limits_{w \to x} \dfrac{(w^2 + 1) - (x^2 + 1)}{w - x} = \lim\limits_{w \to x} \dfrac{w^2 - x^2}{w - x} = \lim\limits_{w \to x} (w + x) = 2x.$

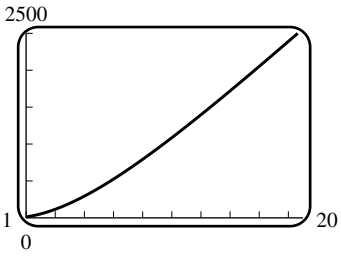
**(b)** $m_{\tan} = 2(2) = 4.$

**4.** To average 60 mi/h one would have to complete the trip in two hours. At 50 mi/h, 100 miles are completed after two hours. Thus time is up, and the speed for the remaining 20 miles would have to be infinite.

**5.** $v_{\text{inst}} = \lim\limits_{h \to 0} \dfrac{3(h + 1)^{2.5} + 580h - 3}{10h} = 58 + \dfrac{1}{10}\dfrac{d}{dx}3x^{2.5}\Big|_{x=1} = 58 + \dfrac{1}{10}(2.5)(3)(1)^{1.5} = 58.75 \text{ ft/s}.$

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**6.** 164 ft/s




**7.** **(a)** $v_{\text{ave}} = \dfrac{[3(3)^2+3]-[3(1)^2+1]}{3-1} = 13$ mi/h.

**(b)** $v_{\text{inst}} = \lim\limits_{t_1\to 1} \dfrac{(3t_1^2+t_1)-4}{t_1-1} = \lim\limits_{t_1\to 1} \dfrac{(3t_1+4)(t_1-1)}{t_1-1} = \lim\limits_{t_1\to 1}(3t_1+4) = 7$ mi/h.

**9.** **(a)** $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{\sqrt{9-4(x+h)}-\sqrt{9-4x}}{h} = \lim\limits_{h\to 0} \dfrac{9-4(x+h)-(9-4x)}{h(\sqrt{9-4(x+h)}+\sqrt{9-4x})} =$

$= \lim\limits_{h\to 0} \dfrac{-4h}{h(\sqrt{9-4(x+h)}+\sqrt{9-4x})} = \dfrac{-4}{2\sqrt{9-4x}} = \dfrac{-2}{\sqrt{9-4x}}.$

**(b)** $\dfrac{dy}{dx} = \lim\limits_{h\to 0} \dfrac{\dfrac{x+h}{x+h+1}-\dfrac{x}{x+1}}{h} = \lim\limits_{h\to 0} \dfrac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)} = \lim\limits_{h\to 0} \dfrac{h}{h(x+h+1)(x+1)} = \dfrac{1}{(x+1)^2}.$

**10.** $f(x)$ is continuous and differentiable at any $x \neq 1$, so we consider $x = 1$.

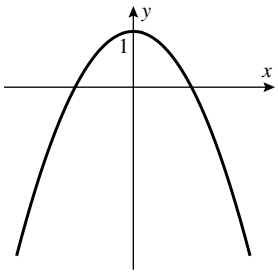
**(a)** $\lim\limits_{x\to 1^-}(x^2-1) = \lim\limits_{x\to 1^+} k(x-1) = 0 = f(1)$, so any value of $k$ gives continuity at $x = 1$.

**(b)** $\lim\limits_{x\to 1^-} f'(x) = \lim\limits_{x\to 1^-} 2x = 2$, and $\lim\limits_{x\to 1^+} f'(x) = \lim\limits_{x\to 1^+} k = k$, so only if $k = 2$ is $f(x)$ differentiable at $x = 1$.

**11.** **(a)** $x = -2, -1, 1, 3$ **(b)** $(-\infty, -2)$, $(-1, 1)$, $(3, +\infty)$ **(c)** $(-2, -1)$, $(1, 3)$

**(d)** $g''(x) = f''(x)\sin x + 2f'(x)\cos x - f(x)\sin x$; $g''(0) = 2f'(0)\cos 0 = 2(2)(1) = 4$

**12.**

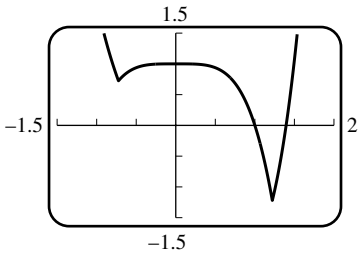



**13.** **(a)** The slope of the tangent line $\approx \dfrac{10-2.2}{2050-1950} = 0.078$ billion, so in 2000 the world population was increasing at the rate of about 78 million per year.

**(b)** $\dfrac{dN/dt}{N} \approx \dfrac{0.078}{6} = 0.013 = 1.3$ %/year

**14.** When $x^4-x-1 > 0$, $f(x) = x^4-2x-1$; when $x^4-x-1 < 0$, $f(x) = -x^4+1$, and $f$ is differentiable in both cases. The roots of $x^4-x-1 = 0$ are $x_1 \approx -0.724492$, $x_2 \approx 1.220744$. So $x^4-x-1 > 0$ on $(-\infty, x_1)$ and $(x_2, +\infty)$, and $x^4-x-1 < 0$ on $(x_1, x_2)$. Then $\lim\limits_{x\to x_1^-} f'(x) = \lim\limits_{x\to x_1^-}(4x^3-2) = 4x_1^3-2$ and $\lim\limits_{x\to x_1^+} f'(x) = \lim\limits_{x\to x_1^+} -4x^3 = -4x_1^3$ which is not equal to $4x_1^3-2$, so $f$ is not differentiable at $x = x_1$; similarly $f$ is not differentiable at $x = x_2$.

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**15.** **(a)** $f'(x) = 2x\sin x + x^2\cos x$ **(c)** $f''(x) = 4x\cos x + (2-x^2)\sin x$

**16.** **(a)** $f'(x) = \dfrac{1 - 2\sqrt{x}\sin 2x}{2\sqrt{x}}$ **(c)** $f''(x) = \dfrac{-1 - 8x^{3/2}\cos 2x}{4x^{3/2}}$

**17.** **(a)** $f'(x) = \dfrac{6x^2 + 8x - 17}{(3x+2)^2}$ **(c)** $f''(x) = \dfrac{118}{(3x+2)^3}$

**18.** **(a)** $f'(x) = \dfrac{(1+x^2)\sec^2 x - 2x\tan x}{(1+x^2)^2}$

**(c)** $f''(x) = \dfrac{(2+4x^2+2x^4)\sec^2 x\tan x - (4x+4x^3)\sec^2 x + (-2+6x^2)\tan x}{(1+x^2)^3}$

**19.** **(a)** $\dfrac{dW}{dt} = 200(t-15)$; at $t=5$, $\dfrac{dW}{dt} = -2000$; the water is running out at the rate of 2000 gal/min.

**(b)** $\dfrac{W(5)-W(0)}{5-0} = \dfrac{10000-22500}{5} = -2500$; the average rate of flow out is 2500 gal/min.

**20.** **(a)** $\dfrac{4^3-2^3}{4-2} = \dfrac{56}{2} = 28$ **(b)** $(dV/d\ell)|_{\ell=5} = 3\ell^2\big|_{\ell=5} = 3(5)^2 = 75$

**21.** **(a)** $f'(x) = 2x, f'(1.8) = 3.6$ **(b)** $f'(x) = (x^2-4x)/(x-2)^2, f'(3.5) = -7/9 \approx -0.777778$

**22.** **(a)** $f'(x) = 3x^2 - 2x, f'(2.3) = 11.27$ **(b)** $f'(x) = (1-x^2)/(x^2+1)^2, f'(-0.5) = 0.48$

**23.** $f$ is continuous at $x=1$ because it is differentiable there, thus $\lim\limits_{h\to 0} f(1+h) = f(1)$ and so $f(1) = 0$ because $\lim\limits_{h\to 0}\dfrac{f(1+h)}{h}$ exists; $f'(1) = \lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0}\dfrac{f(1+h)}{h} = 5$.

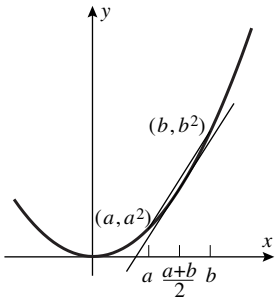
**24.** Multiply the given equation by $\lim\limits_{x\to 2}(x-2) = 0$ to get $0 = \lim\limits_{x\to 2}(x^3 f(x) - 24)$. Since $f$ is continuous at $x=2$, this equals $2^3 f(2) - 24$, so $f(2) = 3$. Now let $g(x) = x^3 f(x)$. Then $g'(2) = \lim\limits_{x\to 2}\dfrac{g(x)-g(2)}{x-2} = \lim\limits_{x\to 2}\dfrac{x^3 f(x) - 2^3 f(2)}{x-2} = \lim\limits_{x\to 2}\dfrac{x^3 f(x) - 24}{x-2} = 28$. But $g'(x) = x^3 f'(x) + 3x^2 f(x)$, so $28 = g'(2) = 2^3 f'(2) + 3\cdot 2^2 f(2) = 8f'(2) + 36$, and $f'(2) = -1$.

**25.** The equation of such a line has the form $y = mx$. The points $(x_0, y_0)$ which lie on both the line and the parabola and for which the slopes of both curves are equal satisfy $y_0 = mx_0 = x_0^3 - 9x_0^2 - 16x_0$, so that $m = x_0^2 - 9x_0 - 16$. By differentiating, the slope is also given by $m = 3x_0^2 - 18x_0 - 16$. Equating, we have $x_0^2 - 9x_0 - 16 = 3x_0^2 - 18x_0 - 16$, or $2x_0^2 - 9x_0 = 0$. The root $x_0 = 0$ corresponds to $m = -16, y_0 = 0$ and the root $x_0 = 9/2$ corresponds to $m = -145/4, y_0 = -1305/8$. So the line $y = -16x$ is tangent to the curve at the point $(0,0)$, and the line $y = -145x/4$ is tangent to the curve at the point $(9/2, -1305/8)$.

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**26.** The slope of the line $x+4y=10$ is $m_1=-1/4$, so we set the negative reciprocal $4=m_2=\dfrac{d}{dx}(2x^3-x^2)=6x^2-2x$ and obtain $6x^2-2x-4=0$ with roots $x=\dfrac{1\pm\sqrt{1+24}}{6}=1,-2/3$.

**27.** The slope of the tangent line is the derivative $y'=2x\Big|_{x=\frac{1}{2}(a+b)}=a+b$. The slope of the secant is $\dfrac{a^2-b^2}{a-b}=a+b$, so they are equal.




**28.** **(a)** $f'(1)g(1)+f(1)g'(1)=3(-2)+1(-1)=-7$ **(b)** $\dfrac{g(1)f'(1)-f(1)g'(1)}{g(1)^2}=\dfrac{-2(3)-1(-1)}{(-2)^2}=-\dfrac{5}{4}$

**(c)** $\dfrac{1}{2\sqrt{f(1)}}f'(1)=\dfrac{1}{2\sqrt{1}}(3)=\dfrac{3}{2}$ **(d)** 0 (because $f(1)g'(1)$ is constant)

**29.** **(a)** $8x^7-\dfrac{3}{2\sqrt{x}}-15x^{-4}$ **(b)** $2\cdot101(2x+1)^{100}(5x^2-7)+10x(2x+1)^{101}=(2x+1)^{100}(1030x^2+10x-1414)$

**30.** **(a)** $\cos x-6\cos^2 x\sin x$ **(b)** $(1+\sec x)(2x-\sec^2 x)+(x^2-\tan x)\sec x\tan x$

**31.** **(a)** $2(x-1)\sqrt{3x+1}+\dfrac{3}{2\sqrt{3x+1}}(x-1)^2=\dfrac{(x-1)(15x+1)}{2\sqrt{3x+1}}$

**(b)** $3\left(\dfrac{3x+1}{x^2}\right)^2\dfrac{x^2(3)-(3x+1)(2x)}{x^4}=-\dfrac{3(3x+1)^2(3x+2)}{x^7}$

**32.** **(a)** $-\csc^2\left(\dfrac{\csc 2x}{x^3+5}\right)\dfrac{-2(x^3+5)\csc 2x\cot 2x-3x^2\csc 2x}{(x^3+5)^2}$ **(b)** $-\dfrac{2+3\sin^2 x\cos x}{(2x+\sin^3 x)^2}$

**33.** Set $f'(x)=0$: $f'(x)=6(2)(2x+7)^5(x-2)^5+5(2x+7)^6(x-2)^4=0$, so $2x+7=0$ or $x-2=0$ or, factoring out $(2x+7)^5(x-2)^4$, $12(x-2)+5(2x+7)=0$. This reduces to $x=-7/2$, $x=2$, or $22x+11=0$, so the tangent line is horizontal at $x=-7/2,2,-1/2$.

**34.** Set $f'(x)=0$: $f'(x)=\dfrac{4(x^2+2x)(x-3)^3-(2x+2)(x-3)^4}{(x^2+2x)^2}$, and a fraction can equal zero only if its numerator equals zero. So either $x-3=0$ or, after factoring out $(x-3)^3$, $4(x^2+2x)-(2x+2)(x-3)=0$, $2x^2+12x+6=0$, whose roots are (by the quadratic formula) $x=\dfrac{-6\pm\sqrt{36-4\cdot3}}{2}=-3\pm\sqrt{6}$. So the tangent line is horizontal at $x=3,-3\pm\sqrt{6}$.

**35.** Suppose the line is tangent to $y=x^2+1$ at $(x_0,y_0)$ and tangent to $y=-x^2-1$ at $(x_1,y_1)$. Since it's tangent to $y=x^2+1$, its slope is $2x_0$; since it's tangent to $y=-x^2-1$, its slope is $-2x_1$. Hence $x_1=-x_0$ and $y_1=-y_0$. Since the line passes through both points, its slope is $\dfrac{y_1-y_0}{x_1-x_0}=\dfrac{-2y_0}{-2x_0}=\dfrac{y_0}{x_0}=\dfrac{x_0^2+1}{x_0}$. Thus $2x_0=\dfrac{x_0^2+1}{x_0}$, so $2x_0^2=x_0^2+1$, $x_0^2=1$, and $x_0=\pm1$. So there are two lines which are tangent to both graphs, namely $y=2x$ and $y=-2x$.

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**36.** **(a)** Suppose $y = mx + b$ is tangent to $y = x^n + n - 1$ at $(x_0, y_0)$ and to $y = -x^n - n + 1$ at $(x_1, y_1)$. Then $m = nx_0^{n-1} = -nx_1^{n-1}$; since $n$ is even this implies that $x_1 = -x_0$. Again since $n$ is even, $y_1 = -x_1^n - n + 1 = -x_0^n - n + 1 = -(x_0^n + n - 1) = -y_0$. Thus the points $(x_0, y_0)$ and $(x_1, y_1)$ are symmetric with respect to the origin and both lie on the tangent line and thus $b = 0$. The slope $m$ is given by $m = nx_0^{n-1}$ and by $m = y_0/x_0 = (x_0^n + n - 1)/x_0$, hence $nx_0^n = x_0^n + n - 1$, $(n-1)x_0^n = n - 1$, $x_0^n = 1$. Since $n$ is even, $x_0 = \pm 1$. One easily checks that $y = nx$ is tangent to $y = x^n + n - 1$ at $(1, n)$ and to $y = -x^n - n + 1$ at $(-1, -n)$, while $y = -nx$ is tangent to $y = x^n + n - 1$ at $(-1, n)$ and to $y = -x^n - n + 1$ at $(1, -n)$.

**(b)** Suppose there is such a common tangent line with slope $m$. The function $y = x^n + n - 1$ is always increasing, so $m \geq 0$. Moreover the function $y = -x^n - n + 1$ is always decreasing, so $m \leq 0$. Thus the tangent line has slope 0, which only occurs on the curves for $x = 0$. This would require the common tangent line to pass through $(0, n-1)$ and $(0, -n+1)$ and do so with slope $m = 0$, which is impossible.

**37.** The line $y - x = 2$ has slope $m_1 = 1$ so we set $m_2 = \dfrac{d}{dx}(3x - \tan x) = 3 - \sec^2 x = 1$, or $\sec^2 x = 2$, $\sec x = \pm\sqrt{2}$ so $x = n\pi \pm \pi/4$ where $n = 0, \pm 1, \pm 2, \ldots$.

**38.** Solve $3x^2 - \cos x = 0$ to get $x = \pm 0.535428$.

**39.** $3 = f(\pi/4) = (M+N)\sqrt{2}/2$ and $1 = f'(\pi/4) = (M-N)\sqrt{2}/2$. Add these two equations to get $4 = \sqrt{2}M$, $M = 2^{3/2}$. Subtract to obtain $2 = \sqrt{2}N$, $N = \sqrt{2}$. Thus $f(x) = 2\sqrt{2}\sin x + \sqrt{2}\cos x$. $f'\left(\dfrac{3\pi}{4}\right) = -3$, so the tangent line is $y - 1 = -3\left(x - \dfrac{3\pi}{4}\right)$.

**40.** $f(x) = M\tan x + N\sec x$, $f'(x) = M\sec^2 x + N\sec x\tan x$. At $x = \pi/4$, $2M + \sqrt{2}N, 0 = 2M + \sqrt{2}N$. Add to get $M = -2$, subtract to get $N = \sqrt{2} + M/\sqrt{2} = 2\sqrt{2}$, $f(x) = -2\tan x + 2\sqrt{2}\sec x$. $f'(0) = -2$, so the tangent line is $y - 2\sqrt{2} = -2x$.

**41.** $f'(x) = 2xf(x)$, $f(2) = 5$

**(a)** $g(x) = f(\sec x)$, $g'(x) = f'(\sec x)\sec x\tan x = 2 \cdot 2f(2) \cdot 2 \cdot \sqrt{3} = 40\sqrt{3}$.

**(b)** $h'(x) = 4\left[\dfrac{f(x)}{x-1}\right]^3 \dfrac{(x-1)f'(x) - f(x)}{(x-1)^2}$, $h'(2) = 4\dfrac{5^3}{1}\dfrac{f'(2) - f(2)}{1} = 4 \cdot 5^3 \dfrac{2 \cdot 2f(2) - f(2)}{1} = 4 \cdot 5^3 \cdot 3 \cdot 5 = 7500$

## Chapter 2 Making Connections

**1.** **(a)** By property (ii), $f(0) = f(0+0) = f(0)f(0)$, so $f(0) = 0$ or 1. By property (iii), $f(0) \neq 0$, so $f(0) = 1$.

**(b)** By property (ii), $f(x) = f\left(\dfrac{x}{2} + \dfrac{x}{2}\right) = f\left(\dfrac{x}{2}\right)^2 \geq 0$. If $f(x) = 0$, then $1 = f(0) = f(x + (-x)) = f(x)f(-x) = 0 \cdot f(-x) = 0$, a contradiction. Hence $f(x) > 0$.

**(c)** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{f(x)f(h) - f(x)}{h} = \lim\limits_{h\to 0} f(x)\dfrac{f(h) - 1}{h} = f(x) \lim\limits_{h\to 0} \dfrac{f(h) - f(0)}{h} =$ $f(x)f'(0) = f(x)$

**2.** **(a)** By the chain rule and Exercise 1(c), $y' = f'(2x) \cdot \dfrac{d}{dx}(2x) = f(2x) \cdot 2 = 2y$.

**(b)** By the chain rule and Exercise 1(c), $y' = f'(kx) \cdot \dfrac{d}{dx}(kx) = kf'(kx) = kf(kx)$.

**(c)** By the product rule and Exercise 1(c), $y' = f(x)g'(x) + g(x)f'(x) = f(x)g(x) + g(x)f(x) = 2f(x)g(x) = 2y$, so $k = 2$.

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**(d)** By the quotient rule and Exercise 1(c), $h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2} = \dfrac{g(x)f(x) - f(x)g(x)}{g(x)^2} = 0$. As we will see in Theorem 4.1.2(c), this implies that $h(x)$ is a constant. Since $h(0) = f(0)/g(0) = 1/1 = 1$ by Exercise 1(a), $h(x) = 1$ for all $x$, so $f(x) = g(x)$.

**3. (a)** For brevity, we omit the "$(x)$" throughout.

$$(f \cdot g \cdot h)' = \frac{d}{dx}[(f \cdot g) \cdot h] = (f \cdot g) \cdot \frac{dh}{dx} + h \cdot \frac{d}{dx}(f \cdot g) = f \cdot g \cdot h' + h \cdot \left(f \cdot \frac{dg}{dx} + g \cdot \frac{df}{dx}\right)$$
$$= f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h'$$

**(b)** $(f \cdot g \cdot h \cdot k)' = \dfrac{d}{dx}[(f \cdot g \cdot h) \cdot k] = (f \cdot g \cdot h) \cdot \dfrac{dk}{dx} + k \cdot \dfrac{d}{dx}(f \cdot g \cdot h)$

$= f \cdot g \cdot h \cdot k' + k \cdot (f' \cdot g \cdot h + f \cdot g' \cdot h + f \cdot g \cdot h') = f' \cdot g \cdot h \cdot k + f \cdot g' \cdot h \cdot k + f \cdot g \cdot h' \cdot k + f \cdot g \cdot h \cdot k'$

**(c)** Theorem: If $n \geq 1$ and $f_1, \cdots, f_n$ are differentiable functions of $x$, then

$$(f_1 \cdot f_2 \cdot \cdots \cdot f_n)' = \sum_{i=1}^{n} f_1 \cdot \cdots \cdot f_{i-1} \cdot f_i' \cdot f_{i+1} \cdot \cdots \cdot f_n.$$

Proof: For $n = 1$ the statement is obviously true: $f_1' = f_1'$. If the statement is true for $n - 1$, then

$$(f_1 \cdot f_2 \cdot \cdots \cdot f_n)' = \frac{d}{dx}[(f_1 \cdot f_2 \cdot \cdots \cdot f_{n-1}) \cdot f_n] = (f_1 \cdot f_2 \cdot \cdots \cdot f_{n-1}) \cdot f_n' + f_n \cdot (f_1 \cdot f_2 \cdot \cdots \cdot f_{n-1})'$$

$$= f_1 \cdot f_2 \cdot \cdots \cdot f_{n-1} \cdot f_n' + f_n \cdot \sum_{i=1}^{n-1} f_1 \cdot \cdots \cdot f_{i-1} \cdot f_i' \cdot f_{i+1} \cdot \cdots \cdot f_{n-1} = \sum_{i=1}^{n} f_1 \cdot \cdots \cdot f_{i-1} \cdot f_i' \cdot f_{i+1} \cdot \cdots \cdot f_n$$

so the statement is true for $n$. By induction, it's true for all $n$.

**4. (a)** $[(f/g)/h]' = \dfrac{h \cdot (f/g)' - (f/g) \cdot h'}{h^2} = \dfrac{h \cdot \frac{g \cdot f' - f \cdot g'}{g^2} - \frac{f \cdot h'}{g}}{h^2} = \dfrac{f' \cdot g \cdot h - f \cdot g' \cdot h - f \cdot g \cdot h'}{g^2 h^2}$

**(b)** $[(f/g)/h]' = [f/(g \cdot h)]' = \dfrac{(g \cdot h) \cdot f' - f \cdot (g \cdot h)'}{(g \cdot h)^2} = \dfrac{f' \cdot g \cdot h - f \cdot (g \cdot h' + h \cdot g')}{g^2 h^2} =$

$= \dfrac{f' \cdot g \cdot h - f \cdot g' \cdot h - f \cdot g \cdot h'}{g^2 h^2}$

**(c)** $[f/(g/h)]' = \dfrac{(g/h) \cdot f' - f \cdot (g/h)'}{(g/h)^2} = \dfrac{\frac{f' \cdot g}{h} - f \cdot \frac{h \cdot g' - g \cdot h'}{h^2}}{(g/h)^2} = \dfrac{f' \cdot g \cdot h - f \cdot g' \cdot h + f \cdot g \cdot h'}{g^2}$

**(d)** $[f/(g/h)]' = [(f \cdot h)/g]' = \dfrac{g \cdot (f \cdot h)' - (f \cdot h) \cdot g'}{g^2} = \dfrac{g \cdot (f \cdot h' + h \cdot f') - f \cdot g' \cdot h}{g^2} =$

$= \dfrac{f' \cdot g \cdot h - f \cdot g' \cdot h + f \cdot g \cdot h'}{g^2}$

**5. (a)** By the chain rule, $\dfrac{d}{dx}\left([g(x)]^{-1}\right) = -[g(x)]^{-2} g'(x) = -\dfrac{g'(x)}{[g(x)]^2}$. By the product rule,

$$h'(x) = f(x) . \frac{d}{dx}\left([g(x)]^{-1}\right) + [g(x)]^{-1} . \frac{d}{dx}[f(x)] = -\frac{f(x)g'(x)}{[g(x)]^2} + \frac{f'(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}.$$

**(b)** By the product rule, $f'(x) = \dfrac{d}{dx}[h(x)g(x)] = h(x)g'(x) + g(x)h'(x)$. So

$$h'(x) = \frac{1}{g(x)}[f'(x) - h(x)g'(x)] = \frac{1}{g(x)}\left[f'(x) - \frac{f(x)}{g(x)}g'(x)\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}.$$

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