Engineering Mechanics Dynamics 4th edition Pytel Solutions Manual

SKU: 9781305579200-SOLUTIONS

Chapter 12

12.1

$$\begin{aligned} y &= -0.16t^4 + 4.9t^3 + 0.14t^2 \text{ ft} \\ v &= \dot{y} = -0.64t^3 + 14.7t^2 + 0.28t \text{ ft/s} \\ a &= \dot{v} = -1.92t^2 + 29.4t + 0.28 \text{ ft/s}^2 \end{aligned}$$

At maximum velocity ($a = 0$):

$$-1.92t^2 + 29.4t + 0.28 = 0 \qquad t = 15.322 \text{ s}$$

$$\begin{aligned} v_{\max} &= -0.64(15.322^3) + 14.7(15.322^2) + 0.28(15.322) \\ &= 1153 \text{ ft/s} \blacktriangleleft \\ y &= -0.16(15.322^4) + 4.9(15.322^3) + 0.14(15.322^2) \\ &= 8840 \text{ ft} \blacktriangleleft \end{aligned}$$

12.2

(a) $x = -\frac{1}{2}gt^2 + v_0t \quad \therefore v = \dot{x} = -gt + v_0 \blacklozenge \quad \therefore a = \ddot{x} = -g \blacklozenge$

When $t = 0$, then $x = 0$ and $v = v_0$. Hence v_0 is the initial velocity.

Since gravity is the only source of acceleration in this problem, g must be the gravitational acceleration.

(b) When $x = x_{max}$, then $v = 0$. $\quad \therefore -gt + v_0 = 0 \quad \therefore t = \frac{v_0}{g}$

$$\therefore x_{max} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g} \blacklozenge$$

At the end of flight $x = 0$. $\quad \therefore -\frac{1}{2}gt^2 + v_0t = 0 \quad \therefore t = \frac{2v_0}{g} \blacklozenge$

(c) $v_0 = \frac{(60 \text{ mi/h})(5280 \text{ ft/mi})}{3600 \text{ s/h}} = 88 \text{ ft/s}$

$$\therefore x_{max} = \frac{(88)^2}{2(32.2)} = 120.2 \text{ ft} \blacklozenge \quad \therefore t = \frac{2(88)}{32.2} = 5.47 \text{ s} \blacklozenge$$

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12.65

$$x = 20\cos\frac{\pi}{2}t \text{ ft} \qquad y = 64.4(4 - t^2) \text{ ft}$$
$$\dot{x} = -10\pi\sin\frac{\pi}{2}t \text{ ft/s} \qquad \dot{y} = -128.8t \text{ ft/s}$$
$$\ddot{x} = -5\pi^2\cos\frac{\pi}{2}t \text{ ft/s}^2 \qquad \ddot{y} = -128.8 \text{ ft/s}^2$$

$$F_x = m\ddot{x} = \frac{4}{32.2}\left(-5\pi^2\cos\frac{\pi}{2}t\right) = -6.130\cos\frac{\pi}{2}t \text{ lb}$$
$$F_y = m\ddot{y} = \frac{4}{32.2}(-128.8) = -16.00 \text{ lb}$$

t (s)	F_x (lb)	F_y (lb)
0	−6.13	−16.0
1	0	−16.0
2	6.13	−16.0

◀

12.66

(a) $x = v_0t - r\sin\dfrac{v_0t}{R}$ $\quad\therefore v_x = \dot{x} = v_0 - \dfrac{v_0r}{R}\cos\dfrac{v_0t}{R}$ $\quad\therefore a_x = \ddot{x} = \dfrac{v_0^2r}{R^2}\sin\dfrac{v_0t}{R}$

$y = R - r\cos\dfrac{v_0t}{R}$ $\quad\therefore v_y = \dot{y} = \dfrac{v_0r}{R}\sin\dfrac{v_0t}{R}$ $\quad\therefore a_y = \ddot{y} = \dfrac{v_0^2r}{R^2}\cos\dfrac{v_0t}{R}$

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v_0^2r}{R^2} = \text{constant} \quad \text{Q.E.D.}$$

(b) $$F = ma = \frac{mv_0^2r}{R^2} = \frac{\left(\frac{2}{16}\frac{1}{32.2}\right)\left(60\frac{5280}{3600}\right)^2(0.8)}{1.25^2} = 15.39 \text{ lb}$$ ◆

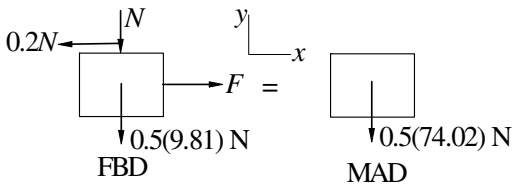
12.67

$$x = b\sin\frac{2\pi t}{t_0} \qquad y = \frac{b}{4}\left(1 + \cos\frac{4\pi t}{t_0}\right)$$
$$v_x = \frac{2\pi b}{t_0}\cos\frac{2\pi t}{t_0} \qquad v_y = -\frac{\pi b}{t_0}\sin\frac{4\pi t}{t_0}$$
$$a_x = -\frac{4\pi^2 b}{t_0^2}\sin\frac{2\pi t}{t_0} \qquad a_y = -\frac{4\pi^2 b}{t_0^2}\cos\frac{4\pi t}{t_0}$$

At point B: $x = 0 \therefore t = 0$

$$\therefore a_x = 0 \qquad \therefore a_y = -\frac{4\pi^2 b}{t_0^2} = -\frac{4\pi^2(1.2)}{0.8^2} = -74.02 \text{ m/s}^2$$

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$$\Sigma F_y = ma_y \quad +\uparrow \qquad -N - 0.5(9.81) = -0.5(74.02)$$

$$N = 32.11 \text{ N}$$

$$\Sigma F_x = 0 \quad \stackrel{+}{\longrightarrow} \qquad F - 0.2N = 0$$

$$F = 0.2N = 0.2(32.11) = 6.42 \text{ N} \quad \blacktriangleleft$$

12.68

$$x = \frac{b}{2}\left(\sin\frac{\pi t}{4t_0} + \sin\frac{3\pi t}{4t_0}\right) \quad \therefore v_x = \dot{x} = \frac{b}{2}\left(\frac{\pi}{4t_0}\cos\frac{\pi t}{4t_0} + \frac{3\pi}{4t_0}\cos\frac{3\pi t}{4t_0}\right)$$

$$\therefore a_x = \ddot{x} = -\frac{b}{2}\left(\frac{\pi^2}{16t_0^2}\sin\frac{\pi t}{4t_0} + \frac{9\pi^2}{16t_0^2}\sin\frac{3\pi t}{4t_0}\right) = -\frac{\pi^2 b}{32t_0^2}\left(\sin\frac{\pi t}{4t_0} + 9\sin\frac{3\pi t}{4t_0}\right)$$

$$y = \frac{b}{2}\left(\cos\frac{\pi t}{4t_0} - \cos\frac{3\pi t}{4t_0}\right) \quad \therefore v_y = \dot{y} = \frac{b}{2}\left(-\frac{\pi}{4t_0}\sin\frac{\pi t}{4t_0} + \frac{3\pi}{4t_0}\sin\frac{3\pi t}{4t_0}\right)$$

$$\therefore a_y = \ddot{y} = \frac{b}{2}\left(-\frac{\pi^2}{16t_0^2}\cos\frac{\pi t}{4t_0} + \frac{9\pi^2}{16t_0^2}\cos\frac{3\pi t}{4t_0}\right) = \frac{\pi^2 b}{32t_0^2}\left(-\cos\frac{\pi t}{4t_0} + 9\cos\frac{3\pi t}{4t_0}\right)$$

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At point A: $t = t_0$

$$\therefore a_x = -\frac{\pi^2 b}{32t_0^2}\left(\sin\frac{\pi}{4} + 9\sin\frac{3\pi}{4}\right) = -\frac{\pi^2(240)}{32(12)^2}(7.071) = -3.635 \text{ m/s}^2$$

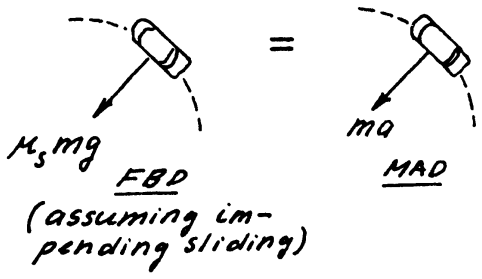
$$\therefore a_y = \frac{\pi^2 b}{32t_0^2}\left(-\cos\frac{\pi}{4} + 9\cos\frac{3\pi}{4}\right) = -\frac{\pi^2(240)}{32(12)^2}(7.071) = -3.635 \text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = 5.141 \text{ m/s}^2$$

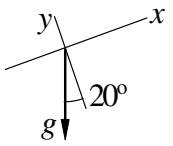
The FBD and MAD show only forces acting in the plane of the motion.

$\Sigma F = ma$: $\mu_s mg = ma$

$$\therefore \mu_s = \frac{a}{g} = \frac{5.141}{9.81} = 0.524 \blacklozenge$$



12.69



From the acceleration diagram of a water droplet:

$$a_x = -g\sin 20^\circ = -32.2\sin 20^\circ = -11.013 \text{ ft/s}^2$$
$$a_y = -g\cos 20^\circ = -32.2\cos 20^\circ = -30.26 \text{ ft/s}^2$$

Initial conditions at $t = 0$:

$$x = y = 0$$
$$v_x = 22\cos 30^\circ = 19.053 \text{ ft/s} \qquad v_y = 22\sin 30^\circ = 11.0 \text{ ft/s}$$

Integrating and using initial conditions:

$$v_x = -11.013t + 19.053 \text{ ft/s} \qquad v_y = -30.26t + 11.0 \text{ ft/s}$$
$$x = -5.507t^2 + 19.053t \text{ ft} \qquad y = -15.13t^2 + 11.0t \text{ ft}$$

Droplet lands when $y = 0$:

$$y = -15.13t^2 + 11.0t = 0 \qquad t = 0.7270 \text{ s}$$
$$R = x|_{t=0.7270\text{s}} = -5.507(0.7270^2) + 19.053(0.7270) = 10.94 \text{ ft} \blacktriangleleft$$

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12.70

From Eqs. (e) of Sample Problem 2.11:

$$\begin{aligned} x &= (v_0 \cos\theta)t = (65\cos 55^\circ)\,t = 37.28t \text{ ft} \\ y &= -\frac{1}{2}gt^2 + (v_0 \sin\theta)t = -\frac{32.2}{2}t^2 + (65\sin 55^\circ)t \\ &= -16.1t^2 + 53.24t \text{ ft} \end{aligned}$$

At point B:

$$\begin{aligned} x &= 60 \text{ ft} \qquad 60 = 37.28t \qquad t = 1.6094 \text{ s} \\ h &= y|_{t=1.6094\text{s}} = -16.1(1.6094^2) + 53.24(1.6094) = 44.0 \text{ ft} \end{aligned}$$ ◀

*12.71

From Sample Problem 12.12:

$$\begin{aligned} x &= C_1 e^{-ct/m} + C_2 \qquad y = C_3 e^{-ct/m} - \frac{mgt}{c} + C_4 \\ v_x &= -C_1 \frac{c}{m} e^{-ct/m} \qquad v_y = -C_3 \frac{c}{m} e^{-ct/m} - \frac{mg}{c} \end{aligned}$$

$$\begin{aligned} \frac{c}{m} &= \frac{0.0025}{1.2/32.2} = 0.06708 \text{ s}^{-1} \qquad \frac{mg}{c} = \frac{1.2}{0.0025} = 480.0 \text{ ft/s} \\ x &= C_1 e^{-0.06708t} + C_2 \qquad y = C_3 e^{-0.06708t} - 480t + C_4 \\ v_x &= -C_1 \left(0.06708 e^{-0.06708t}\right) \\ v_y &= -C_3 \left(0.06708 e^{-0.06708t}\right) - 480.0 \end{aligned}$$

$$\begin{aligned} v_0 \sin\theta &= 70 \sin 65^\circ = 63.44 \text{ ft/s} \\ v_0 \cos\theta &= 70 \cos 65^\circ = 29.58 \text{ ft/s} \end{aligned}$$

Initial conditions at $t = 0$:

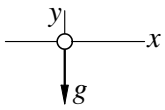
$$\begin{aligned} x &= 0 \quad \therefore C_2 = -C_1 \\ y &= 0 \quad \therefore C_4 = -C_3 \\ v_x &= v_0 \cos\theta \quad \therefore -C_1(0.06708) = 29.58 \qquad C_1 = -441.0 \text{ ft} \\ v_y &= v_0 \sin\theta \quad \therefore -C_3(0.06708) - 480.0 = 63.44 \qquad C_3 = -8101 \text{ ft} \end{aligned}$$

When $x = 60$ ft:

$$\begin{aligned} 60 &= -441.0 e^{-0.06708t} + 441.0 \qquad t = 2.180 \text{ s} \\ h &= y|_{t=2.180} = -8101 e^{-(0.06708)(2.180)} - 480(2.180) + (8101) = 55.7 \text{ ft} \end{aligned}$$ ◀

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12.72



Acceleration diagram

At $t = 0$ (initial conditions):

$$\begin{aligned} x &= 0 \qquad v_x = 200 \sin 30^\circ = 100 \text{ m/s} \\ y &= 1200 \text{ m} \qquad v_y = -200 \cos 30^\circ = -173.21 \text{ m/s} \end{aligned}$$

Integrating acceleration and applying initial conditions:

$$\begin{aligned} a_x &= 0 & a_y &= -9.81 \text{ m/s}^2 \\ v_x &= 100 \text{ m/s} & v_y &= -9.81t - 173.21 \text{ m/s} \\ x &= 100t \text{ m} & y &= -4.905t^2 - 173.21t + 1200 \text{ m} \end{aligned}$$

When $y = 0$:

$$-4.905t^2 - 173.21t + 1200 = 0 \qquad t = 5.932 \text{ s}$$

$$x = 100(5.932) = 593.2 \text{ m}$$

$$d = 1200 \tan 30^\circ - 593.2 = 99.6 \text{ m} \quad \blacktriangleleft$$

12.73

Eqs. (d) and (e) of Sample Problem 12.11:

$$\begin{aligned} x &= v_0 t \cos\theta = 2500t \cos\theta \text{ ft} \\ y &= v_0 t \sin\theta - \frac{1}{2} g t^2 = 2500t \sin\theta - 16.1t^2 \text{ ft} \end{aligned}$$

Setting $x = R = 5280$ ft and solving for t:

$$5280 = 2500t \cos\theta \qquad t = \frac{5280}{2500 \cos\theta} = \frac{2.112}{\cos\theta} \text{ s}$$

Setting $y = 0$, we get after dividing by t:

$$\begin{aligned} 2500 \sin\theta - 16.1t &= 0 \qquad 2500 \sin\theta - 16.1 \left(\frac{2.112}{\cos\theta} \right) = 0 \\ \sin\theta \cos\theta &= 16.1 \left(\frac{2.112}{2500} \right) = 0.013\ 601 \\ \frac{1}{2} \sin 2\theta &= 0.013\ 601 \qquad \sin 2\theta = 2(0.013\ 601) = 0.02720 \end{aligned}$$

$$\theta = \frac{1}{2} \sin^{-1}(0.02720) = 0.779^\circ \quad \blacktriangleleft$$

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12.74

From Eqs. (e) of Sample Problem 12.11:

$$x = (v_0 \cos\theta)\, t \qquad y = -\frac{1}{2} g t^2 + (v_0 \sin\theta)\, t$$

(a)

$$\begin{aligned} x &= (42\cos 28^\circ)\, t = 37.08t \text{ ft} \qquad \therefore t = \frac{x}{37.08} \text{ s} \\ y &= -\frac{1}{2}(32.2)t^2 + (42\sin 28^\circ)\, t = -16.1t^2 + 19.718t \end{aligned}$$

Substituting for t:

$$\begin{aligned} y &= -16.1\left(\frac{x}{37.08}\right)^2 + 19.718\left(\frac{x}{37.08}\right) \\ &= -0.01171x^2 + 0.5318x \text{ ft} \quad \blacktriangleleft \end{aligned}$$

(b) Check if ball hits the ceiling.

$$\begin{aligned} \frac{dy}{dx} &= -0.02342x + 0.5318 = 0 \qquad x = 22.71 \text{ ft} \\ y_{\max} &= -0.01171(22.71)^2 + 0.5318(22.71) = 6.04 \text{ ft} \end{aligned}$$

Since $y_{\max} < 25$ ft, the ball will not hit the ceiling.
Check if ball clears the net. When $x = 22$ ft:

$$y = -0.01171(22)^2 + 0.5318(22) = 6.03 \text{ ft}$$

Since $y > 5$ ft, the ball clears the net. $\blacktriangleleft$
When $x = 42$ ft:

$$y = -0.01171(42)^2 + 0.5318(42) = 1.679 \text{ ft}$$

Since $y > 0$, the ball lands behind the baseline. $\blacktriangleleft$

12.75

From Eqs. (e) of Sample Problem 12.11:

$$x = (v_0 \cos\theta)\, t \qquad y = -\frac{1}{2} g t^2 + (v_0 \sin\theta)\, t$$

$$\begin{aligned} y &= -\frac{1}{2}(32.2)t^2 + (v_0 \sin 70^\circ)t = -16.1t^2 + 0.9397v_0 t \text{ ft} \\ v_y &= \dot{y} = -32.2t + 0.9397v_0 \text{ ft/s} \end{aligned}$$

When $y = y_{\max}$

$$\begin{aligned} v_y = 0 \qquad & -32.2t + 0.9397v_0 = 0 \qquad t = 0.02918v_0 \text{ s} \\ y_{\max} = 27 \text{ ft} \qquad & -16.1(0.02918v_0)^2 + 0.9397v_0(0.02918v_0) = 27 \\ & 0.013712v_0^2 = 27 \qquad v_0 = 44.4 \text{ ft/s} \quad \blacktriangleleft \end{aligned}$$

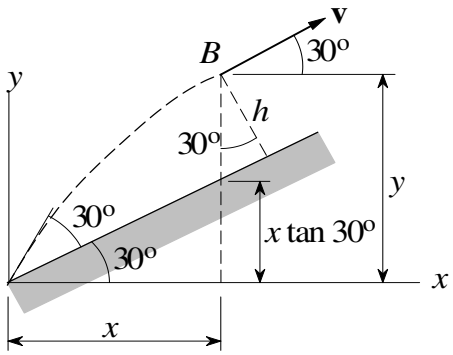
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12.76

Equations (e) of Sample Problem 12.10:

$$\begin{aligned} x &= (v_0 \cos\theta)t = (30\cos 60^\circ)t = 15t \text{ ft} \\ y &= -\frac{1}{2}gt^2 + (v_0 \sin\theta)t = -\frac{1}{2}(32.2)t^2 + (30\sin 60^\circ)t \\ &= -16.1t^2 + 25.98t \text{ ft} \end{aligned}$$

$$v_x = \dot{x} = 15 \text{ ft/s} \qquad v_y = \dot{y} = -32.2t + 25.98 \text{ ft/s}$$



At point B:

$$\mathbf{v} \text{ is parallel to the inclined surface} \quad \therefore \frac{v_y}{v_x} = \tan 30^\circ$$

$$\frac{-32.2t + 25.98}{15} = \tan 30^\circ \qquad t = 0.5379 \text{ s}$$

$$\begin{aligned} x &= 15(0.5379) = 8.069 \text{ ft} \\ y &= -16.1(0.5379^2) + 25.98(0.5379) = 9.316 \text{ ft} \\ h &= (y - x\tan 30^\circ)\cos 30^\circ = (9.316 - 8.069\tan 30^\circ)\cos 30^\circ \\ &= 4.03 \text{ ft} \blacktriangleleft \end{aligned}$$

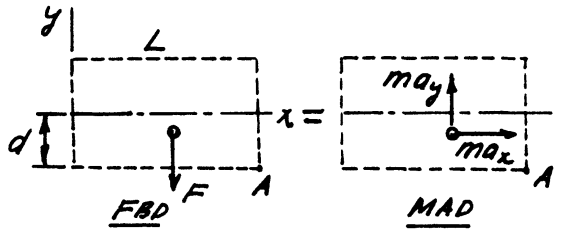
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12.77

$\Sigma F_x = ma_x$: $\stackrel{+}{\rightarrow}$ $0 = ma_x$ $\therefore a_x = 0$

$\Sigma F_y = ma_y$: $+\uparrow$ $-F = ma_y$

$$\therefore a_y = -\frac{F}{m} = -\frac{q\,\Delta V}{2md}$$



Horizontal motion

$$v_x = \int a_x\,dt + C_1 = 0 + C_1 \qquad x = \int v_x\,dt + C_2 = C_1 t + C_2$$

Initial conditions: $x = 0$, $v_x = v_0$ when $t = 0$. $\therefore C_1 = v_0$ $\therefore C_2 = 0$ $\therefore x = v_0 t$

Vertical motion

$$v_y = \int a_y\,dt + C_3 = -\frac{q\,\Delta V}{2md}t + C_3 \qquad y = \int v_y\,dt + C_4 = -\frac{q\,\Delta V}{4md}t^2 + C_3 t + C_4$$

Initial conditions: $y = 0$, $v_y = 0$ when $t = 0$. $\therefore C_3 = C_4 = 0$ $\therefore y = -\dfrac{q\,\Delta V}{4md}t^2$

When particle is at A:

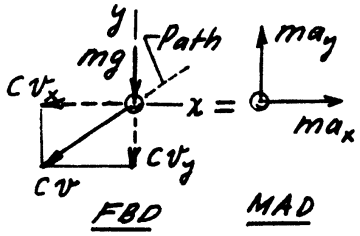
$x = L$ $\therefore v_0 t = L$ $\therefore t = L/v_0$

$$y = -d \quad \therefore -\frac{q\,\Delta V}{4md}(L/v_0)^2 = -d \qquad \therefore \Delta V = \frac{4m}{q}\left(v_0\frac{d}{L}\right)^2 \blacklozenge$$

*12.78

$\Sigma F_x = ma_x$: $\stackrel{+}{\rightarrow}$ $-cv_x = ma_x$ $\therefore a_x = -\dfrac{c}{m}v_x$

$\Sigma F_y = ma_y$: $+\uparrow$ $-mg - cv_y = ma_y$ $\therefore a_y = -g - \dfrac{c}{m}v_y$



(a) Vertical motion

$$\frac{dv_y}{dt} = a_y = -g - \frac{c}{m}v_y \qquad \therefore -dt = \frac{dv_y}{g + (c/m)v_y} \qquad \therefore -t = \frac{m}{c}\ln\left(g + \frac{c}{m}v_y\right) + C_1$$

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Initial condition: $v_y = v_0 \sin\alpha$ when $t = 0$. $\therefore C_1 = -\frac{m}{c}\ln\left(g + \frac{c}{m}v_0 \sin\alpha\right)$

$$\therefore t = \frac{m}{c}\ln\frac{g + (c/m)v_0 \sin\alpha}{g + (c/m)v_y}$$

At point A: $v_y = 0$ $\quad \therefore t = \frac{m}{c}\ln\left(1 + \frac{c}{mg}v_0 \sin\alpha\right)$ ◆

(b) Horizontal motion

$$\frac{dv_x}{dt} = a_x = -\frac{c}{m}v_x \qquad \therefore \frac{1}{v_x}dv_x = -\frac{c}{m}dt \qquad \therefore -\frac{c}{m}t = \ln v_x + C_2$$

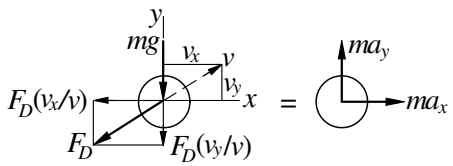
Initial condition: $v_x = v_0 \cos\alpha$ when $t = 0$. $\therefore C_2 = -\ln(v_0 \cos\alpha)$

$$\therefore -\frac{c}{m}t = \ln\frac{v_x}{v_0 \cos\alpha} \qquad \therefore v_x = (v_0 \cos\alpha)e^{-(c/m)t}$$

At point A: $v = v_x = (v_0 \cos\alpha)\left(1 + \frac{c}{mg}v_0 \sin\alpha\right)^{-1} = \frac{v_0 \cos\alpha}{1 + \frac{c}{mg}v_0 \sin\alpha}$ ◆

■12.79

(a)



FBD MAD

$$\Sigma F_x = ma_x \quad \stackrel{+}{\longrightarrow} \quad -F_D\frac{v_x}{v} = ma_x$$

$$a_x = -\frac{F_D}{m}\frac{v_x}{v} = -\frac{0.0005v^2}{0.1}\frac{v_x}{v} = -0.005vv_x$$
$$= -0.005v_x\sqrt{v_x^2 + v_y^2}\ \text{m/s}^2 \quad ◄$$

$$\Sigma F_y = ma_y \quad +\uparrow \qquad -F_D\frac{v_y}{v} - mg = ma_y$$

$$a_y = -\frac{F_D}{m}\frac{v_y}{v} - g = -\frac{0.0005v^2}{0.1}\frac{v_y}{v} - 9.81 = -0.005vv_y - 9.81$$
$$= -0.005v_y\sqrt{v_x^2 + v_y^2} - 9.81\ \text{m/s}^2 \quad ◄$$

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(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4$$

$$\dot{x}_3 = -0.005x_3\sqrt{x_3^2 + x_4^2} \qquad \dot{x}_4 = -0.005x_4\sqrt{x_3^2 + x_4^2} - 9.81$$

The initial conditions are

$$x_1(0) = 0 \qquad x_2(0) = 2 \text{ m/s}$$

$$x_3(0) = 30\cos 50^\circ = 19.284 \text{ m/s} \qquad x_4(0) = 30\sin 50^\circ = 22.981 \text{ m/s}$$

The following MATLAB program was used to integrate the equations:

```
function problem12_79
[t,x] = ode45(@f,(0:0.05:2),[0 2 19.284 22.981]);
printSol(t,x)
    function dxdt = f(t,x)
    v = sqrt(x(3)^2 + x(4)^2);
    dxdt = [x(3)
            x(4)
            -0.005*x(3)*v
            -0.005*x(4)*v-9.81];
    end
end
```

The two lines of output that span $x = 30$ m are

```
  t            x1            x2            x3            x4
1.7000e+000  2.9607e+001  2.3944e+001  1.5990e+001  3.6997e+000
1.7500e+000  3.0405e+001  2.4117e+001  1.5925e+001  3.1952e+000
```

Linear interpolation for h:

$$\frac{30.405 - 29.607}{24.117 - 23.944} = \frac{30 - 29.607}{h - 23.944} \qquad h = 24.0 \text{ m} \quad \blacktriangleleft$$

Linear interpolation for v_x and v_y:

$$\frac{30.405 - 29.607}{15.925 - 15.990} = \frac{30 - 29.607}{v_x - 15.990} \qquad v_x = 15.958 \text{ m/s}$$

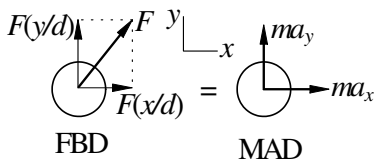
$$\frac{30.405 - 29.607}{3.1952 - 3.6997} = \frac{30 - 29.607}{v_y - 3.6997} \qquad v_y = 3.451 \text{ m/s}$$

$$\therefore v = \sqrt{15.958^2 + 3.451^2} = 16.33 \text{ m/s} \quad \blacktriangleleft$$

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■12.80

(a)



$$\Sigma F_x = ma_x \quad \stackrel{+}{\longrightarrow} \qquad F\frac{x}{d} = ma_x$$

$$a_x = \frac{1}{m}F\frac{x}{d} = \frac{1}{0.01}\frac{0.005}{d^2}\frac{x}{d} = 0.5\frac{x}{d^3} = \frac{0.5x}{(x^2+y^2)^{3/2}} \text{ m/s}^2 \quad \blacktriangleleft$$

$$\Sigma F_y = ma_y \quad +\uparrow \qquad F\frac{y}{d} = ma_y$$

$$a_y = \frac{1}{m}F\frac{y}{d} = \frac{1}{0.01}\frac{0.005}{d^2}\frac{y}{d} = 0.5\frac{y}{d^3} = \frac{0.5y}{(x^2+y^2)^{3/2}} \text{ m/s}^2 \quad \blacktriangleleft$$

The initial conditions are:

$$x = 0.3 \text{ m} \quad y = 0.4 \text{ m} \quad v_x = 0 \quad v_y = -2 \text{ m/s} \quad \text{at } t = 0 \quad \blacktriangleleft$$

(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4 \qquad \dot{x}_3 = \frac{0.5x_1}{(x_1^2+x_2^2)^{3/2}} \qquad \dot{x}_4 = \frac{0.5x_2}{(x_1^2+x_2^2)^{3/2}}$$

The MATLAB program for solving the equations is

```
function problem12_80
[t,x] = ode45(@f,(0:0.005:0.25),[0.3 0.4 0 -2]);
printSol(t,x)
    function dxdt = f(t,x)
    d3 = (sqrt(x(1)^2 + x(2)^2))^3;
    dxdt = [x(3)
            x(4)
            0.5*x(1)/d3
            0.5*x(2)/d3];
    end
end
```

The two output lines spanning $y = 0$ are shown below.

```
  t           x1           x2           x3           x4
2.2000e-001  3.5879e-001  3.0450e-003  6.5959e-001 -1.6667e+000
2.2500e-001  3.6213e-001 -5.2884e-003  6.7883e-001 -1.6668e+000
```


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Linear interpolation for x at $y = 0$:

$$\frac{0.36213 - 0.35879}{-0.0052884 - 0.0030450} = \frac{x - 0.35879}{0 - 0.0030450} \qquad x = 0.360 \text{ m} \blacktriangleleft$$

Linear interpolation for v_x:

$$v_x : \frac{0.67883 - 0.65959}{-0.0052884 - 0.0030450} = \frac{v_x - 0.65959}{0 - 0.0030450} \qquad v_x = 0.6666 \text{ m/s}$$

By inspection $v_y = -1.6667$ m/s.

$$\therefore v = \sqrt{0.6666^2 + 1.6667^2} = 1.795 \text{ m/s} \blacktriangleleft$$

■12.81

(a) The signs of a_x and a_y in the solution of Prob. 12.80 must be reversed.

$$a_x = -\frac{0.5x}{(x^2 + y^2)^{3/2}} \text{ m/s}^2 \blacktriangleleft \qquad a_y = -\frac{0.5y}{(x^2 + y^2)^{3/2}} \text{ m/s}^2 \blacktriangleleft$$

The initial conditions are the same as in Problem 12.80:

$$x = 0.3 \text{ m} \quad y = 0.4 \text{ m} \quad v_x = 0 \quad v_y = -2 \text{ m/s} \quad \text{at } t = 0. \blacktriangleleft$$

(b) MATLAB program:

```
function problem12_81
[t,x] = ode45(@f,(0:0.005:0.2),[0.3 0.4 0 -2]);
printSol(t,x)
    function dxdt = f(t,x)
    d3 = (sqrt(x(1)^2 + x(2)^2))^3;
    dxdt = [x(3)
            x(4)
            -0.5*x(1)/d3
            -0.5*x(2)/d3];
    end
end
```

The two lines of output spanning $y = 0$ are:

```
  t           x1           x2           x3           x4
1.8000e-001  2.5952e-001  8.5117e-003 -6.3935e-001 -2.3329e+000
1.8500e-001  2.5623e-001 -3.1544e-003 -6.7692e-001 -2.3333e+000
```

Linear interpolation for x at $y = 0$:

$$\frac{0.25623 - 0.25952}{-0.0031544 - 0.0085117} = \frac{x - 0.25952}{0 - 0.0085177} \qquad x = 0.257 \text{ m} \blacktriangleleft$$

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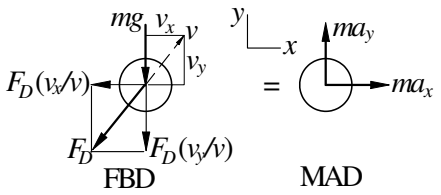
Linear interpolation for v_x:

$$\frac{-0.67692-(-0.63935)}{-0.0031544-0.0085117}=\frac{v_x-(-0.63935)}{0-0.0085177} \qquad v_x=-0.6668 \text{ m/s}$$

By inspection, $v_y=-2.3332$ m/s.

$$v=\sqrt{0.6668^2+2.3332^2}=2.43 \text{ m/s} \blacktriangleleft$$

■12.82



$$\Sigma F_x=ma_x \quad \stackrel{+}{\longrightarrow} \qquad -F_D\frac{v_x}{v}=ma_x \qquad -c_Dv^{1.5}\frac{v_x}{v}=ma_x$$

$$\therefore a_x=-\frac{c_D}{m}v_x\sqrt{v}$$

$$\frac{c_D}{m}=\frac{0.0012}{(9/16)(32.2)}=0.06869 \text{ (ft}\cdot\text{s)}^{-0.5}$$

$$\therefore a_x=-0.06869v_x\left(v_x^2+v_y^2\right)^{0.25} \text{ ft/s}^2 \blacktriangleleft$$

$$\Sigma F_y=ma_y \quad +\uparrow \qquad -F_D\frac{v_y}{y}-mg=ma_y \qquad -c_Dv^{1.5}\frac{v_y}{y}-mg=ma_y$$

$$\therefore a_y=-\frac{c_D}{m}v_y\sqrt{v}-g=-0.06869v_y\left(v_x^2+v_y^2\right)^{0.25}-32.2 \text{ ft/s}^2 \blacktriangleleft$$

The initial conditions are:

$$x=0 \quad y=6 \text{ ft} \quad v_x=120 \text{ ft/s} \quad v_y=0 \quad \text{at } t=0 \blacktriangleleft$$

(b) Letting $x_1=x$, $x_2=y$, $x_3=v_x$ and $x_4=v_y$, the equivalent first-order equations are

$$\begin{aligned}\dot{x}_1 &= x_3 \qquad \dot{x}_2=x_4\\ \dot{x}_3 &= -0.06869x_3\left(x_3^2+x_4^2\right)^{0.25}\\ \dot{x}_4 &= -0.06869x_4\left(x_3^2+x_4^2\right)^{0.25}-32.2\end{aligned}$$

The MATLAB program is

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```
function problem12_82
[t,x] = ode45(@f,(0:0.02:0.7),[0 6 120 0]);
printSol(t,x)
    function dxdt = f(t,x)
    v25 = sqrt((sqrt(x(3)^2 + x(4)^2)));
    dxdt = [x(3)
            x(4)
            -0.06869*x(3)*v25
            -0.06869*x(4)*v25-32.2];
    end
end
```

The two lines of output that span $y = 0$ are:

```
  t            x1            x2            x3            x4
6.4000e-001  6.1878e+001  2.6073e-001  7.7840e+001 -1.6851e+001
6.6000e-001  6.3426e+001 -8.0650e-002  7.6894e+001 -1.7286e+001
```

Linear interpolation for x at $y = 0$:

$$\frac{63.426 - 61.878}{-0.080650 - 0.26073} = \frac{R - 61.878}{0 - 0.26072} \qquad R = 63.1 \text{ ft} \quad \blacktriangleleft$$

Linear interpolation for t at $y = 0$:

$$\frac{0.66 - 0.64}{-0.080650 - 0.26073} = \frac{t - 0.64}{0 - 0.26072} \qquad t = 0.655 \text{ s} \quad \blacktriangleleft$$

■12.83

$$a_x = -10 - 0.5v_x \text{ m/s}^2 \qquad a_y = -9.81 - 0.5v_y \text{ m/s}^2$$

(a) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4 \qquad \dot{x}_3 = -10 - 0.5x_3 \qquad \dot{x}_4 = -9.81 - 0.5x_4$$

At $t = 0$ (initial conditions):

$$x_1 = x_2 = 0 \qquad x_3 = 30\cos 40^\circ = 22.98 \text{ m/s} \qquad x_4 = 30\sin 40^\circ = 19.284 \text{ m/s}$$

MATLAB program:

```
function problem12_83
[t,x] = ode45(@f,(0:0.05:3.5),[0 0 22.98 19.284]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
```


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```
grid on
xlabel('x (ft)'); ylabel('y (ft)')
    function dxdt = f(t,x)
    dxdt = [x(3)
            x(4)
            -10-0.5*x(3)
            -9.81-0.5*x(4)];
    end
end
```

Two lines of output that span $y = 0$:

```
 t            x1            x2            x3            x4
3.1000e+000  5.7152e+000  4.7141e-001 -1.0878e+001 -1.1363e+001
3.1500e+000  5.1656e+000 -1.0184e-001 -1.1103e+001 -1.1567e+001
```

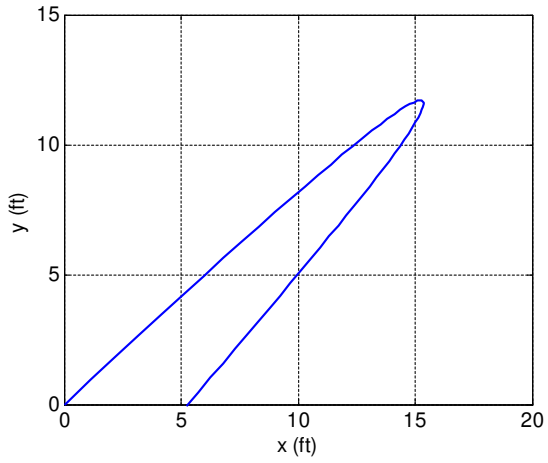
Linear interpolation for x at $y = 0$:

$$\frac{5.1656 - 5.7152}{-0.10184 - 0.47141} = \frac{b - 5.7152}{0 - 0.47141} \qquad b = 5.26 \text{ m} \blacktriangleleft$$

Linear interpolation for t at $y = 0$:

$$\frac{3.15 - 3.10}{-0.10184 - 0.47141} = \frac{t - 3.10}{0 - 0.47141} \qquad t = 3.14 \text{ s} \blacktriangleleft$$

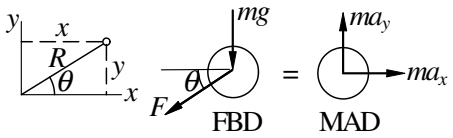
(b)



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■12.84

(a)



Spring force is $F = k(R - L_0)$, where $R = \sqrt{x^2 + y^2}$.

$$\Sigma F_x = ma_x \quad \stackrel{+}{\longrightarrow} \qquad -F\cos\theta = ma_x$$

$$\begin{aligned} a_x &= -\frac{F}{m}\cos\theta = -\frac{k(R-L_0)}{m}\frac{x}{R} = -\frac{k}{m}\left(1 - \frac{L_0}{R}\right)x \\ &= -\frac{10}{0.25}\left(1 - \frac{0.5}{R}\right)x = -40\left(1 - \frac{0.5}{R}\right)x \text{ m/s}^2 \quad \blacktriangleleft \end{aligned}$$

$$\Sigma F_y = ma_y \quad + \uparrow \qquad -F\sin\theta - mg = ma_y$$

$$\begin{aligned} a_y &= -\frac{F}{m}\sin\theta - g = -\frac{k(R-L_0)}{m}\frac{y}{R} - g = -\frac{k}{m}\left(1 - \frac{L_0}{R}\right)y - g \\ &= -40\left(1 - \frac{0.5}{R}\right)y - 9.81 \text{ m/s}^2 \quad \blacktriangleleft \end{aligned}$$

The initial conditions are:

$$x = 0.5 \text{ m} \quad y = -0.5 \text{ m} \quad v_x = v_y = 0 \quad \text{at } t = 0 \quad \blacktriangleleft$$

(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

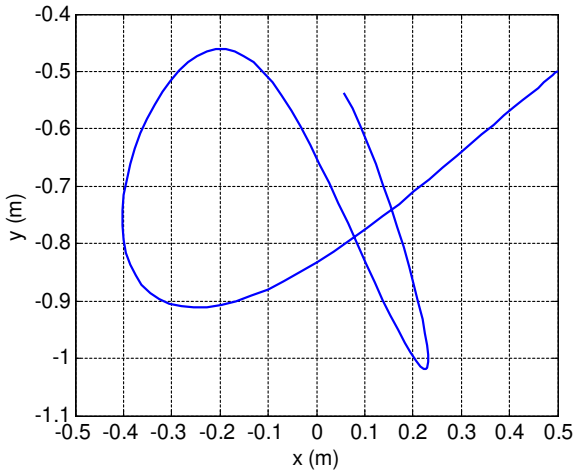
$$\begin{aligned} \dot{x}_1 &= x_3 \qquad \dot{x}_2 = x_4 \\ \dot{x}_3 &= -40\left(1 - \frac{0.5}{R}\right)x_1 \qquad \dot{x}_4 = -40\left(1 - \frac{0.5}{R}\right)x_2 - 9.81 \end{aligned}$$

where $R = \sqrt{x_1^2 + x_2^2}$.
The MATLAB program is:

```
function problem12_84
[t,x] = ode45(@f,(0:0.02:2),[0.5 -0.5 0 0]);
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (m)'); ylabel('y (m)')
    function dxdt = f(t,x)
```


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```
    rr = 1-0.5/sqrt(x(1)^2 + x(2)^2);
    dxdt = [x(3)
            x(4)
            -40*rr*x(1)
            -40*rr*x(2)-9.81];
    end
end
```



■12.85

(a) The expressions for the accelerations in Prob. 12.84 are now valid only when the spring is in tension. If the spring is not in tension, the spring force is zero. Therefore, we have

$$a_x = \begin{cases} -40\left(1 - \dfrac{0.5}{R}\right) x \text{ m/s}^2 & \text{if } R > 0.5 \text{ m} \\ 0 & \text{if } R \leq 0.5 \text{ m} \end{cases} \quad \blacktriangleleft$$

$$a_y = \begin{cases} -40\left(1 - \dfrac{0.5}{R}\right) y - 9.81 \text{ m/s}^2 & \text{if } R > 0.5 \text{ m} \\ -9.81 \text{ m/s}^2 & \text{if } R \leq 0.5 \text{ m} \end{cases} \quad \blacktriangleleft$$

The initial conditions are:

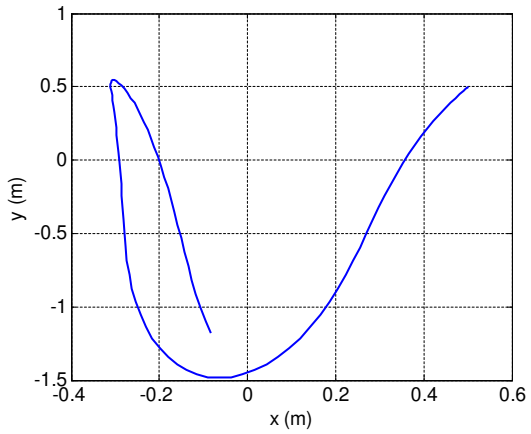
$$x = y = 0.5\text{m} \quad v_y = v_y = 0 \quad \text{at } t = 0 \quad \blacktriangleleft$$

(b) MATLAB program:

```
function problem12_85
[t,x] = ode45(@f,(0:0.02:2),[0.5 0.5 0 0]);
```


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```
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (m)'); ylabel('y (m)')
    function dxdt = f(t,x)
    rr = 1-0.5/sqrt(x(1)^2 + x(2)^2);
    if rr < 0; rr = 0; end
    dxdt = [x(3)
            x(4)
            -40*rr*x(1)
            -40*rr*x(2)-9.81];
    end
end
```



■12.86

(a)

$$a_x = -a_D \cos\theta + a_L \sin\theta \qquad a_y = -a_D \sin\theta - a_L \cos\theta - g$$

Substitute

$$\sin\theta = \frac{v_y}{v} \qquad \cos\theta = \frac{v_x}{v} \qquad a_D = 0.05v^2$$
$$a_L = 0.16\omega v = 0.16(10)v = 1.6v$$

where $v = v_x^2 + v_y^2$.

$$a_x = -0.05v^2\frac{v_x}{v} + 1.6v\frac{v_y}{v} = -0.05vv_x + 1.6v_y \text{ ft/s}^2 \quad \blacktriangleleft$$

$$a_y = -0.05v^2\frac{v_y}{v} - 1.6v\frac{v_x}{v} - 32.2 = -0.05vv_y - 1.6v_x - 32.2 \text{ ft/s}^2 \quad \blacktriangleleft$$

The initial conditions at $t = 0$ are:

$$x = y = 0 \quad v_x = 60\cos 60^\circ = 30 \text{ ft/s} \quad v_y = 60\sin 60^\circ = 51.96 \text{ ft/s}$$

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(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\begin{aligned} \dot{x}_1 &= x_3 \qquad \dot{x}_2 = x_4 \\ \dot{x}_3 &= -0.05vx_3 + 16x_4 \qquad \dot{x}_4 = -0.05vx_4 - 16x_3 - 32.2 \end{aligned}$$

MATLAB program:

```
function problem12_86
[t,x] = ode45(@f,(0:0.02:1.2),[0 0 30 51.96]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (ft)'); ylabel('y (ft)')
    function dxdt = f(t,x)
    v = sqrt(x(3)^2 + x(4)^2);
    dxdt = [x(3)
            x(4)
            -0.05*v*x(3)+1.6*x(4)
            -0.05*v*x(4)-1.6*x(3)-32.2];
    end
end
```

The two lines of output that span $y = 0$ are

```
  t            x1            x2            x3            x4
1.0400e+000  1.9377e+001  2.5868e-001  9.6888e-001 -2.2523e+001
1.0600e+000  1.9388e+001 -1.9335e-001  2.3210e-001 -2.2675e+001
```

Linear interpolation for t at $y = 0$:

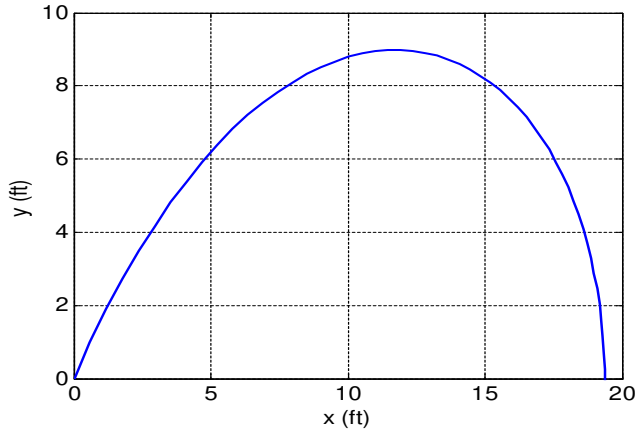
$$\frac{1.06 - 1.04}{-0.19335 - 0.25868} = \frac{t - 1.04}{0 - 0.25868} \qquad t = 1.051 \text{ s} \quad \blacktriangleleft$$

Linear interpolation for x at $y = 0$:

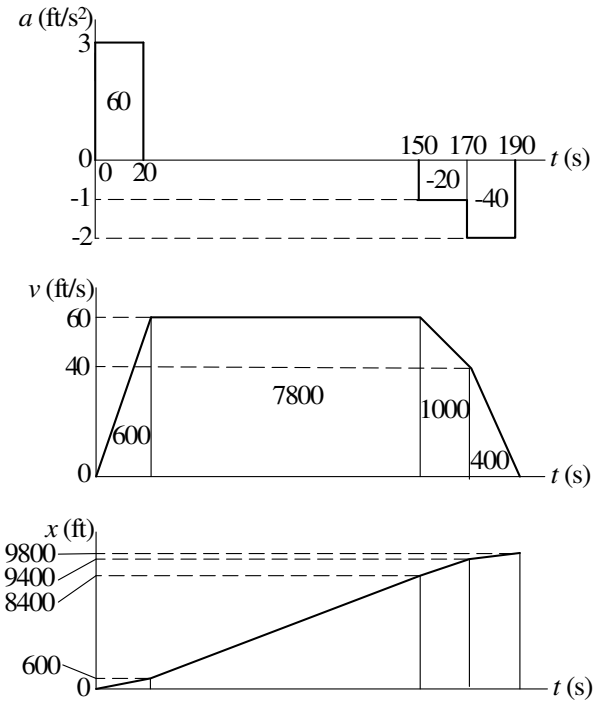
$$\frac{19.388 - 19.377}{-0.19335 - 0.25868} = \frac{x - 19.377}{0 - 0.25868} \qquad x = 19.38 \text{ ft} \quad \blacktriangleleft$$

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(c)



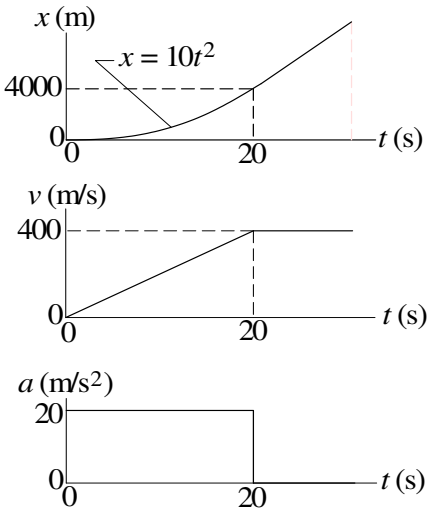
12.87



$d = 9800$ ft ◄

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12.88



$$v = \text{slope of } x \text{ diagram} \quad v|_{20\text{s}} = \left.\frac{dx}{dy}\right|_{20\text{s}} = 400 \text{ m/s}$$

$$a = \text{slope of } v \text{ diagram} \quad a = \frac{400}{20} = 20 \text{ m/s}^2$$

12.89

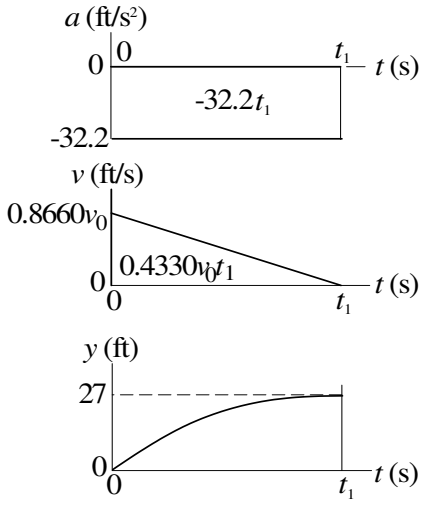
It is sufficient to consider vertical motion only:

$$a = -g = -32.2 \text{ ft/s}^2$$

Initial conditions:

$$v|_{t=0} = v_0 \sin 60^\circ = 0.8660 v_0 \text{ ft/s} \qquad y|_{t=0} = 0$$

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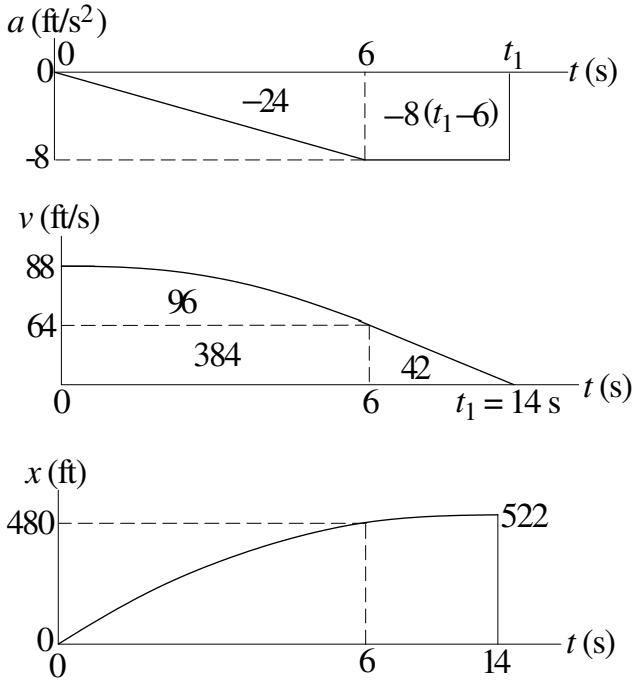


End conditions (t_1 is the time when the ball is at its maximum height):

$$v|_{t=t_1} = 0 \qquad 0.8660v_0 - 32.2t = 0 \qquad t_1 = 0.026\ 89v_0$$
$$y|_{t=t_1} = 27\ \text{ft} \qquad 0.4330(0.026\ 89)v_0^2 = 27 \qquad v_0 = 48.2\ \text{ft/s} \quad \blacktriangleleft$$

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12.90



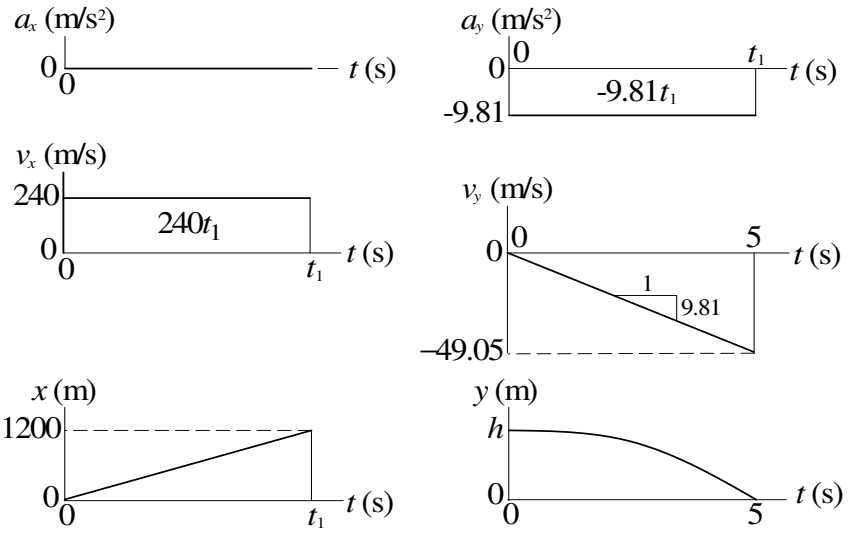
From a and v diagrams $\quad -8(t_1 - 6) = -64 \quad t_1 = 14.0$ s ◄

From x diagram $\quad x|_{t_1} = 522$ ft ◄

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12.91

Horizontal motion ($a_x = 0$) Vertical motion ($a_y = -9.81$ m/s^2)



End condition:

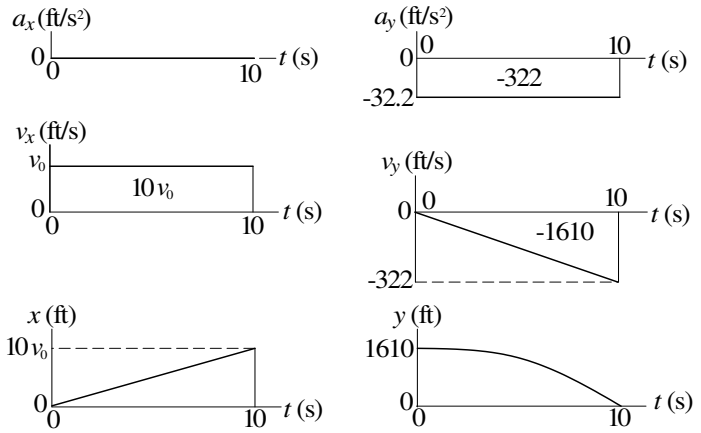
$$x|_{t=t_1} = 1200 \text{ m} \qquad 240t_1 = 1200 \qquad t_1 = 5.0 \text{ s}$$

$$h = -[\text{area under } v_y \text{ diagram}] = \frac{1}{2}(49.05)(5) = 122.6 \text{ m} \quad \blacktriangleleft$$

12.92

Horizontal motion: $a_x = 0$ Vertical motion: $a_y = -32.2$ ft/s^2

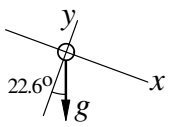
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$$-\left.\frac{v_y}{v_x}\right|_{t=10} = \tan 20^\circ \qquad \frac{322}{v_0} = \tan 20^\circ \qquad v_0 = 885 \text{ ft/s} \blacktriangleleft$$

$$R = x|_{t=10} = 10v_0 = 8850 \text{ ft} \blacktriangleleft \qquad h = y|_{t=0} = 1610 \text{ ft} \blacktriangleleft$$

12.93



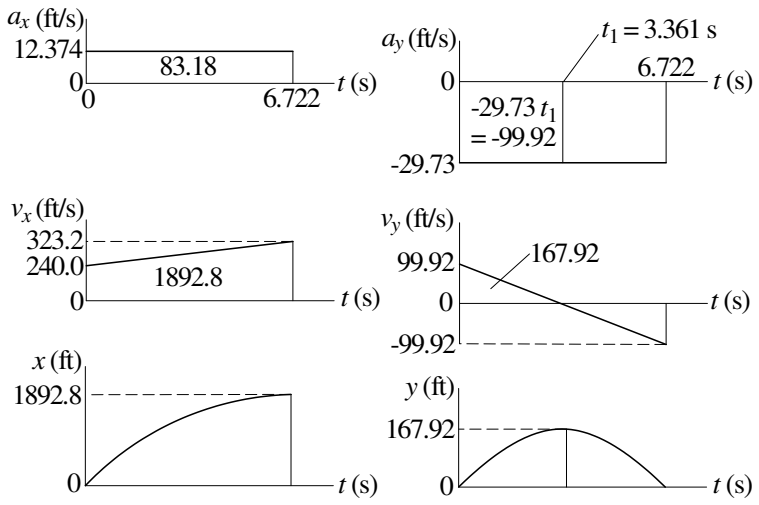
Acceleration diagram

$$\begin{aligned} a_x &= g\sin 22.6^\circ = 32.2\sin 22.6^\circ = 12.374 \text{ ft/s}^2 \\ a_y &= -g\cos 22.6^\circ = -32.2\cos 22.6^\circ = -29.73 \text{ ft/s}^2 \end{aligned}$$

At $t = 0$ (initial conditions):

$$\begin{aligned} x &= 0 \qquad v_x = 260\cos 22.6^\circ = 240.0 \text{ ft/s} \\ y &= 0 \qquad v_y = 260\sin 22.6^\circ = 99.92 \text{ ft/s} \end{aligned}$$

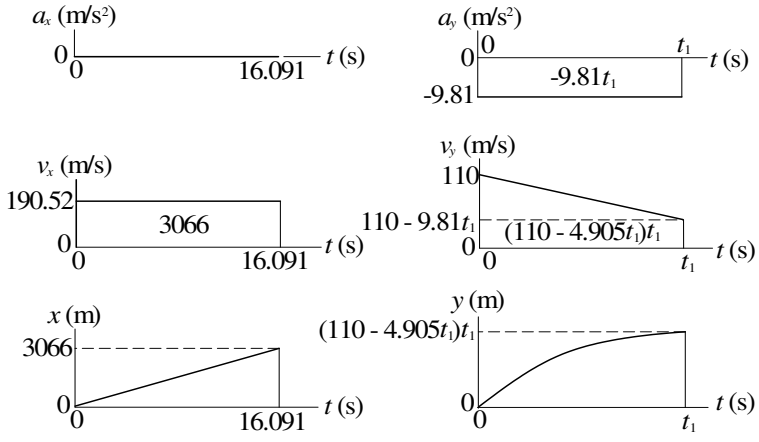
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$v_y = 0$ at $t = t_1$. $\therefore 99.92 - 29.73t_1 = 0$ $\therefore t_1 = 3.361$ s

$h = 167.9$ ft ◀ $R = 1893$ ft ◀ time of flight $= 6.72$ s ◀

12.94



End condition: $y|_{t=t_1} = 500$ m. $\therefore (110 - 4.905t_1)t_1 = 500$
The larger root is $t_1 = 16.091$ s ◀
$R = x|_{t=16.091} = 3066$ m ◀

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12.95

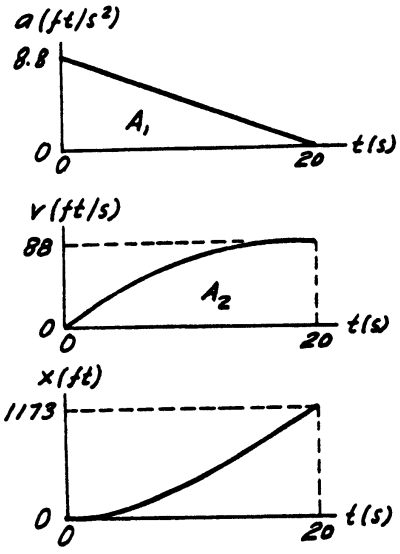
Maximum speed is reached when the acceleration becomes zero, i.e. at t = 20 s.

(a) $A_1 = \frac{1}{2}(8.8)(20) = 88$ ft/s

$$v\big|_{20s} = v\big|_0 + A_1 = 0 + 88 = 88 \text{ ft/s} \blacklozenge$$

(b) $A_2 = \frac{2}{3}(88)(20) = 1173$ ft

$$x\big|_{20s} = x\big|_0 + A_2 = 0 + 1173 = 1173 \text{ ft} \blacklozenge$$

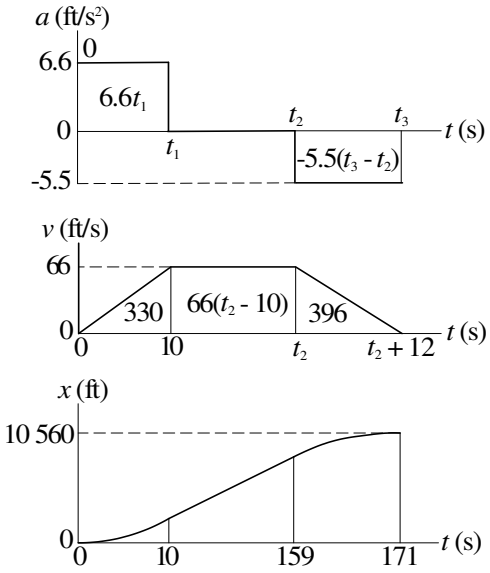


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12.96

$$2 \text{ mi } = 2(5280) = 10\ 560 \text{ ft} \qquad 45 \text{ mi/h } = 45\frac{5280}{3600} = 66 \text{ ft/s}$$

Accelerate at the maximum rate (6.6 ft/s^2) until maximum allowable speed (66 ft/s) is reached at time t_1. Then maintain this speed until time t_2. Finally, decelerate at the maximum rate (5.5 ft/s^2) until the train stops at time t_3. The distance traveled during this time must be 10 560 ft.



From a and v diagrams:

$$\begin{aligned} 6.6t_1 &= 66 \text{ ft/s} \quad \therefore t_1 = 10 \text{ s} \\ 5.5(t_3 - t_2) &= 66 \text{ ft/s} \quad \therefore t_3 - t_2 = 12 \text{ s} \end{aligned}$$

From v and x diagrams:

$$330 + 66(t_2 - 10) + 396 = 10\ 560 \quad \therefore t_2 = 159.0 \text{ s}$$

$$t_3 = t_2 + 12 = 159.0 + 12 = 171.0 \text{ s} \blacktriangleleft$$

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12.97

In order to compute the area A_3 under the v-diagram, we must analyze the fictitious motion between 16 s and t_1 (time when the slope of v-diagram becomes zero).

From similar triangles: $\dfrac{t_1 - 16}{0.75} = \dfrac{16}{1.75 - 0.75}$

$\therefore\ t_1 = 28$ s

$A_1 = -\dfrac{1}{2}(1.75 + 0.75)(16) = -20$ m/s

$A_2 = -\dfrac{1}{2}(0.75)(28 - 16) = -4.5$ m/s

$v|_{16s} = v|_0 + A_1 = v_0 - 20$ m/s

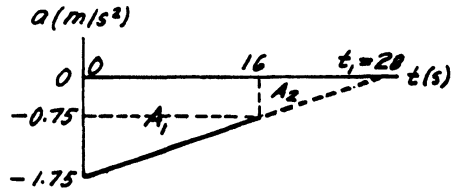
But $v|_{16s} = 0$ $\quad\therefore\ v_0 = 20$ m/s ◆

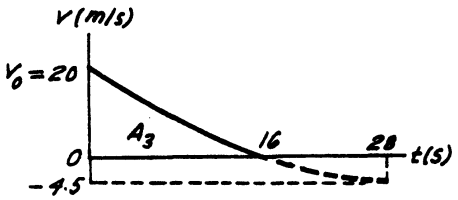
$v|_{28s} = v|_{16s} + A_2 = 0 - 4.5 = -4.5$ m/s

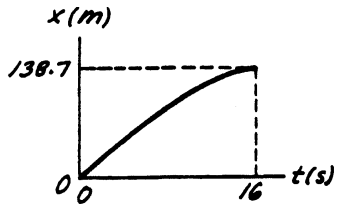
$A_3 = \dfrac{1}{3}(24.5)(28) - (4.5)(28) + \dfrac{2}{3}(12)(4.5)$

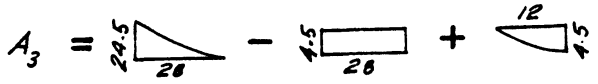
$= 138.7$ m (see diagram below)

$x|_{16s} = x|_0 + A_3 = 0 + 138.7 = 138.7$ m ◆



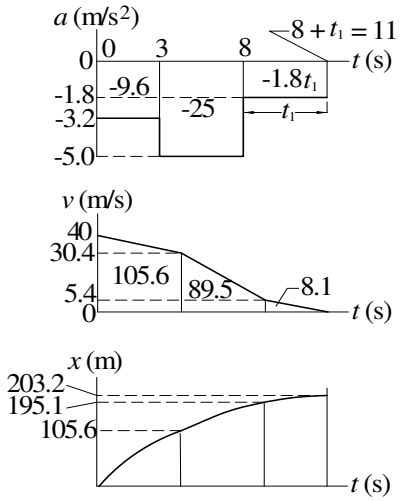






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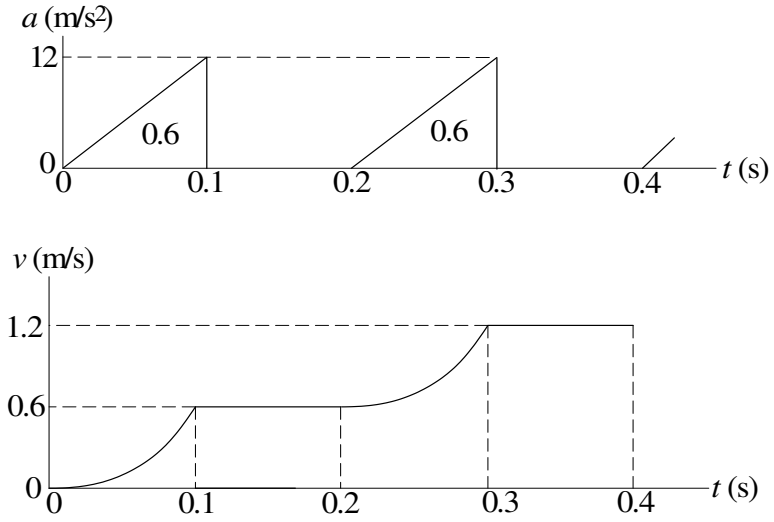
12.98



$$v = 0 \text{ when } 40 - 9.6 - 25 - 1.8t_1 = 0 \qquad \therefore t_1 = 3.0\text{s}$$

After touchdown, the plane travels 203 m ◄

12.99



From the v diagram:

$$v|_{0.4\text{s}} = 1.2 \text{ m/s} \text{ ◄}$$

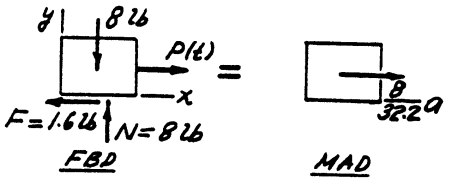
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$$x|_{0.4s} = \text{area under } v \text{ diagram}$$
$$= 2\left[\frac{1}{3}(0.6)(0.1)\right] + 4(0.6)(0.1) = 0.28 \text{ m} \blacktriangleleft$$

12.100

$\Sigma F_x = ma_x$: $\stackrel{+}{\rightarrow}$ $P(t) - 1.6 = \frac{8}{32.2}a$

$\therefore a = 4.025\,P(t) - 6.44 \text{ ft/s}^2$



Consider only one cycle of acceleration.

$A_1 = (4.025P_0 - 6.44)(0.2) =$

$= 0.8050P_0 - 1.288 \text{ ft/s}$

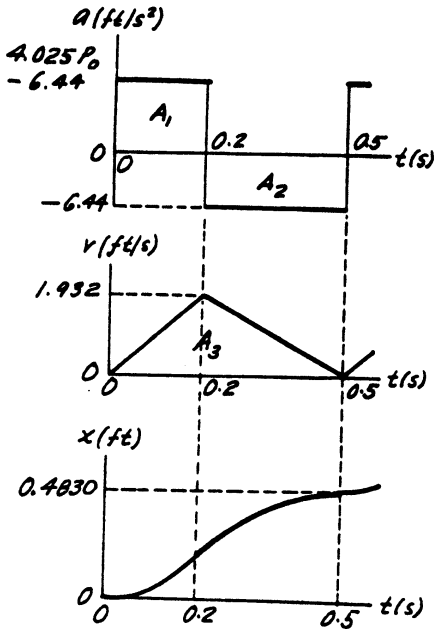
$A_2 = -(6.44)(0.3) = -1.932 \text{ ft/s}$

For the average speed to be constant, the change of speed over the cycle must be zero.

$\therefore \Delta v = A_1 + A_2 = 0$

$\therefore 0.8050P_0 - 1.288 - 1.932 = 0$

$\therefore P_0 = 4.00 \text{ lb} \blacklozenge$



$v|_{0.2s} = v|_0 + A_1 = 0 + 1.932 = 1.932 \text{ ft/s}$

$A_3 = \frac{1}{2}(1.932)(0.5) = 0.4830 \text{ ft}$

$x|_{0.5s} = x|_0 + A_3 = 0 + 0.4830 = 0.4830 \text{ ft}$

This is also the distance traveled during the 0.5 second cycle.

$\therefore v_{aver} = \frac{\Delta x}{\Delta t} = \frac{0.4830}{0.5} = 0.966 \text{ ft/s} \blacklozenge$

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12.101

$\Sigma F_x = ma_x$: $\overset{+}{\rightarrow}$ $P(t) - 1.6 = \dfrac{8}{32.2}a$

$\therefore a = 4.025\,P(t) - 6.44\ \text{ft/s}^2$

$= \begin{cases} 4.025(6) - 6.44 \\ -6.44 \end{cases} = \begin{cases} 17.71\ \text{ft/s}^2 \\ -6.44\ \text{ft/s}^2 \end{cases}$

$A_1 = A_3 = (17.71)(0.2) = 3.542\ \text{ft/s}$

$A_2 = -(6.44)(0.3) = -1.932\ \text{ft/s}$

$v|_{0.2s} = v|_0 + A_1 = 2 + 3.542 = 5.542\ \text{ft/s}$

$v|_{0.5s} = v|_{0.2s} + A_2 = 5.542 - 1.932 = 3.610\ \text{ft/s}$

$v|_{0.7s} = v|_{0.5s} + A_3 = 3.610 + 3.542 = 7.152\ \text{ft/s}$

$A_4 = \dfrac{1}{2}(2 + 5.542)(0.2) = 0.754\ \text{ft}$

$A_5 = \dfrac{1}{2}(5.542 + 3.610)(0.3) = 1.373\ \text{ft}$

$A_6 = \dfrac{1}{2}(3.610 + 7.152)(0.2) = 1.076\ \text{ft}$

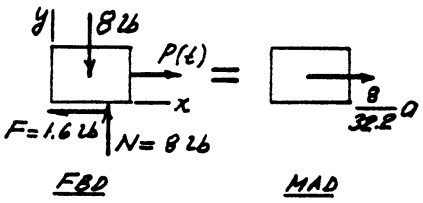
$x|_{0.2s} = x|_0 + A_4 = 0 + 0.754 = 0.754\ \text{ft}$

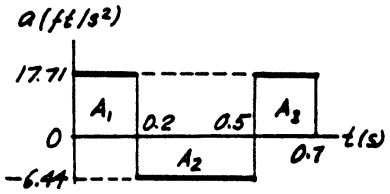
$x|_{0.5s} = x|_{0.2s} + A_5 = 0.754 + 1.373 = 2.127\ \text{ft}$

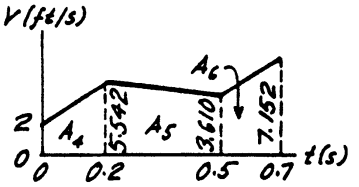
$x|_{0.7s} = x|_{0.5s} + A_6 = 2.127 + 1.076 = 3.203\ \text{ft}$

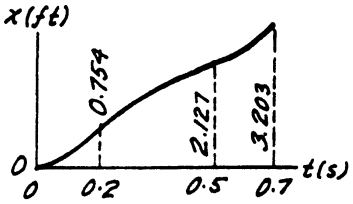
At t = 0.7 s:

(a) v = 7.15 ft/s ◆ **(b)** x = 3.20 ft ◆



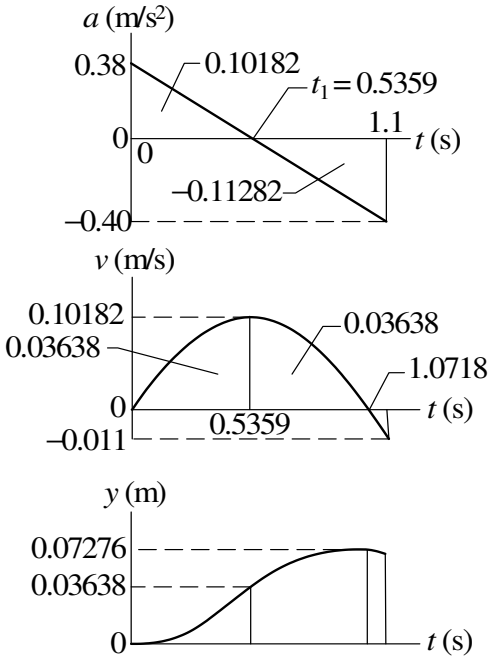






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12.102



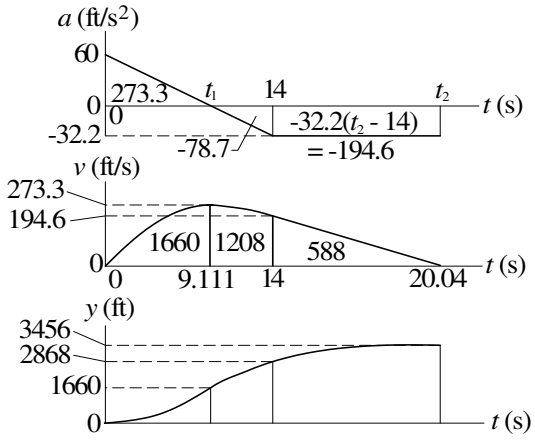
From similar triangles on the a-diagram:

$$\frac{t_1}{0.38} = \frac{1.1}{0.78} \qquad t_1 = 0.5359 \text{ s}$$

$$v_{\max} = 0.1018 \text{ m/s} \blacktriangleleft \qquad y_{\max} = 0.0728 \text{ m} \blacktriangleleft$$

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12.103



From similar triangles on a-diagram:

$$\frac{t_1}{60} = \frac{14}{60 + 32.2} \qquad \therefore t_1 = 9.111 \text{ s}$$

Let t_2 be the time when $v = 0$. Therefore,

$$194.6 - 32.2(t_2 - 14) = 0 \qquad \therefore t_2 = 20.04 \text{ s}$$

$v_{\max} = 273$ ft/s ◀ $\quad y_{\max} = 3460$ ft ◀ occurring at $t = 20.0$ s ◀

12.104

$$v = 2x^3 - 8x^2 + 12x \text{ mm/s}$$

$$a = v\frac{dv}{dx} = \left(2x^3 - 8x^2 + 12x\right)\left(6x^2 - 16x + 12\right)$$

$$a|_{x=2} = (2(8) - 8(4) + 12(2))\,(6(4) - 16(2) + 12) = 32.0 \text{ mm/s}^2 \quad ◀$$

12.105

$$a = At + B$$

When $t = 0$: $a = 0 \quad \therefore B = 0$
When $t = 6$ ft/s: $a = 8$ ft/s $\quad \therefore 8 = A(6) \quad \therefore A = \frac{4}{3}$ ft/s^3

$$\therefore a = \frac{4}{3}t \text{ ft/s}^2 \qquad v = \int a\, dt + C = \frac{2}{3}t^2 + C$$

When $t = 0$: $v = v_0 \quad \therefore C = v_0$
When $t = 6$ s: $v = 16$ ft/s

$$\therefore \frac{2}{3}(36) + v_0 = 16 \quad \therefore v_0 = -8.0 \text{ ft/s} \quad ◀$$

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12.106

Car A	Car B
$a = 4$ ft/s^2	$a = -2$ ft/s^2
$v = 4t + C_1$ ft/s	$v = -2t + C_3$ ft/s
$v\|_{t=0} = 30$ ft/s $\therefore C_1 = 30$ ft/s	$v\|_{t=0} = 60$ ft/s $\therefore C_3 = 60$ ft/s
$v = 4t + 30$ ft/s	$v = -2t + 60$ ft/s
$x = 2t^2 + 30t + C_2$ ft	$x = -t^2 + 60t + C_4$ ft
$x\|_{t=0} = -400$ ft $\therefore C_2 = -400$ ft	$x\|_{t=0} = 0$ $\therefore C_4 = 0$
$x = 2t^2 + 30t - 400$ ft	$x = -t^2 + 60t$ ft

Car A overtakes car B when $x_A = x_B$:

$$\begin{aligned} 2t^2 + 30t - 400 &= -t^2 + 60t \\ 3t^2 - 30t - 400 &= 0 \qquad t = 17.58 \text{ s} \blacktriangleleft \end{aligned}$$

12.107

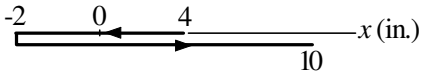
(a)

$$\begin{aligned} x &= 3t^3 - 9t + 4 \text{ in.} \\ \Delta x &= x|_{t=2} - x|_{t=0} = [3(8) - 9(2) + 4] - 4 = 6.0 \text{ in.} \blacktriangleleft \end{aligned}$$

(b)

$$v = \dot{x} = 9t^2 - 9 \text{ in./s} \qquad \therefore v = 0 \text{ when } t = 1.0 \text{ s}$$

$$x|_{t=0} = 4 \text{ in.} \qquad x|_{t=1} = -2 \text{ in.} \qquad x|_{t=2} = 10 \text{ in.}$$



$$d = 6 + 12 = 18 \text{ in.} \blacktriangleleft$$

12.108

Fall of the stone:

$$\begin{aligned} a &= 32.2 \text{ ft/s}^2 \qquad \therefore v = 32.2t + C_1 \text{ ft/s} \\ \therefore y &= 16.1t^2 + C_1 t + C_2 \text{ ft} \end{aligned}$$

When $t = 0$: $v = x = 0$ $\therefore C_1 = C_2 = 0$
Let t_1 be the time of fall and h the depth of the well. $\therefore h = 16.1t_1^2$ ft
Travel of the sound:
Let t_2 be the time for sound to travel the distance h. $\therefore h = 1120t_2$ ft

$$t_1 + t_2 = 3 \qquad \therefore t_2 = 3 - t_1$$

$$16.1t_1^2 = 1120t_2 \qquad 16.1t_1^2 = 1120(3 - t_1) \qquad t_1 = 2.881 \text{ s}$$

$$h = y|_{t=2.881\text{s}} = 16.1(2.881)^2 = 133.6 \text{ ft} \blacktriangleleft$$

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12.109

$$\begin{aligned} a &= 12t - 6t^2 \text{ ft/s}^2 \\ v &= \int a\, dt = 6t^2 - 2t^3 + C_1 \text{ ft/s} \\ x &= \int v\, dt = 2t^3 - 0.5t^4 + C_1 t + C_2 \text{ ft} \end{aligned}$$

When $t = 0$: $x = v = 0$. $\therefore C_1 = C_2 = 0$

$$\therefore x = 2t^3 - 0.5t^4 \text{ ft} \qquad \therefore v = 6t^2 - 2t^3 \text{ ft/s}$$

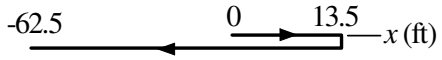
(a)

$$\Delta x = x|_{t=5} - x|_{t=0} = \left[2(5)^3 - 0.5(5)^4\right] - 0 = -62.5 \text{ ft} \blacktriangleleft$$

(b) When $v = 0$:

$$6t^2 - 2t^3 = 0 \qquad t = 3.0 \text{ s}$$

$$x|_{t=0} = 0 \quad x|_{t=3} = 2(3)^3 - 0.5(3)^4 = 13.5 \text{ ft} \quad x|_{t=5} = -62.5 \text{ ft}$$



$$d = 2(13.5) + 62.5 = 89.5 \text{ ft} \blacktriangleleft$$

12.110

$$\frac{dv}{dt} = -cv^2 \qquad \frac{1}{v^2}dv = -c\, dt \qquad \int \frac{1}{v^2}\, dv = -ct + C_1 \qquad -\frac{1}{v} = -ct + C_1$$

Initial condition: $v|_{t=0} = 800$ ft/s $\quad \therefore \dfrac{1}{800} = -C_1 \quad \therefore -\dfrac{1}{v} = -0.8t - \dfrac{1}{800}$

When $v = 400$ ft/s: $\quad \dfrac{1}{800} - \dfrac{1}{400} = -0.8t \qquad t = 1.5625 \times 10^{-3}$ s $\blacktriangleleft$

$$\begin{aligned} \frac{1}{v} &= 0.8t + \frac{1}{800} \qquad v = \frac{1}{0.8t + 1/800} \\ x &= \int \frac{dt}{0.8t + 1/800} + C_2 = \frac{1}{0.8} \ln\left(0.8t + \frac{1}{800}\right) + C_2 \end{aligned}$$

Initial condition: $\quad x|_{t=0} = 0 \quad \therefore 0 = \dfrac{1}{0.8} \ln\left(\dfrac{1}{800}\right) + C_2$

$$\begin{aligned} C_2 &= -\frac{1}{0.8} \ln\left(\frac{1}{800}\right) = 8.356 \text{ ft}^{-1} \\ x &= \frac{1}{0.8} \ln\left(0.8t + \frac{1}{800}\right) + 8.356 \end{aligned}$$

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$$\text{When } v = 400 \text{ ft/s} \quad \left(t = 1.5625 \times 10^{-3} \text{ s}\right):$$

$$x = \frac{1}{0.8} \ln \left[0.8(1.5625 \times 10^{-3}) + \frac{1}{800}\right] + 8.356 = 0.867 \text{ ft} \quad \blacktriangleleft$$

12.111

$$v = 75e^{-0.05t} \text{ ft/s}$$

$$x = \int 75e^{-0.05t} dt = -1500e^{-0.05t} + C \text{ ft}$$

Initial condition: $x|_{t=0} = 0 \quad \therefore C = 1500 \text{ ft}$

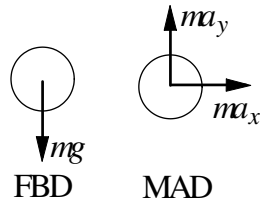
$$x = 1500(1 - e^{-0.05t}) \text{ ft}$$

Setting $v = 5$ ft/s and solving for t:

$$5 = 75e^{-0.05t} \qquad t = 54.16 \text{ s} \quad \blacktriangleleft$$

$$x|_{v=5 \text{ ft/s}} = 1500 \left[1 - e^{-0.05(54.16)}\right] = 1400 \text{ ft} \quad \blacktriangleleft$$

12.112



$$\Sigma F_x = ma_x \qquad a_x = 0$$

$$\Sigma F_y = ma_y \qquad a_y = -g = -9.81 \text{ m/s}^2$$

$$v_x = \int a_x dt = C_1$$

Initial condition: $v_x|_{t=0} = v_0 \cos 50^\circ$

$$\therefore C_1 = v_0 \cos 50^\circ = 0.6428 v_0 \qquad v_x = 0.6428 v_0$$

$$x = \int v_x dt = 0.6428 v_0 t + C_2$$

Initial condition: $x|_{t=0} = 0$

$$\therefore C_2 = 0 \qquad x = 0.6428 v_0 t$$

$$v_y = \int a_y dt = -9.81t + C_3$$

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Initial condition: $v_y|_{t=0} = v_0 \sin 50°$

$$\therefore C_3 = v_0 \sin 50° = 0.7660 v_0 \qquad v_y = -9.81t + 0.7660 v_0$$

$$y = \int v_y dt = -4.905t^2 + 0.7660 v_0 t + C_4$$

Initial condition: $y|_{t=0} = 0$

$$\therefore C_4 = 0 \qquad y = -4.905t^2 + 0.7660 v_0 t$$

At point B:

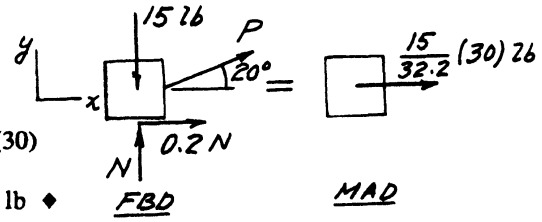
$$\begin{aligned} x &= 18 \text{ m} \quad \therefore 0.6428 v_0 t = 18 \qquad v_0 t = 28.0 \text{ m} \\ y &= 18 \text{ m} \quad \therefore -4.905t^2 + 0.7660(28.0) = 18 \qquad t = 0.8384 \text{ s} \\ v_0 &= \frac{28.0}{0.8384} = 33.4 \text{ m/s} \blacktriangleleft \end{aligned}$$

12.113

$\Sigma F_y = 0 \quad +\uparrow \quad N + P \sin 20° - 15 = 0$

$\Sigma F_x = ma \quad \overset{+}{\rightarrow} \quad P \cos 20° + 0.2N = \frac{15}{32.2}(30)$

The solution is: N = 10.69 lb P = 12.60 lb ♦



12.114

$$F = (50 - t) \times 10^3 \text{ N}$$

$$\begin{aligned} +\uparrow \quad a &= \frac{F}{m} - g = \frac{(50 - t) \times 10^3}{1400} - 9.81 = 25.90 - 0.7143t \text{ m/s}^2 \\ v &= \int a \, dt = 25.90t - 0.3571t^2 + C_1 \\ y &= \int v \, dt = 12.95t^2 - 0.11903t^3 + C_1 t + C_2 \end{aligned}$$

Initial conditions: $y = v = 0$ when $t = 0$. $\therefore C_1 = C_2 = 0$.

$$y|_{t=20} = 12.95(20)^2 - 0.11903(20)^3 = 4230 \text{ m} \blacktriangleleft$$

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12.115

$$\Sigma F_x = ma_x \quad -1.6e^{4x} = 0.2a \quad \therefore a = -\frac{1.6e^{4x}}{0.2} = -8e^{4x}$$

$$a = v\frac{dv}{dx} \quad \therefore v\,dv = a\,dx \quad \therefore \int v\,dv = \int -8e^{4x}\,dx \quad \therefore \frac{1}{2}v^2 = -2e^{4x} + C$$

Initial condition: $v = 6$ m/s when $x = 0$ $\quad \therefore \frac{1}{2}(6)^2 = -2 + C \quad \therefore C = 20\ (\text{m/s})^2$

$$\therefore \frac{1}{2}v^2 = -2e^{4x} + 20$$

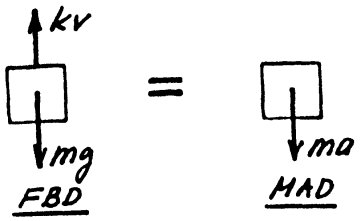
When $v = 0$: $0 = -2e^{4x} + 20 \quad \therefore e^{4x} = 10 \quad \therefore x = \frac{\ln 10}{4} = 0.576$ m ◆

12.116

(a) $\Sigma F = ma$: $+\downarrow \quad mg - kv = ma \quad \therefore a = g - \frac{k}{m}v$

$v = v_\infty$ when $a = 0$. $\therefore g - \frac{k}{m}v_\infty = 0$

$$\therefore k = \frac{mg}{v_\infty} = \frac{(10)(9.81)}{60} = 1.635\ \text{N·s/m}$$ ◆



(b) $a = \frac{dv}{dt} = g - \frac{k}{m}v \quad \therefore \frac{dv}{g - (k/m)v} = dt \quad \therefore -\frac{m}{k}\ln\left(g - \frac{k}{m}v\right) = t + C$

Initial condition: $v = 0$ when $t = 0$. $\therefore C = -\frac{m}{k}\ln g$

$$\therefore t = \frac{m}{k}\ln\frac{g}{g - (k/m)v} = \frac{m}{k}\ln\frac{1}{1 - [k/(mg)]v} = \frac{m}{k}\ln\frac{1}{1 - v/v_\infty}$$

When $v = 59$ m/s: $t = \frac{10}{1.635}\ln\frac{1}{1 - (59/60)} = 25.0$ s ◆

12.117

From Eqs. (d) and (e) of Sample Problem12.11:

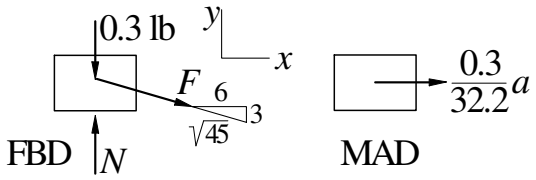
$$x = v_0 t\cos\theta = 8t\cos 30^\circ = 6.928t$$
$$y = -\frac{1}{2}gt^2 + v_0 t\sin\theta = -\frac{1}{2}(9.81)t^2 + 8t\sin 30^\circ = -4.905t^2 + 4t$$

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At the landing point

$$y = -x\tan 20^\circ \qquad -4.905t^2 + 4t = -6.928t\tan 20^\circ$$
$$4.905t = 6.522 \qquad t = 1.330 \text{ s} \blacktriangleleft$$
$$x = 6.928(1.330) = 9.214 \text{ m} \qquad d = \frac{9.214}{\cos 20^\circ} = 9.81 \text{ m} \blacktriangleleft$$

12.118



When $x = 6$ in., the elongation of the spring and the spring force are

$$\delta = \sqrt{3^2 + 6^2} - 3 = 3.708 \text{ in.} \qquad F = k\delta = 5(3.708) = 18.540 \text{ lb}$$

$$\Sigma F_x = ma \qquad \frac{6}{\sqrt{45}}F = \frac{0.3}{32.2}a \qquad \frac{6}{\sqrt{45}}(18.540) = \frac{0.3}{32.2}a$$
$$a = 1780 \text{ ft/s}^2 \blacktriangleleft$$

12.119

$\mathbf{x = b\sin\omega t} \qquad \therefore \dot{x} = b\omega\cos\omega t \qquad \therefore \ddot{x} = -b\omega^2\sin\omega t$

$\mathbf{y = b\exp(-\omega t/2)} \qquad \therefore \dot{y} = -\frac{1}{2}b\omega\exp(-\omega t/2) \qquad \therefore \ddot{y} = \frac{1}{4}b\omega^2\exp(-\omega t/2)$

(a) At point A $\mathbf{x = 0} \qquad \therefore \omega t = 0$

$\therefore \ddot{x} = 0 \qquad \therefore \ddot{y} = \frac{1}{4}b\omega^2 \qquad \therefore a = \frac{1}{4}b\omega^2 \uparrow$ ◆

(b) At point B $\mathbf{x = b} \qquad \therefore \omega t = \frac{\pi}{2}$

$\therefore \ddot{x} = -b\omega^2 \qquad \therefore \ddot{y} = \frac{1}{4}b\omega^2\exp(-\pi/4) = 0.11398b\omega^2$

$\therefore a = b\omega^2\sqrt{1^2 + 0.11398^2} = 1.006\,b\omega^2 \qquad \theta = 6.50°$ ◆

$\therefore \theta = \tan^{-1} 0.11398 = 6.50°$

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12.120

When x = 6 ft:

$\Sigma F_y = 0$

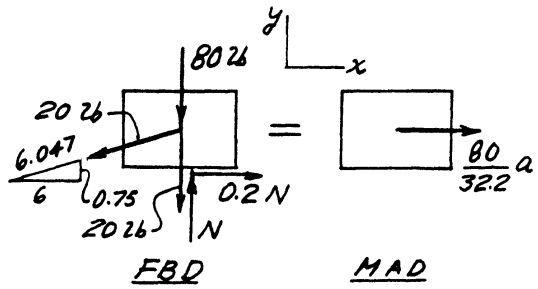
$+\uparrow \quad N - 80 - 20 - \frac{0.75}{6.047}(20) = 0$

$\therefore N = 102.48$ lb

$\Sigma F_x = ma$

$\overset{+}{\rightarrow} \quad -\frac{6}{6.047}(20) + 0.2(102.48) = \frac{80}{32.2}a$

$\therefore a = 0.262 \text{ ft/s}^2 \rightarrow$ ◆



12.121

From Eqs. (d) and (e) of Sample Problem 12.11:

$$x = v_0 t \cos\theta_0 \qquad y = -\frac{1}{2}gt^2 + v_0 t \sin\theta_0$$

(a) Let $t = t_1$ when the ball hits the fairway at $y = -8$ m, $x = R$.

$$\therefore \quad -8 = -\frac{1}{2}(9.81)t_1^2 + 45t_1 \sin 40^\circ \qquad t_1 = 6.162 \text{ s}$$

$$\therefore \quad R = 45(6.162)\cos 40^\circ = 212 \text{ m} \blacktriangleleft$$

(b) At $t = 6.162$ s:

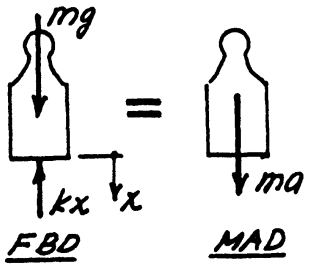
$$v_x = \dot{x} = v_0 \cos\theta_0 = 45\cos 40^\circ = 34.47 \text{ m/s}$$

$$v_y = \dot{y} = -gt + v_0 \sin\theta_0 = -9.81(6.162) + 45\sin 40^\circ = -31.52 \text{ m/s}$$

$$v = \sqrt{34.47^2 + 31.52^2} = 46.7 \text{ m/s} \blacktriangleleft$$

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12.122



(a) $\Sigma F = ma$: $+\downarrow$ $mg - kx = ma$ $\quad \therefore a = g - \frac{k}{m}x$

$$a = \frac{dv}{dx}v \qquad \therefore \int v\,dv = \int a\,dx$$

$$\therefore \frac{1}{2}v^2 = gx - \frac{k}{2m}x^2 + C$$

Initial condition: $v = 0$ when $x = 0$. $\therefore C = 0$

Spring force is maximized when x is maximized, i.e. when $v = 0$.

$$\therefore gx_{max} - \frac{k}{2m}x_{max}^2 = 0 \qquad \therefore x_{max} = \frac{2mg}{k} \qquad \therefore F_{max} = kx_{max} = 2mg \ \blacklozenge$$

(b) v is maximized when $a = 0$. $\quad \therefore g - \frac{k}{m}x = 0 \qquad \therefore x = \frac{mg}{k}$

$$\therefore \frac{1}{2}v_{max}^2 = g\left(\frac{mg}{k}\right) - \frac{k}{2m}\left(\frac{mg}{k}\right)^2 = g^2\frac{m}{2k} \qquad \therefore v_{max} = g\sqrt{m/k} \ \blacklozenge$$

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12.3

$$\begin{aligned} x &= 6\left(1-e^{-t/2}\right) \text{ m} \\ v &= \dot{x} = 6\left(\frac{1}{2}e^{-t/2}\right) = 3e^{-t/2} \text{ m/s} \\ a &= \dot{v} = -3\left(\frac{1}{2}e^{-t/2}\right) = -1.5e^{-t/2} \text{ m/s}^2 \end{aligned}$$

(a) $x_{\max} = 6$ m at $t = \infty$ ◄

$v_{\max} = 3$ m/s and $|a|_{\max} = 1.5$ m/s^2 both occurring at $t = 0$ ◄

(b) When $x = 3$ m: $3 = 6(1 - e^{-t/2})$ $\quad e^{-t/2} = 0.5$

$t = -2\ln(0.5) = 1.3863$ s ◄

$v = 3(0.5) = 1.5$ m/s ◄ $\quad a = -1.5(0.5) = -0.75$ m/s^2 ◄

12.4

$$\begin{aligned} x &= t^3 - 6t^2 - 32t \text{ in.} \\ v &= \dot{x} = 3t^2 - 12t - 32 \text{ in./s} \\ a &= \dot{v} = 6t - 12 \text{ in./s}^2 \end{aligned}$$

At $t = 10$ s:

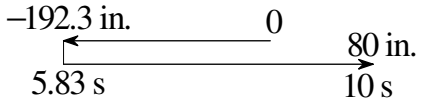
$$\begin{aligned} x &= 10^3 - 6(10^2) - 32(10) = 80 \text{ in.} \quad ◄ \\ v &= 3(10^2) - 12(10) - 32 = 148 \text{ in./s} \quad ◄ \\ a &= 6(10) - 12 = 48 \text{ in./s}^2 \quad ◄ \end{aligned}$$

Reversal of velocity occurs when $v = 0$ $(t \neq 0)$:

$$\begin{aligned} v &= 3t^2 - 12t - 32 = 0 \qquad t = 5.830 \text{ s} \\ x &= 5.830^3 - 6(5.830^2) - 32(5.830) = -192.3 \text{ in.} \end{aligned}$$

At $t = 10$ s the distance travelled is

$$s = 192.3 + (193.3 + 80) = 466 \text{ in.} \quad ◄$$



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12.5

(a) $x = t^2 - \dfrac{t^3}{90}$ ft $\qquad v = \dot{x} = 2t - \dfrac{t^2}{30}$ ft/s $\qquad v = 0$ when $t = 60$ s

$$x_{\max} = 60^2 - \frac{60^3}{90} = 1200 \text{ ft} \quad \blacktriangleleft$$

(b) $a = \dot{v} = 2 - \dfrac{t}{15}$ ft/s^2 $\qquad a = 0$ when $t = 30$ s

$$v_{\max} = 2(30) - \frac{30^2}{30} = 30 \text{ ft/s} \quad \blacktriangleleft$$

12.6

(a) $x = v_0(t - t_0 + t_0 e^{-t/t_0}) \quad \therefore \; v = \dot{x} = v_0(1 - e^{-t/t_0})$ ◆

Since $v \to v_0$ as $t \to \infty$, v_0 is the limiting or terminal velocity.

(b) $a = \dot{v} = \dfrac{v_0}{t_0} e^{-t/t_0}$ ◆ But from part (a): $v_0 - v = v_0 e^{-t/t_0} \quad \therefore \; a = \dfrac{v_0 - v}{t_0}$ ◆

12.7

$$x = 3t^2 - 12t \text{ in.} \qquad v = \dot{x} = 6t - 12 \text{ in./s}$$

(a) The bead leaves the wire when $x = 40$ in.

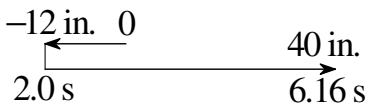
$$3t^2 - 12t = 40 \qquad t = 6.16 \text{ s} \quad \blacktriangleleft$$

(b) Reversal of velocity occurs when $v = 0 \quad (t \neq 0)$:

$$\begin{aligned} v &= 6t - 10 \qquad t = 2.0 \text{ s} \\ x &= 3(2.0^2) - 12(2.0) = -12.0 \text{ in.} \end{aligned}$$

The distance travelled is

$$s = 2(12) + 40 = 64.0 \text{ in.} \quad \blacktriangleleft$$



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12.8

$$x = 4t^2 - 2 \text{ mm}$$
$$y = \frac{x^2}{12} = \frac{16t^4 - 16t^2 + 4}{12} = \frac{4t^4 - 4t^2 + 1}{3} \text{ mm}$$

When $t = 2$ s:

$$v_x = \dot{x} = 8t = 8(2) = 16 \text{ mm/s}$$
$$v_y = \dot{y} = \frac{16t^3 - 8t}{3} = \frac{16(2)^3 - 8(2)}{3} = 37.33 \text{ mm/s}$$
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 37.33^2} = 40.6 \text{ mm/s} \blacktriangleleft$$

$$a_x = \dot{v}_x = 8 \text{ mm/s}^2$$
$$a_y = \dot{v}_y = \frac{48t^2 - 8}{3} = \frac{48(2)^2 - 8}{3} = 61.33 \text{ mm/s}^2$$
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 61.33^2} = 61.9 \text{ mm/s}^2 \blacktriangleleft$$

12.9

(a) $x = R\left(1 + \frac{1}{2}\cos \omega t\right)$ $\quad \therefore v = \dot{x} = -\frac{1}{2} R\omega \sin \omega t \blacklozenge$ $\quad \therefore a = \dot{v} = -\frac{1}{2} R\omega^2 \cos \omega t \blacklozenge$

(b) $|v|_{max} = \frac{1}{2} R\omega \quad |a|_{max} = \frac{1}{2} R\omega^2$

$\therefore$ Doubling ω would <u>double</u> $|v|_{max}$ and <u>quadruple</u> $|a|_{max}$.

12.10

$$y = 50 - 2t \text{ s} \qquad v_y = \dot{y} = -2 \text{ in./s} \qquad a_y = \dot{v}_y = 0$$
$$x = \frac{6}{y} = \frac{6}{50 - 2t} \text{ in.} \qquad v_x = \dot{x} = \frac{3}{(25 - t)^2} \text{ in./s} \qquad a_x = v_x = \frac{6}{(25 - t)^3}$$

At $t = 20$ s:

$$v_x = \frac{3}{(25 - 20)^2} = 0.12 \text{ in./s} \qquad v_y = -2 \text{ in./s} \qquad \mathbf{v} = 0.12\mathbf{i} - 2\mathbf{j} \text{ in./s} \blacktriangleleft$$
$$a_x = \frac{6}{(25 - 20)^3} = 0.048 \text{ in./s}^2 \qquad a_y = 0 \qquad \mathbf{a} = 0.048\mathbf{i} \text{ in./s}^2 \blacktriangleleft$$

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12.11

(a) $v^2 = 2gr_0(r_0/r - 1) + v_0^2$

Differentiate with respect to time: $2v\dot{v} = 2gr_0(-r_0/r^2)\dot{r}$ or $2va = -2g(r_0/r)^2 v$

$\therefore a = -g(r_0/r)^2$ ♦

(b) v_0 is the escape velocity if $v \to 0$ when $r \to \infty$.

$\therefore 0 = \lim_{r \to \infty}[2gr_0(r_0/r - 1) + v_0^2]$ $\therefore 0 = 2gr_0(0 - 1) + v_0^2$ $\therefore v_0 = \sqrt{2gr_0}$ ♦

(c) For earth: $v_0 = \sqrt{2(32.2)(3960 \times 5280)} = 36\,700$ ft/s ♦

12.12

$$x = 15 - 2t^2 \text{ m} \qquad v_x = \dot{x} = -4t \text{ m/s} \qquad a_x = \dot{v}_x = -4 \text{ m/s}^2$$
$$y = 15 - 10t + t^2 \text{ m} \qquad v_y = \dot{y} = -10 + 2t \text{ m/s} \qquad a_y = \dot{v}_y = 2 \text{ m/s}^2$$

(a) At $t = 0$: $\mathbf{v} = -10\mathbf{j}$ m/s ◄ $\mathbf{a} = -4\mathbf{i} + 2\mathbf{j}$ m/s^2 ◄

(b) At $t = 5$ s: $\mathbf{v} = -20\mathbf{i}$ m/s ◄ $\mathbf{a} = -4\mathbf{i} + 2\mathbf{j}$ m/s^2 ◄

12.13

$$x = 58t \text{ m} \qquad v_x = \dot{x} = 58 \text{ m/s} \qquad a_x = \dot{v}_x = 0$$
$$y = 78t - 4.91t^2 \text{ m} \qquad v_y = \dot{y} = 78 - 9.82t \text{ m/s} \qquad a_y = \dot{v}_y = -9.82 \text{ m/s}^2$$

(a) $\mathbf{a} = -9.82\mathbf{j}$ m/s^2 ◄

(b) $\mathbf{v}|_{t=0} = 58\mathbf{i} + 78\mathbf{j}$ m/s ◄

(c) $y = h$ when $v_y = 0$:

$$v_y = 78 - 9.82t = 0 \qquad t = 7.943 \text{ s}$$
$$h = 78(7.943) - 4.91(7.943^2) = 310 \text{ m} \text{ ◄}$$

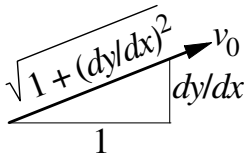
(d) $x = L$ when $y = -140$ m:

$$y = 78t - 4.91t^2 = -140 \qquad t = 17.514 \text{ s}$$
$$L = 58(17.514) = 1016 \text{ m} \text{ ◄}$$

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12.14

$$y = \frac{x^2}{1000} \qquad \frac{dy}{dx} = \frac{x}{500}$$



$$\begin{aligned} v_x &= v_0 \frac{1}{\sqrt{1+(dy/dx)^2}} = \frac{v_0}{\sqrt{1+(x/500)^2}} = \frac{500v_0}{\sqrt{500^2+x^2}} \\ v_y &= v_0 \frac{dy/dx}{\sqrt{1+(dy/dx)^2}} = \frac{x/500}{\sqrt{1+(x/500)^2}} = \frac{xv_0}{\sqrt{500^2+x^2}} \end{aligned}$$

When $x = 100$ m:

$$\begin{aligned} v_x &= \frac{500(6)}{\sqrt{500^2+100^2}} = 5.883 \text{ m/s} \\ v_y &= \frac{100(6)}{\sqrt{500^2+100^2}} = 1.1767 \text{ m/s} \\ \mathbf{v} &= 5.88\mathbf{i} + 1.177\mathbf{j} \text{ m/s} \quad \blacktriangleleft \end{aligned}$$

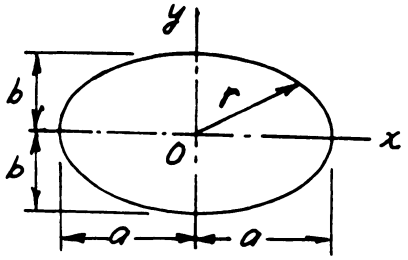
$$\begin{aligned} a_x &= \dot{v}_x = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{dv_x}{dx}v_x = -\frac{500xv_0}{\left(500^2+x^2\right)^{3/2}}v_x \\ a_y &= \dot{v}_y = \frac{dv_y}{dx}\frac{dx}{dt} = \frac{dv_y}{dx}v_x = \frac{500^2v_0}{\left(500^2+x^2\right)^{3/2}}v_x \end{aligned}$$

When $x = 100$ m:

$$\begin{aligned} a_x &= -\frac{500(100)(6)}{\left(500^2+100^2\right)^{3/2}}(5.883) = -0.013\ 31 \text{ m/s}^2 \\ a_y &= \frac{500^2(6)}{\left(500^2+100^2\right)^{3/2}}(5.883) = 0.0666 \text{ m/s}^2 \\ \mathbf{a} &= -0.013\ 32\mathbf{i} + 0.666\mathbf{j} \text{ m/s}^2 \quad \blacktriangleleft \end{aligned}$$

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12.15



(a) $x = a\cos\omega t \qquad y = b\sin\omega t$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2\omega t + \sin^2\omega t = 1$$

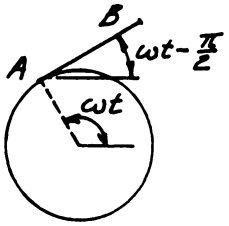
which is the equation of ellipse $\therefore$ Q.E.D.

(b) $v_x = \dot{x} = -a\omega\sin\omega t \qquad a_x = \ddot{x} = -a\omega^2\cos\omega t = -\omega^2 x$

$v_y = \dot{y} = -b\omega\cos\omega t \qquad a_y = \ddot{y} = -b\omega^2\sin\omega t = -\omega^2 y$

$\therefore \mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = -\omega^2(x\mathbf{i} + y\mathbf{j}) = -\omega^2\mathbf{r}$ $\therefore$ a and r are collinear $\therefore$ Q.E.D.

12.16



$x = R\cos\omega t + R\omega t\sin\omega t \qquad y = R\sin\omega t - R\omega t\cos\omega t$

$\therefore v_x = \dot{x} = -R\omega\sin\omega t + R\omega\sin\omega t + R\omega^2 t\cos\omega t = R\omega^2 t\cos\omega t$

$\therefore v_y = \dot{y} = R\omega\cos\omega t - R\omega\cos\omega t + R\omega^2 t\sin\omega t = R\omega^2 t\sin\omega t$

$\therefore \mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = R\omega^2 t(\mathbf{i}\cos\omega t + \mathbf{j}\sin\omega t) \qquad \therefore v = R\omega^2 t$ ◆

$\overrightarrow{AB} = \overline{AB}[\mathbf{i}\cos(\omega t - \pi/2) + \mathbf{j}\sin(\omega t - \pi/2)]$

$= \overline{AB}(\mathbf{i}\sin\omega t - \mathbf{j}\cos\omega t)$

By inspection: $\mathbf{v}\cdot\overrightarrow{AB} = 0$ $\therefore$ v is perpendicular to AB $\therefore$ Q.E.D.

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12.17

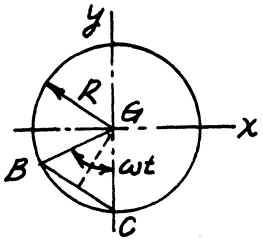
(a) $x = R(\omega t - \sin \omega t) \quad \therefore v_x = \dot{x} = R\omega(1 - \cos \omega t)$

$y = R(1 - \cos \omega t) \quad \therefore v_y = \dot{y} = R\omega \sin \omega t$

$$\therefore v^2 = v_x^2 + v_y^2 = R^2\omega^2\left[(1 - \cos \omega t)^2 + \sin^2 \omega t\right] = 2R^2\omega^2(1 - \cos \omega t) = 4R^2\omega^2 \sin^2 \frac{\omega t}{2}$$

$$\therefore v = 2R\omega \sin \frac{\omega t}{2}$$

But $\overline{BC} = 2R \sin \dfrac{\omega t}{2} \quad \therefore v = \omega \overline{BC} \quad \therefore$ Q.E.D.



(b) $a_x = \ddot{x} = R\omega^2 \sin \omega t \qquad a_y = \ddot{y} = R\omega^2 \cos \omega t$

$$\therefore \mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = (R\omega^2 \sin \omega t)\mathbf{i} + (R\omega^2 \cos \omega t)\mathbf{j}$$

$$\overrightarrow{BG} = (R \sin \omega t)\mathbf{i} + (R \cos \omega t)\mathbf{j} = \frac{1}{\omega^2}\mathbf{a} \quad \therefore \overrightarrow{BG} \text{ and } \mathbf{a} \text{ are parallel} \quad \therefore \text{Q.E.D.}$$

12.18

$$x = R \cos \omega t \qquad y = R \sin \omega t \qquad z = -\frac{h}{2\pi}\omega t$$

$$\therefore v_x = \dot{x} = -R\omega \sin \omega t \qquad v_y = \dot{y} = R\omega \cos \omega t \qquad v_z = \dot{z} = -\frac{h\omega}{2\pi}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 = R^2\omega^2 \sin^2 \omega t + R^2\omega^2 \cos^2 \omega t + \left(\frac{h\omega}{2\pi}\right)^2 = (R\omega)^2 + \left(\frac{h\omega}{2\pi}\right)^2$$

$$\therefore v = R\omega\sqrt{1 + \left[h/(2\pi R)\right]^2} = \text{constant} \quad \therefore \text{Q.E.D.}$$

$$a_x = \ddot{x} = -R\omega^2 \cos \omega t \qquad a_y = \ddot{y} = -R\omega^2 \sin \omega t \qquad a_z = \ddot{z} = 0$$

$$a^2 = a_x^2 + a_y^2 + a_z^2 = R^2\omega^4(\cos^2 \omega t + \sin^2 \omega t) = R^2\omega^4 \quad \therefore a = R\omega^2 = \text{constant} \quad \therefore \text{Q.E.D.}$$

Using the given data:

$$v = (1.2)(4\pi)\sqrt{1 + \left[\frac{0.75}{2\pi(1.2)}\right]^2} = 15.15 \text{ m/s} \blacklozenge \qquad a = (1.2)(4\pi)^2 = 189.5 \text{ m/s}^2 \blacklozenge$$

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12.19

$x = 0.8v_0t \qquad y = 0.6v_0t \qquad z = -0.04v_0^2t^2$

$\therefore v_x = \dot{x} = 0.8v_0 \qquad v_y = \dot{y} = 0.6v_0 \qquad v_z = \dot{z} = -0.08v_0^2t$

$\therefore a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = 0 \qquad a_z = \ddot{z} = -0.08v_0^2$

(a) At point B: $x = 4$ in $\quad \therefore 4 = 0.8v_0t \quad \therefore t = 5/v_0 \quad \therefore v_z = -0.4v_0$

$$\therefore v = \sqrt{v_x^2 + v_y^2 + v_z^2} = v_0\sqrt{0.8^2 + 0.6^2 + 0.4^2} = 1.0770v_0 \blacklozenge$$

$$a = |a_z| = 0.08v_0^2 \blacklozenge$$

(b) Let θ be the angle between the path and the z-axis at point B.

$$\therefore \cos\theta = \frac{\mathbf{v}\cdot\mathbf{k}}{|\mathbf{v}|} = \frac{-0.4v_0}{1.0770v_0} = -0.3714 \qquad \therefore \theta = 111.8°$$

$\therefore$ The angle between the path and the x-y plane is: $\theta - 90° = 21.8°$ $\blacklozenge$

12.20

(a) $\mathbf{r} = (3t^2 + 4t)\mathbf{i} + (-4t^2 + 3t)\mathbf{j} + (-6t + 9)\mathbf{k}$ ft

$\therefore \mathbf{v} = \dot{\mathbf{r}} = (6t + 4)\mathbf{i} + (-8t + 3)\mathbf{j} - 6\mathbf{k}$ ft/s $\blacklozenge$

$\therefore \mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} - 8\mathbf{j}$ ft/s^2 $\blacklozenge$

(b) The vector normal to the plane formed by $\mathbf{v}$ and $\mathbf{a}$ (the instantaneous plane of motion) is

$$\mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6t + 4 & -8t + 3 & -6 \\ 6 & -8 & 0 \end{vmatrix} = -48\mathbf{i} - 36\mathbf{j} - 50\mathbf{k}$$

and the corresponding unit vector is

$$\mathbf{n} = \pm\frac{48\mathbf{i} + 36\mathbf{j} + 50\mathbf{k}}{\sqrt{48^2 + 36^2 + 50^2}} = \pm(0.615\mathbf{i} + 0.461\mathbf{j} + 0.640\mathbf{k})$$

Since this vector is independent of t, the orientation of the plane does not vary with the location of the particle. Thus the particle is in plane motion on an inclined plane. Q.E.D.

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12.21

$$x = R\cos\omega t \qquad y = R\sin\omega t \qquad z = \frac{R}{2}\sin 2\omega t$$

$$\therefore\ v_x = \dot{x} = -R\omega\sin\omega t \qquad v_y = \dot{y} = R\omega\cos\omega t \qquad v_z = \dot{z} = R\omega\cos 2\omega t$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2 + v_z^2} = R\omega\sqrt{\cos^2\omega t + \sin^2\omega t + \cos^2 2\omega t} = R\omega\sqrt{1 + \cos^2 2\omega t}$$

$$\therefore\ v_{max} = \sqrt{2}\,R\omega \ \blacklozenge$$

$$a_x = \ddot{x} = -R\omega^2\cos\omega t \qquad a_y = \ddot{y} = -R\omega^2\sin\omega t \qquad a_z = \ddot{z} = -2R\omega^2\sin 2\omega t$$

$$\therefore\ a = \sqrt{a_x^2 + a_y^2 + a_z^2} = R\omega^2\sqrt{\cos^2\omega t + \sin^2\omega t + 4\sin^2 2\omega t} = R\omega^2\sqrt{1 + 4\sin^2 2\omega t}$$

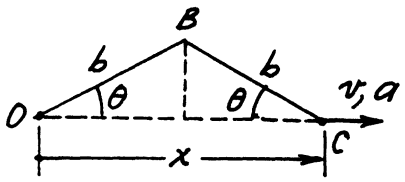
$$\therefore\ a_{max} = \sqrt{5}\,R\omega^2 \ \blacklozenge$$

12.22

(a) From geometry: $x = 2b\cos\theta$

$\therefore\ v = \dot{x} = -2b\dot{\theta}\sin\theta \rightarrow \ \blacklozenge$

(b) $\therefore\ a = \dot{v} = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \rightarrow \ \blacklozenge$

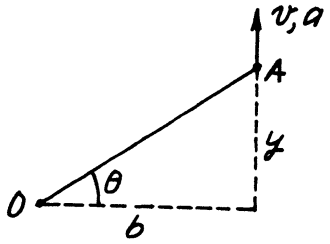


12.23

(a) Geometry: $y = b\tan\theta \qquad \therefore\ v = \dot{y} = b\dot{\theta}\sec^2\theta\uparrow \ \blacklozenge$

(b) $\therefore\ a = \dot{v} = b\left[\ddot{\theta}\sec^2\theta + 2\dot{\theta}\sec\theta(\sec\theta\tan\theta\,\dot{\theta})\right]$

$= b\sec^2\theta(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)\uparrow \ \blacklozenge$



12.24

$$\begin{aligned} x &= R\cos\theta \qquad v_x = \dot{x} = (-R\sin\theta)\,\dot{\theta} \\ y &= R\sin\theta \qquad v_y = \dot{y} = (R\cos\theta)\dot{\theta} \end{aligned}$$

$$v_y = v_0 \quad \text{yields} \quad \dot{\theta} = \frac{v_0}{R\cos\theta}$$

$$\therefore\ v_x = (-R\sin\theta)\,\frac{v_0}{R\cos\theta} = -v_0\tan\theta$$

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$$a_x = \dot{v}_x = \left(-v_0 \sec^2\theta\right)\dot{\theta} = \left(-v_0 \sec^2\theta\right)\frac{v_0}{R\cos\theta} = -\frac{v_0^2}{R}\sec^3\theta$$

With $R = 6$ ft, $v_0 = 2.5$ ft/s and $\theta = 60°$ we get

$$\begin{aligned} v_y &= 2.5 \text{ ft/s} \qquad v_x = -2.5\tan 60° = -4.330 \text{ ft/s} \\ \mathbf{v} &= -4.33\mathbf{i} + 2.5\mathbf{j} \text{ ft/s} \quad \blacktriangleleft \end{aligned}$$

$$\begin{aligned} a_y &= 0 \qquad a_x = -\frac{2.5^2}{6}\sec^3 60° = -8.333 \text{ ft/s}^2 \\ \mathbf{a} &= -8.33\mathbf{i} \text{ ft/s}^2 \quad \blacktriangleleft \end{aligned}$$

12.25

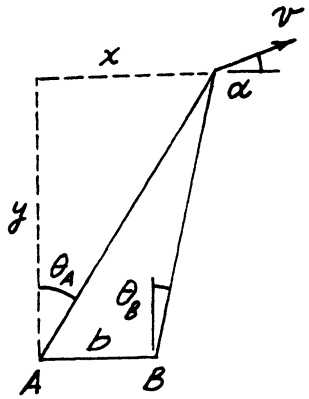
$$\dot{\theta} = \frac{1200 \text{ rev}}{1.0 \text{ min}} \times \frac{2\pi \text{ rad}}{1.0 \text{ rev}} \times \frac{1.0 \text{ min}}{60 \text{ s}} = 125.66 \text{ rad/s}$$

$$\begin{aligned} r &= 55 + 10\cos\theta + 5\cos 2\theta \text{ mm} \\ v &= \dot{r} = \frac{dr}{d\theta}\dot{\theta} = (-10\sin\theta - 10\sin 2\theta)(125.66) \text{ mm/s} \\ a &= \dot{v} = \frac{dv}{d\theta}\dot{\theta} = (-10\cos\theta - 20\cos 2\theta)(125.66)^2 \text{ mm/s}^2 \\ |a|_{\max} &= 30(125.66)^2 = 474\,000 \text{ mm/s}^2 = 474 \text{ m/s}^2 \text{ (at } \theta = 0) \quad \blacktriangleleft \end{aligned}$$

*12.26

(a) Geometry: $x = y\tan\theta_A = y\tan\theta_B + b$ . . . . . (a)

$$\therefore y = \frac{b}{\tan\theta_A - \tan\theta_B} = \frac{1000}{\tan 30° - \tan 22°} = 5770 \text{ m} \quad \blacklozenge$$



(b) Differentiate Eq. (a):

$$\dot{x} = y\dot{\theta}_A \sec^2\theta_A + \dot{y}\tan\theta_A = y\dot{\theta}_B \sec^2\theta_B + \dot{y}\tan\theta_B \quad \text{(b)}$$

$$\therefore \dot{y} = -y\frac{\dot{\theta}_A \sec^2\theta_A - \dot{\theta}_B \sec^2\theta_B}{\tan\theta_A - \tan\theta_B}$$

$$= -(5770)\frac{(0.026)\sec^2 30° - (0.032)\sec^2 22°}{\tan 30° - \tan 22°} = 85.11 \text{ m/s}$$

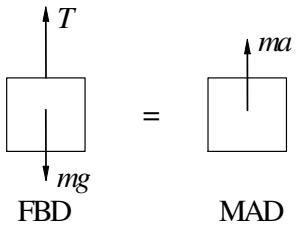
From Eq. (b): $\dot{x} = (5770)(0.026)\sec^2 30° + (85.11)\tan 30° = 249.2$ m/s

$$\therefore v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{249.2^2 + 85.11^2} = 263 \text{ m/s} \quad \blacklozenge$$

(c) $\therefore \alpha = \tan^{-1}(\dot{y}/\dot{x}) = \tan^{-1}\dfrac{85.11}{249.2} = 18.86° \quad \blacklozenge$

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12.27

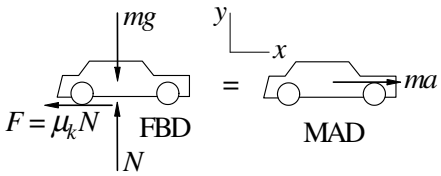


$$v = 4t \text{ m/s} \qquad a = \dot{v} = 4 \text{ m/s}^2$$

$$\Sigma F = ma \quad +\uparrow \qquad T - mg = ma$$

$$T = m(g + a) = 50(9.81 + 4) = 691 \text{ N} \quad \blacktriangleleft$$

12.28



$$v_0 = 100 \text{ km/h} = 100 \text{ km/h} \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 \quad +\uparrow \qquad N - mg = 0 \qquad \therefore N = mg$$

$$\Sigma F_x = ma \quad \overset{+}{\longrightarrow} \qquad -\mu_k N = ma \qquad \therefore a = -\frac{\mu_k N}{m} = -\mu_k g$$

$$v = \int a \, dt = -\mu_k g t + C_1$$

$$x = \int v \, dt = -\frac{1}{2}\mu_k g t^2 + C_1 t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \qquad v = v_0 \quad \therefore C_1 = v_0$$

$$\therefore x = -\frac{1}{2}\mu_k g t^2 + v_0 t \qquad v = -\mu_k g t + v_0$$

When $v = 0$:

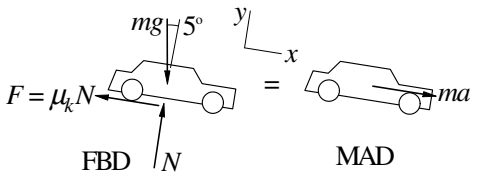
$$-\mu_k g t + v_0 = 0 \qquad \therefore t = \frac{v_0}{\mu_k g}$$

$$x = -\frac{1}{2}\mu_k g \left(\frac{v_0}{\mu_k g} \right)^2 + v_0 \left(\frac{v_0}{\mu_k g} \right) = \frac{v_0^2}{2\mu_k g}$$

$$= \frac{27.78^2}{2(0.65)(9.81)} = 60.5 \text{ m} \quad \blacktriangleleft$$

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12.29



$$v_0 = 100 \text{ km/h} = 27.78 \text{ m/s}$$

$$\begin{aligned}
\Sigma F_y &= 0 \quad +\uparrow \qquad N - mg\cos 5^\circ = 0 \qquad \therefore N = mg\cos 5^\circ \\
\Sigma F_x &= ma \quad \xrightarrow{+} \qquad -\mu_k N + mg\sin 5^\circ = ma \\
\therefore a &= -\frac{\mu_k N}{m} + g\sin 5^\circ = (\sin 5^\circ - \mu_k \cos 5^\circ)g \\
&= (\sin 5^\circ - 0.65\cos 5^\circ)9.81 = -5.497 \text{ m/s}^2
\end{aligned}$$

$$\begin{aligned}
v &= \int a\, dt = -5.497t + C_1 \\
x &= \int v\, dt = -2.749t^2 + C_1 t + C_2
\end{aligned}$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \qquad v = v_0 \quad \therefore C_1 = v_0 = 27.78 \text{ m/s}$$

$$\begin{aligned}
x &= -2.749t^2 + 27.78t \text{ m} \\
v &= -5.497t + 27.78 \text{ m/s}
\end{aligned}$$

When $v = 0$:

$$-5.497t + 27.78 = 0 \qquad \therefore t = 5.054 \text{ s}$$

$$x = -2.749(5.054)^2 + 27.78(5.054) = 70.2 \text{ m} \quad \blacktriangleleft$$

12.30

$$\begin{aligned}
a &= \frac{F}{m} = \frac{-1.2t}{0.1} = -12t \text{ m/s}^2 \\
v &= \int a_x dt = -6t^2 + C_1 \text{ m/s} \\
x &= \int v_x dt = -2t^3 + C_1 t + C_2 \text{ m}
\end{aligned}$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \qquad v = 64 \text{ m/s} \quad \therefore C_1 = 64 \text{ m/s}$$

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$$\therefore x = -2t^3 + 64t \text{ m} \qquad v = -6t^2 + 64 \text{ m/s}$$

When $t = 4$ s:

$$x = -2(4)^3 + 64(4) = 128 \text{ m}$$

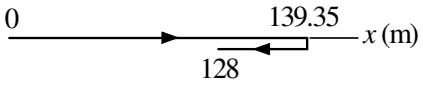
When $v = 0$:

$$-6t^2 + 64 = 0 \qquad t = 3.266 \text{ s}$$

$$x = -2(3.266)^3 + 64(3.266) = 139.35 \text{ m}$$

Distance traveled:

$$d = 2(139.35) - 128 = 150.7 \text{ m} \blacktriangleleft$$



12.31

$$\begin{aligned} a &= \frac{F}{m} = \frac{0.06\sqrt{v}}{0.012} = 5\sqrt{v} \text{ m/s}^2 \\ dt &= \frac{dv}{a} = \frac{dv}{5\sqrt{v}} \qquad t = \int \frac{dv}{5\sqrt{v}} = \frac{2}{5}\sqrt{v} + C_1 \end{aligned}$$

Given $v = 0.25$ m/s when $t = 0.8$ s:

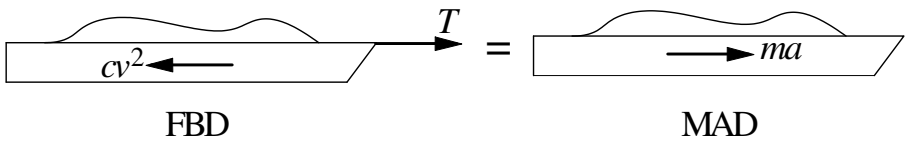
$$0.8 = \frac{2}{5}\sqrt{0.25} + C_1 \qquad C_1 = 0.6 \text{ s}$$

$$\begin{aligned} t &= \frac{2}{5}\sqrt{v} + 0.6 \text{ s} \\ v &= (2.5t - 1.5)^2 = 6.25t^2 - 7.5t + 2.25 \text{ m/s} \\ x &= \int v\, dt = \frac{6.25}{3}t^3 - \frac{7.5}{2}t^2 + 2.25t + C_2 \end{aligned}$$

Initial condition: $x = 0$ when $t = 0$ $\quad \therefore C_2 = 0$

$$x|_{t=1.2\text{s}} = \frac{6.25}{3}(1.2)^3 - \frac{7.5}{2}\left(1.2^2\right) + 2.25(1.2) = 0.90 \text{ m} \blacktriangleleft$$

12.32



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$$ma = T - F_D = T - cv^2 \qquad a = \frac{T - cv^2}{m}$$

$$x = \int \frac{v}{a}\,dv = m\int \frac{v}{T - cv^2}\,dv + C = -\frac{m}{2c}\ln\left(T - cv^2\right) + C$$

Initial condition: $v = 0$ at $x = 0$:

$$0 = -\frac{m}{2c}\ln(T) + C \qquad C = \frac{m}{2c}\ln(T)$$

$$\therefore \quad x = -\frac{m}{2c}\ln(T - cv^2) + \frac{m}{2c}\ln(T) = \frac{m}{2c}\ln\frac{T}{(T - cv^2)}$$

Solve for v:

$$\frac{T}{(T - cv^2)} = \exp\left(\frac{2c}{m}x\right) \qquad T - cv^2 = T\exp\left(-\frac{2c}{m}x\right)$$

$$v^2 = \frac{T}{c}\left[1 - \exp\left(-\frac{2c}{m}x\right)\right] \qquad v = \sqrt{\frac{T}{c}\left[1 - \exp\left(-\frac{2c}{m}x\right)\right]} \quad \blacktriangleleft$$

Terminal velocity:

$$v_\infty = \lim_{x\to\infty} v(x) = \sqrt{\frac{T}{c}} \quad \blacktriangleleft$$

12.33

$$a = \frac{F}{m} = \frac{4t - 4}{4} = t - 1 \text{ m/s}^2$$

$$v = \int a\,dt = \frac{1}{2}t^2 - t + C_1$$

$$y = \int v\,dt = \frac{1}{6}t^3 - \frac{1}{2}t^2 + C_1 t + C_2$$

When $t = 0$ (initial condition):

$$y = 0 \quad \therefore C_2 = 0 \qquad v = -8 \text{ m/s} \quad \therefore C_1 = -8 \text{ m/s}$$

$$\therefore y = \frac{1}{6}t^3 - \frac{1}{2}t^2 - 8t \text{ m} \qquad v = \frac{1}{2}t^2 - t - 8 \text{ m/s}$$

When $t = 8$ s:

$$y = \frac{1}{6}(8)^3 - \frac{1}{2}(8)^2 - 8(8) = -10.67 \text{ m}$$

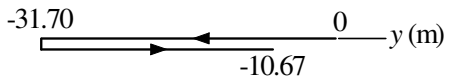
When $v = 0$:

$$\frac{1}{2}t^2 - t - 8 = 0 \qquad t = 5.123 \text{ s}$$

$$y = \frac{1}{6}(5.123)^3 - \frac{1}{2}(5.123)^2 - 8(5.123) = -31.70 \text{ m}$$

Distance traveled:

$$d = 2(31.70) - 10.67 = 52.7 \text{ m} \quad \blacktriangleleft$$



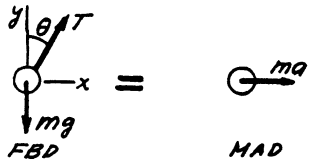
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12.34

$\Sigma F_y = ma_y$: $+\uparrow$ $T\cos\theta - mg = 0$

$\Sigma F_x = ma_x$: $\pm\rightarrow$ $T\sin\theta = ma$

The solution is: $\theta = \tan^{-1}\dfrac{a}{g}$ ◆



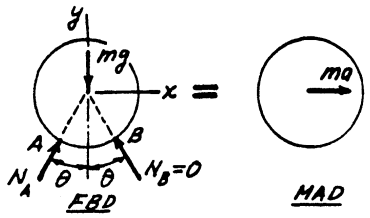
12.35

Assume impending climbing ($N_B = 0$).

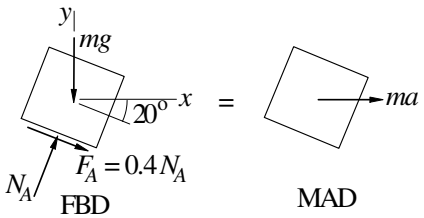
$\Sigma F_y = ma_y$: $+\uparrow$ $N_A\cos\theta - mg = 0$

$\Sigma F_x = ma_x$: $\pm\rightarrow$ $N_A\sin\theta = ma$

The solution is: $a = g\tan\theta$ ◆



12.36



Assume impending sliding ($F_A = 0.4N_A$)

$$\Sigma F_y = 0 \quad +\uparrow \qquad N_A\cos 20^\circ - 0.4N_A\sin 20^\circ - mg = 0$$

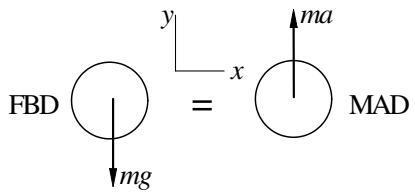
$$N_A = 1.2455\ mg$$

$$\Sigma F_x = ma_x \quad \pm\rightarrow \qquad N_A\sin 20^\circ + 0.4N_A\cos 20^\circ = ma$$

$$1.2455\ mg(\sin 20^\circ + 0.4\cos 20^\circ) = ma \qquad a = 0.894g \ \blacktriangleleft$$

12.37

Let y be measured up from the base of the cliff.



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$$\Sigma F_y = ma \quad +\uparrow \qquad -mg = ma \qquad \therefore a = -g$$

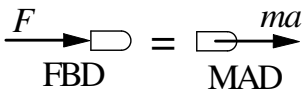
$$a = \frac{dv}{dy}v \qquad \therefore v\ dv = -g\ dy \qquad \frac{1}{2}v^2 = -gy + C$$

Initial condition: $v = v_0$ when $y = h$. $\therefore C = \frac{1}{2}v_0^2 + gh$

$$\therefore \frac{1}{2}\left(v^2 - v_0^2\right) = g(h - y)$$

At impact $y = 0 \quad \therefore \frac{1}{2}\left(v^2 - v_0^2\right) = gh \quad \therefore v = \sqrt{v_0^2 + 2gh}$ ◄

12.38



$$F = ma \qquad a = \frac{F}{m} = \frac{F_0}{m}e^{-x/b} \qquad v\ dv = \frac{F_0}{m}e^{-x/b}dx$$

$$\frac{1}{2}v^2 = \frac{F_0}{m}\int e^{-x/b}\ dx = -\frac{F_0 b}{m}e^{-x/b} + C$$

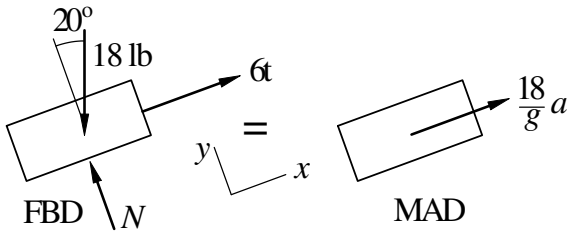
$$\text{Initial condition} \quad : \quad v = 0 \text{ at } x = 0 \qquad \therefore C = \frac{F_0 b}{m}$$

$$\frac{1}{2}v^2 = \frac{F_0 b}{m}\left(1 - e^{-x/b}\right)$$

$$v = \sqrt{\frac{2F_0 b}{m}\left(1 - e^{-x/b}\right)}$$

$$v|_{x=1.8\text{ ft}} = \sqrt{\frac{2(1576)(2)}{6.62 \times 10^{-3}/32.2}\left(1 - e^{-1.8/2.0}\right)} = 4270 \text{ ft/s}$$ ◄

12.39



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$$\begin{aligned}
\Sigma F_x &= ma \qquad 6t - 18\sin 20^\circ = \frac{18}{32.2}a \\
a &= \frac{32.2}{18}(6t - 18\sin 20^\circ) = 10.733t - 11.013 \text{ ft/s}^2 \\
v &= \int a\, dt = 5.367t^2 - 11.013t + C_1 \text{ ft/s} \\
x &= \int v\, dt = 1.789t^3 - 5.507t^2 + C_1 t + C_2 \text{ ft}
\end{aligned}$$

Initial conditions: $x = v = 0$ at $t = 0$ $\quad \therefore C_1 = C_2 = 0$

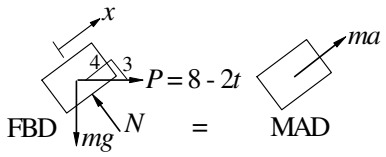
(a) When $v = 0$:

$$\begin{aligned}
v &= 5.367t^2 - 11.013t = 0 \qquad t = 2.052 \text{ s} \\
x &= 1.789\left(2.052^3\right) - 5.507(2.052^2) = -7.73 \text{ ft} \quad \blacktriangleleft
\end{aligned}$$

(b) When $x = 0$:

$$\begin{aligned}
x &= 1.789t^3 - 5.507t^2 = 0 \qquad t = 3.078 \text{ s} \\
v &= 5.367(3.078^2) - 11.013(3.078) = 16.95 \text{ ft} \quad \blacktriangleleft
\end{aligned}$$

12.40



$$\Sigma F_x = ma \quad +\nearrow \qquad \frac{4}{5}P - \frac{3}{5}mg = ma$$

$$a = \frac{4}{5}\frac{P}{m} - \frac{3}{5}g = \frac{4}{5}\frac{8-2t}{5/32.2} - \frac{3}{5}(32.2) = 21.90 - 10.304t \text{ ft/s}^2$$

$$\begin{aligned}
v &= \int a\, dt = 21.90t - 5.152t^2 + C_1 \\
x &= \int v\, dt = 10.95t^2 - 1.7173t^3 + C_1 t + C_2
\end{aligned}$$

Initial conditions: $v = -10$ ft/s, $x = 0$ at $t = 0$. $\quad \therefore C_1 = -10$ ft/s $\quad C_2 = 0$

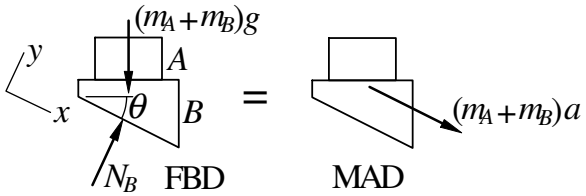
When $x = 0$:

$$10.95t^2 - 1.7173t^3 - 10t = 0 \qquad t = 1.1049 \text{ s} \quad \blacktriangleleft$$

$$v = 21.90(1.1049\ ) - 5.152(1.1049\ )^2 - 10 = 7.91 \text{ ft/s} \quad \blacktriangleleft$$

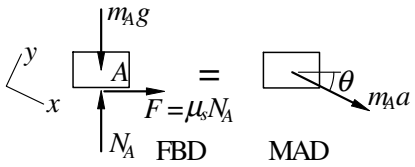
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12.41



$$\Sigma F_x = ma \quad +\searrow \qquad (m_A + m_B)g\sin\theta = (m_A + m_B)a$$
$$a = g\sin\theta$$

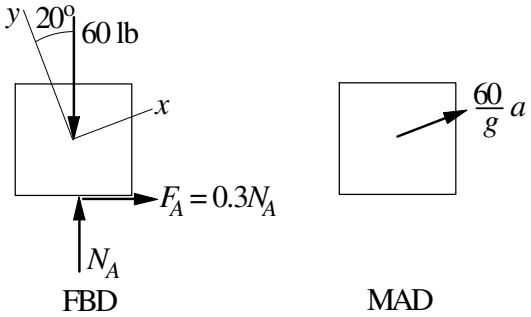
Assume impending slipping between A and B.



$$\Sigma F_x = ma \quad +\searrow \qquad (m_Ag - N_A)\sin\theta + \mu_s N_A\cos\theta = m_Ag\sin\theta$$
$$\mu_s = \tan\theta \blacktriangleleft$$

12.42

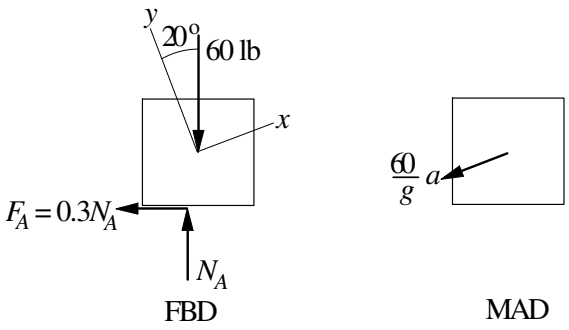
(a) Assume impending sliding of crate to the left.



$$\Sigma F_y = 0 \qquad N_A\cos 20^\circ - 0.3N_A\sin 20^\circ - 60\cos 20^\circ = 0$$
$$N_A = 67.35 \text{ lb}$$
$$\Sigma F_x = ma_x \qquad N_A\sin 20^\circ + 0.3N_A\cos 20^\circ - 60\sin 20^\circ = \frac{60}{32.2}a$$
$$67.35(\sin 20^\circ + 0.3\cos 20^\circ) - 60\sin 20^\circ = \frac{60}{32.2}a$$
$$a = 11.54 \text{ ft/s}^2 \blacktriangleleft$$

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(b) Assume impending sliding of crate to the right.



$$\Sigma F_y = 0 \qquad N_A \cos 20^\circ + 0.3N_A \sin 20^\circ - 60\cos 20^\circ = 0$$

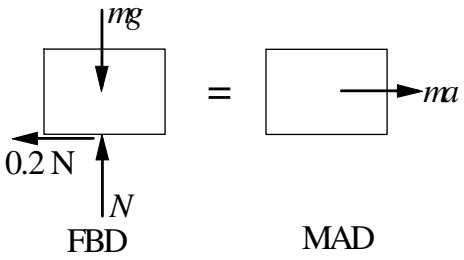
$$N_A = 54.09 \text{ lb}$$

$$\Sigma F_x = ma_x \qquad N_A \sin 20^\circ - 0.3N_A \cos 20^\circ - 60 \sin 20^\circ = -\frac{60}{32.2}a$$

$$54.09\,(\sin 20^\circ - 0.3\cos 20^\circ) - 60\sin 20^\circ = -\frac{60}{32.2}a$$

$$a = 9.27 \text{ ft/s}^2 \quad \blacktriangleleft$$

12.43



$$\Sigma F_y = 0 \qquad N - mg = 0 \qquad N = mg$$

$$\Sigma F_x = 0 \qquad -0.2N = ma \qquad -0.2mg = ma$$

$$a = -0.2g = -1.962 \text{ m/s}^2$$

$$v\,dv = a\,dx \qquad \frac{1}{2}v^2 = -1.962\int dx = -1.962x + C$$

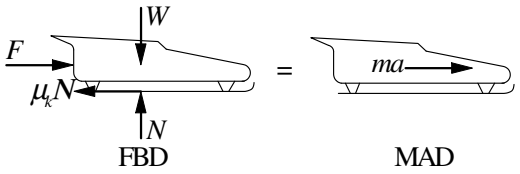
Initial condition: $v|_{x=0} = 6$ m/s $\qquad \frac{1}{2}(6^2) = C \qquad C = 18 \text{ m}^2/\text{s}^2$

$$\therefore \quad \frac{1}{2}v^2 = -1.962x + 18$$

$$v = 0 \text{ when } -1.962x + 18 = 0 \qquad x = 9.17 \text{ m} \quad \blacktriangleleft$$

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12.44



$$\begin{aligned}
\Sigma F_y &= 0 \qquad +\uparrow \qquad N - W = 0 \qquad N = W = 3000 \text{ lb} \\
\Sigma F_x &= 0 \qquad +\rightarrow \qquad F - \mu_k N = ma \\
a &= \frac{F - \mu_k N}{m} = \frac{1000e^{-0.2t} - 0.05(3000)}{3000/32.2} \\
&= 10.733e^{-0.2t} - 1.61 \text{ ft/s}^2
\end{aligned}$$

$$\begin{aligned}
v &= \int a\, dt = \int (10.733e^{-0.2t} - 1.61)dt \\
&= -53.67e^{-0.2t} - 1.61t + C \text{ ft/s}
\end{aligned}$$

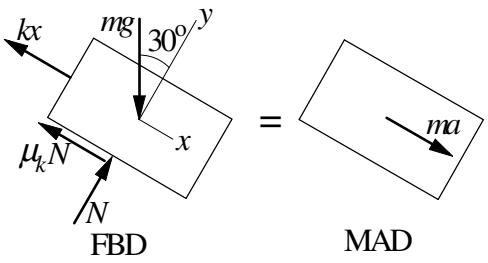
Initial condition: $v = 0$ when $t = 0$. $\therefore C = 53.67$ ft/s

$$v = 53.67(1 - e^{-0.2t}) - 1.61t \text{ ft/s}$$

Maximum velocity occurs at $t = 4$ s (end of powered travel)

$$v_{\max} = 53.67\left[1 - e^{-0.2(4)}\right] - 1.61(4) = 23.1 \text{ ft/s} \quad \blacktriangleleft$$

12.45



$$\begin{aligned}
\Sigma F_y &= 0 \qquad mg\cos 30^\circ - N = 0 \qquad N = mg\cos 30^\circ \\
\Sigma F_x &= ma \qquad mg\sin 30^\circ - \mu_k N - kx = ma \\
a &= g\left(\sin 30^\circ - \mu_k \cos 30^\circ\right) - \frac{k}{m}x \\
a &= 9.81(\sin 30^\circ - 0.3\cos 30^\circ) - \frac{25}{2.5}x = 2.356 - 10x \text{ m/s}^2
\end{aligned}$$

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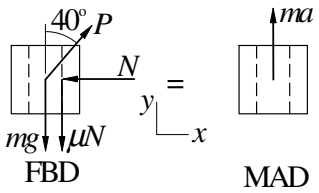
$$v\,dv = a\,dx \qquad \frac{1}{2}v^2 = \int (2.356 - 10x)dx$$

$$\frac{1}{2}v^2 = 2.356x - 5x^2 + C$$

Initial condition: $v|_{x=0} = 0 \qquad \therefore C = 0$

$v = 0$ when $2.356x - 5x^2 = 0 \qquad \therefore x = 0.471$ m ◄

12.46



$$\Sigma F_x = 0 \quad \stackrel{+}{\longrightarrow} \qquad P\sin 40° - N = 0 \qquad \therefore N = P\sin 40°$$

$$\Sigma F_y = ma_y \quad + \uparrow \qquad P\cos 40° - mg - \mu(P\sin 40°) = ma$$

$$a = \frac{P}{m}(\cos 40° - \mu \sin 40°) - g$$

When motion impends: $a = 0$ and $\mu = \mu_s = 0.5$

$$0 = \frac{P}{5}(\cos 40° - 0.5\sin 40°) - 9.81 \qquad P = 110.31 \text{ N}$$

When collar begins to slide: $P = 110.31$ N and $\mu = \mu_k = 0.4$

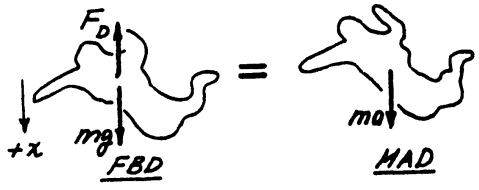
$$a = \frac{110.31}{5}(\cos 40° - 0.4\sin 40°) - 9.81 = 1.418 \text{ m/s}^2 \text{ ◄}$$

12.47

$\Sigma F_x = ma_x$: $+\downarrow \quad mg - F_D = ma$

$\therefore a = g - F_D/m = g - (c/m)v^2$

where $c = 0.00436$



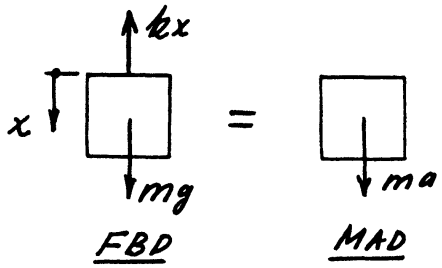
When a = 0: $v = v_\infty = \sqrt{\frac{mg}{c}} = \sqrt{\frac{135}{0.00436}} = 175.96$ ft/s ♦

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12.48

$\Sigma F_x = ma_x$: $+\downarrow \quad mg - kx = ma$

$$\therefore a = g - \frac{k}{m}x$$



$$\text{But } a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$$

$$\therefore v\,dv = a\,dx = \left(g - \frac{k}{m}x\right)dx \quad \therefore \frac{1}{2}v^2 = gx - \frac{k}{2m}x^2 + C$$

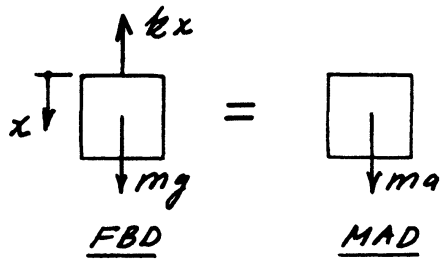
Initial condition: $v = 0$ when $x = 0 \quad \therefore C = 0 \quad \therefore v^2 = 2gx - \frac{k}{m}x^2$

When $x = 0.3$ m: $v^2 = 2(9.81)(0.3) - \frac{80}{1.8}(0.3)^2 = 1.8860\ (\text{m/s})^2 \quad \therefore v = 1.373$ m/s ◆

12.49

$\Sigma F_x = ma_x$: $+\downarrow \quad mg - kx = ma$

$$\therefore a = g - \frac{k}{m}x$$



$$\text{But } a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$$

$$\therefore v\,dv = a\,dx = \left(g - \frac{k}{m}x\right)dx$$

$$\therefore \frac{1}{2}v^2 = gx - \frac{k}{2m}x^2 + C = 9.81x - \frac{80}{2(1.8)}x^2 + C = 9.81x - 22.22x^2 + C$$

Initial condition: $v = 0$ when $x = 0.3$ m $\quad \therefore 0 = 9.81(0.3) - 22.22(0.3)^2 + C$

$$\therefore C = -0.9432\ (\text{m/s})^2 \quad \therefore \frac{1}{2}v^2 = 9.81x - 22.22x^2 - 0.9432$$

When $v = 0$: $22.22x^2 - 9.81x + 0.9432 = 0$

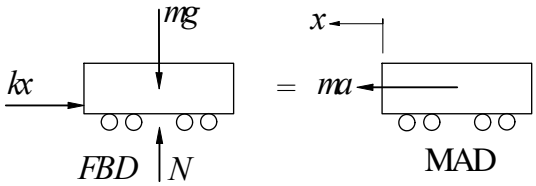
$$\therefore x = \frac{9.81 \pm \sqrt{(9.81)^2 - 4(22.22)(0.9432)}}{2(22.22)} = \frac{9.81 \pm 3.522}{44.44}$$

The smaller root is $x = 0.14149$ m $\quad \therefore h = 0.3 - x = 0.3 - 0.14149 = 0.1585$ m ◆

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12.50

$$v_0 = 10 \text{ km/h} = 10\frac{1000}{3600} = 2.778 \text{ m/s}$$



$$\Sigma F_x = ma_x \qquad -kx = ma \qquad a = -\frac{k}{m}x$$

$$a = \frac{dv}{dx}v \qquad v\,dv = a\,dx \qquad v\,dv = -\frac{k}{m}x\,dx$$

$$\frac{1}{2}v^2 = -\frac{k}{2m}x^2 + C$$

Initial condition: $v = v_0$ when $x = 0$. $\quad \therefore C_1 = \frac{1}{2}v_0^2$

$$\therefore v^2 = v_0^2 - \frac{k}{m}x^2$$

Stopping condition: $v = 0$ when $x = 0.5$ m:

$$2.778^2 - \frac{k}{18 \times 10^3}(0.5)^2 = 0 \qquad k = 5.56 \times 10^5 \text{ N/m} \blacktriangleleft$$

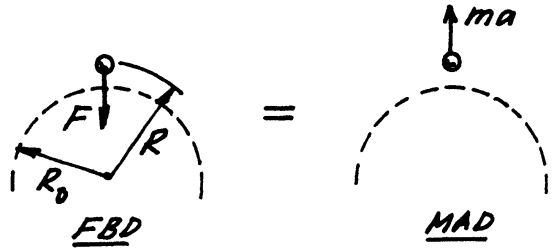
12.51

$\Sigma F = ma$: $+\uparrow \quad -F = ma$

$\therefore a = -F/m = -g(R_0/R)^2$

But $a = \frac{dv}{dt} = \frac{dv}{dR}\frac{dR}{dt} = \frac{dv}{dR}v$

$\therefore v\,dv = a\,dR = -g(R_0/R)^2\,dR \qquad \therefore \frac{1}{2}v^2 = gR_0^2/R + C$



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Initial condition: $v = v_0$ when $R = R_0$ $\quad \therefore C = \frac{1}{2}v_0^2 - gR_0 \quad \therefore v^2 = v_0^2 - 2gR_0(1 - R_0/R)$

When $v = 0$ $(R = R_{max})$: $v_0^2 - 2gR_0(1 - R_0/R_{max}) = 0$

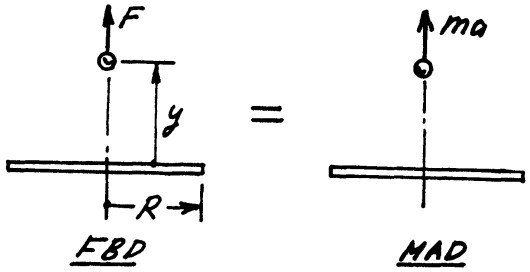
$$\therefore R_{max} = \frac{R_0}{1 - \dfrac{v_0^2}{2gR_0}} = \frac{3960}{1 - \dfrac{5000^2}{2(32.2)(3960 \times 5280)}} = 4034.9 \text{ mi}$$

$$\therefore h_{max} = R_{max} - R_0 = 4034.9 - 3960 = 74.9 \text{ mi} \ \blacklozenge$$

12.52

$\Sigma F = ma$: $+\uparrow \quad F = ma$

$$\therefore a = \frac{F}{m} = \frac{Vq}{mR}\left[1 - y(R^2 + y^2)^{-1/2}\right]$$



FBD MAD

$$\text{But } a = \frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt} = \frac{dv}{dy}v$$

$$\therefore v\,dv = a\,dy = \frac{Vq}{mR}\left[1 - y(R^2 + y^2)^{-1/2}\right]dy \quad \therefore \frac{1}{2}v^2 = \frac{Vq}{mR}\left[y - (R^2 + y^2)^{1/2}\right] + C$$

Initial condition: $v = 0$ when $y = 0$ $\quad \therefore C = \dfrac{Vq}{m} \quad \therefore v^2 = \dfrac{2Vq}{mR}\left[y + R - (R^2 + y^2)^{1/2}\right]$

When $y = R$: $v^2 = \dfrac{2Vq}{mR}(2 - \sqrt{2})R = 1.1716\dfrac{Vq}{m} \quad \therefore v = 1.082\sqrt{\dfrac{Vq}{m}}$ $\blacklozenge$

*12.53

$$\Sigma F = ma \qquad -T - c_D v^2 = ma$$

$$a = v\frac{dv}{dx} = -\frac{1}{m}(T + c_D v^2) \qquad -\frac{mv}{T + c_D v^2}dv = dx$$

$$x = -\int \frac{mv}{T + c_D v^2}dv = -\frac{m}{2c_D}\ln\frac{T + c_D v^2}{c_D} + C$$

Initial condition: $v = v_0$ when $x = 0$. $\quad \therefore C = \dfrac{m}{2c_D}\ln\dfrac{T + c_D v_0^2}{c_D}$

$$\therefore x = \frac{m}{2c_D}\ln\frac{T + c_D v_0^2}{T + c_D v^2}$$

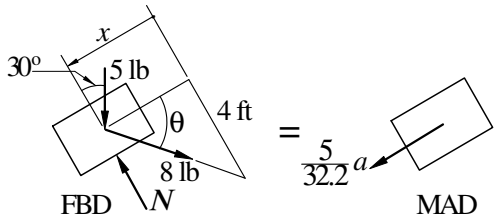
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$$m = \frac{2500}{32.2} = 77.64 \text{ slugs} \qquad v_0 = (90)\frac{5280}{3600} = 132 \text{ ft/s}$$

When $v = 0$:

$$x = \frac{m}{2c_D}\ln\frac{T + c_D v_0^2}{T} = \frac{77.64}{2(0.006)}\ln\frac{450 + 0.006(132)^2}{450} = 1352 \text{ ft} \quad \blacktriangleleft$$

*12.54



$$\Sigma F_x = ma \quad +\swarrow \qquad 5\sin 30° - 8\cos\theta = \frac{5}{32.2}a$$

$$\therefore a = \frac{32.2}{5}(5\sin 30° - 8\cos\theta) = 16.1 - 51.52\cos\theta \text{ ft/s}^2$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + 4^2}} = \frac{x}{\sqrt{x^2 + 16}}$$

$$a = v\frac{dv}{dx} = 16.1 - \frac{51.52x}{\sqrt{x^2 + 16}} \qquad v\,dv = \left(16.1 - \frac{51.52x}{\sqrt{x^2 + 16}}\right)dx$$

$$\frac{1}{2}v^2 = 16.1x - 51.52\sqrt{x^2 + 16} + C$$

Initial condition: $v = 0$ when $x = 0$. $\quad \therefore C = 51.51(4) = 206.0 \text{ (ft/s)}^2$

When $a = 0$ ($v = v_{\max}$):

$$16.1 - \frac{51.52x}{\sqrt{x^2 + 16}} = 0 \qquad \therefore x = 1.3159 \text{ ft} \quad \blacktriangleleft$$

$$\begin{aligned}\frac{1}{2}v_{\max}^2 &= 16.1(1.3159) - 51.52\sqrt{(1.3159)^2 + 16} + 206.0 \\ &= 10.241 \text{ (ft/s)}^2\end{aligned}$$

$$v_{\max} = \sqrt{2(10.241)} = 4.53 \text{ ft/s} \quad \blacktriangleleft$$

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***12.55**

$$\Sigma F = ma \quad +\downarrow \qquad mg - c_D v = ma$$

$$a = \frac{dv}{dt} = g - \frac{c_d}{m} v \qquad dt = \frac{dv}{g - (c_d/m)\, v}$$

$$t = -\frac{1}{c_d/m} \ln \left[-\frac{g - (c_d/m)v}{c_d/m} \right] + C = -\frac{m}{c_D} \ln \left(-\frac{mg}{c_D} + v \right) + C$$

Initial condition: $v = 0$ when $t = 0$. $\quad \therefore C = \dfrac{m}{c_D} \ln \left(-\dfrac{mg}{c_D} \right)$

$$\therefore t = \frac{m}{c_D} \ln \frac{mg/c_D}{mg/c_d - v}$$

When $v = v_\infty$ (terminal velocity), $a = 0$. $\quad \therefore v_\infty = \dfrac{mg}{c_d}$

When $v = 0.9 v_\infty = 0.9 \dfrac{mg}{c_d}$:

$$t = \frac{m}{c_d} \ln \frac{1}{1 - 0.9} = \frac{m}{c_D} \ln 10 = 2.30 \frac{m}{c_d} \quad \blacktriangleleft$$

■12.56

$$a = -\frac{7}{4} + \frac{v}{16} \text{ m/s}^2$$

Letting $x_1 = x$, $x_2 = v$, the equivalent first-order equations and initial conditions are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -\frac{7}{4} + \frac{x_2}{16}$$

$$x_1(0) = 0 \qquad x_2(0) = 20 \text{ m/s}$$

The MATLAB program that integrates the equations is

```
function problem12_56
[t,x] =ode45(@f,[0:0.5:25],[0 20]);
printSol(t,x)
    function dxdt = f(t,x)
    dxdt = [x(2)
           -7/4 + x(2)/16];
    end
end
```

The 2 lines of output that span the instant where $v = 0$ are

```
    t           x1           x2
 2.0000e+001  2.4124e+002  7.7255e-002
 2.0500e+001  2.4105e+002 -8.0911e-001
```

By inspection of output, the stopping distance is $x = 241$ m ◄

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■12.57

$$a = -\frac{v^2}{10}$$

Letting $x_1 = x$, $x_2 = v$, the equivalent first-order equations and initial conditions are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -\frac{x_2^2}{10}$$

$$x_1(0) = 0 \qquad x_2(0) = 20 \text{ in.}$$

The MATLAB program that integrates the equations is

```
function problem12_57
[t,x] =ode45(@f,(0:0.01:0.51),[0 20]);
printSol(t,x)
    function dxdt = f(t,x)
    dxdt = [x(2)
           -x(2)^2/10];
    end
end
```

The 3 lines of output that span the instant where $v = 10$ in./s are

```
    t            x1            x2
  4.9000e-001  6.8310e+000  1.0101e+001
  5.0000e-001  6.9315e+000  1.0000e+001
  5.1000e-001  7.0310e+000  9.9010e+000
```

By inspection, when $v = 10$ in./s, $t = 0.500$ s ◀

■12.58

Letting $x_1 = x$, $x_2 = v$, the equivalent first-order equations and initial conditions are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = 32.2(1 - 32.3 \times 10^{-6} x_2^2)$$

$$x_1(0) = 0 \qquad x_2(0) = 0$$

The MATLAB program that integrates the equations is

```
function problem12_58
[t,x] =ode45(@f,(0:0.1:7),[0 0]);
printSol(t,x)
    function dxdt = f(t,x)
    dxdt = [x(2)
           32.2*(1-32.3e-6*x(2)^2)];
    end
end
```

The 2 lines of output that span the instant where $v = 100$ mi/h $= 146.67$ ft/s are

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```
    t           x1           x2
 6.5000e+000  5.6244e+002  1.4612e+002
 6.6000e+000  5.7710e+002  1.4710e+002
```

Linear interpolation:

$$\frac{6.6 - 6.5}{147.10 - 146.12} = \frac{t - 6.5}{146.67 - 146.12} \qquad t = 6.56 \text{ s} \blacktriangleleft$$

■12.59

$$a = -5796\left(1 - \frac{5}{\sqrt{x^2 + 9}}\right) x \text{ in./s}^2$$

Letting $x_1 = x$, $x_2 = v$, the equivalent first-order equations and initial conditions are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -5796\left(1 - \frac{5}{\sqrt{x_1^2 + 9}}\right) x_1$$

$$x_1(0) = 8 \text{ in.} \qquad x_2(0) = 0$$

The MATLAB program that integrates the equations is

```
function problem12_59
[t,x] =ode45(@f,(0:0.001:0.042),[8 0]);
printSol(t,x)
    function dxdt = f(t,x)
    dxdt = [x(2)
           -5796*(1 - 5/sqrt(x(1)^2 + 9))*x(1)];
    end
end
```

The 2 lines of output that span the instant where $x = 0$ are

```
    t           x1           x2
  3.9000e-002  3.7400e-002 -2.2275e+002
  4.0000e-002 -1.8542e-001 -2.2304e+002
```

By inspection, when $x = 0$, $v = 223$ in./s ◄

■12.60

$$a = -32.2\left[1 - 6.24 \times 10^{-4} v^2 \exp(-3.211 \times 10^{-5} x)\right] \text{ ft/s}^2$$

(a) Letting $x_1 = x$, $x_2 = v$, the equivalent first-order equations and initial conditions are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -32.2\left(1 - 6.24 \times 10^{-4} x_2^2 e^{-3.211 \times 10^{-5} x_1}\right)$$

$$x_1(0) = 30\,000 \text{ ft} \qquad x_2(0) = 0$$

The MATLAB program that integrates the equations is

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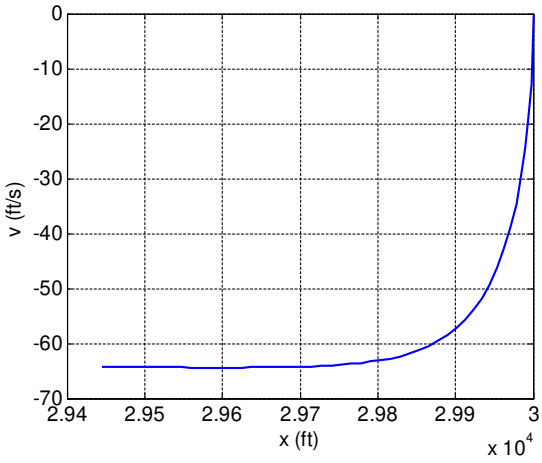
```
function problem12_60
[t,x] = ode45(@f,(0:0.2:10),[30000 0]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (ft)'); ylabel('v (ft/s)')
    function dxdt = f(t,x)
    dxdt = [x(2)
           -32.2*(1-6.24e-4*x(2)^2*exp(-3.211e-5*x(1)))];
    end
end
```

The 3 lines of output that span the instant where $v = v_{\max}$ are

```
    t            x1            x2
  7.4000e+000  2.9612e+004 -6.4384e+001
  7.6000e+000  2.9599e+004 -6.4385e+001
  7.8000e+000  2.9586e+004 -6.4384e+001
```

By inspection, $v_{\max} = 64.4$ ft/s ◀ at $x = 29\ 600$ ft ◀

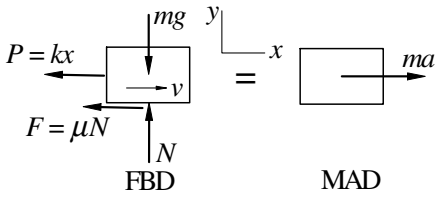
(b)



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■12.61

(a)



The FBD shown is valid only if $v > 0$ (block is moving to the right.) If $v < 0$ (block is moving to the left), the direction of the friction force F must be reversed.

$$\begin{aligned} \Sigma F_y &= 0 \quad +\uparrow \qquad N - mg = 0 \qquad \therefore N = mg \\ \Sigma F_x &= ma \quad \overset{+}{\longrightarrow} \qquad -kx - \mu N \text{ sign}(v) = ma \end{aligned}$$

$$\therefore \quad a = -\frac{k}{m}x - \mu\frac{N}{m} \text{ sign}(v) = -\frac{k}{m} - \mu g \text{ sign}(v)$$

$$= \quad -\frac{30}{1.6}x - 0.2(9.81) \text{ sign}(v) = -18.75x - 1.962 \text{ sign}(v) \text{ m/s}^2 \quad \blacktriangleleft$$

(b) With the notation $x_1 = x$ and $x_2 = v$, the equivalent first-order equations are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -18.75x_1 - 1.962 \text{ sign}(x_2)$$

subject to the initial conditions $x_1 = 0$, $x_2 = 6$ m/s at $t = 0$.
The corresponding MATLAB program is:

```
function problem12_61
[t,x] =ode45(@f,[0:0.02:1.2],[0 6]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (m)'); ylabel('v (m/s)')
    function dxdt = f(t,x)
    dxdt = [x(2)
           -18.75*x(1)-1.962*sign(x(2))];
    end
end
```

The block stops twice during the period $0 < t < 1.2$ s. Only the lines of output that span the instant where $v = 0$ are shown below.

```
    t           x1           x2
  3.4000e-001  1.2844e+000  1.3160e-001
  3.6000e-001  1.2822e+000 -3.5272e-001
```


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Linear interpolation:

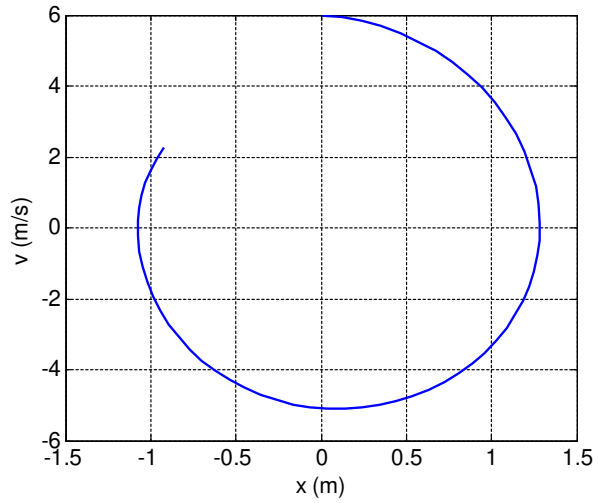
$$\frac{3.6-3.4}{-0.35272-0.13160}=\frac{t_1-3.4}{0-0.13160} \qquad t_1=3.45 \text{ s} \blacktriangleleft$$

```
1.0600e+000 -1.0745e+000 -2.1421e-001
1.0800e+000 -1.0745e+000  2.0038e-001
```

Linear interpolation:

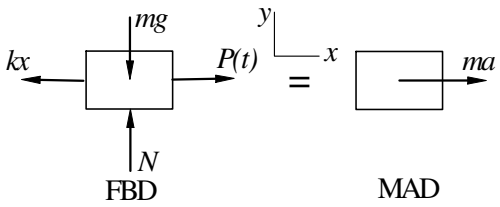
$$\frac{1.08-1.06}{0.20038-(-0.21421)}=\frac{t_2-1.06}{0-(-0.21421)} \qquad t_2=1.070 \text{ s} \blacktriangleleft$$

(c)



■12.62

(a)



$$\Sigma F_x=ma \quad \stackrel{+}{\longrightarrow} \quad P(t)-kx=ma \qquad a=\frac{P(t)}{m}-\frac{k}{m}x$$

$$\begin{aligned} P(t) &= \begin{cases} 25t \text{ N when } t\le 1 \text{ s} \\ 25 \text{ N when } t\ge 1 \text{ s} \end{cases} \\ &= (12.5t+12.5)+(12.5t-12.5)\ \text{sgn}\ (1-t) \end{aligned}$$

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$$\therefore \quad a = \frac{(12.5t + 12.5) + (12.5t - 12.5)\ \text{sgn}\ (1-t)}{2} - \frac{25}{2}x$$
$$= \quad 6.25\left[(t+1) + (t-1)\ \text{sgn}\ (1-t)\right] - 12.5x\ \text{m/s}^2 \quad \blacktriangleleft$$

(b) With the notation $x_1 = x$ and $x_2 = v$, the equivalent first-order equations are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = 6.25\left[(t+1) + (t-1)\ \text{sgn}\ (1-t)\right] - 12.5x_1$$

subject to the initial conditions $x_1 = x_2 = 0$ at $t = 0$.
The corresponding MATLAB program is:

```
function problem12_62
[t,x] = ode45(@f,(0:0.05:3),[0 0]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (m)'); ylabel('v (m/s)')
    function dxdt = f(t,x)
    dxdt = [x(2)
           6.25*(t+1 + (t-1)*sign(1-t)) - 12.5*x(1)];
    end
end
```

Below are partial printouts that span $v_{\max}$ and $x_{\max}$.

```
    t           x1           x2
  8.0000e-001  7.1290e-001  1.9518e+000
  9.0000e-001  9.1150e-001  2.0003e+000
  1.0000e+000  1.1087e+000  1.9205e+000
```

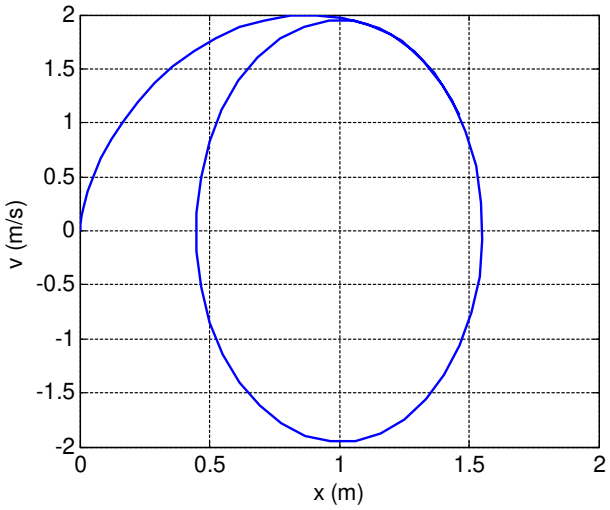
By inspection, $v_{\max} = 2.00$ m/s ◀

```
  1.3000e+000  1.5271e+000  6.0188e-001
  1.4000e+000  1.5535e+000 -8.0559e-002
  1.5000e+000  1.5113e+000 -7.5304e-001
```

By inspection, $x_{\max} = 1.554$ m ◀

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(c)



■12.63

$$a = 80 - 16v^{1.5} \text{ ft/s}^2$$

With the notation $x_1 = x$ and $x_2 = v$, the equivalent first-order equations are

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = 80 - 16x_1$$

subject to the initial conditions $x_1 = x_2 = 0$ at $t = 0$.
The corresponding MATLAB program is:

```
function problem12_63
[t,x] =ode45(@f,[0:0.005:0.15],[0 0]);
printSol(t,x)
    function dxdt = f(t,x)
    dxdt = [x(2)
           80 - 16*x(2)^1.5];
    end
end
```

Only the 4 lines of output that span $x = 0.25$ ft are shown below.

```
    t            x1            x2
  1.0500e-001  2.2384e-001  2.8693e+000
  1.1000e-001  2.3822e-001  2.8794e+000
  1.1500e-001  2.5263e-001  2.8876e+000
  1.2000e-001  2.6709e-001  2.8944e+000
```

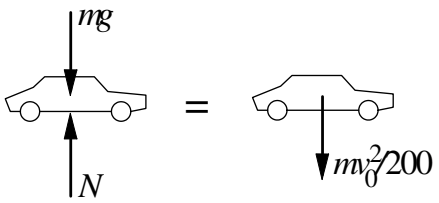
Use linear interpolation to find v at $x = 0.25$ ft:

$$\frac{2.8876 - 2.8794}{0.25263 - 0.23822} = \frac{v - 2.8794}{0.25 - 0.23822} \qquad v = 2.89 \text{ ft/s} \quad \blacktriangleleft$$

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12.64

$$y = -\frac{x^2}{400} \qquad \dot{y} = -\frac{x}{200}\dot{x} = -\frac{x}{200}v_0 \qquad \ddot{y} = -\frac{1}{200}v_0\dot{x} = -\frac{v_0^2}{200}$$

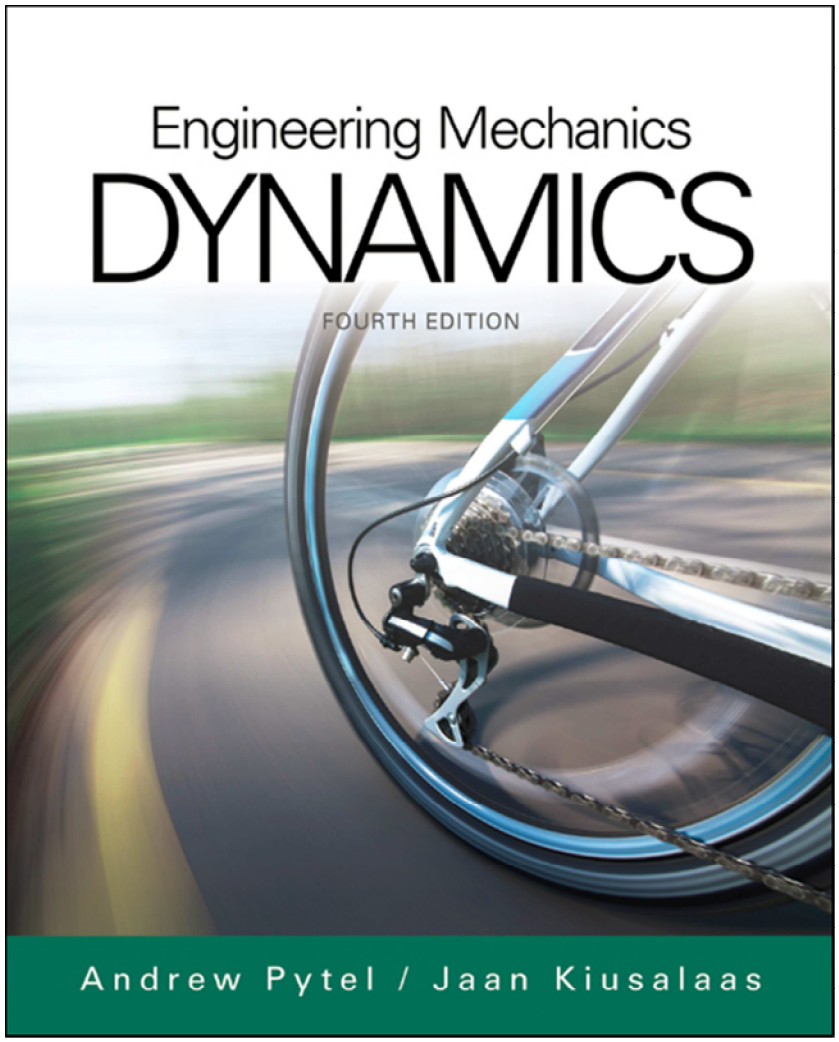


$$\Sigma F_y = ma_y \qquad N - mg = -m\frac{v_0^2}{200}$$

$$\text{Contact is lost when } N = 0\text{:} \quad g = \frac{v_0^2}{200}$$

$$v_0 = \sqrt{200g} = \sqrt{200(32.2)} = 80.3 \text{ ft/s} \quad \blacktriangleleft$$

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Chapter 12

Dynamics of a Particle: Rectangular Coordinates

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Kinematics

- We examine a single particle relative to fixed Cartesian coordinates.
- Position, velocity and acceleration can be broken down into their three base directions x, y and z.
  - These are represented by the unit vectors **i**, **j** and **k**.
- So any **v**, for example, is $v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$. The same is true for **a** and **r**.
- A special case: plane motion. This is movement through the *x* and *y* only ($\Delta z = 0$).
  - There are two angles of interest, θ and β.
  - Theta is the angle between the x-axis and the velocity vector and is always tangent to the path.
  - Beta is the angle between the x-axis and the acceleration vector and does not have to be tangent to the path.
- A further simplification, rectilinear motion, is movement in one dimension. For example, $a = a\mathbf{j}$.

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Force-Mass-Acceleration

- Equations of motion of the particle are $\sum F_i = ma_i$ where i is x, y or z.
- Uses two diagrams: Free-body (FBD) and mass-acceleration (MAD).
- Free-body diagrams are the same as in statics.
  - Review: An FBD shows an object with all forces acting on it indicated.
- Mass-acceleration diagrams display the inertia vector of an object.

- Step 1: Draw the FBD, as in statics.
- Step 2: Determine the acceleration of the particle.
- Step 3: Draw the MAD (acceleration found in step 2).
- Step 4: Set the sum of the forces from the FBD equal to the mass-acceleration from the MAD. Solve for the unknown.

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Rectilinear Motion Dynamics

- Movement in a straight line only, so $\sum F = ma$ in that direction and $\sum F = 0$ in the others.
- Acceleration is a function of position (x), velocity (v) and time (t).
- Frequently we can only find an analytical solution to the 2nd order ODE that results because the ODE is nonlinear.

- There are three cases where the ODE can be integrated because it is explicit in one variable only (x, v or t). These can be solved analytically if initial conditions are known.
  - Initial conditions are usually – but do not have to be – where the particle is at t_0 or when the particle is at $x = 0$.

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Curvilinear Motion

- Curvilinear motion is motion along a path that is not rectilinear.
- Superposition occurs when the accelerations in the three Cartesian directions are not “coupled”: they are dependent on velocities and positions that are independent of each other.
  - Another way of saying this: the acceleration in any one direction can be changed without altering the others.
- In general curvilinear motion, the equations may be coupled.
  - For example, both a_x and a_y may be functions of the same equation for **v**.
  - In this case, analytical solutions are impossible and numerical methods must be resorted to.

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Area Method of Motion Analysis

- By plotting the acceleration of a particle as a function of time, we are graphically solving the equation $\int a\, dt$. The area under the curve between t_0 and t is the velocity of the particle at t.
- Performing the same operation with velocity as a function of time solves $\int v\, dt$ for the position.
- Note that if the acceleration or velocity go negative, the area beneath the x-axis is negative and subtracted from the total.

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