Principles of Corporate Finance 12th edition Brealey Solutions Manual

SKU: 9781259144387-SOLUTIONS

CHAPTER 2

How to Calculate Present Values

The values shown in the solutions may be rounded for display purposes. However, the answers were derived using a spreadsheet without any intermediate rounding.

Answers to Problem Sets

1. $C_t = PV \times (1 + r)^t$
   $C_8 = \$100 \times 1.15^8$
   $C_8 = \$305.90$

Est time: 01-05

2. $PV = C_t / DF_t$
   $DF_t = \$125 / \$139$
   $DF_t = .8993$

Est time: 01-05

3. $PV = C_t / (1 + r)^t$
   $PV = \$374 / 1.09^9$
   $PV = \$172.20$

Est time: 01-05

4. a. $PV = C_1 / (1 + r)^1 + C_2 / (1 + r)^2 + C_3 / (1 + r)^3$
   $PV = \$432 / 1.15 + \$137 / 1.15^2 + \$797 / 1.15^3$
   $PV = \$1,003.28$

   b. NPV = PV – investment
   NPV = $1,003.28 – 1,200
   NPV = –$196.72

Est time: 01-05

5. a. False. The opportunity cost of capital varies with the risks associated with each individual project or investment. The cost of borrowing is unrelated to these risks.

   b. True. The opportunity cost of capital depends on the risks associated with each project and its cash flows.

   c. True. The opportunity cost of capital is dependent on the rates of returns shareholders can earn on the own by investing in projects with similar risks

   d. False. Bank accounts, within FDIC limits, are considered to be risk-free. Unless an investment is also risk-free, its opportunity cost of capital must be adjusted upward to account for the associated risks.

Est time: 01-05

6. $NPV = C / r$ – investment

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NPV = $138 / .09 − $1,548
NPV = −$14.67

Est time: 01-05

7. PV = C / (r – g)
PV = $4 / (.14 − .04)
PV = $40

Est time: 01-05

8. a. PV = C / r
PV = $1 / .10
PV = $10

b. $PV_7 = (C_8 / r)$
$PV_{0\ approx} = (C_8 / r) / 2$
$PV_{0\ approx}$ = ($1 / .10) / 2
$PV_{0\ approx}$ = $5

c. A perpetuity paying $1 starting now would be worth $10 (part a), whereas a perpetuity starting in year 8 would be worth roughly $5 (part b). Thus, a payment of $1 for the next seven years would also be worth approximately $5 (= $10 – 5).

d. PV = C / (r − g)
PV = $10,000 / (.10 − .05)
PV = $200,000

Est time: 06-10

9. The basic present value formula is: PV = $C / (1 + r)^t$

a. PV = $100 / 1.01^{10}
PV = $90.53

b. PV = $100 / 1.13^{10}
PV = $29.46

c. PV = $100 / 1.25^{15}
PV = $3.52

d. $PV = C_1 / (1 + r) + C_2 / (1 + r)^2 + C_3 / (1 + r)^3$
PV = $100 / 1.12 + $100 / 1.12^2 + $100 / 1.12^3
PV = $240.18

Est time: 01-05

10. a. $FV = C \times e^{rt}$
FV = $1,000 × $e^{.12 \times 5}$
FV = $1,822.12

b. $PV = C / e^{rt}$

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PV = \$5,000,000 / $e^{.12 \times 8}$
PV = \$1,914,464

c. PV = C (1 / r – 1 / re^{rt})
PV = \$2,000 (1 / .12 – 1 / .12$e^{.12 \times 15}$)
PV = \$13,911.69

Est time: 01-05

11.

a. C_t = PV × $(1 + r)^t$
C_t = \$10,000,000 x $(1.06)^4$
C_t = \$12,624,770

b. C_t = PV × $[1+ (r / m)^{mt}$
C_t = \$10,000,000 × $[1 + (.06 / 12)]^{12 \times 4}$
C_t = \$12,704,892

b. C_t = PV × e^{rt}
C_t = \$10,000,000 × $e^{.06 \times 4}$
C_t = \$12,712,492

Est time: 01-05

12. a. PV = C_t / $(1 + r)^t$
PV = \$10,000 / 1.05^5
PV = \$7,835.26

b. PV = $C((1 / r) – \{1 / [r(1 + r)^t]\})$
PV = \$12,000((1 / .08) – {1 / [.08$(1.08)^6$]})
PV = \$55,474.56

c. C_t = PV × $(1 + r)^t$
C_t = (\$60,476 − 55,474.56) × 1.08^6
C_t = \$7,936.66

Est time: 06-10

13. a. DF_1 = 1 / (1 + r)
r = (1 – .905) / .905
r = .1050, or 10.50%

b. DF_2 = 1 / $(1 + r)^2$
DF_2 = 1 / 1.105^2
DF_2 = .8190

c. $PVAF_2$ = DF_1 + DF_2
$PVAF_2$ = .905 + .819
$PVAF_2$ = 1.7240

d. PVA = C × $PVAF_3$

$PVAF_3$ = \$24.65 / \$10

$PVAF_3$ = 2.4650

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e. $PVAF_3 = PVAF_2 + DF_3$

$DF_3 = 2.465 - 1.7240$

$DF_3 = .7410$

Est time: 06-10

14. a. $NPV = -\text{Investment} + C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$NPV = -\$800{,}000 + \$170{,}000 \times ((1 / .14) - \{1 / [.14(1.14)^{10}]\})$

$NPV = \$86{,}739.66$

b. After five years, the factory's value will be the present value of the five remaining year's of cash flows.

$PV = \$170{,}000 \times ((1 / .14) - \{1 / [.14(1.14)^{(10-5)}]\})$

$PV = \$583{,}623.76$

Est time: 01-05

15. $NPV = \sum_{t=0}^{10} \frac{C_t}{(1.12)^t}$

$NPV = -\$380{,}000 + \$50{,}000 / 1.12 + \$57{,}000 / 1.12^2 + \$75{,}000 / 1.12^3 + \$80{,}000 / 1.12^4 + \$85{,}000 / 1.12^5 + \$92{,}000 / 1.12^6 + \$92{,}000 / 1.12^7 + \$80{,}000 / 1.12^8 + \$68{,}000 / 1.12^9 + \$50{,}000 / 1.12^{10}$

$NPV = \$23{,}696.15$

Est time: 01-05

16. The opportunity cost of capital refers to the rate of return a firm's shareholders could earn on their own by investing at the same level of risk. Thus, when a firm considers a new project, it is the risk level of the project that determines opportunity cost of capital for that project.

Est time: 06-10

17. $NPV = C_0 + C_1 / (1 + r) + C_2 / (1 + r)^2 + C_3 / (1 + r)^3$
$NPV = -\$400{,}000 + \$100{,}000 / 1.12 + \$200{,}000 / 1.12^2 + \$300{,}000 / 1.12^3$
$NPV = \$62{,}258.56$

Period		Present Value
0		–$400,000.00
1	$100,000 / 1.12 =	89,285.71
2	$200,000 / 1.12² =	159,438.78
3	$300,000 / 1.12³	213,534.07

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		$ 62,258.56

Est time: 01-05

18. a. $NPV = -Investment + PVA_{operating\ cash\ flows} - PV_{refits} + PV_{scrap\ value}$

$NPV = -\$8{,}000{,}000 + (\$5{,}000{,}000 - 4{,}000{,}000) \times ((1 / .08) - \{1 / [.08(1.08)^{15}]\}) -$

$(\$2{,}000{,}000 / 1.08^{5} + \$2{,}000{,}000 / 1.08^{10}) + \$1{,}500{,}000 / 1.08^{15}$

$NPV = -\$8{,}000{,}000 + 8{,}559{,}479 - 2{,}287{,}553 + 472{,}863$

$NPV = -\$1{,}255{,}212$

b. The cost of borrowing does not affect the NPV because the opportunity cost of capital depends on the use of the funds, not the source.

Est time: 06-10

19. a. $PV = C_0$
$PV = \$100{,}000$

b. $PV = C_t / (1 + r)^t$
$PV = \$180{,}000 / 1.12^{5}$
$PV = \$102{,}136.83$

c. $PV = C / r$
$PV = \$11{,}400 / .12$
$PV = \$95{,}000$

d. $PV = C \times ((1 / r) - \{1 / - [r(1 + r)^t]\})$
$PV = \$19{,}000 \times ((1 / .12) - \{1 / - [.12(1.12)^{10}]\})$
$PV = \$107{,}354.24$

e. $PV = C / (r - g)$
$PV = \$6{,}500 / (.12 - .05)$
$PV = \$92{,}857.14$

Prize (d) is the most valuable because it has the highest present value.

Est time: 01-05

20. $C = PVA / ((1 / r) - \{1 / [r(1 + r)^t]\})$

$C = \$20{,}000 / ((1 / .08) - \{1 / [.08(1 + .08)^{12}]\})$

$C = \$2{,}653.90$

Est time: 01-05

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21. $PV = C_t / (1 + r)^t$

$PV = \$20{,}000 / 1.10^5$

$PV = \$12{,}418.43$

$C = PVA / ((1 / r) - \{1 / [r(1 + r)^t]\})$

$C = \$12{,}418.43 / ((1 / .10) - \{1 / [.10 (1 + .10)^5]\})$

$C = \$3{,}275.95$

Est time: 06-10

22. The fact that Kangaroo Autos is offering "free credit" tells us what the cash payments are; it does not change the fact that money has time value.

Present value of payments to Kangaroo Auto:

$PV = \text{Down payment} + C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
$PV = \$1{,}000 + \$300 \times ((1 + .0083) - \{1 / [.0083(1 + .0083)^{30}]\})$

$PV = \$8{,}938.02$

Present value of car at Turtle Motors:

PV = price of car – discount
PV = $10,000 – 1,000

PV = $9,000

Kangaroo Autos offers the best deal because it has the lower present value of costs.

Est time: 01-05

23. Use the formula: $NPV = -C_0 + C_1 / (1 + r) + C_2 / (1 + r)^2$

$NPV_{5\%} = -\$700{,}000 + \$30{,}000 / 1.05 + \$870{,}000 / 1.05^2$

$NPV_{5\%} = \$117{,}687.07$

$NPV_{10\%} = -\$700{,}000 + \$30{,}000 / 1.10 + \$870{,}000 / 1.10^2$

$NPV_{10\%} = \$46{,}280.99$

$NPV_{15\%} = -\$700{,}000 + \$30{,}000 / 1.15 + \$870{,}000 / 1.15^2$

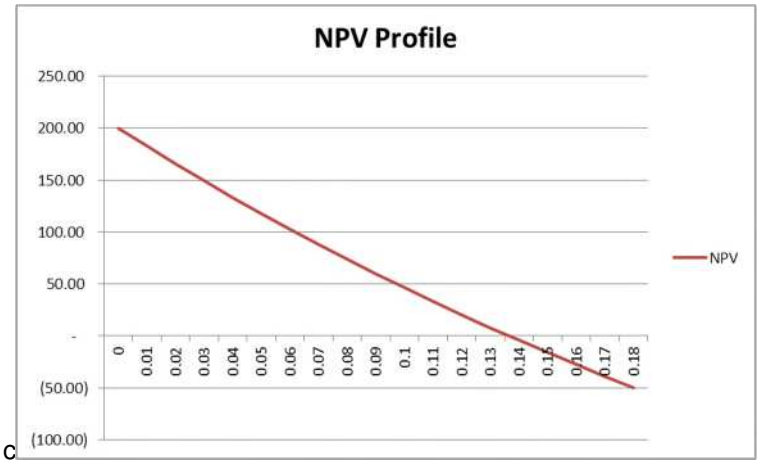
$NPV_{15\%} = -\$16{,}068.05$

The figure below shows that the project has a zero NPV at about 13.65%.

$NPV_{13.65\%} = -\$700{,}000 + \$30{,}000 / 1.1365 + \$870{,}000 / 1.1365^2$

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$NPV_{13.65\%} = -\$36.83$

c 

Est time: 06-10

24. a. $PV_{OA} = C / r$

$PV_{OA} = \$100 / .07$

$PV_{OA} = \$1{,}428.57$

b. $PV_{AD} = C / r \times (1 + r)$

$PV_{AD} = \$100 / .07 \times (1 + .07)$

$PV_{AD} = \$1{,}528.57$

c. To find the present value with payments spread evenly over the year, use the continuously compounded rate that equates to 7% compounded annually. This rate is found using natural logarithms.

$PV_{CC} = C / r_{CC}$

$PV_{CC} = \$100 / \text{Ln}(1 + .07)$

$PV_{CC} = \$1{,}478.01$

For informational purposes, the continuously compounded rate is:

$\text{Ln}(1 + .07) = .0677$, or 6.77%

The sooner payments are received, the more valuable they are.

Est time: 06-10

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25. a. PV = C / r
PV = $1 billion / .08
PV = $12.5 billion

b. PV = $C / (r - g)$
PV = $1 billion / (.08 – .04)
PV = $25.0 billion

c. PV = $C \times ((1 / r) - \{1 / [r (1 + r)^t]\})$
PV = $1 billion $\times ((1 / .08) - \{1 / [.08(1 + .08)^{20}]\})$
PV = $9.818 billion

d. The continuously compounded equivalent to an annually compounded rate of 8% is approximately 7.7%, which is computed as:

Ln(1.08) = .077, or 7.7%

PV = $C \times \{(1 / r) - [1 / (r \times e^{rt})]\}$

PV = $1 billion $\times \{(1 / .077) - [1 / (.077 - e^{.077 \times 20})]\}$

PV = $10.206 billion

This result is greater than the answer in Part (c) because the endowment is now earning interest during the entire year.

Est time: 06-10

26. Annual compounding:

$C_t = PV \times (1 + r)^t$
$C_{20} = \$100 \times 1.15^{20}$
$C_{20} = \$1,636.65$

Continuous compounding:

$C_t = PV \times e^{rt}$
$C_{20} = \$100 \times e^{.15 \times 20}$
$C_{20} = \$2,008.55$

Est time: 01-05

27. One way to approach this problem is to solve for the present value of:

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(1) $100 per year for 10 years, and
(2) $100 per year in perpetuity, with the first cash flow at year 11.

If this is a fair deal, these present values must be equal, and thus we can solve for the interest rate (r).

The present value of $100 per year for 10 years is:

$$PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$$

$$PV = \$100 \times ((1 / r) - \{1 / [r(1 + r)^{10}]\})$$

The present value, as of year 0, of $100 per year forever, with the first payment in year 11, is:

$$PV = (C / r) / (1 + r)^t$$

$$PV = (\$100 / r) / (1 + r)^{10}$$

Equating these two present values, we have:

$$\$100 \times ((1 / r) - \{1 / [r(1 + r)^{10}]\}) = (\$100 / r) / (1 + r)^{10}$$

Using trial and error or algebraic solution, we find that $r = 7.18\%$.

Est time: 06-10

28. **a.** $C_t = PV \times (1 + r)^t$

$C_1 = \$1 \times 1.12^1 = \$1.1200$
$C_5 = \$1 \times 1.12^5 = \$1.7623$
$C_{10} = \$1 \times 1.12^{10} = \$9.6463$

b. $C_t = PV \times (1 + r / m)^{mt}$

$C_1 = \$1 \times [1 + (.117 / 2)^{2 \times 1} = \$1.1204$
$C_5 = \$1 \times [1 + (.117 / 2)^{2 \times 5} = \$1.7657$
$C_{10} = \$1 \times [1 + (.117 / 2)^{2 \times 20} = \$9.7193$

c. $C_t = PV \times e^{mt}$

$C_1 = \$1 \times e^{(.115 \times 1)} = \$1.1219$
$C_5 = \$1 \times e^{(.115 \times 5)} = \$1.7771$
$C_{10} = \$1 \times e^{(.115 \times 20)} = \$9.9742$

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The preferred investment is (c) because it compounds interest faster and produces the highest future value at any point in time.

Est time: 06-10

29. First, find the semiannual rate that is equivalent to the annual rate:

$1 + r = (1 + r / 2)^2$

$1.08 = (1 + r / 2)^2$

$r / 2 = 1.08^{.5} - 1$

$r = .039230$, or 3.9230%

$PV = C_0 + C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$PV = \$100{,}000 + \$100{,}000 \times ((1 / .039230) - \{1 / [.039230(1 + .039230)^9]\})$

$PV = \$846{,}147.28$

Est time: 06-10

30. a. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$PV = (\$9{,}420{,}713 / 19) \times ((1 / .08) - \{1 / [.08(1 + .08)^{19}]\})$

$PV = \$4{,}761{,}724$

b. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

$\$4{,}200{,}000 = (\$9{,}420{,}713 / 19) \times ((1 / r) - \{1 / [r(1 + r)^t]\})$

Using Excel or a financial calculator, we find that $r = 9.81\%$.

Est time: 06-10

31. a. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
$PV = \$70{,}000 \times ((1 / .08) - \{1 / [.08(1 + .08)^8]\})$
$PV = \$402{,}264.73$

b.

Year	Beginning Balance	Interest Payment	Principal Payment	Ending Balance
1	$402,264.73	$32,181.18	$37,818.82	$364,445.90
2	364,445.90	29,155.67	40,844.33	323,601.58
3	323,601.58	25,888.13	44,111.87	279,489.70
4	279,489.70	22,359.18	47,640.82	231,848.88
5	231,848.88	18,547.91	51,452.09	180,396.79
6	180,396.79	14,431.74	55,568.26	124,828.53
7	124,828.53	9,986.28	60,013.72	64,814.81
8	64,814.81	5,185.19	64,814.81	0.00

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Est time: 06-10

32. a. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
$C = \$2{,}000{,}000 / ((1 / .08) - \{1 / [.08(1 + .08)^{15}]\})$
$C = \$233{,}659.09$

b. $r = (1 + R) / (1 + h) - 1$

$r = 1.08 / 1.04 - 1$

$r = .0385$, or 3.85%

$PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
$C = \$2{,}000{,}000 / ((1 / .0385) - \{1 / [.0385(1 + .0385)^{15}]\})$
$C = \$177{,}952.49$

The retirement expenditure amount will increase by 4% annually.

Est time: 06-10

33. a. $PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
$PV = \$50{,}000 \times ((1 / .055) - \{1 / [.055(1 + .055)^{12}]\})$
$PV = \$430{,}925.89$

b. Since the payments now arrive six months earlier than previously:

$PV = \$430{,}925.89 \times \{1 + [(1 + .055)^{.5} - 1]\}$
$PV = \$442{,}617.74$

Est time: 06-10

34. $C_t = PV \times (1 + r)^t$

$C_t = \$1{,}000{,}000 \times (1.035)^3$

$C_t = \$1{,}108{,}718$

Annual retirement shortfall = 12 × (monthly aftertax pension + monthly aftertax Social Security – monthly living expenses)
Annual retirement shortfall = 12 × (\$7,500 + 1,500 – 15,000)
Annual retirement shortfall = –\$72,000

The withdrawals are an annuity due, so:

$PV = C \times ((1 / r) - \{1 / [r(1 + r)^t]\}) \times (1 + r)$

$\$1{,}108{,}718 = \$72{,}000 \times ((1 / .035) - \{1 / [.035(1 + .035)^t]\}) \times (1 + .035)$

$14.878127 = (1 / .035) - \{1 / [.035(1 + .035)^t]\}$

$13.693302 = 1 / [.035(1 + .035)^t]$

$.073028 / .035 = 1.035t$

$t = \ln 2.086514 / \ln 1.035$

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t = 21.38 years

Est time: 06-10

35. a. PV = C / r
PV = $2,000,000 / .12
PV = $16,666,667

b. PV = $C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
PV = $2,000,000 × ((1 / .12) – {1 / [.12(1 + .12)^{20}]})
PV = $14,938,887

c. PV = C / $(r - g)$
PV = $2,000,000 / (.12 – .03)
PV = $22,222,222

d. PV = $C \times ([1 / (r - g)] - \{(1 + g)^t / [(r - g) \times (1 + r)^t]\})$
PV = $2,000,000 × ([1 / (.12 – .03)] – {(1 + .03)^{20} / [(.12 – .03) × (1 + .12)^{20}]})
PV = $18,061,473

Est time: 06-10

36. a. PV = $C \times ((1 / r) - \{1 / [r(1 + r)^t]\})$
C = PV / ((1 / r) – {1 / [r (1 + r)^t]})
C = $200,000 / ((1 / .06) – {1 / [.06(1 + .06)^{20}]})
C = $17,436.91

b.

Year	Beginning-of-Year Balance	Year-end Interest on Balance	Total Year-end Payment	Amortization of Loan	End-of-Year Balance
1	$200,000.00	$12,000.00	$17,436.91	$5,436.91	194,563.09
2	194,563.09	11,673.79	17,436.91	5,763.13	188,799.96
3	188,799.96	11,328.00	17,436.91	6,108.91	182,691.05
4	182,691.05	10,961.46	17,436.91	6,475.45	176,215.60
5	176,215.60	10,572.94	17,436.91	6,863.98	169,351.63
6	169,351.63	10,161.10	17,436.91	7,275.81	162,075.81
7	162,075.81	9,724.55	17,436.91	7,712.36	154,363.45
8	154,363.45	9,261.81	17,436.91	8,175.10	146,188.34
9	146,188.34	8,771.30	17,436.91		137,522.73

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				8,665.61	
10	137,522.73	8,251.36	17,436.91	9,185.55	128,337.19
11	128,337.19	7,700.23	17,436.91	9,736.68	118,600.51
12	118,600.51	7,116.03	17,436.91	10,320.88	108,279.62
13	108,279.62	6,496.78	17,436.91	10,940.13	97,339.49
14	97,339.49	5,840.37	17,436.91	11,596.54	85,742.95
15	85,742.95	5,144.58	17,436.91	12,292.33	73,450.61
16	73,450.61	4,407.04	17,436.91	13,029.87	60,420.74
17	60,420.74	3,625.24	17,436.91	13,811.67	46,609.07
18	46,609.07	2,796.54	17,436.91	14,640.37	31,968.71
19	31,968.71	1,918.12	17,436.91	15,518.79	16,449.92
20	16,449.92	986.99	17,436.91	16,449.92	0.00

c. Interest percent of first payment = Interest_1 / Payment

Interest percent of first payment = (.06 × $200,000) / $17,436.91

Interest percent of first payment = .6882, or 68.82%

Interest percent of last payment = Interest_{20} / Payment

Interest percent of last payment = $986.99 / $17,436.91

Interest percent of last payment = .0566, or 5.66%

Without creating an amortization schedule, the interest percent of the last payment can be computed as:

Interest percent of last payment = 1 – {[Payment / (1 + r)] / Payment}

Interest percent of last payment = 1 – [($17,436.91 / 1.06) / $17,436.91]

Interest percent of last payment = .0566, or 5.66%

After 10 years, the balance is:

$PV_{10} = C \times ((1 + r) - \{1 / [r \times (1 + r)^t]\})$

$PV_{10} = \$17,436.91 \times \{1.06 - [1 / (.06 \times 1.06^{10})]\}$

$PV_{10} = \$128,337.19$

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Fraction of loan paid off = (Loan amount – PV_{10}) / Loan amount

Fraction of loan paid off = ($200,000 – 128,337.19) / $200,000

Fraction of loan paid off = .3583, or 35.83%

Est time: 16-20

37. a. Rule of 72 estimate:

Time to double = 72 / r

Time to double = 72 / 12

Time to double = 6 years

Exact time to double:

$C_t = PV \times (1 + r)^t$
$t = \ln 2 / \ln 1.12$
$t = 6.12$ years

b. With continuous compounding for interest rate r and time period t:

$e^{rt} = 2$

$rt = \ln 2$

Solving for t when r is expressed as a decimal:

$rt = .693$

$t = .693 / r$

Est time: 06-10

38. Spreadsheet exercise

Est time: 11-15

39. a. $PV = C / (r - g)$

$PV = \$2,000,000 / [.10 - (-.04)]$

$PV = \$14,285,714$

b. $PV_{20} = C_{21} / (r - g)$
$PV_{20} = \{\$2,000,000 \times [1 + (-.04)]^{20}\} / [.10 - (-.04)]$
$PV_{20} = \$6,314,320$

$PV_{\text{cash flows 1-20}} = PV - PV_{20} / (1 + r)^{20}$

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$$PV_{\text{cash flows 1-20}} = \$14{,}285{,}714 - (\$6{,}314{,}320 / 1.10^{20})$$
$$PV_{\text{cash flows 1-20}} = \$13{,}347{,}131$$

Est time: 06-10

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Chapter 2 How to Calculate Present Values

OVERVIEW

This chapter introduces the concept of present value and shows why a firm should maximize the market value of the stockholders' stake in it. It describes the mechanics of calculating present values of lump sum amounts, perpetuities, annuities, growing perpetuities, growing annuities and unequal cash flows. Other related topics like simple interest, frequent compounding, continuous compounding, and nominal and effective interest rates are discussed. The net present value rule and the rate of return rule are explained in great detail.

LEARNING OBJECTIVES

- To learn how to calculate present value of lump sum cash flows.
- To understand and use the formulas associated with the present value of perpetuities; growth perpetuities; annuities; and growing annuities.
- To understand more frequent compounding including continuous compounding.
- To understand the important difference between nominal and effective interest rates.
- To understand value-additive property and the concept of arbitrage.
- To understand the net present value rule and the rate of return rule.

CHAPTER OUTLINE

Future values and present values

The concepts of future value, present value, net present value (NPV) and the opportunity cost of capital (hurdle rate) are introduced. The authors show, using several numerical examples, that simple projects with rates of return exceeding the opportunity cost of capital have positive net present values. The "Net present value rule" and the "Rate of return rule" are stated here.

This chapter also extends the concept of discounting to assets, which produce a series of cash flows. The possibility of arbitrage restricts the relative values of discount factors DF_1, DF_2,…. DF_t –.The main point is that money machines cannot exist in well-functioning financial markets. Using numerical examples it shows how to calculate PV and NPV of a series of cash flows over a number of periods (years).

Looking for shortcuts – perpetuities and annuities

This section is devoted to developing formulae for perpetuities and annuities. It explains the difference between an ordinary annuity and an annuity due. It also explains how the future value of an annuity is calculated. The present value of an annuity can be thought of as the difference between two perpetuities

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beginning at different times. Using this simple idea, the formula for the present value of an annuity is derived. The future value of an annuity formula is also derived. These have numerous applications in pension funds, mortgages and valuation of financial assets.

More shortcuts – growing perpetuities and annuities

Some applications need the present value of a perpetual cash flow growing at a constant rate, as well as annuities that grow at a constant rate. The formula for the present value of a growing perpetuity is derived. The present value of a growing annuity can be thought of as the difference between two growing perpetuities starting at different times. Using this simple idea, the formula for the present value of a growing annuity is also derived. These formulas have many applications in the valuation of assets.

How Interest Is Paid and Quoted

This section explains the differences between compound interest and simple interest, as well as the differences between effective annual rates and annual percentage rates. It deals with how each interest rate is used in the market place and the math necessary to move between the two kinds of interest rates.

TEACHING TIPS FOR POWERPOINT SLIDES

Slide 1 - Title slide

Slide 2 – Topics covered

Slide 3
Explain the terms "future value" and "present value". The concept must be emphasized at this point. Consequently, it may be necessary to spend some time explaining real world examples of how present value and future value relate. A good example to use is retirement planning.

Slide 4
$FV = PV \times [(1 + r)^{t}]$

Define the terms:

FV = Future value
PV = Present value
r = interest rate
t = number of years (Periods)

Explain the time value of money and its importance to financial decision making.

Slide 5

Walk through each step in the math process and show how the value increases. If you plan to have your students use a financial calculator, you can skip the details of the basic math. Be aware, students often stumble when doing simple math calculations.

Slide 6
The longer the funds are invested, the greater the advantage with compound interest. Discuss the four examples and be sure to use the phrase "power of compounding."

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Slide 7
This slide contains the Present Value formula.

Slide 8
The Discount Factor (DF) is the present value of $1 expected to be received in the future. Here it is appropriate to introduce the use of the financial calculator to solve these problems.

Slide 9
Here we reverse the future value process from earlier. Show students how they can easily move between future value and present value with the basic formulae.

Slide 10
For visual learners, this graph illustrates the reverse of the future value compounding chart shown earlier. It is downwardly sloping, which can confuse students, so it may be necessary to spend time explaining the concept.

Slide 11
Explain how the present value concept discussed earlier is useful in valuing assets.

Cost of the building = $700,000
Sale price in Year –1 = $800,000
Opportunity cost of capital = 7%

Slide 12
Discount future cash flows at the opportunity cost of capital

PV = 800,000/(1.07) = 747,664

NPV = PV – required investment
NPV = 747,664 – 700,000 = 47,664

Explain the difference between PV and NPV. Explain sign conventions for cash flows.

Slide 13
Explain each variable in the equation. It is easy to tell the students that all present values come at a cost. That cost is the initial investment. This may help them easily transition from present value to net present value.
C_0 = initial investment for the project. Normally it is a cash outflow and has a negative sign (-)
C_1 = cash inflow from the project. Normally it has a positive sign (+)
r = opportunity cost of capital

Positive NPVs increase the value of a firm. Negative NPVs lower the value of a firm.

Slide 14
The concept of risk is introduced here. Briefly explain the idea of risk (lottery vs. bank deposit). Generally, investors do not like risk. In order to induce the investors invest in risky projects, a higher rate of return is needed. Higher rate of return causes lower PVs. Explain the relationship between discount rates and present values. Higher the discount rate, lower the present value.

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Slide 15 & 16
Explain the relationship between discount rates and net present values. Higher the discount rate, the lower the net present value.

NPV at 12% : NPV = 714,286 - 700,000 = 14,286
NPV at 7% : NPV = 747,664 - 700,000 = 47,664
NPV at 5% : NPV = 761 - 700,000 = 61,904

Slide 17
Net Present Value Rule
Accept if NPV > 0: A very powerful financial decision-making rule. It looks simple but can get complicated quickly. This project is acceptable as the NPV > 0. Make sure that students understand this rule clearly.

Slide 18
This slide explains the rate of return rule.

Slide 19
Multiple cash flows occurring at different time periods can be evaluated using the DCF formula. It is a simple extension of the NPV formula, but can intimidate students because of the extra equations. Show how it is a minor extension of the prior Basic Point Value formula.

Slide 20
The graphic presentation of the net present value of multiple cash flows or sequential cash flows is given here. Here we extend the concept of PV to a series of cash flows by applying value-additive property of present values. These cash flows can be positive (cash inflows) or negative (cash outflows). We merely add the initial cost to make it NPV.

Slide 21
Depending on the type of cash flow you can use the formulas to simplify the calculations. There are formulas that can be used for finding the present values for cash flows with a pattern; for example perpetuities and annuities. Define perpetuity (same cash flow each year forever) and give an example of perpetuity.

Slide 22
Introduce the perpetuity concept as one in which you earn money forever. In doing so, you can easily demonstrate the return an investor earns.

Slide 23
Now, manipulate the formula to get the value of the infinite cash flow given a discount rate. Provide the formula for calculating the present value perpetuity. This formula is obtained using an algebraic technique; sum of an infinite geometric series.

Slide 24
Present value of $1 billion received forever at 10% is: PV = $1/0.1 = $10 billion. Using the formula will simplify the calculations.

Slide 25
The same example is used as in the previous slide, except the modification of time is added. Show the students how the value is reduced if you get the money later. This reinforces the time value of money concepts introduced earlier.

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Slide 26
This slide provides the formula for the present value of an annuity. An annuity can be thought of as the difference between two perpetuities starting at different times. A slight derivation is presented, can be ignored if it is beyond the scope of the course, with no harm in understanding the broader concept.

Slide 27
This is the PVAF formula. Take some time to explain the variables. If a financial calculator is to be used in class, there is no need to cover the use in detail.

Slide 28
This slide is a more comprehensive example of an annuity and its relationship to perpetuities.

Slide 29
An asset that pays a fixed sum each period for a specified number of periods is called an annuity. For example: the present value of annual payments of $5,000 per year for five years is presented. In addition to the formula, using a financial calculator: PMT = 5,000; I = 7; N = 5; FV = 0 and compute PV = 20,501.

Slide 30
The state lottery pays the jackpot prize in 30 annual installments. If Internet access is available, it might be helpful to pull up an actual national lottery and determine how the lump sum payout was determined. The instructor should first calculate the internal rate of return and provided as a given number to the students.

Slide 31
This is the formula that reverses the PVAF math for future values.

Slide 32
This is an example of the Future Value of an annuity. It is highly recommended this example also be provided with a financial calculator.

Slide 33
This is another example of an annuity. In this case we are determining the payment necessary on a loan.

Slide 34
Has was done for the present value of an annuity earlier, this slide presents the future value of an annuity.

Slide 35
Continuing the theme of the prior slide, I hear we have an example of the future value of an annuity.

Slide 36
This formula is the present value of a perpetuity that is growing at a constant rate, where g is the annual growth rate of the cash flow; and r > g. It is useful and should be explained as a formula the students will use often.

Slide 37
The formula can be used to evaluate a perpetuity starting at any point in time, where g is the annual growth rate of the cash flow; and r > g.

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Slide 38
The present value of $1 billion (first payment starting one year from today) perpetuity that is growing at a constant rate of 4% and requires a rate of return of 10% is: $PV_0 = C_1/(r-g) = 1/(0.1 - 0.04) = 16.667$ billion. This is $6.667 billion more that the perpetuity without growth.

Slide 39
Go over each definition: EAR and APR. Students need to know how each is used in the market and why APR is quoted rather than EAR.

Slide 40
The basic formula for APR and EAR are presented. In reality, students are more likely to use the spreadsheet or financial calculator.

Slide 41 & 42
Go over the math of the problem. If students are comfortable with the use of a financial calculator, use the following method as a substitute for the formula. Using the financial calculator: PMT = 0; I = 1%; N = 12; PV = 1; FV = -1.1268. By dropping the (-1) the answer is arrived at of 12.68%.

KEY TERMS AND CONCEPTS

Present value, discount factor, discount rate, hurdle rate, opportunity cost of capital, net present value, net present value rule, rate of return rule, discounted cash flow, perpetuity, growing perpetuity, annuity, growing annuity, compound interest, simple interest, continuous compounding, annual percentage rate, effective annual rate.

CHALLENGE AREAS

The link between financial markets and NPV

WEB LINKS

finance.yahoo.com
www.maxigrade.com/CorpFin1/corpfin1samplequestions2.php
www.empowerosity.com/FinancePractice/

ADDITIONAL REFERENCES

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White, M. "Financial Analysis with a Calculator." 5th ed. Burr Ridge Ill, McGraw-Hill/Irwin, 2004.

Copeland,T.E, J.F. Weston, and K. Shastri, "*Financial Theory and Corporate Policy*," Fourth Edition, Pearson Addison Wesley, Boston, 2005.

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Benninga, S. *Financial Modeling*. 3nd Edition, Cambridge, MA: The MIT Press, 2008.

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REEBY SPORTS
Minicase solution, Chapter 4

Principles of Corporate Finance, 12th Edition
R. A. Brealey, S. C. Myers and F. Allen

What is Reeby Sports worth per share? We will value the company using George Reeby's forecasts.

The spreadsheet accompanying this solution sets out a forecast in the same general format as Table 4.5. Historical results from 2011 to 2016 are also shown. Earnings per share (EPS) equals return on equity (ROE) times starting book value per share (BVPS). EPS is divided between dividends and retained earnings, depending on the dividend payout ratio. BVPS grows as retained earnings are reinvested.

The keys to Reeby Sports' future value and growth are profitability (ROE) and the reinvestment of retained earnings. Retained earnings are determined by dividend payout. The spreadsheet sets ROE at 15% for the six years from 2018 to 2022. If Reeby Sports will lose its competitive edge by 2022, then it cannot continue earning more than its 10% cost of capital. Therefore ROE is reduced to 10% starting in 2023.[1]

The payout ratio is set at .30 from 2018 onwards. Notice that the long-term growth rate, which settles in after 2023, is ROE × (1 – dividend payout ratio) = .10 × (1 - .30) = .07.

The spreadsheet allows you to vary ROE and the dividend payout ratio separately for 2018-2022 and for 2023-2024.[2] But let's start with the initial input values. To calculate share value, we have to estimate a horizon value at H = 2022 and add its PV to the PV of dividends from 2017 to 2022. Using the constant-growth DCF formula,

$$PV_H = \frac{0.71}{.10 - .07} = 23.72$$

The PV of dividends from 2017 to 2022 is $3.43 at the start of 2017, so share value is:[3]

[1] A more sophisticated spreadsheet could distinguish ROE on existing assets from ROE on new capital investment. For example, ROE on new investment could drop all the way to 10% in 2020, but ROE for existing assets could decline gradually.

[2] You can vary these inputs, but be careful not to enter ROEs and dividend payout ratios that generate long-term growth rates close to or above the cost of capital. As the growth rate approaches the cost of capital, the DCF formula explodes. If the growth rate exceeds the cost of capital, the DCF formula says stock price is negative, which is impossible. The spreadsheet reports "Formula not applicable" in this case.

[3] This DCF calculation assumes that the first dividend will be received after one year, at the end of 2017. Normally dividends are paid quarterly, so it would be more realistic to assume receipt at the middle of the year. This "mid-year convention" would move all cash flows 6 months closer and therefore increase PV by $1.1^{.5}$. (In finance, "mid-year convention" does *not* mean June in Las Vegas.)

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$$PV = 3.43 + \frac{23.72}{(1.1)^6} = \$16.82$$

The spreadsheet also calculates the PV of dividends through 2024 and the horizon value at 2024. Notice that the PV at the start of 2017 remains at $16.82. This makes sense, since the value of a firm should not depend on the investment horizon chosen to calculate PV. (If you calculate a value that does depend on the horizon, you have made a mistake.)

We have reduced ROE to the 10% cost of capital after 2022, assuming that Reeby Sports will have exhausted valuable growth opportunities by that date. With PVGO = 0, PV = EPS/r. [4] So we could discard the constant-growth DCF formula and just divide EPS in 2023 by the cost of capital:

$$PV_H = \frac{2.37}{.10} = \$23.72$$

This PV is identical to the PV from the constant-growth DCF formula. It doesn't matter how fast a company grows after the horizon date H if it only earns its cost of capital.

How much of Reeby Sports' value is due to PVGO? You can check by setting ROE = .10 for 2018 and all later years. You should get PV = $13.82. Thus PVGO = 16.82 – 13.82 = $3.00 per share for investments made in 2017 onward.

George Reeby has also identified a "comparable," Molly Sports. We could use its P/E ratio of 13.1 to calculate horizon value in 2022 and PV at the start of 2017. Using the original inputs for ROE, EPS in 2023 is 2.37.[5]

$$PV_H = 13.1 \times 2.37 = \$31.05$$

$$PV = 3.43 + \frac{31.05}{(1.10)^6} = \$20.96$$

We could also use Molly's P/E ratio to calculate Reeby Sports' PV at the start of 2017 directly from 2017 EPS:

$$PV = 13.1 \times 2.03 = \$26.59$$

---

[4] See Section 4.5.

[5] Notice that the P/E ratio applies to the next year's forecasted earnings. Sometimes "trailing P/Es" are used instead. Trailing P/Es are based on earnings over the previous year.

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Both values based on Molly's P/E are higher than our DCF calculations. Is Molly significantly more profitable than Reeby Sports, or does our spreadsheet understate Reeby Sports' prospects?

What if Reeby Sports could continue to earn ROE = .15 for two extra years, until 2024? You can check by changing ROE for 2023-2024 from .10 to .15. (The ROE for 2025 and 2026 is hard-wired at .10.) You should get NPV of $18.04, somewhat higher than our original DCF calculations, but not enough for Reeby Sports to match Molly's P/E. You may wish to experiment to find inputs that generate P/E = 13 for Reeby Sports at the start of 2017. Do you think these inputs are reasonable?

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CHAPTER 2

HOW TO CALCULATE PRESENT VALUES

Brealey, Myers, and Allen

Principles of Corporate Finance

12th Edition

Slides by Matthew Will

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Topics Covered

- Future Values and Present Values
- Looking for Shortcuts—Perpetuities and Annuities
- More Shortcuts—Growing Perpetuities and Annuities
- How Interest Is Paid and Quoted

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Present Value and Future Value

Present Value

Value today of a future cash flow.

Future Value

Amount to which an investment will grow after earning interest

Future Values

Future Value of $100 = FV

$$FV = \$100 \times (1 + r)^t$$

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Future Values

$$FV = \$100 \times (1 + r)^t$$

$$\$\$\$\$\$\$\$$$

Example - FV

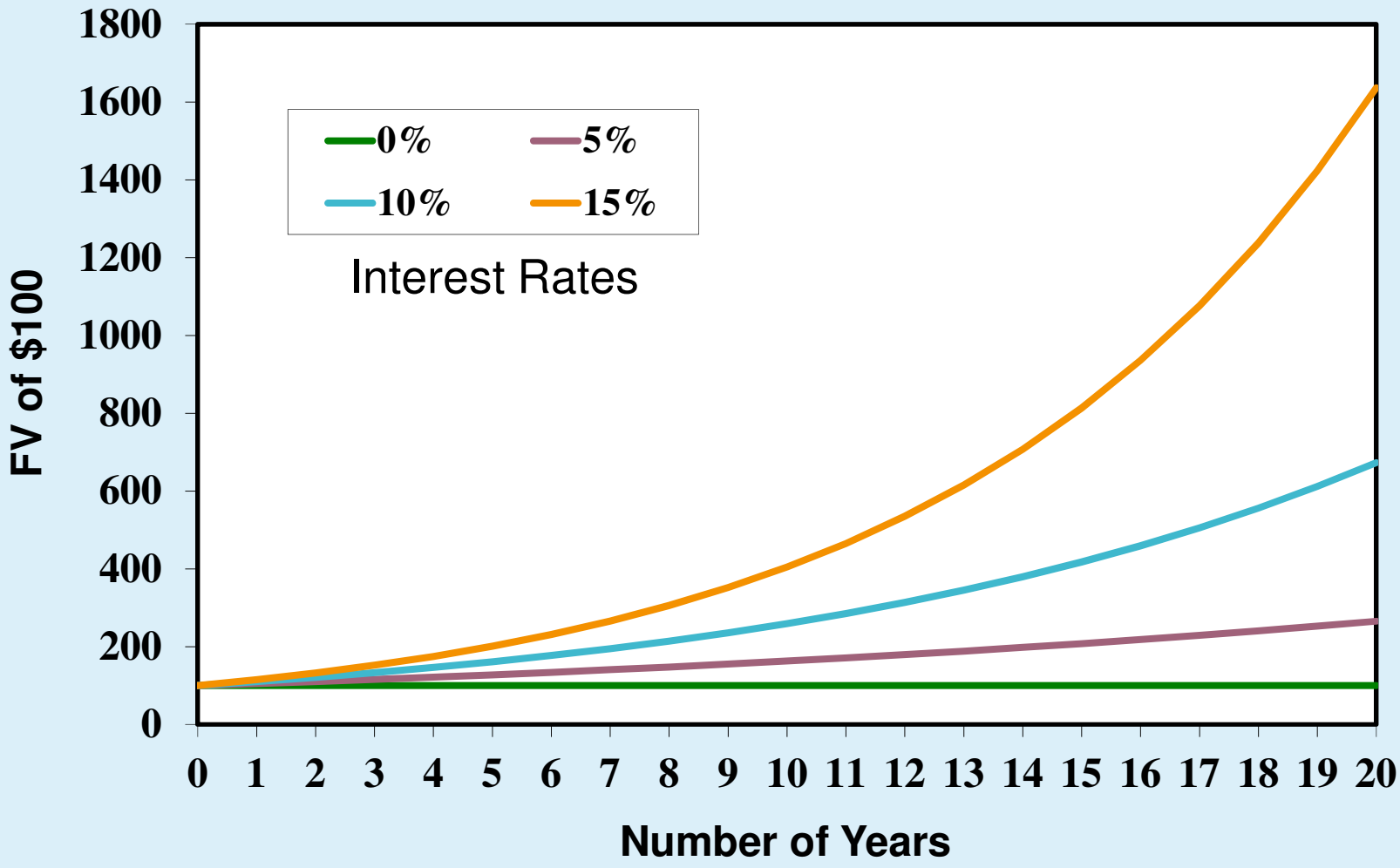
What is the future value of $100 if interest is compounded annually at a rate of 7% for two years?

$$FV = \$100 \times (1.07) \times (1.07) = 114.49$$

$$FV = \$100 \times (1 + .07)^2 = \$114.49$$

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Future Values with Compounding



Present Value

Present value = PV

$$\text{PV} = \text{discount factor} \times C_1$$

Present Value

Discount factor = DF = PV of $1

$$DF = \frac{1}{(1+r)^t}$$

Discount factors can be used to compute the present value of any cash flow

Present Value

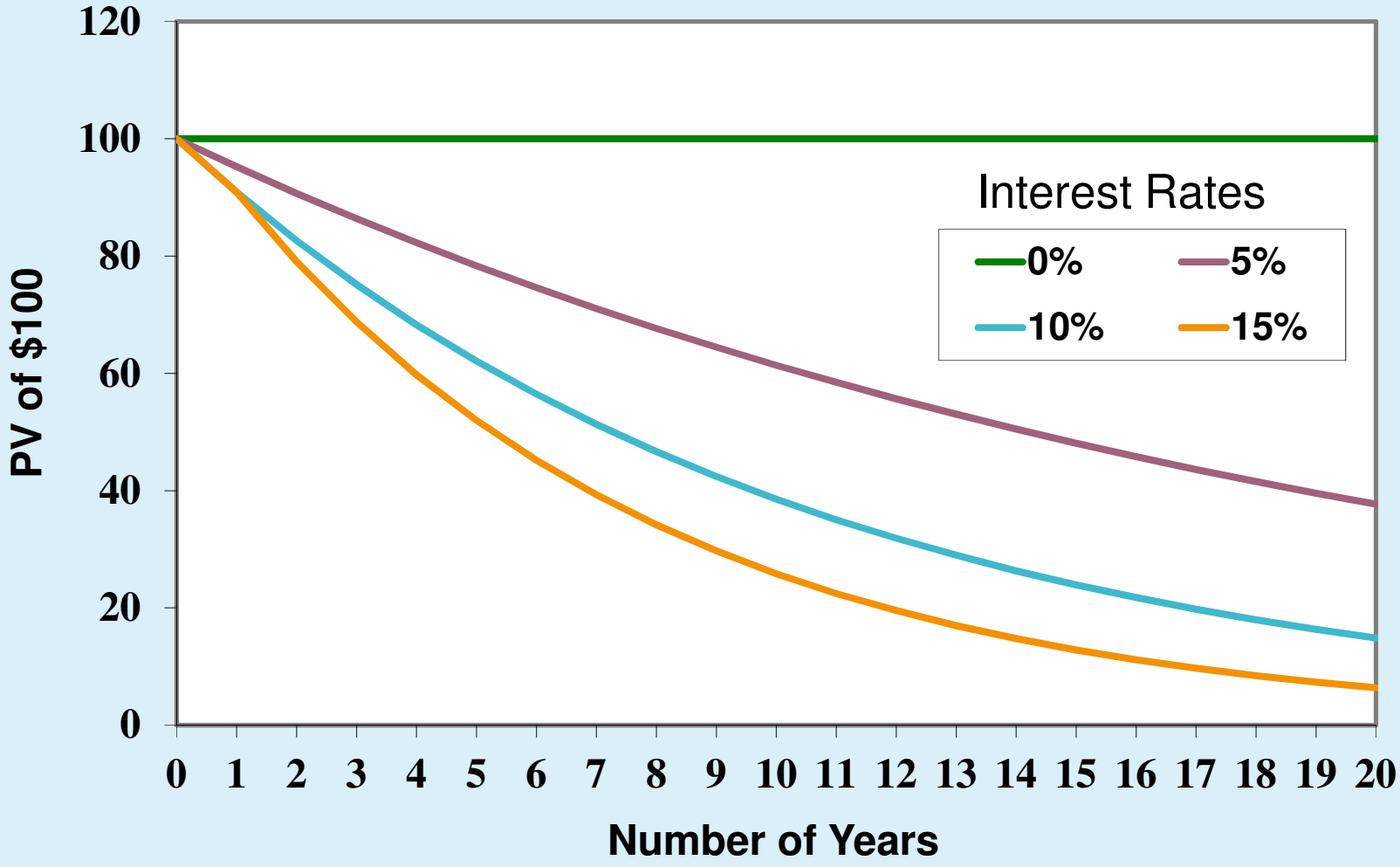
- The PV formula has many applications. Given any variables in the equation, you can solve for the remaining variable. Also, you can reverse the prior example.

$$PV = DF_2 \times C_2$$

$$PV = \frac{1}{(1+.07)^2} \times 114.49 = 100$$

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Present Values with Compounding



Valuing an Office Building

Step 1: Forecast cash flows

Cost of building = C_0 = 700,000

Sale price in Year 1 = C_1 = 800,000

Step 2: Estimate opportunity cost of capital

If equally risky investments in the capital market offer a return of 7%, then

Cost of capital = r = 7%

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Valuing an Office Building

Step 3: Discount future cash flows

$$\text{PV} = \frac{C_1}{(1+r)} = \frac{800,000}{(1+.07)} = 747,664$$

Step 4: Go ahead if PV of payoff exceeds investment

$$\text{NPV} = 747,664 - 700,000$$

$$= 47,664$$

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Net Present Value

$$\text{NPV} = \text{PV} - \text{required investment}$$

$$\text{NPV} = C_0 + \frac{C_1}{1+r}$$

Risk and Present Value

- Higher risk projects require a higher rate of return
- Higher required rates of return cause lower PVs

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 7\%$$

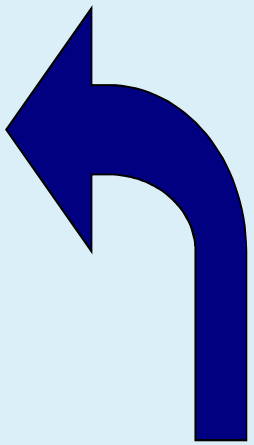
$$\text{PV} = \frac{800{,}000}{1 + .07} = 747{,}664$$

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Risk and Present Value

$$\text{PV of } C_1 = \$800{,}000 \text{ at } 12\%$$

$$\text{PV} = \frac{800{,}000}{1 + .12} = 714{,}286$$



$$\text{PV of } C_1 = \$800{,}000 \text{ at } 7\%$$

$$\text{PV} = \frac{800{,}000}{1 + .07} = 747{,}664$$

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Risk and Net Present Value

$$NPV = PV - \text{required investment}$$

$$NPV = 714{,}286 - 700{,}000$$

$$= \$14{,}286$$

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Net Present Value Rule

- Accept investments that have positive net present value

Example

Use the original example. Should we accept the project given a 10% expected return?

$$NPV = -700{,}000 + \frac{800{,}000}{1.10} = \$27{,}273$$

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Rate of Return Rule

- Accept investments that offer rates of return in excess of their opportunity cost of capital

Example

In the project listed below, the foregone investment opportunity is 12%. Should we do the project?

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{800{,}000 - 700{,}000}{700{,}000} = .143 \text{ or } 14.3\%$$

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Multiple Cash Flows

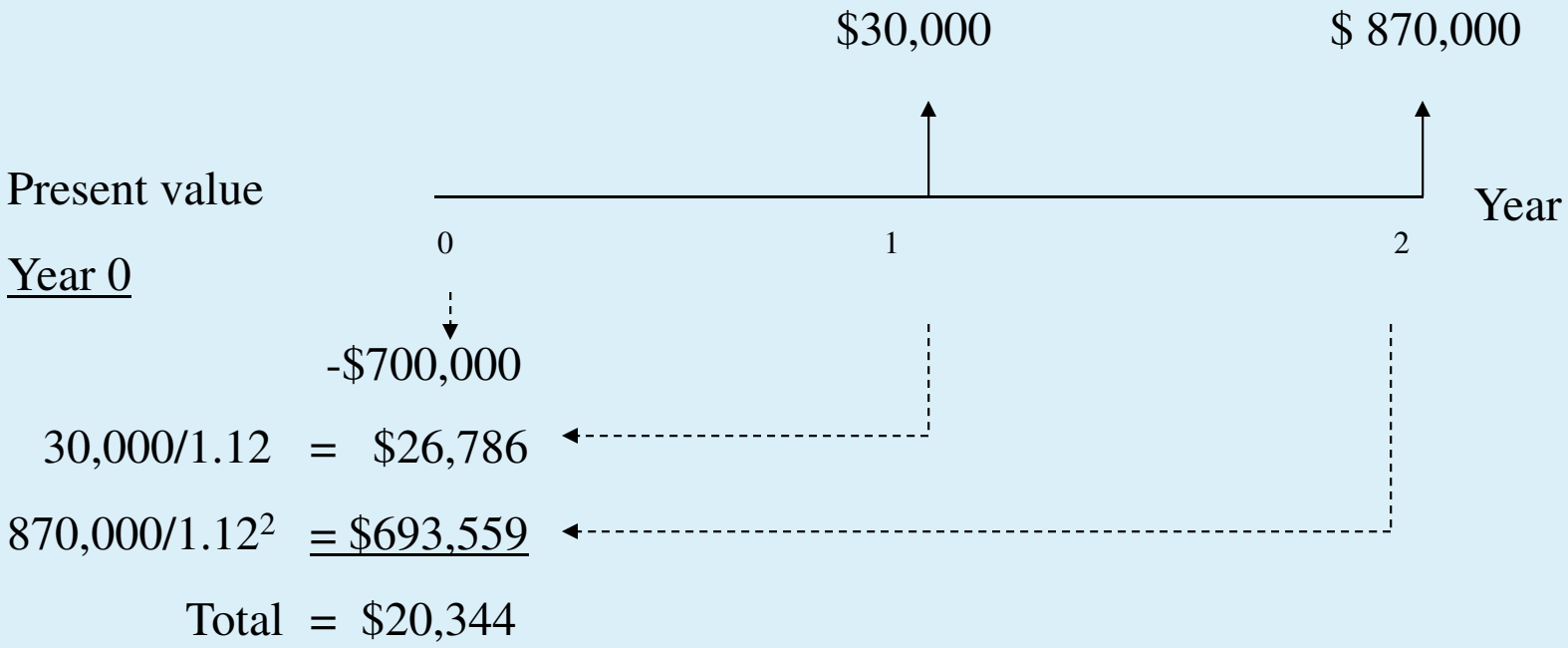
For multiple periods we have the discounted cash flow (DCF) formula

$$PV_0 = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + ... + \frac{C_t}{(1+r)^t}$$

$$NPV_0 = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

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Net Present Values



Shortcuts

- Sometimes there are shortcuts that make it very easy to calculate the present value of an asset that pays off in different periods. These tools allow us to cut through the calculations quickly.

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Shortcuts

Perpetuity - Financial concept in which a cash flow is theoretically received forever.

$$\text{Return} = \frac{\text{cash flow}}{\text{present value}}$$

$$r = \frac{C}{PV}$$

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Shortcuts

Perpetuity - Financial concept in which a cash flow is theoretically received forever.

$$\text{PV of cash flow} = \frac{\text{cash flow}}{\text{discount rate}}$$

$$PV_0 = \frac{C_1}{r}$$

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Present Values

Example

What is the present value of $1 billion every year, for all eternity, if you estimate the perpetual discount rate to be 10%?

$$PV = \frac{\$1\,\text{bil}}{0.10} = \$10\,\text{billion}$$

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Present Values

Example - continued

What if the investment does not start making money for 3 years?

$$PV = \frac{\$1 \text{ bil}}{0.10} \times \left(\frac{1}{1.10^{3}}\right) = \$7.51 \text{ billion}$$

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How to Value Annuities

Annuity - An asset that pays a fixed sum each year for a specified number of years

$$\text{PV of annuity} = C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

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Perpetuities & Annuities

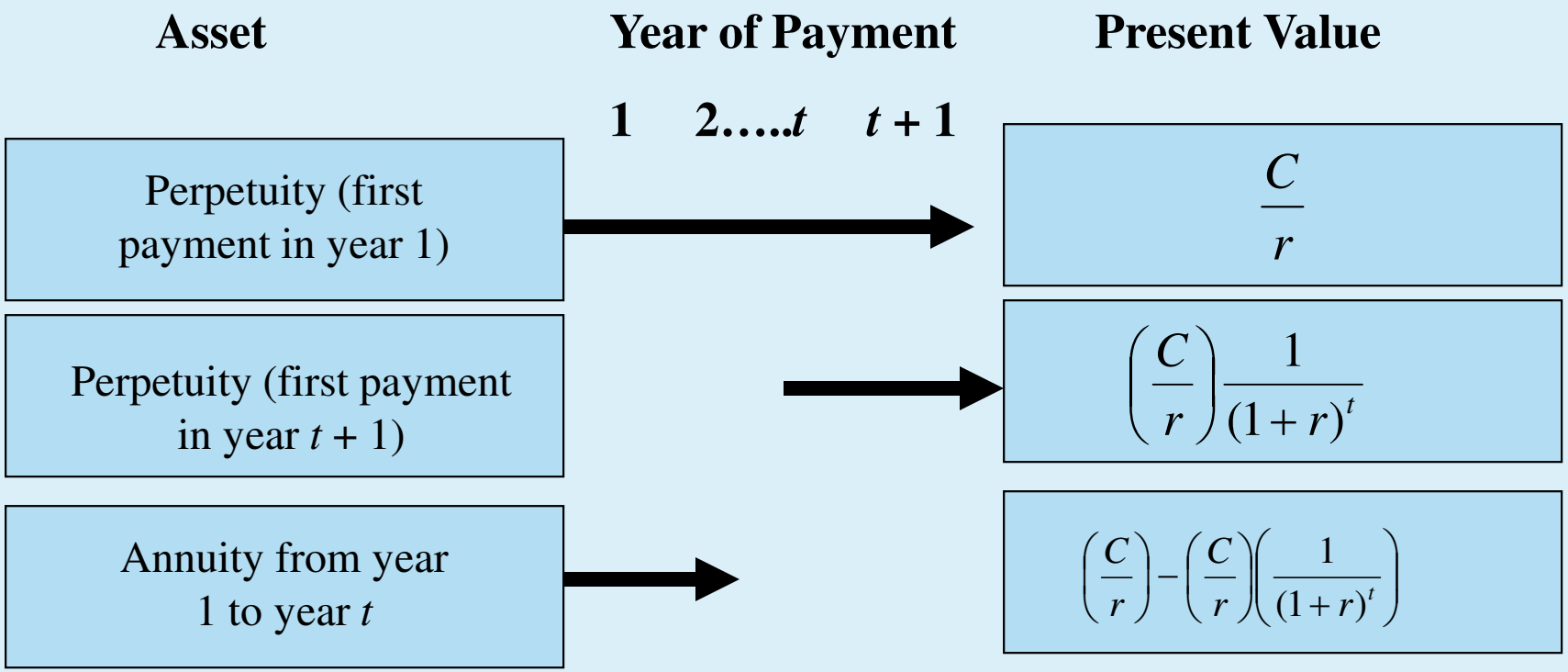
PV Annuity Factor (PVAF) - The present value of $1 a year for each of *t* years

$$PVAF = \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

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Short Cuts

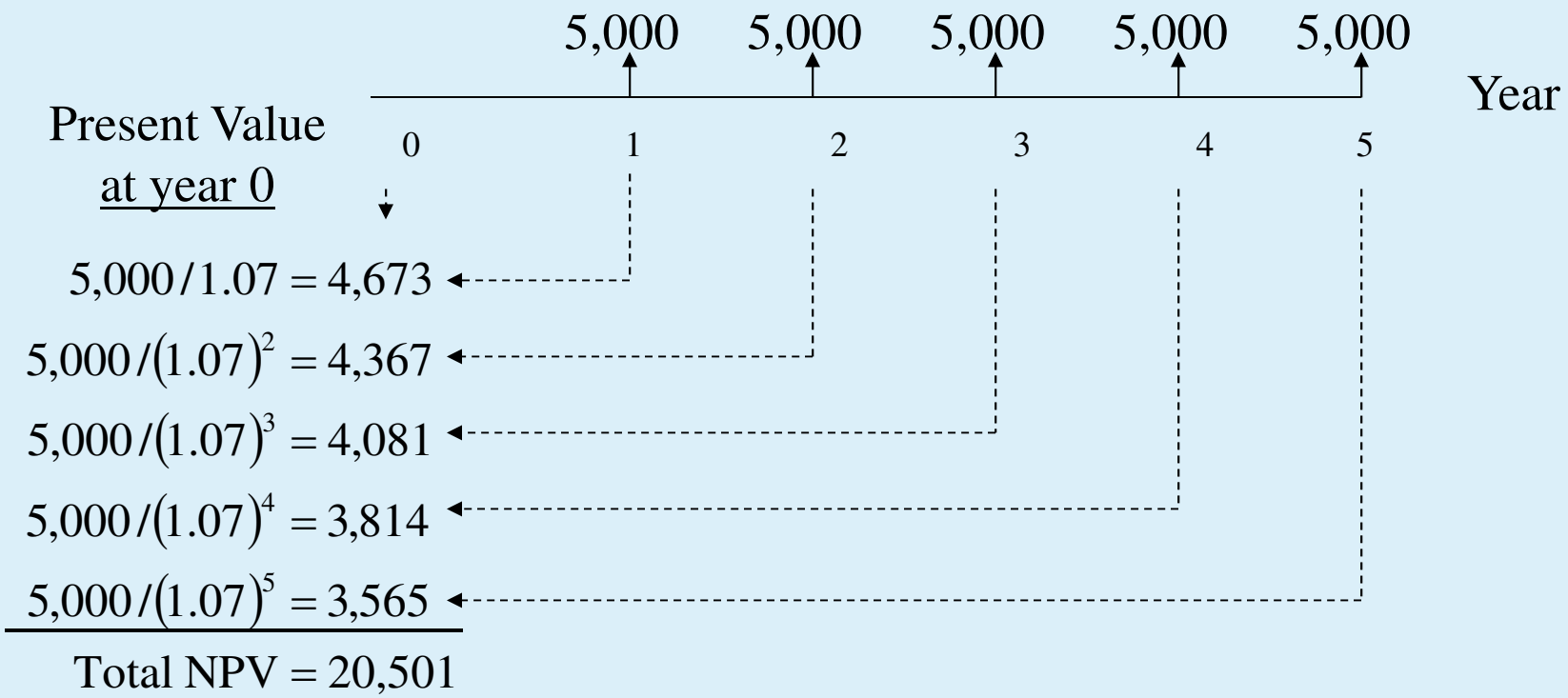
Annuity - An asset that pays a fixed sum each year for a specified number of years.



Costing an Installment Plan

Example

Tiburon Autos offers you "easy payments" of $5,000 per year, at the end of each year for 5 years. If interest rates are 7%, per year, what is the cost of the car?



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Winning Big at the Lottery

Example

The state lottery advertises a jackpot prize of $590.5 million, paid in 30 installments over 30 years of $19.683 million per year, at the end of each year. If interest rates are 3.6% what is the true value of the lottery prize?

$$\text{Lottery value} = 19.683 \times \left[\frac{1}{.036} - \frac{1}{.036(1+.036)^{30}} \right]$$

$$\text{Value} = \$357.5 \text{ million}$$

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Annuity Due

Annuity due - Level stream of cash flows starting immediately

How does it differ from an ordinary annuity?

$$PV_{\text{Annuity due}} = PV_{\text{Annuity}} \times (1 + r)$$

How does the future value differ from an ordinary annuity?

$$FV_{\text{Annuity due}} = FV_{\text{Annuity}} \times (1 + r)$$

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Annuities Due: Example

$$FV_{AD} = FV_{Annuity} \times (1 + r)$$

Example: Suppose you invest $429.59 annually at the beginning of each year at 10% interest. After 50 years, how much would your investment be worth?

$$FV_{AD} = \$429.59 \times \left[\frac{1}{.10} - \frac{1}{.10(1+.10)^{50}}\right] \times 1.10^{50} \times 1.10$$
$$= \$550{,}003.81$$

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Paying Off a Bank Loan

Example - Annuity

You are purchasing a TV for $1,000. You are scheduled to make 4 annual installments. Given a rate of interest of 10%, what is the annual payment?

$$\$1{,}000 = \text{PMT} \times \left[\frac{1}{.10} - \frac{1}{.10(1+.10)^4}\right]$$

$$\text{PMT} = \$315.47$$

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FV Annuity Short Cut

Future Value of an Annuity – The future value of an asset that pays a fixed sum each year for a specified number of years.

$$\text{FV of annuity} = C \times \left[\frac{(1+r)^t - 1}{r} \right]$$

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FV Annuity Short Cut

Example

What is the future value of $20,000 paid at the end of each of the following 5 years, assuming your investment returns 8% per year?

$$FV = 20{,}000 \times \left[\frac{(1+.08)^5 - 1}{.08} \right]$$

$$= \$117{,}332$$

Constant Growth Perpetuity

$$PV_0 = \frac{C_1}{r - g}$$

g = the annual growth rate of the cash flow

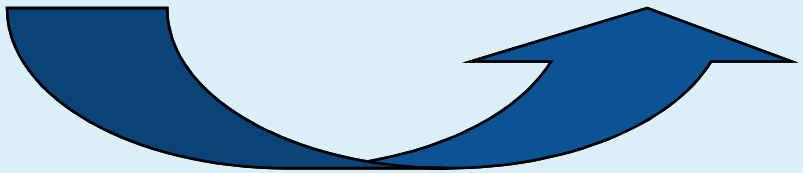
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Constant Growth Perpetuity

NOTE: This formula can be used to value a perpetuity at any point in time.

$$PV_0 = \frac{C_1}{r - g}$$

$$PV_t = \frac{C_{t+1}}{r - g}$$



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Constant Growth Perpetuity

Example

What is the present value of $1 billion paid at the end of every year in perpetuity, assuming a rate of return of 10% and a constant growth rate of 4%?

$$PV_0 = \frac{1}{.10 - .04}$$
$$= \$16.667 \text{ billion}$$

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Effective Interest Rates

Effective Annual Interest Rate - Interest rate that is annualized using compound interest

Annual Percentage Rate - Interest rate that is annualized using simple interest

EAR & APR Calculations

Annual Percentage Rate (APR):

$$\text{APR} = \text{MR} \times 12$$

Effective Annual Interest Rate (EAR):

$$\text{EAR} = (1 + \text{MR})^{12} - 1$$

**where MR = monthly interest rate*

Effective Interest Rates

Example:

Given a monthly rate of 1%, what is the effective annual rate (EAR)? What is the annual percentage rate (APR)?

Effective Interest Rates

Example:

Given a monthly rate of 1%, what is the effective annual rate (EAR)? What is the annual percentage rate (APR)?

$$\text{EAR} = (1+.01)^{12} - 1 = r$$

$$\text{EAR} = (1+.01)^{12} - 1 = .1268 \text{ or } 12.68\%$$

$$\text{APR} = .01 \times 12 = .12 \text{ or } 12.00\%$$