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Solutions for Chapter 2

2.1. The returns are given by $R_t = (P_t - P_{t-1})/P_{t-1}$, $t = 1, 2, 3, 4$. Hence,

$$R_1 = 0.0400, \quad R_2 = -0.0192, \quad R_3 = 0.0784, \quad R_4 = 0.0091.$$

The log-returns are given by $r_t = \log(1 + R_t)$, $t = 1, 2, 3, 4$. Hence,

$$r_1 = 0.0392, \quad r_2 = -0.0194, \quad r_3 = 0.0076, \quad r_4 = 0.0091.$$

2.2. Returns are given by $R_t = (P_t - P_{t-1})/P_{t-1}$ so that

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} - 1 = \frac{a\exp(bt) - a\exp\left(b(t-1)\right)}{a\exp\left(b(t-1)\right)} = \exp(b) - 1.$$

The log-returns are given by $r_t = \log(1 + R_t)$ so that

$$r_t = \log(\exp(b)) = b.$$

2.3. Using a Taylor's series approximation, $\log(1 + x) \doteq x$ for small $|x|$. Hence,

$$r_t = \log(1 + R_t) \doteq R_t \quad \text{for small} \quad |R_t|.$$

Including an additional term in the Taylor's series,

$$\log(1 + x) \doteq x - \frac{1}{2}x^2$$

for small $|x|$. Hence,

$$r_t = \log(1 + R_t) \doteq R_t - \frac{1}{2}R_t^2 \quad \text{for small} \quad |R_t|.$$

2.4. Returns are given by $R_t = (P_t + D_t)/P_{t-1} - 1$. Hence,

$$R_1 = 0.200, \quad R_2 = 0.125, \quad R_3 = 0.080.$$

2.5. Adjusted prices are given by $\bar{P}_3 = P_3 = \$5.40$,

$$\bar{P}_2 = \left(1 - \frac{D_3}{P_2}\right) P_2 = P_2 = \$4.80$$

and

$$\bar{P}_1 = \left(1 - \frac{D_2}{P_1}\right)\left(1 - \frac{D_3}{P_2}\right) P_1 = \left(1 - \frac{0.40}{4.00}\right) 4.00 = \$3.60.$$

2.6. (a) The single-period return at time t is given by

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1 = \frac{P_t + \alpha P_{t-1}}{P_{t-1}} - 1 = \frac{P_t}{P_{t-1}} - 1 + \alpha.$$

(b) Let $\bar{P}_t$, $t = 0, 1, 2, \ldots, T$ denote the sequence of adjusted prices. Then $\bar{P}_T = P_T$,

$$\bar{P}_{T-1} = \left(1 - \frac{D_T}{P_{T-1}}\right) P_{T-1} = (1 - \alpha)\, P_{T-1},$$

$$\bar{P}_{T-2} = \left(1 - \frac{D_T}{P_{T-1}}\right)\left(1 - \frac{D_{T-1}}{P_{T-2}}\right) P_{T-2} = (1 - \alpha)^2 P_{T-2}$$

and so on. The general relationship is

$$\bar{P}_{T-k} = (1 - \alpha)^k P_{T-k}.$$

2.7. Consider

$$\mathrm{Cov}(Y_t + X_t, Y_s + X_s) = \mathrm{Cov}(Y_t, Y_s) + \mathrm{Cov}(X_t, X_s) + \mathrm{Cov}(Y_t, X_s) + \mathrm{Cov}(Y_s, X_t).$$

Let γ_Y denote the covariance function of $\{Y_t : t = 1, 2, \ldots\}$ and let γ_X denote the covariance function of $\{X_t : t = 1, 2, \ldots\}$. Since these processes are both weakly stationary,

$$\mathrm{Cov}(Y_t + X_t, Y_s + X_s) = \gamma_Y(|t - s|) + \gamma_X(|t - s|) + \mathrm{Cov}(Y_t, X_s) + \mathrm{Cov}(Y_s, X_t).$$

However, since we do not know anything about the covariance of Y_t and X_s, it does not follow that the process $Y_1 + X_1, \ldots$ is weakly stationary. For instance, if Y_t and X_s are uncorrelated for all t, s, then it is weakly stationary. However, if the correlation of Y_t and X_s is $1/2$ if $t = s = 1$ and 0 otherwise, then the process is not weakly stationary.

2.8. (a) The mean function is given by

$$\mathrm{E}(Y_t) = \mathrm{E}(X_t - X_{t-1}) = \mathrm{E}(X_t) - \mathrm{E}(X_{t-1}) = 0$$

and the variance function is given by

$$\mathrm{Var}(Y_t) = \mathrm{Var}(X_t - X_{t-1}) = \mathrm{Var}(X_t) + \mathrm{Var}(X_{t-1}) - 2\mathrm{Cov}(X_t, X_{t-1}) = 2\sigma^2 - 2\gamma(1).$$

(b) The covariance function is given by

$$\begin{aligned}
\mathrm{Cov}(Y_t, Y_s) &= \mathrm{Cov}(X_t - X_{t-1}, X_s - X_{s-1}) \\
&= \mathrm{Cov}(X_t, X_s) + \mathrm{Cov}(X_{t-1}, X_{s-1}) - \mathrm{Cov}(X_{t-1}, X_s) - \mathrm{Cov}(X_t, X_{s-1}) \\
&= 2\gamma(|t - s|) - \gamma(|t - s - 1|) - \gamma(|t - s + 1|).
\end{aligned}$$

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(c) The mean and variance functions of the process are constant. Consider the term in the covariance function

$$\gamma(|t-s-1|)+\gamma(|t-s+1|).$$

If $t=s$ this is $2\gamma(|1|)=2\gamma(|t-s|+1)$. If $t\geq s+1$, then

$$\gamma(|t-s-1|)+\gamma(|t-s+1|)=\gamma(t-s-1)+\gamma(t-s+1)=\gamma(|t-s|-1)+\gamma(|t-s|+1).$$

Similarly, if $t\leq s-1$,

$$\gamma(|t-s-1|)+\gamma(|t-s+1|)=\gamma(s+1-t)+\gamma(s-1-t)=\gamma(|t-s|+1)+\gamma(|t-s|-1).$$

It follows that the covariance of Y_t, Y_s is a function of $|t-s|$ and, hence, the process is weakly stationary.

2.9. (a) $\mathrm{E}(X_t)=\mathrm{E}(ZZ_t)=\mathrm{E}(Z)\mathrm{E}(Z_t)=0$; hence, the mean function is 0. Let $\mu=\mathrm{E}(Z_t)$ and $\sigma^2=\mathrm{Var}(Z_t)$. Since $\mathrm{E}(X_t^2)=\mathrm{E}(Z^2Z_t^2)=\mathrm{E}(Z^2)\mathrm{E}(Z_t^2)=(\sigma^2+\mu^2)$, the variance function of the process is $\sigma^2+\mu^2$.

(b) Since $\mathrm{E}(X_t)=0$ for all t,

$$\mathrm{Cov}(X_t,X_s)=\mathrm{E}(X_tX_s)=\mathrm{E}(Z^2Z_tZ_s)=\mathrm{E}(Z^2)\mathrm{E}(Z_tZ_s)=0.$$

(c) Since the mean and variance functions are constant and the $X_1, X_2, \ldots$ are uncorrelated, it follows that $\{X_t : t=1,2,\ldots\}$ is a white noise process. Hence, it is also weakly stationary.

2.10. Since $\mathrm{E}(r_t)$ does not depend on t, clearly $\mathrm{E}(\tilde{r}_t)$ does not depend on t. Let $\sigma^2=\mathrm{Var}(r_t)$ and consider $\mathrm{Var}(\tilde{r}_t)$. Using the fomula for the variance of a sum,

$$\mathrm{Var}(\tilde{r}_t)=21\sigma^2+2\sum_{i<j}\mathrm{Cov}(\tilde{r}_{21(t-1)+i},\tilde{r}_{21(t-1)+j})$$

where the sum in this expression is over all i, j from 1 to 21 such that $i<j$. Note that, since $\{r_t : t=1,2,\ldots\}$ is weakly stationary,

$$\mathrm{Cov}(\tilde{r}_{21(t-1)+i},\tilde{r}_{21(t-1)+j})=\gamma(|i-j|)$$

where $\gamma(\cdot)$ is the autocovariance function of $\{r_t : t=1,2,\ldots\}$. It follows that $\mathrm{Var}(\tilde{r}_t)$ does not depend on t.

Now consider $\mathrm{Cov}(\tilde{r}_t,\tilde{r}_s)$ for $t\neq s$. Note that

$$\mathrm{Cov}(\tilde{r}_t,\tilde{r}_s)=\sum_{j=1}^{21}\sum_{i=1}^{21}\mathrm{Cov}(r_{21(t-1)+j},r_{21(s-1)+i}).$$

Since, for any i, j,

$$\mathrm{Cov}(r_{21(t-1)+j},r_{21(s-1)+i})=\gamma(|21(t-s)+j-i|)$$

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for any $j = 1, 2, \ldots, 21$,

$$\mathrm{Cov}(\tilde{r}_t, \tilde{r}_s) = \sum_{j=1}^{21} \sum_{i=1}^{21} \gamma(|21(t-s) + j - i|),$$

which clearly depends on t, s only through $t - s$. By symmetry of the covariance operator,

$$\sum_{j=1}^{21} \sum_{i=1}^{21} \gamma(|21(t-s) + j - i|) = \sum_{j=1}^{21} \sum_{i=1}^{21} \gamma(|21(s-t) + j - i|)$$

so that $\mathrm{Cov}(\tilde{r}_t, \tilde{r}_s)$ depends on t, s only through $|t - s|$. It follows that $\{\tilde{r}_t : t = 1, 2, \ldots\}$ is weakly stationary.

2.11. Since $\mathrm{E}(X_j) = \mu$, $j = 1, \ldots, n$,

$$\mathrm{E}(Y_k) = \frac{1}{w} \sum_{j=k+1}^{k+w} \mathrm{E}(X_j) = \frac{1}{w} w\mu = \mu, \quad k = 1, \ldots, n - w.$$

Since $X_1, \ldots, X_n$ are independent with $\mathrm{Var}(X_j) = \sigma^2$, $j = 1, \ldots, n$,

$$\mathrm{Var}(Y_k) = \frac{1}{w^2} \sum_{j=k+1}^{k+w} \mathrm{Var}(X_j) = \frac{1}{w^2} w\sigma^2 = \frac{1}{w}\sigma^2, \quad k = 1, \ldots, n - w.$$

Consider $\mathrm{Cov}(Y_i, Y_k)$, where $i < k$. If $k > i + w$, then Y_i and Y_k have no terms in common so that $\mathrm{Cov}(Y_i, Y_k) = 0$. Otherwise, the sums

$$\sum_{j=i+1}^{i+w} X_j \quad \text{and} \quad \sum_{\ell=k+1}^{k+w} X_\ell$$

have terms $X_{k+1}, \ldots, X_{i+w}$ in common so that

$$\mathrm{Cov}(Y_i, Y_k) = \frac{i - k + w}{w^2}\sigma^2.$$

Since $\mathrm{E}(Y_k)$ and $\mathrm{Var}(Y_k)$ are constant and $\mathrm{Cov}(Y_i, Y_k)$ depends only on $k - i$, it follows that the process $Y_1, \ldots, Y_{n-w}$ is weakly stationary with mean function μ and variance function σ^2/w.

The correlation of Y_i and Y_k is

$$\frac{((i - k + w)/w^2)\sigma^2}{\sigma^2/w} = 1 - \frac{k - i}{w}$$

so that the correlation function of the process is

$$\rho(h) = 1 - \frac{|h|}{w}, \quad h = 1, 2, \ldots.$$

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2.12. (a) Let $\mu_X = \mathrm{E}(X_t)$, $\sigma_X^2 = \mathrm{Var}(X_t)$, $\mu_Y = \mathrm{E}(Y_t)$, and $\sigma_Y^2 = \mathrm{Var}(Y_t)$. Then

$$\mathrm{E}(X_t + Y_t) = \mathrm{E}(X_t) + \mathrm{E}(Y_t) = \mu_X + \mu_Y,\ t = 1, 2, \ldots,$$

$$\mathrm{Var}(X_t + Y_t) = \mathrm{Var}(X_t) + \mathrm{Var}(Y_t) = \sigma_X^2 + \sigma_Y^2,\ t = 1, 2, \ldots$$

and for $t \neq s$,

$$\mathrm{Cov}(X_t + Y_t, X_s + Y_s) = \mathrm{Cov}(X_t, X_s) + \mathrm{Cov}(X_t, Y_s) + \mathrm{Cov}(Y_t, X_s) + \mathrm{Cov}(Y_t, Y_s) = 0.$$

It follows that $\{X_t + Y_t : t = 1, 2, \ldots\}$ is a weak white noise process.

(b) Using the same notation as in part (a),

$$\mathrm{E}(X_t Y_t) = \mathrm{E}(X_t)\mathrm{E}(Y_t) = \mu_X \mu_Y,\ t = 1, 2, \ldots;$$

note that

$$\mathrm{E}\{(X_t Y_t)^2\} = \mathrm{E}(X_t^2)\mathrm{E}(Y_t^2) = (\mu_X^2 + \sigma_X^2)(\mu_Y^2 + \sigma_Y^2)$$

so that

$$\mathrm{Var}(X_t Y_t) = (\mu_X^2 + \sigma_X^2)(\mu_Y^2 + \sigma_Y^2) - \mu_X^2 \mu_Y^2,\ t = 1, 2, \ldots.$$

Similarly, for $t \neq s$,

$$\mathrm{E}(X_t Y_t X_s Y_s) = \mu_X \mu_Y \mu_X \mu_Y = \mu_X^2 \mu_Y^2$$

so that

$$\mathrm{Cov}(X_t Y_t, X_s Y_s) = \mu_X^2 \mu_Y^2 - (\mu_X \mu_Y)^2 = 0.$$

It follows that $\{X_t Y_t : t = 1, 2, \ldots\}$ is a weak white noise process.

2.13. (a) The following R commands may be used to download the necessary price data.

```
> library(tseries)
> x<-get.hist.quote(instrument="PZZA", start="2012-12-31", end="2015-12-31",
+     quote="AdjClose", compression="d")
> pzza0<-as.vector(x)
```

(b) The returns corresponding to the prices downloaded in part (a) may be calculated using the commands

```
> length(pzza0)
[1] 757
> pzza<-(pzza0[-1]-pzza0[-757])/pzza0[-757]
```

(c) The summary statistics for the returns are

```
> summary(pzza)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.1210 -0.0074  0.0010  0.0011  0.0097  0.0804
```

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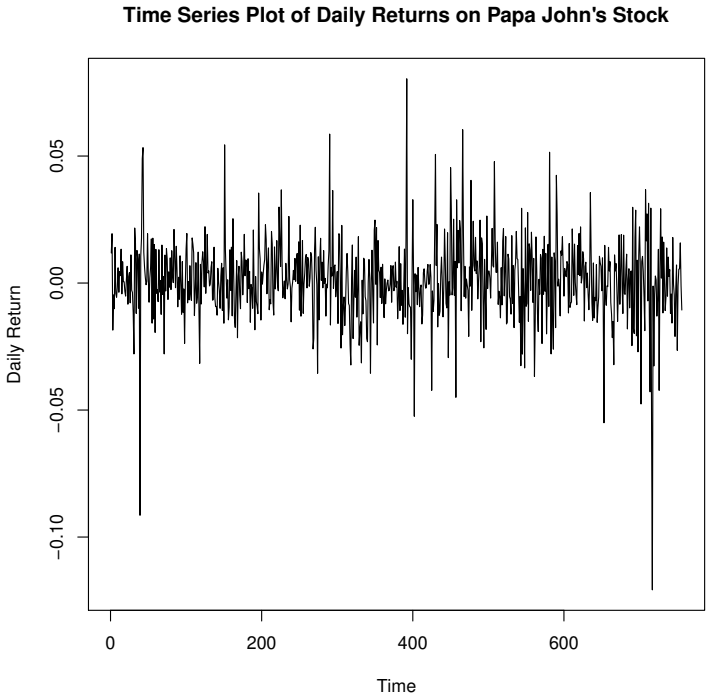


FIGURE 2.1
Plot in Exercise 2.13

(d) The time series plot of the returns may be constructed using the commands

```
> plot(pzza, type="l", xlab="Time", ylab="Daily Return")
> title(main="Time Series Plot of Daily Returns on Papa John's Stock")
```

The plot is given in Figure 2.1.

2.14. (a) The following R commands may be used to download the necessary price data.

```
> library(tseries)
> x<-get.hist.quote(instrument="PZZA", start="2010-12-31", end="2015-12-31",
+     quote="AdjClose", compression="m")
> pzza0<-as.vector(x)
```

(b) The returns corresponding to the prices downloaded in part (a) may be calculated using the commands

```
> length(pzza0)
[1] 61
> pzza.m<-(pzza0[-1]-pzza0[-61])/pzza0[-61]
```

(c) The summary statistics are

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```
> summary(pzza.m)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.1780 -0.0072  0.0216  0.0265  0.0696  0.1890
```

(d) The time series plot of the returns may be constructed using the commands

```
> plot(pzza.m, type="l", xlab="Time", ylab="Monthly Return")
> title(main="Time Series Plot of Monthly Returns on Papa John's Stock")
```

The plot is given in Figure 2.2.

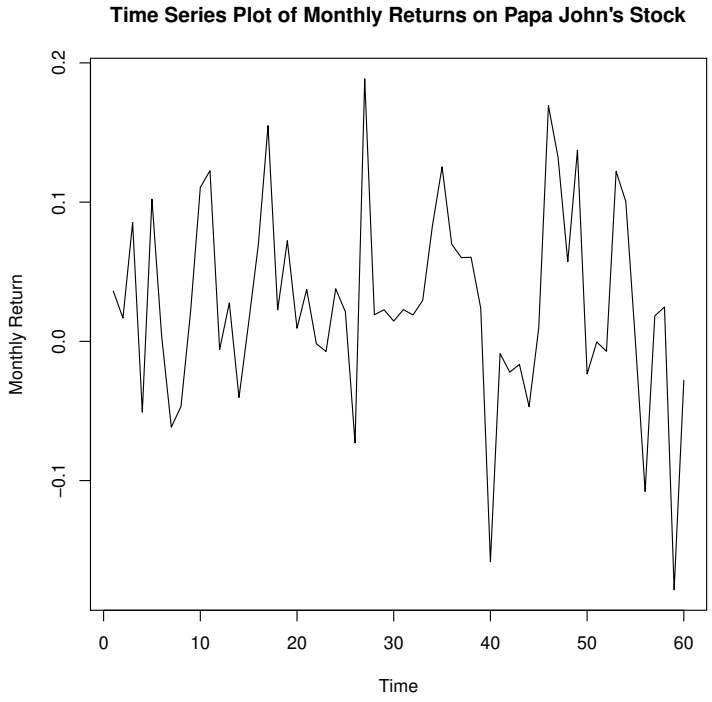


FIGURE 2.2
Plot in Exercise 2.14

2.15. The running means may be calculated and the plot constructed using the following commands.

```
> library(gtools)
>  pzza.rmean<-running(pzza.m, fun=mean, width=12)
>  mean(pzza.m) + 2*sd(pzza.m)/(12^.5)
[1] 0.0677
>  mean(pzza.m) - 2*sd(pzza.m)/(12^.5)
[1] -0.0147
>  plot(pzza.rmean, type="l", ylim=c(-.02, .07), xlab="Time", ylab="Return")
```

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```
> title(main="Running Means for Monthly Returns on Papa John's Stock")
> lines(1:49, rep(0.0677,49), lty=2)
> lines(1:49, rep(-0.0147,49), lty=2)
```

The plot is given in Figure 2.3. According to this plot, there is no evidence of non-stationarity in the returns.

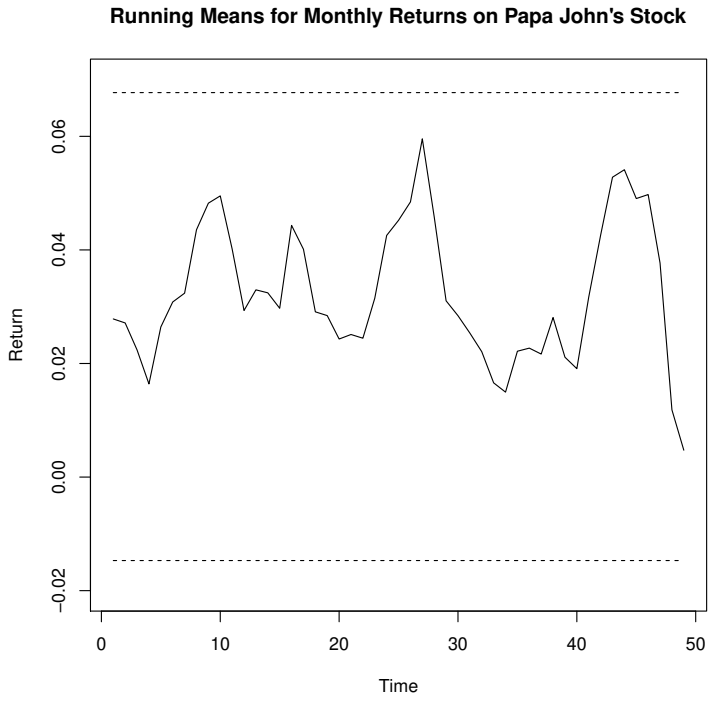


FIGURE 2.3
Plot in Exercise 2.15

2.16. The running standard deviations may be calculated and the plot may be constructed using the following commands.

```
> pzza.rsd<-running(pzza.m, fun=sd, width=12)
> log(sd(pzza.m)) + (2/11)^.5
[1] -2.21
> log(sd(pzza.m)) - (2/11)^.5
[1] -3.07
> plot(log(pzza.rsd), type="l", ylim=c(-3.6, -2), ylab="log of running sd",
+  xlab="time")
> title(main="Log of Running SDs of Returns on Papa John's Stock")
> lines(1:49, rep(-2.21, 49), lty=2)
> lines(1:49, rep(-3.07, 49), lty=2)
```

The plot is given in Figure 2.4. According to this plot, there is some evidence of non-stationarity of the returns. There is a relatively long period of relatively small variability, as well as brief periods of relatively large variability.

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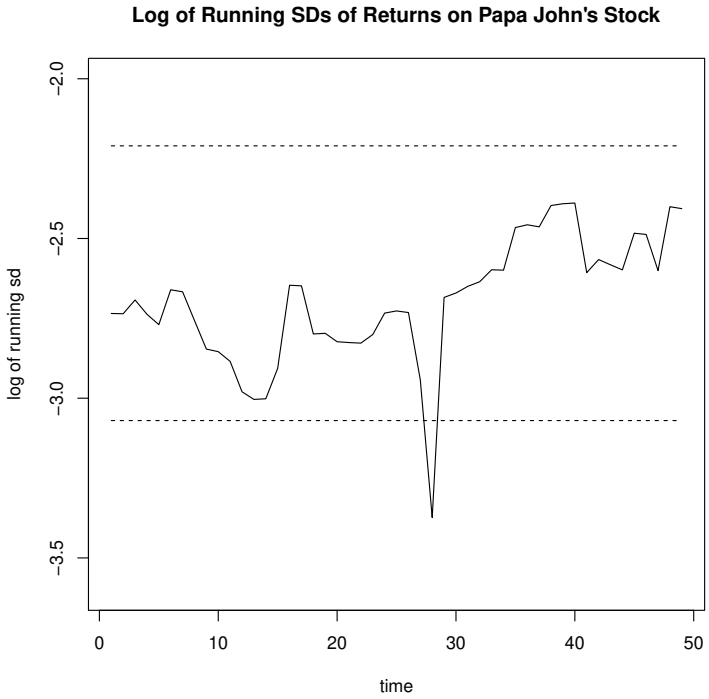


FIGURE 2.4
Plot in Exercise 2.16

2.17. The autocorrelation function based on the daily returns may be calculated using the command

```
>  print(acf(pzza, lag.max=20))

Autocorrelations of series pzza, by lag

     0      1      2      3      4      5      6      7      8      9     10
 1.000 -0.011 -0.013  0.050 -0.013 -0.061  0.005 -0.036  0.062  0.052 -0.007
    11     12     13     14     15     16     17     18     19     20
-0.027  0.044 -0.016 -0.023  0.027  0.029 -0.018  0.032  0.008 -0.028
```

The plot is given in Figure 2.5.

The estimated autocorrelation function based on the monthly returns is given by

```
> print(acf(pzza.m, lag.max=12))

Autocorrelations of series pzza.m, by lag

     0      1      2      3      4      5      6      7      8      9     10
 1.000  0.167 -0.009 -0.061 -0.057 -0.179 -0.146 -0.053  0.103 -0.218 -0.064
    11     12
 0.049  0.015
```

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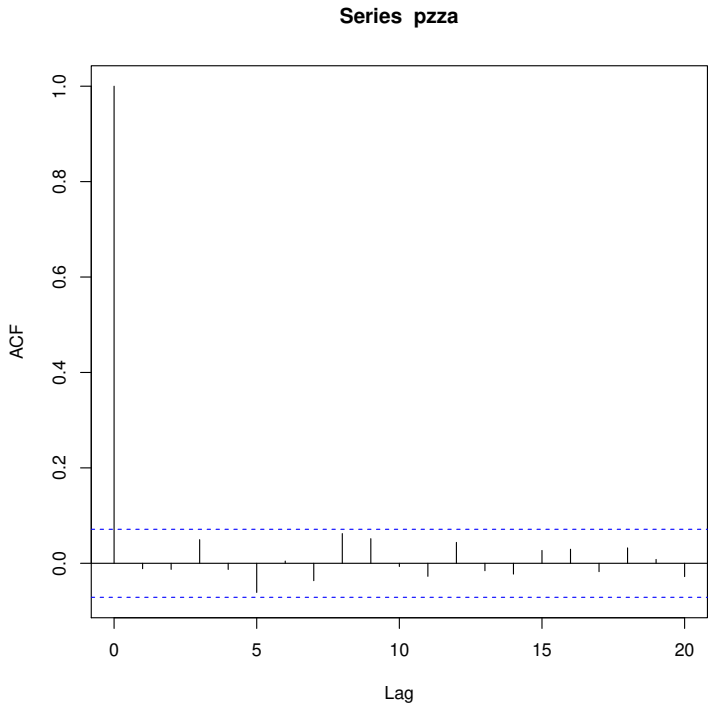


FIGURE 2.5
ACF for Daily Returns in Exercise 2.17

The plot is given in Figure 2.6. The autocorrelations are all small and, hence, the results are consistent with the assumption that the returns are uncorrelated random variables.

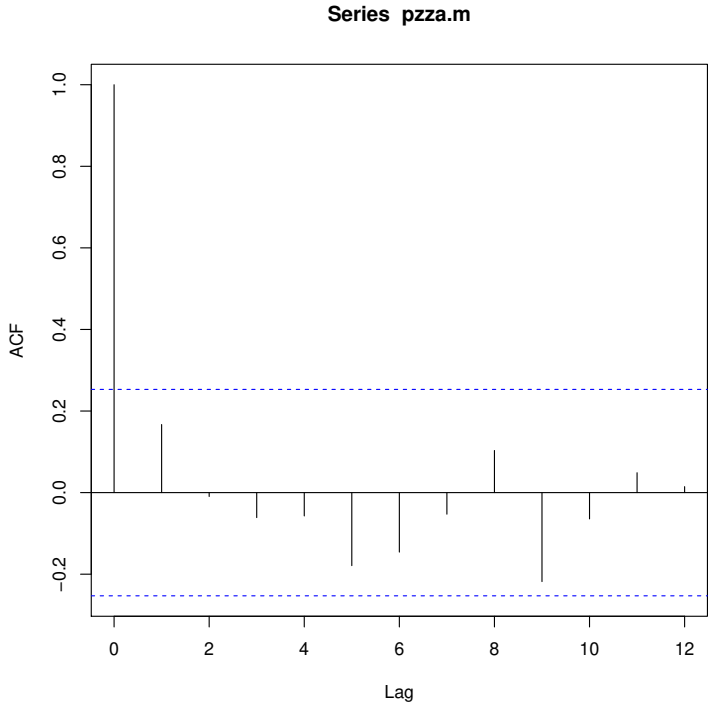


FIGURE 2.6
ACF for Monthly Returns in Exercise 2.17

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2.18. The daily returns on Papa John's stock are stored in the R variable `pzza`. To construct a normal probability plot of these data, we may use the commands

```
> qqnorm(pzza)
> abline(a=mean(pzza), b=sd(pzza))
```

The plot is given in Figure 2.7. The plot is very similar to the one in Figure 2.11 in the text; it suggests that the distribution of the daily returns on Papa John's stock is long-tailed relative to a normal distribution.

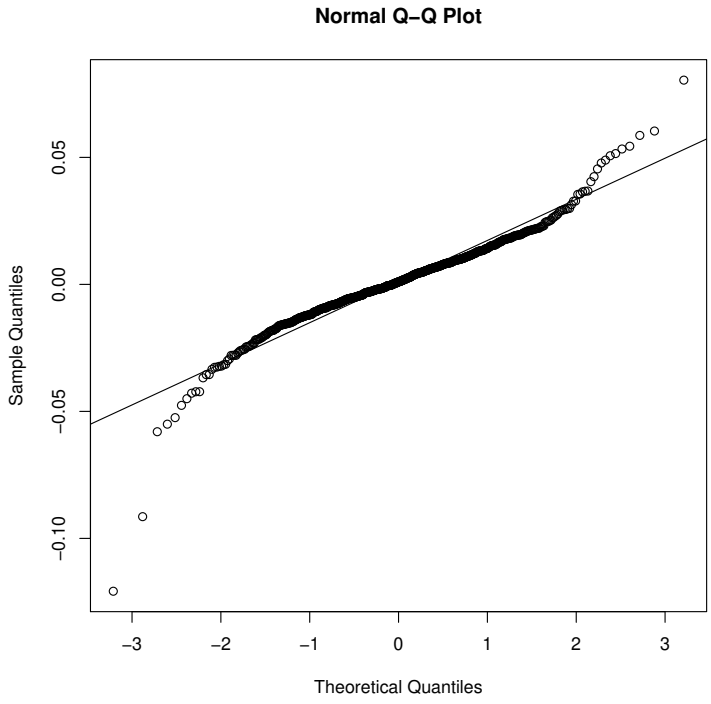


FIGURE 2.7
Q-Q Plot for Daily Returns in Exercise 2.18

The monthly returns on Papa John's stock are stored in the R variable `pzza.m`. To construct a normal probability plot of these data, we may use the commands

```
> qqnorm(pzza.m)
> abline(a=mean(pzza.m), b=sd(pzza.m))
```

The plot is given in Figure 2.8. The plot suggests that the distribution of the monthly returns on Papa John's stock is more nearly normal than is the distribution of daily returns, although there is some evidence of asymmetry, with a slightly-long left tail.

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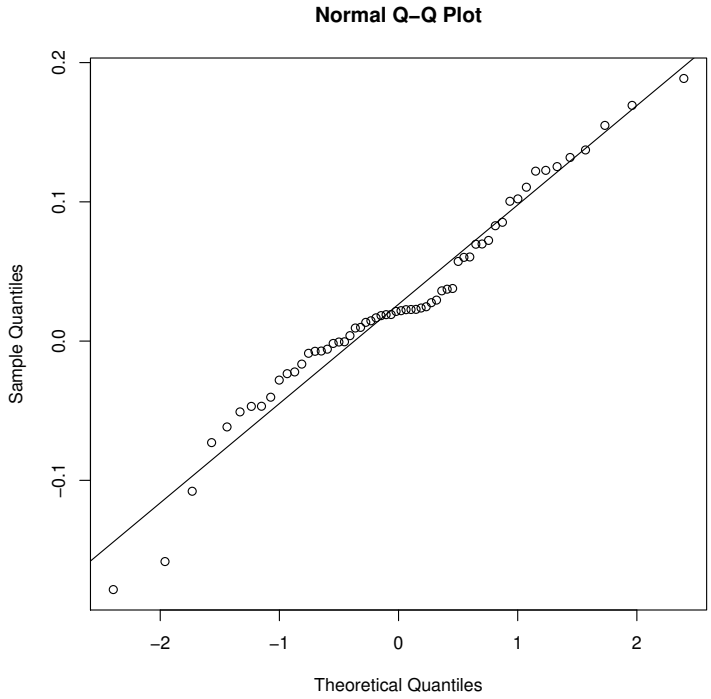


FIGURE 2.8
Q-Q Plot for Monthly Returns in Exercise 2.18

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