# *Elementary Linear Algebra 8th edition Larson Solutions Manual*

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# C H A P T E R 2
# Matrices


**Section 2.1** Operations with Matrices ..... **30**

**Section 2.2** Properties of Matrix Operations ..... **36**

**Section 2.3** The Inverse of a Matrix ..... **41**

**Section 2.4** Elementary Matrices ..... **46**

**Section 2.5** Markov Chains ..... **53**

**Section 2.6** More Applications of Matrix Operations ..... **62**

**Review Exercises** ..... **66**

**Project Solutions** ..... **75**



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# CHAPTER 2
# Matrices

## Section 2.1 Operations with Matrices

**2.** $x = 13, y = 12$

**4.** $x + 2 = 2x + 6 \qquad 2y = 18$
$-4 = x \qquad y = 9$

$2x = -8 \qquad y + 2 = 11$
$x = -4 \qquad y = 9$

**6.** (a) $A + B = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 6+1 & -1+4 \\ 2+(-1) & 4+5 \\ -3+1 & 5+10 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 1 & 9 \\ -2 & 15 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 6-1 & -1-4 \\ 2-(-1) & 4-5 \\ -3-1 & 5-10 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 3 & -1 \\ -4 & -5 \end{bmatrix}$

(c) $2A = 2\begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 2(6) & 2(-1) \\ 2(2) & 2(4) \\ 2(-3) & 2(5) \end{bmatrix} = \begin{bmatrix} 12 & -2 \\ 4 & 8 \\ -6 & 10 \end{bmatrix}$

(d) $2A - B = \begin{bmatrix} 12 & -2 \\ 4 & 8 \\ -6 & 10 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 12-1 & -2-4 \\ 4-(-1) & 8-5 \\ -6-1 & 10-10 \end{bmatrix} = \begin{bmatrix} 11 & -6 \\ 5 & 3 \\ -7 & 0 \end{bmatrix}$

(e) $B + \frac{1}{2}A = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} + \begin{bmatrix} 3 & -\frac{1}{2} \\ 1 & 2 \\ -\frac{3}{2} & \frac{5}{2} \end{bmatrix} = \begin{bmatrix} 4 & \frac{7}{2} \\ 0 & 7 \\ -\frac{1}{2} & \frac{25}{2} \end{bmatrix}$

**8.** (a) $A + B = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & 2+2 & -1+1 \\ 2+5 & 4+4 & 5+2 \\ 0+2 & 1+1 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 0 \\ 7 & 8 & 7 \\ 2 & 2 & 2 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3-0 & 2-2 & -1-1 \\ 2-5 & 4-4 & 5-2 \\ 0-2 & 1-1 & 2-0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -2 \\ -3 & 0 & 3 \\ -2 & 0 & 2 \end{bmatrix}$

(c) $2A = 2\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(2) & 2(-1) \\ 2(2) & 2(4) & 2(5) \\ 2(0) & 2(1) & 2(2) \end{bmatrix} = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 8 & 10 \\ 0 & 2 & 4 \end{bmatrix}$

(d) $2A - B = 2\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 8 & 10 \\ 0 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -3 \\ -1 & 4 & 8 \\ -2 & 1 & 4 \end{bmatrix}$

(e) $B + \frac{1}{2}A = \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & 1 & -\frac{1}{2} \\ 1 & 2 & \frac{5}{2} \\ 0 & \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 3 & \frac{1}{2} \\ 6 & 6 & \frac{9}{2} \\ 2 & \frac{3}{2} & 1 \end{bmatrix}$


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**10.** (a) $A + B$ is not possible. $A$ and $B$ have different sizes.

(b) $A - B$ is not possible. $A$ and $B$ have different sizes.

(c) $2A = 2\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -2 \end{bmatrix}$

(d) $2A - B$ is not possible. $A$ and $B$ have different sizes.

(e) $B + \frac{1}{2}A$ is not possible. $A$ and $B$ have different sizes.

**12.** (a) $c_{23} = 5a_{23} + 2b_{23} = 5(2) + 2(11) = 32$

(b) $c_{32} = 5a_{32} + 2b_{32} = 5(1) + 2(4) = 13$

**14.** Simplifying the right side of the equation produces

$$\begin{bmatrix} w & x \\ y & x \end{bmatrix} = \begin{bmatrix} -4 + 2y & 3 + 2w \\ 2 + 2z & -1 + 2x \end{bmatrix}.$$

By setting corresponding entries equal to each other, you obtain four equations.

$$\begin{aligned} w &= -4 + 2y \\ x &= 3 + 2w \\ y &= 2 + 2z \\ x &= -1 + 2x \end{aligned} \Rightarrow \begin{cases} -2y + w = -4 \\ x - 2w = 3 \\ y - 2z = 2 \\ x = 1 \end{cases}$$

The solution to this linear system is: $x = 1$, $y = \frac{3}{2}$, $z = -\frac{1}{4}$, and $w = -1$.

**16.** (a) $AB = \begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 2(4) + (-2)(2) & 2(1) + (-2)(-2) \\ -1(4) + 4(2) & -1(1) + 4(-2) \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 4 & -9 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4(2) + 1(-1) & 4(-2) + 1(4) \\ 2(2) + (-2)(-1) & 2(-2) + (-2)(4) \end{bmatrix} = \begin{bmatrix} 7 & -4 \\ 6 & -12 \end{bmatrix}$

**18.** (a) $AB = \begin{bmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1(1) + (-1)(2) + 7(1) & 1(1) + (-1)(1) + 7(-3) & 1(2) + (-1)(1) + 7(2) \\ 2(1) + (-1)(2) + 8(1) & 2(1) + (-1)(1) + 8(-3) & 2(2) + (-1)(1) + 8(2) \\ 3(1) + 1(2) + (-1)(1) & 3(1) + 1(1) + (-1)(-3) & 3(2) + 1(1) + (-1)(2) \end{bmatrix} = \begin{bmatrix} 6 & -21 & 15 \\ 8 & -23 & 19 \\ 4 & 7 & 5 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1(1) + 1(2) + 2(3) & 1(-1) + 1(-1)1 + 2(1) & 1(7) + 1(8) + 2(-1) \\ 2(1) + 1(2) + 1(3) & 2(-1) + 1(-1) + 1(1) & 2(7) + 1(8) + 1(-1) \\ 1(1) + (-3)(2) + 2(3) & 1(-1) + (-3)(-1) + 2(1) & 1(7) + (-3)(8) + 2(-1) \end{bmatrix} = \begin{bmatrix} 9 & 0 & 13 \\ 7 & -2 & 21 \\ 1 & 4 & -19 \end{bmatrix}$

**20.** (a) $AB = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 3(1) + 2(2) + 1(1) & 3(2) + 2(-1) + 1(-2) \\ -3(1) + 0(2) + 4(1) & -3(2) + 0(-1) + 4(-2) \\ 4(1) + (-2)(2) + (-4)(1) & 4(2) + (-2)(-1) + (-4)(-2) \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 1 & -14 \\ -4 & 18 \end{bmatrix}$

(b) $BA$ is not defined because $B$ is $3 \times 2$ and $A$ is $3 \times 3$.

**22.** (a) $AB = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -1(2) & -1(1) & -1(3) & -1(2) \\ 2(2) & 2(1) & 2(3) & 2(2) \\ -2(2) & -2(1) & -2(3) & -2(2) \\ 1(2) & 1(1) & 1(3) & 1(2) \end{bmatrix} = \begin{bmatrix} -2 & -1 & -3 & -2 \\ 4 & 2 & 6 & 4 \\ -4 & -2 & -6 & -4 \\ 2 & 1 & 3 & 2 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 2 & 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1) + 1(2) + 3(-2) + 2(1) \end{bmatrix} = \begin{bmatrix} -4 \end{bmatrix}$

**24.** (a) $AB$ is not defined because $A$ is $2 \times 2$ and $B$ is $3 \times 2$.

(b) $BA = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2(2) + 1(5) & 2(-3) + 1(2) \\ 1(2) + 3(5) & 1(-3) + 3(2) \\ 2(2) + (-1)(5) & 2(-3) + (-1)(2) \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ 17 & 3 \\ -1 & -8 \end{bmatrix}$

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**26.** (a) $AB = \begin{bmatrix} 2 & 1 & 2 \\ 3 & -1 & -2 \\ -2 & 1 & -2 \end{bmatrix}\begin{bmatrix} 4 & 0 & 1 & 3 \\ -1 & 2 & -3 & -1 \\ -2 & 1 & 4 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 2(4)+1(-1)+2(-2) & 2(0)+1(2)+2(1) & 2(1)+1(-3)+2(4) & 2(3)+1(-1)+2(3) \\ 3(4)+(-1)(-1)+(-2)(-2) & 3(0)+(-1)(2)+(-2)(1) & 3(1)+(-1)(-3)+(-2)(4) & 3(3)+(-1)(-1)+(-2)(3) \\ -2(4)+1(-1)+(-2)(-2) & -2(0)+1(2)+(-2)(1) & -2(1)+1(-3)+(-2)(4) & -2(3)+1(-1)+(-2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 7 & 11 \\ 17 & -4 & -2 & 4 \\ -5 & 0 & -13 & -13 \end{bmatrix}$$

(b) $BA$ is not defined because $B$ is $3 \times 4$ and $A$ is $3 \times 3$.

**28.** (a) $AB$ is not defined because $A$ is $2 \times 5$ and $B$ is $2 \times 2$.

(b) $BA = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 & -2 & 4 \\ 6 & 13 & 8 & -17 & 20 \end{bmatrix}$

$$= \begin{bmatrix} 1(1)+6(6) & 1(0)+6(13) & 1(3)+6(8) & 1(-2)+6(-17) & 1(4)+6(20) \\ 4(1)+2(6) & 4(0)+2(13) & 4(3)+2(8) & 4(-2)+2(-17) & 4(4)+2(20) \end{bmatrix}$$

$$= \begin{bmatrix} 37 & 78 & 51 & -104 & 124 \\ 16 & 26 & 28 & -42 & 56 \end{bmatrix}$$

**30.** $C + E$ is not defined because $C$ and $E$ have different sizes.

**32.** $-4A$ is defined and has size $3 \times 4$ because $A$ has size $3 \times 4$.

**34.** $BE$ is defined. Because $B$ has size $3 \times 4$ and $E$ has size $4 \times 3$, the size of $BE$ is $3 \times 3$.

**36.** $2D + C$ is defined and has size $4 \times 2$ because $2D$ and $C$ have size $4 \times 2$.

**38.** As a system of linear equations, $A\mathbf{x} = \mathbf{0}$ is

$$\begin{aligned} x_1 + 2x_2 + x_3 + 3x_4 &= 0 \\ x_1 - x_2 \qquad + x_4 &= 0. \\ x_2 - x_3 + 2x_4 &= 0 \end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Choosing $x_4 = t$, the solution is $x_1 = -2t, x_2 = -t, x_3 = t$, and $x_4 = t$, where $t$ is any real number.

**40.** In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & 4 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

**42.** In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -13 \\ 12 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} -4 & 9 & -13 \\ 1 & -3 & 12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -23 \\ 0 & 1 & -\frac{35}{3} \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -23 \\ -\frac{35}{3} \end{bmatrix}$.

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**44.** In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 1 & -3 & -1 \\ -1 & 2 & 0 & 1 \\ 1 & -1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$.

**46.** In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & 0 \\ 0 & -6 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -11 \\ 40 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & -1 & 4 & 17 \\ 1 & 3 & 0 & -11 \\ 0 & -6 & 5 & 40 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$.

**48.** In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ -1 & 1 & -1 & 1 & -1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

**50.** The augmented matrix row reduces as follows.

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ -1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

There are an infinite number of solutions. For example, $x_3 = 0$, $x_2 = 2$, $x_1 = -3$.

So, $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = -3\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$.

**52.** The augmented matrix row reduces as follows.

$$\begin{bmatrix} -3 & 5 & -22 \\ 3 & 4 & 4 \\ 4 & -8 & 32 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -3 & 10 \\ 0 & 9 & -18 \\ 0 & -4 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & 10 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}.$$

So,

$$\mathbf{b} = \begin{bmatrix} -22 \\ 4 \\ 32 \end{bmatrix} = 4\begin{bmatrix} -3 \\ 3 \\ 4 \end{bmatrix} + (-2)\begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}.$$

**54.** Expanding the left side of the equation produces

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 2a_{11} - a_{21} & 2a_{12} - a_{22} \\ 3a_{11} - 2a_{21} & 3a_{12} - 2a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and you obtain the system

$$\begin{aligned} 2a_{11} - a_{21} &= 1 \\ 2a_{12} - a_{22} &= 0 \\ 3a_{11} - 2a_{21} &= 0 \\ 3a_{12} - 2a_{22} &= 1. \end{aligned}$$

Solving by Gauss-Jordan elimination yields

$a_{11} = 2$, $a_{12} = -1$, $a_{21} = 3$, and $a_{22} = -2$.

So, you have $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$.

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**56.** Expanding the left side of the matrix equation produces

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2a + 3b & a + b \\ 2c + 3d & c + d \end{bmatrix} = \begin{bmatrix} 3 & 17 \\ 4 & -1 \end{bmatrix}.$$

You obtain two systems of linear equations (one involving $a$ and $b$ and the other involving $c$ and $d$).

$$\begin{aligned} 2a + 3b &= 3 \\ a + b &= 17, \end{aligned}$$

and

$$\begin{aligned} 2c + 3d &= 4 \\ c + d &= -1. \end{aligned}$$

Solving by Gauss-Jordan elimination yields $a = 48$, $b = -31$, $c = -7$, and $d = 6$.

**58.** $$AA = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

**60.** $$AB = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -7 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 3(-7) + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + (-5)4 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} -21 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Similarly,

$$BA = \begin{bmatrix} -21 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

**62.** (a) $$AB = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & a_{11}b_{13} \\ a_{22}b_{21} & a_{22}b_{22} & a_{22}b_{23} \\ a_{33}b_{31} & a_{33}b_{32} & a_{33}b_{33} \end{bmatrix}$$

The $i$th row of $B$ has been multiplied by $a_{ii}$, the $i$th diagonal entry of $A$.

(b) $$BA = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{22}b_{12} & a_{33}b_{13} \\ a_{11}b_{21} & a_{22}b_{22} & a_{33}b_{23} \\ a_{11}b_{31} & a_{22}b_{32} & a_{33}b_{33} \end{bmatrix}$$

The $i$th column of $B$ has been multiplied by $a_{ii}$, the $i$th diagonal entry of $A$.

(c) If $a_{11} = a_{22} = a_{33}$, then $AB = a_{11}B = BA$.

**64.** The trace is the sum of the elements on the main diagonal.

$$1 + 1 + 1 = 3$$

**66.** The trace is the sum of the elements on the main diagonal.

$$1 + 0 + 2 + (-3) = 0$$

**68.** Let $AB = [c_{ij}]$, where $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$. Then, $Tr(AB) = \sum_{i=1}^{n} c_{ii} = \sum_{i=1}^{n}\left(\sum_{k=1}^{n} a_{ik}b_{ki}\right)$.

Similarly, if $BA = [d_{ij}]$, $d_{ij} = \sum_{k=1}^{n} b_{ik}a_{kj}$. Then $Tr(BA) = \sum_{i=1}^{n} d_{ii} = \sum_{i=1}^{n}\left(\sum_{k=1}^{n} b_{ik}a_{ki}\right) = Tr(AB)$.

**70.** $$AB = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}\begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha(-\sin\beta) - \sin\alpha\cos\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & \sin\alpha(-\sin\beta) + \cos\alpha\cos\beta \end{bmatrix}$$

$$BA = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta\cos\alpha - \sin\beta\sin\alpha & \cos\beta(-\sin\alpha) - \sin\beta\cos\alpha \\ \sin\beta\cos\alpha + \cos\beta\sin\alpha & \sin\beta(-\sin\alpha) + \cos\beta\cos\alpha \end{bmatrix}$$

So, you see that $AB = BA = \begin{bmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix}$.

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**72.** Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$.

Then the matrix equation $AB - BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is equivalent to

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

This equation implies that

$$a_{11}b_{11} + a_{12}b_{21} - b_{11}a_{11} - b_{12}a_{21} = a_{12}b_{21} - b_{12}a_{21} = 1$$
$$a_{21}b_{12} + a_{22}b_{22} - b_{21}a_{12} - b_{22}a_{22} = a_{21}b_{12} - b_{21}a_{12} = 1$$

which is impossible. So, the original equation has no solution.

**74.** Assume that $A$ is an $m \times n$ matrix and $B$ is a $p \times q$ matrix. Because the product $AB$ is defined, you know that $n = p$. Moreover, because $AB$ is square, you know that $m = q$. Therefore, $B$ must be of order $n \times m$, which implies that the product $BA$ is defined.

**76.** Let rows $s$ and $t$ be identical in the matrix $A$. So, $a_{sj} = a_{ij}$ for $j = 1, \ldots, n$. Let $AB = [c_{ij}]$, where $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$. Then, $c_{sj} = \sum_{k=1}^{n} a_{sk}b_{kj}$, and $c_{tj} = \sum_{k=1}^{n} a_{tk}b_{kj}$. Because $a_{sk} = a_{tk}$ for $k = 1, \ldots, n$, rows $s$ and $t$ of $AB$ are the same.

**78.** (a) No, the matrices have different sizes.

(b) No, the matrices have different sizes.

(c) Yes; No, $BA$ is undefined.

**80.** $1.2\begin{bmatrix} 70 & 50 & 25 \\ 35 & 100 & 70 \end{bmatrix} = \begin{bmatrix} 84 & 60 & 30 \\ 42 & 120 & 84 \end{bmatrix}$

**82.** (a) Multiply the matrix for 2010 by $\frac{1}{3090}$. This produces a matrix giving the information as percents of the total population.

$$A = \frac{1}{3090}\begin{bmatrix} 12{,}306 & 35{,}240 & 7830 \\ 16{,}095 & 41{,}830 & 9051 \\ 27{,}799 & 72{,}075 & 14{,}985 \\ 5698 & 13{,}717 & 2710 \\ 12{,}222 & 31{,}867 & 5901 \end{bmatrix} \approx \begin{bmatrix} 3.98 & 11.40 & 2.53 \\ 5.21 & 13.54 & 2.93 \\ 9.00 & 23.33 & 4.85 \\ 1.84 & 4.44 & 0.88 \\ 3.96 & 10.31 & 1.91 \end{bmatrix}$$

Multiply the matrix for 2013 by $\frac{1}{3160}$. This produces a matrix giving the information as percents of the total population.

$$B = \frac{1}{3160}\begin{bmatrix} 12{,}026 & 35{,}471 & 8446 \\ 15{,}772 & 41{,}985 & 9791 \\ 27{,}954 & 73{,}703 & 16{,}727 \\ 5710 & 14{,}067 & 3104 \\ 12{,}124 & 32{,}614 & 6636 \end{bmatrix} \approx \begin{bmatrix} 3.81 & 11.23 & 2.67 \\ 4.99 & 13.29 & 3.10 \\ 8.85 & 23.32 & 5.29 \\ 1.81 & 4.45 & 0.98 \\ 3.84 & 10.32 & 2.10 \end{bmatrix}$$

(b)
$$B - A = \begin{bmatrix} 3.81 & 11.23 & 2.67 \\ 4.99 & 13.29 & 3.10 \\ 8.85 & 23.32 & 5.29 \\ 1.81 & 4.45 & 0.98 \\ 3.84 & 10.32 & 2.10 \end{bmatrix} - \begin{bmatrix} 3.98 & 11.40 & 2.53 \\ 5.21 & 13.54 & 2.93 \\ 9.00 & 23.33 & 4.85 \\ 1.84 & 4.44 & 0.88 \\ 3.96 & 10.31 & 1.91 \end{bmatrix} = \begin{bmatrix} -0.18 & -0.18 & 0.14 \\ -0.22 & -0.25 & 0.17 \\ -0.15 & -0.001 & 0.44 \\ -0.04 & 0.01 & 0.11 \\ -0.12 & 0.01 & 0.19 \end{bmatrix}$$

(c) The 65+ age group is projected to show relative growth from 2010 to 2013 over all regions because its column in $B - A$ contains all positive percents.

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**84.** $AB = \left[\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]\left[\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline -1 & -2 & -3 & -4 \\ -5 & -6 & -7 & -8 \end{array}\right]$

**86.** (a) True. The number of elements in a row of the first matrix must be equal to the number of elements in a column of the second matrix. See page 43 of the text.

(b) True. See page 45 of the text.

## Section 2.2 Properties of Matrix Operations

**2.** $\begin{bmatrix} 6 & 8 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -11 & -7 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 6 + 0 + (-11) & 8 + 5 + (-7) \\ -1 + (-3) + 2 & 0 + (-1) + (-1) \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ -2 & -2 \end{bmatrix}$

**4.** $\frac{1}{2}\left(\begin{bmatrix} 5 & -2 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 6 & -18 & 9 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 5 + 14 & -2 + 6 & 4 + (-18) & 0 + 9 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 19 & 4 & -14 & 9 \end{bmatrix} = \begin{bmatrix} \frac{19}{2} & 2 & -7 & \frac{9}{2} \end{bmatrix}$

**6.** $-1\begin{bmatrix} 4 & 11 \\ -2 & -1 \\ 9 & 3 \end{bmatrix} + \frac{1}{6}\left(\begin{bmatrix} -5 & -1 \\ 3 & 4 \\ 0 & 13 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ -9 & -1 \\ 6 & -1 \end{bmatrix}\right) = \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} -5 + 7 & -1 + 5 \\ 3 + (-9) & 4 + (-1) \\ 0 + 6 & 13 + (-1) \end{bmatrix}$

$= \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 & 4 \\ -6 & 3 \\ 6 & 12 \end{bmatrix} = \begin{bmatrix} -4 & -11 \\ 2 & 1 \\ -9 & -3 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -1 & \frac{1}{2} \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -4 + \frac{1}{3} & -11 + \frac{2}{3} \\ 2 + (-1) & 1 + \frac{1}{2} \\ -9 + 1 & -3 + 2 \end{bmatrix} = \begin{bmatrix} -\frac{11}{3} & -\frac{31}{3} \\ 1 & \frac{3}{2} \\ -8 & -1 \end{bmatrix}$

**8.** $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

**10.** $(a + b)B = (3 + (-4))\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = (-1)\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$

**12.** $(ab)O = (3)(-4)\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = (-12)\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**14.** (a) $X = 3A - 2B$

$= \begin{bmatrix} -6 & -3 \\ 3 & 0 \\ 9 & -12 \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ 4 & 0 \\ -8 & -2 \end{bmatrix}$

$= \begin{bmatrix} 6 & -9 \\ -1 & 0 \\ 17 & -10 \end{bmatrix}$

(b) $2X = 2A - B$

$2X = \begin{bmatrix} -4 & -2 \\ 2 & 0 \\ 6 & -8 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$

$2X = \begin{bmatrix} -4 & -5 \\ 0 & 0 \\ 10 & -7 \end{bmatrix}$

$X = \begin{bmatrix} -2 & -\frac{5}{2} \\ 0 & 0 \\ 5 & -\frac{7}{2} \end{bmatrix}$

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(c) $2X + 3A = B$

$$2X + \begin{bmatrix} -6 & -3 \\ 3 & 0 \\ 9 & -12 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 6 & 6 \\ -1 & 0 \\ -13 & 11 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 3 \\ -\frac{1}{2} & 0 \\ -\frac{13}{2} & \frac{11}{2} \end{bmatrix}$$

(d) $2A + 4B = -2X$

$$\begin{bmatrix} -4 & -2 \\ 2 & 0 \\ 6 & -8 \end{bmatrix} + \begin{bmatrix} 0 & 12 \\ 8 & 0 \\ -16 & -4 \end{bmatrix} = -2X$$

$$\begin{bmatrix} -4 & 10 \\ 10 & 0 \\ -10 & -12 \end{bmatrix} = -2X$$

$$\begin{bmatrix} 2 & -5 \\ -5 & 0 \\ 5 & 6 \end{bmatrix} = X$$

**16.** $c(CB) = (-2)\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\right)$

$$= (-2)\begin{bmatrix} -1 & 2 \\ -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 \\ 2 & 6 \end{bmatrix}$$

**18.** $C(BC) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\right)$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 3 & -1 \end{bmatrix}$$

**20.** $B(C + O) = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right)$

$$= \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$$

**22.** $B(cA) = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\left((-2)\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}\right)$

$$= \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 & -4 & -6 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -10 & 0 \\ 2 & 0 & 10 \end{bmatrix}$$

**24.** (a) $(AB)C = \left(\begin{bmatrix} -4 & 2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 & -5 & 0 \\ -2 & 3 & 3 \end{bmatrix}\right)\begin{bmatrix} -3 & 4 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -8 & 26 & 6 \\ 7 & -14 & -9 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 0 \\ -12 & 5 \end{bmatrix}$$

(b) $A(BC) = \begin{bmatrix} -4 & 2 \\ 1 & -3 \end{bmatrix}\left(\begin{bmatrix} 1 & -5 & 0 \\ -2 & 3 & 3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}\right)$

$$= \begin{bmatrix} -4 & 2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} -3 & -1 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 0 \\ -12 & 5 \end{bmatrix}$$

**26.** $AB = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{8} \end{bmatrix}$

$$BA = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix} \neq AB$$

**28.** $AC = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 4 \\ 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -6 & 9 \\ 16 & -8 & 12 \\ 4 & -2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4 & -6 & 3 \\ 5 & 4 & 4 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 4 & -2 & 3 \end{bmatrix} = BC$$

But $A \neq B$.

**30.** $AB = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

But $A \neq O$ and $B \neq O$.

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**32.** $AT = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

**34.** $A + IA = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0 & -2 \end{bmatrix}$

**36.** $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

So, $A^4 = \left(A^2\right)^2 = I_2^2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

**38.** In general, $AB \neq BA$ for matrices.

**40.** $D^T = \begin{bmatrix} 6 & -7 & 19 \\ -7 & 0 & 23 \\ 19 & 23 & -32 \end{bmatrix}^T = \begin{bmatrix} 6 & -7 & 19 \\ -7 & 0 & 23 \\ 19 & 23 & -32 \end{bmatrix}$

**42.** $(AB)^T = \left(\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 1 \\ -4 & -2 \end{bmatrix}^T = \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} -3 & -1 \\ 2 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix}$

**44.** $(AB)^T = \left(\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 0 & 1 & 3 \end{bmatrix}\right)^T = \begin{bmatrix} 4 & 0 & -7 \\ 2 & 4 & 7 \\ 4 & 2 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 4 \\ 0 & 4 & 2 \\ -7 & 7 & 2 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 0 & 1 & 3 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 4 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & 4 \\ 1 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 4 \\ 0 & 4 & 2 \\ -7 & 7 & 2 \end{bmatrix}$

**46.** (a) $A^T A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 3 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 11 & 21 \end{bmatrix}$

(b) $AA^T = \begin{bmatrix} 1 & -1 \\ 3 & 4 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 \\ -1 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 25 & -8 \\ 2 & -8 & 4 \end{bmatrix}$

**48.** (a) $A^T A = \begin{bmatrix} 4 & 2 & 14 & 6 \\ -3 & 0 & -2 & 8 \\ 2 & 11 & 12 & -5 \\ 0 & -1 & -9 & 4 \end{bmatrix}\begin{bmatrix} 4 & -3 & 2 & 0 \\ 2 & 0 & 11 & -1 \\ 14 & -2 & 12 & -9 \\ 6 & 8 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 252 & 8 & 168 & -104 \\ 8 & 77 & -70 & 50 \\ 168 & -70 & 294 & -139 \\ -104 & 50 & -139 & 98 \end{bmatrix}$

(b) $AA^T = \begin{bmatrix} 4 & -3 & 2 & 0 \\ 2 & 0 & 11 & -1 \\ 14 & -2 & 12 & -9 \\ 6 & 8 & -5 & 4 \end{bmatrix}\begin{bmatrix} 4 & 2 & 14 & 6 \\ -3 & 0 & -2 & 8 \\ 2 & 11 & 12 & -5 \\ 0 & -1 & -9 & 4 \end{bmatrix} = \begin{bmatrix} 29 & 30 & 86 & -10 \\ 30 & 126 & 169 & -47 \\ 86 & 169 & 425 & -28 \\ -10 & -47 & -28 & 141 \end{bmatrix}$

**50.** $A^{17} = \begin{bmatrix} (1)^{17} & 0 & 0 & 0 & 0 \\ 0 & (-1)^{17} & 0 & 0 & 0 \\ 0 & 0 & (1)^{17} & 0 & 0 \\ 0 & 0 & 0 & (-1)^{17} & 0 \\ 0 & 0 & 0 & 0 & (1)^{17} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

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**52.**
$$A^{20} = \begin{bmatrix} (1)^{20} & 0 & 0 & 0 & 0 \\ 0 & (-1)^{20} & 0 & 0 & 0 \\ 0 & 0 & (1)^{20} & 0 & 0 \\ 0 & 0 & 0 & (-1)^{20} & 0 \\ 0 & 0 & 0 & 0 & (1)^{20} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

**54.** Because $A^3 = \begin{bmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 27 \end{bmatrix} = \begin{bmatrix} 2^3 & 0 & 0 \\ 0 & (-1)^3 & 0 \\ 0 & 0 & (3)^3 \end{bmatrix}$, you have $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

**56.** (a) False. In general, for $n \times n$ matrices $A$ and $B$ it is *not* true that $AB = BA$. For example, let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$.

Then $AB = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = BA$.

(b) False. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$. Then $AB = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = AC$, but $B \neq C$.

(c) True. See Theorem 2.6, part 2 on page 57.

**58.**

| | |
|---|---|
| $aX + A(bB) = b(AB + IB)$ | Original equation |
| $aX + (Ab)B = b(AB + B)$ | Associative property; property of the identity matrix |
| $aX + bAB = bAB + bB$ | Property of scalar multiplication; distributive property |
| $aX + bAB + (-bAB) = bAB + bB + (-bAB)$ | Add $-bAB$ to both sides. |
| $aX = bAB + bB + (-bAB)$ | Additive inverse |
| $aX = bAB + (-bAB) + bB$ | Commutative property |
| $aX = bB$ | Additive inverse |
| $X = \frac{b}{a}B$ | Divide by $a$. |

**60.**
$$f(A) = -10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 5\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix} - 2\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}^2 + \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}^3$$

$$= -\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} + \begin{bmatrix} 10 & 5 & -5 \\ 5 & 0 & 10 \\ -5 & 5 & 15 \end{bmatrix} - 2\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}^2$$

$$= \begin{bmatrix} 0 & 5 & -5 \\ 5 & -10 & 10 \\ -5 & 5 & 5 \end{bmatrix} - 2\begin{bmatrix} 6 & 1 & -3 \\ 0 & 3 & 5 \\ -4 & 2 & 12 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 6 & 1 & -3 \\ 0 & 3 & 5 \\ -4 & 2 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 & -5 \\ 5 & -10 & 10 \\ -5 & 5 & 5 \end{bmatrix} - \begin{bmatrix} 12 & 2 & -6 \\ 0 & 6 & 10 \\ -8 & 4 & 24 \end{bmatrix} + \begin{bmatrix} 16 & 3 & -13 \\ -2 & 5 & 21 \\ -18 & 8 & 44 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & -12 \\ 3 & -11 & 21 \\ -15 & 9 & 25 \end{bmatrix}$$

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**62.** $(cd)A = (cd)[a_{ij}] = [(cd)a_{ij}] = [c(da_{ij})] = c[da_{ij}] = c(dA)$

**64.** $(c + d)A = (c + d)[a_{ij}] = [(c + d)a_{ij}] = [ca_{ij} + da_{ij}] = [ca_{ij}] + [da_{ij}] = c[a_{ij}] + d[a_{ij}] = cA + dA$

**66.** (a) To show that $A(BC) = (AB)C$, compare the $ij$th entries in the matrices on both sides of this equality. Assume that $A$ has size $n \times p$, $B$ has size $p \times r$, and $C$ has size $r \times m$. Then the entry in the $k$th row and the $j$th column of $BC$ is $\sum_{l=1}^{r} b_{kl}c_{lj}$. Therefore, the entry in $i$th row and $j$th column of $A(BC)$ is

$$\sum_{k=1}^{p} a_{ik} \sum_{l=1}^{r} b_{kl}c_{lj} = \sum_{k,l} a_{ik}b_{kl}c_{lj}.$$

The entry in the $i$th row and $j$th column of $(AB)C$ is $\sum_{l=1}^{r} d_{il}c_{lj}$, where $d_{il}$ is the entry of $AB$ in the $i$th row and the $l$th column.

So, $d_{il} = \sum_{k=1}^{p} a_{ik}b_{kl}$ for each $l = 1, \ldots, r$. So, the $ij$th entry of $(AB)C$ is

$$\sum_{i=1}^{r} \sum_{k=1}^{p} a_{ik}b_{kl}c_{lj} = \sum_{k,l} a_{ik}b_{kl}c_{ij}.$$

Because all corresponding entries of $A(BC)$ and $(AB)C$ are equal and both matrices are of the same size $(n \times m)$, you conclude that $A(BC) = (AB)C$.

(b) The entry in the $i$th row and $j$th column of $(A + B)C$ is $(a_{il} + b_{il})c_{1j} + (a_{i2} + b_{i2})c_{2j} + \cdots + (a_{in} + b_{in})c_{nj}$, whereas the entry in the $i$th row and $j$th column of $AC + BC$ is $(a_{i1}c_{1j} + \cdots + a_{in}c_{nj}) + (b_{i1}c_{1j} + \cdots + b_{in}c_{nj})$, which are equal by the distributive law for real numbers.

(c) The entry in the $i$th row and $j$th column of $c(AB)$ is $c[a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}]$. The corresponding entry for $(cA)B$ is $(ca_{i1})b_{1j} + (ca_{i2})b_{2j} + \cdots + (ca_{in})b_{nj}$ and the corresponding entry for $A(cB)$ is $a_{i1}(cb_{1j}) + a_{i2}(cb_{2j}) + \cdots + a_{in}(cb_{nj})$. Because these three expressions are equal, you have shown that $c(AB) = (cA)B = A(cB)$.

**68.** (2) $(A + B)^T = ([a_{ij}] + [b_{ij}])^T = [a_{ij} + b_{ij}]^T = [a_{ji} + b_{ji}] = [a_{ji}] + [b_{ji}] = A^T + B^T$

(3) $(cA)^T = (c[a_{ij}])^T = [ca_{ij}]^T = [ca_{ji}] = c[a_{ji}] = c(A^T)$

(4) The entry in the $i$th row and $j$th column of $(AB)^T$ is $a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots a_{jn}b_{ni}$. On the other hand, the entry in the $i$th row and $j$th column of $B^TA^T$ is $b_{1i}a_{j1} + b_{2i}a_{j2} + \cdots + b_{ni}a_{jn}$, which is the same.

**70.** (a) Answers will vary. *Sample answer*: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

(b) Let $A$ and $B$ be symmetric.

If $AB = BA$, then $(AB)^T = B^TA^T = BA = AB$ and $AB$ is symmetric.

If $(AB)^T = AB$, then $AB = (AB)^T = B^TA^T = BA$ and $AB = BA$.

**72.** Because $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = A^T$, the matrix is symmetric.

**74.** Because $-A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix} = A^T$, the matrix is skew-symmetric.

**76.** If $A^T = -A$ and $B^T = -B$, then $(A + B)^T = A^T + B^T = -A - B = -(A + B)$, which implies that $A + B$ is skew-symmetric.

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**78.** Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$.

$$A - A^T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{21} & a_{31} & \cdots & a_{n1} \\ a_{12} & a_{22} & a_{32} & \cdots & a_{n2} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & a_{3n} & \cdots & a_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a_{12} - a_{21} & a_{13} - a_{31} & \cdots & a_{1n} - a_{n1} \\ a_{21} - a_{12} & 0 & a_{23} - a_{32} & \cdots & a_{2n} - a_{n2} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} - a_{1n} & a_{n2} - a_{2n} & a_{n3} - a_{3n} & \cdots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a_{12} - a_{21} & a_{13} - a_{31} & \cdots & a_{1n} - a_{n1} \\ -(a_{12} - a_{21}) & 0 & a_{23} - a_{32} & \cdots & a_{2n} - a_{n2} \\ \vdots & \vdots & \vdots & & \vdots \\ -(a_{1n} - a_{n1}) & -(a_{2n} - a_{n2}) & -(a_{3n} - a_{n3}) & \cdots & 0 \end{bmatrix}$$

So, $A - A^T$ is skew-symmetric.

## Section 2.3 The Inverse of a Matrix

**2.** $AB = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2-1 & 1-1 \\ -2+2 & -1+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & -2+2 \\ 1-1 & -1+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**4.** $AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**6.** $AB = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 3 & 6 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**8.** Use the formula

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}.$$

So, the inverse is

$$A^{-1} = \frac{1}{2(2) - (-2)(2)}\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}.$$

**10.** Use the formula

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}.$$

So, the inverse is

$$A^{-1} = \frac{1}{(1)(-3) - (-2)(2)}\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}.$$

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**12.** Using the formula

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$$

you see that $ad - bc = (-1)(-3) - (1)(3) = 0$. So, the matrix has no inverse.

**14.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$[I \;\; A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{bmatrix}$$

**16.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} 10 & 5 & -7 & 1 & 0 & 0 \\ -5 & 1 & 4 & 0 & 1 & 0 \\ 3 & 2 & -2 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$[I \;\; A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & -10 & -4 & 27 \\ 0 & 1 & 0 & 2 & 1 & -5 \\ 0 & 0 & 1 & -13 & -5 & 35 \end{bmatrix}$$

Therefore, the inverse is

$$A^{-1} = \begin{bmatrix} -10 & -4 & 27 \\ 2 & 1 & -5 \\ -13 & -5 & 35 \end{bmatrix}.$$

**18.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} 3 & 2 & 5 & 1 & 0 & 0 \\ 2 & 2 & 4 & 0 & 1 & 0 \\ -4 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, you cannot form the identity matrix on the left side. Therefore, the matrix has no inverse.

**20.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} -\frac{5}{6} & \frac{1}{3} & \frac{11}{6} & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 2 & 0 & 1 & 0 \\ 1 & -\frac{1}{2} & -\frac{5}{2} & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, you cannot form the identity matrix on the left side. Therefore, the matrix has no inverse.

**22.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 1 & 0 & 0 \\ -0.3 & 0.2 & 0.2 & 0 & 1 & 0 \\ 0.5 & 0.5 & 0.5 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$[I \;\; A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0.8 \\ 0 & 1 & 0 & -10 & 4 & 4.4 \\ 0 & 0 & 1 & 10 & -2 & -3.2 \end{bmatrix}$$

Therefore, the inverse is

$$A^{-1} = \begin{bmatrix} 0 & -2 & 0.8 \\ -10 & 4 & 4.4 \\ 10 & -2 & -3.2 \end{bmatrix}.$$

**24.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 \\ 2 & 5 & 5 & 0 & 0 & 1 \end{bmatrix}$$

Using elementary row operations, you cannot form the identity matrix on the left side. Therefore, the matrix has no inverse.

**26.** Adjoin the identity matrix to form

$$[A \;\; I] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$[I \;\; A^{-1}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{3} \end{bmatrix}$$

Therefore, the inverse is

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix}.$$

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**28.** Adjoin the identity matrix to form

$$[A\ \ I] = \begin{bmatrix} 4 & 8 & -7 & 14 & 1 & 0 & 0 & 0 \\ 2 & 5 & -4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & -7 & 0 & 0 & 1 & 0 \\ 3 & 6 & -5 & 10 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$[A\ \ I] = \begin{bmatrix} 1 & 0 & 0 & 0 & 27 & -10 & 4 & -29 \\ 0 & 1 & 0 & 0 & -16 & 5 & -2 & 18 \\ 0 & 0 & 1 & 0 & -17 & 4 & -2 & 20 \\ 0 & 0 & 0 & 1 & -7 & 2 & -1 & 8 \end{bmatrix}$$

Therefore the inverse is

$$A^{-1} = \begin{bmatrix} 27 & -10 & 4 & -29 \\ -16 & 5 & -2 & 18 \\ -17 & 4 & -2 & 20 \\ -7 & 2 & -1 & 8 \end{bmatrix}.$$

**30.** Adjoin the identity matrix to form

$$[A\ \ I] = \begin{bmatrix} 1 & 3 & -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$\left[I\ \ A^{-1}\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1.5 & -4 & 2.6 \\ 0 & 1 & 0 & 0 & 0 & 0.5 & 1 & -0.8 \\ 0 & 0 & 1 & 0 & 0 & 0 & -0.5 & 0.1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.2 \end{bmatrix}.$$

Therefore, the inverse is

$$A^{-1} = \begin{bmatrix} 1 & -1.5 & -4 & 2.6 \\ 0 & 0.5 & 1 & -0.8 \\ 0 & 0 & -0.5 & 0.1 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}.$$

**32.** $A = \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$

$ad - bc = (1)(2) - (-2)(-3) = -4$

$$A^{-1} = -\frac{1}{4}\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{4} & -\frac{1}{4} \end{bmatrix}$$

**34.** $A = \begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}$

$ad - bc = (-12)(-2) - 3(5) = 24 - 15 = 9$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} -2 & -3 \\ -5 & -12 \end{bmatrix} = \begin{bmatrix} -\frac{2}{9} & -\frac{1}{3} \\ -\frac{5}{9} & -\frac{4}{3} \end{bmatrix}$$

**36.** $A = \begin{bmatrix} -\frac{1}{4} & \frac{9}{4} \\ \frac{5}{3} & \frac{8}{9} \end{bmatrix}$

$ad - bc = \left(-\frac{1}{4}\right)\left(\frac{8}{9}\right) - \left(\frac{9}{4}\right)\left(\frac{5}{3}\right) = -\frac{143}{36}$

$$A^{-1} = -\frac{36}{143}\begin{bmatrix} \frac{8}{9} & -\frac{9}{4} \\ -\frac{5}{3} & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} -\frac{32}{143} & \frac{81}{143} \\ \frac{60}{143} & \frac{9}{143} \end{bmatrix}$$

**38.** $A^{-2} = \left(A^{-1}\right)^2 = \left(\frac{1}{47}\begin{bmatrix} 6 & -7 \\ 5 & 2 \end{bmatrix}\right)^2 = \frac{1}{2209}\begin{bmatrix} 1 & -56 \\ 40 & -31 \end{bmatrix}$

$$A^{-2} = \left(A^2\right)^{-1} = \begin{bmatrix} -31 & 56 \\ -40 & 1 \end{bmatrix}^{-1} = \frac{1}{2209}\begin{bmatrix} 1 & -56 \\ 40 & -31 \end{bmatrix}$$

The results are equal.

**40.** $A^{-2} = \left(A^{-1}\right)^2 = \left(\frac{1}{2}\begin{bmatrix} -15 & -4 & 28 \\ -1 & 0 & 2 \\ 23 & 6 & -42 \end{bmatrix}\right)^2 = \frac{1}{4}\begin{bmatrix} 873 & 228 & -1604 \\ 61 & 16 & -112 \\ -1317 & -344 & 2420 \end{bmatrix}$

$$A^{-2} = \left(A^2\right)^{-1} = \begin{bmatrix} 48 & 4 & 32 \\ -29 & 48 & -17 \\ 22 & 9 & 15 \end{bmatrix}^{-1} = \frac{1}{4}\begin{bmatrix} 873 & 228 & -1604 \\ 61 & 16 & -112 \\ -1317 & -344 & 2420 \end{bmatrix}$$

The results are equal.

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**42.** (a) $(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} \frac{5}{11} & \frac{2}{11} \\ \frac{3}{11} & -\frac{1}{11} \end{bmatrix} \begin{bmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

$$= \frac{1}{77} \begin{bmatrix} -4 & 9 \\ -9 & 1 \end{bmatrix}$$

(b) $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T = \begin{bmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix}^T = \begin{bmatrix} -\frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{2}{7} \end{bmatrix}$

(c) $(2A)^{-1} = \frac{1}{2}A^{-1} = \frac{1}{2}\begin{bmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{1}{14} \\ \frac{3}{14} & \frac{1}{7} \end{bmatrix}$

**44.** (a) $(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} 6 & 5 & -3 \\ -2 & 4 & -1 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -25 & 24 \\ -6 & 10 & 7 \\ 17 & 7 & 15 \end{bmatrix}$$

(b) $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 4 \\ -4 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

(c) $(2A)^{-1} = \frac{1}{2}A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -4 & 2 \\ 0 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -2 & 1 \\ 0 & \frac{1}{2} & \frac{3}{2} \\ 2 & 1 & \frac{1}{2} \end{bmatrix}$

**46.** The coefficient matrix for each system is

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

and the formula for the inverse of a $2 \times 2$ matrix produces

$$A^{-1} = \frac{1}{2+2}\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

(a) $\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

The solution is: $x = 1$ and $y = 5$.

(b) $\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

The solution is: $x = -1$ and $y = -1$.

**48.** The coefficient matrix for each system is

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & -1 & -1 \end{bmatrix}.$$

Using the algorithm to invert a matrix, you find that the inverse is

$$A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ \frac{2}{3} & \frac{1}{3} & -1 \\ \frac{1}{3} & \frac{2}{3} & -1 \end{bmatrix}.$$

(a) $\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 1 & 1 & -1 \\ \frac{2}{3} & \frac{1}{3} & -1 \\ \frac{1}{3} & \frac{2}{3} & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

The solution is: $x_1 = 1$, $x_2 = 1$, and $x_3 = 1$.

(b) $\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 1 & 1 & -1 \\ \frac{2}{3} & \frac{1}{3} & -1 \\ \frac{1}{3} & \frac{2}{3} & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

The solution is: $x_1 = 1$, $x_2 = 0$, and $x_3 = 1$.

**50.** Using a graphing utility or software program, you have

$A\mathbf{x} = \mathbf{b}$

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 1 & 1 & -1 & 3 & -1 \\ 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 2 & -1 \\ 2 & 1 & 4 & 1 & -1 \\ 3 & 1 & 1 & -2 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 3 \\ 4 \\ 3 \\ -1 \\ 5 \end{bmatrix}.$$

The solution is: $x_1 = 1$, $x_2 = 2$, $x_3 = -1$, $x_4 = 0$, and $x_5 = 1$.

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**52.** Using a graphing utility or software program, you have

$A\mathbf{x} = \mathbf{b}$

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 4 & -2 & 4 & 2 & -5 & -1 \\ 3 & 6 & -5 & -6 & 3 & 3 \\ 2 & -3 & 1 & 3 & -1 & -2 \\ -1 & 4 & -4 & -6 & 2 & 4 \\ 3 & -1 & 5 & 2 & -3 & -5 \\ -2 & 3 & -4 & -6 & 1 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \text{ and}$$

$$\mathbf{b} = \begin{bmatrix} 1 \\ -11 \\ 0 \\ -9 \\ 1 \\ -12 \end{bmatrix}.$$

The solution is: $x_1 = -1$, $x_2 = 2$, $x_3 = 1$, $x_4 = 3$, $x_5 = 0$, and $x_6 = 1$.

**54.** The inverse of $A$ is given by

$$A^{-1} = \frac{1}{x-4}\begin{bmatrix} -2 & -x \\ 1 & 2 \end{bmatrix}.$$

Letting $A^{-1} = A$, you find that $\dfrac{1}{x-4} = -1$.

So, $x = 3$.

**56.** The matrix $\begin{bmatrix} x & 2 \\ -3 & 4 \end{bmatrix}$ will be singular if

$ad - bc = (x)(4) - (-3)(2) = 0$, which implies that

$4x = -6$ or $x = -\frac{3}{2}$.

**58.** First, find $4A$.

$$4A = \left[(4A)^{-1}\right]^{-1} = \frac{1}{4+12}\begin{bmatrix} 2 & -4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{8} & -\frac{1}{4} \\ \frac{3}{16} & \frac{1}{8} \end{bmatrix}$$

Then, multiply by $\frac{1}{4}$ to obtain

$$A = \tfrac{1}{4}(4A) = \tfrac{1}{4}\begin{bmatrix} \frac{1}{8} & -\frac{1}{4} \\ \frac{3}{16} & \frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{32} & -\frac{1}{16} \\ \frac{3}{64} & \frac{1}{32} \end{bmatrix}.$$

**60.** Using the formula for the inverse of a $2 \times 2$ matrix, you have

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{\sec^2\theta - \tan^2\theta}\begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}.$$

**62.** Adjoin the identity matrix to form

$$\begin{bmatrix} F & I \end{bmatrix} = \begin{bmatrix} 0.017 & 0.010 & 0.008 & 1 & 0 & 0 \\ 0.010 & 0.012 & 0.010 & 0 & 1 & 0 \\ 0.008 & 0.010 & 0.017 & 0 & 0 & 1 \end{bmatrix}.$$

Using elementary row operations, reduce the matrix as follows.

$$\begin{bmatrix} I & F^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 115.56 & -100 & 4.44 \\ 0 & 1 & 0 & -100 & 250 & -100 \\ 0 & 0 & 1 & 4.44 & -100 & 115.56 \end{bmatrix}$$

So, $F^{-1} = \begin{bmatrix} 115.56 & -100 & 4.44 \\ -100 & 250 & -100 \\ 4.44 & -100 & 115.56 \end{bmatrix}$ and

$$\mathbf{w} = F^{-1}\mathbf{d} = \begin{bmatrix} 115.56 & -100 & 4.44 \\ -100 & 250 & -100 \\ 4.44 & -100 & 115.56 \end{bmatrix}\begin{bmatrix} 0 \\ 0.15 \\ 0 \end{bmatrix} = \begin{bmatrix} -15 \\ 37.5 \\ -15 \end{bmatrix}.$$

**64.** $A^T\left(A^{-1}\right)^T = \left(A^{-1}A\right)^T = I_n^T = I_n$ and

$\left(A^{-1}\right)^T A^T = \left(AA^{-1}\right)^T = I_n^T = I_n$

So, $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$.

**66.** $(I - 2A)(I - 2A) = I^2 - 2IA - 2AI + 4A^2$

$= I - 4A + 4A^2$

$= I - 4A + 4A \quad (\text{because } A = A^2)$

$= I$

So, $(I - 2A)^{-1} = I - 2A$.

**68.** Because $ABC = I$, $A$ is invertible and $A^{-1} = BC$.

So, $ABC\ A = A$ and $BC\ A = I$.

So, $B^{-1} = CA$.

**70.** Let $A^2 = A$ and suppose $A$ is nonsingular. Then, $A^{-1}$ exists, and you have the following.

$A^{-1}\left(A^2\right) = A^{-1}A$

$\left(A^{-1}A\right)A = I$

$A = I$

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**72.** (a) True. See Theorem 2.8, part 1 on page 67.

(b) False. For example, consider the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, which is not invertible, but $1 \cdot 1 - 0 \cdot 0 = 1 \neq 0$.

(c) False. If $A$ is a square matrix then the system $A\mathbf{x} = \mathbf{b}$ has a unique solution if and only if $A$ is a *nonsingular* matrix.

**74.** $A$ has an inverse if $a_{ii} \neq 0$ for all $i = 1 \ldots n$ and

$$A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 & \ldots & 0 \\ 0 & \frac{1}{a_{22}} & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{a_{nn}} \end{bmatrix}.$$

**76.** $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

(a) $A^2 - 2A + 5I = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $A\left(\frac{1}{5}(2I - A)\right) = \frac{1}{5}(2A - A^2) = \frac{1}{5}(5I) = I$

Similarly, $\left(\frac{1}{5}(2I - A)\right)A = I$. Or, $\frac{1}{5}(2I - A) = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = A^{-1}$ directly.

(c) The calculation in part (b) did not depend on the entries of $A$.

**78.** Let $C$ be the inverse of $(I - AB)$, that is $C = (I - AB)^{-1}$. Then $C(I - AB) = (I - AB)C = I$.

Consider the matrix $I + BCA$. Claim that this matrix is the inverse of $I - BA$. To check this claim, show that $(I + BCA)(I - BA) = (I - BA)(I + BCA) = I$.

First, show $(I - BA)(I + BCA) = I - BA + BCA - BABCA$

$$= I - BA + B(C - ABC)A$$

$$= I - BA + B\underbrace{((I - AB)C)}_{I}A$$

$$= 1 - BA + BA = 1$$

Similarly, show $(I + BCA)(I - BA) = I$.

**80.** Answers will vary. *Sample answer*:

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} \text{ or } A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

**82.** $AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\left(\frac{1}{ad - bc}\right)\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^{-1}A = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad - bc}\begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

## Section 2.4 Elementary Matrices

**2.** This matrix is *not* elementary, because it is not square.

**4.** This matrix is elementary. It can be obtained by interchanging the two rows of $I_2$.

**6.** This matrix is elementary. It can be obtained by multiplying the first row of $I_3$ by 2, and adding the result to the third row.

**8.** This matrix is *not* elementary, because two elementary row operations are required to obtain it from $I_4$.

**10.** $C$ is obtained by adding the third row of $A$ to the first row. So,

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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**12.** $A$ is obtained by adding $-1$ times the third row of $C$ to the first row. So,

$$E = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

**14.** Answers will vary. Sample answer:

| Matrix | Elementary Row Operation | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & 3 & -3 & 6 \\ 0 & 0 & 2 & 2 \end{bmatrix}$ | $R_1 \leftrightarrow R_2$ | $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$ | $\left(\frac{1}{3}\right)R_2 \rightarrow R_2$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ | $\left(\frac{1}{2}\right)R_3 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ |

So, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 & -3 & 6 \\ 1 & -1 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$

**16.** Answers will vary. Sample answer:

| Matrix | Elementary Row Operation | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & -1 \\ 3 & -2 & -4 \end{bmatrix}$ | $(-2)R_1 + R_2 \rightarrow R_2$ | $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & -1 \\ 0 & -11 & -4 \end{bmatrix}$ | $(-3)R_1 + R_3 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & -11 & -4 \end{bmatrix}$ | $(-1)R_2 \rightarrow R_2$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{bmatrix}$ | $(11)R_2 + R_3 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 11 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ | $\left(\frac{1}{7}\right)R_3 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$ |

So, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 11 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 \\ 2 & 5 & -1 \\ 3 & -2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$

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**18.**

| Matrix | Elementary Row Operations | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & -3 & 3 & 9 \\ 0 & 17 & -1 & -3 \\ 4 & 8 & -5 & 1 \end{bmatrix}$ | $R_3 + (-2)R_1 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & -3 & 3 & 9 \\ 0 & 17 & -1 & -3 \\ 0 & 32 & -5 & -7 \end{bmatrix}$ | $R_4 + (-4)R_2 \rightarrow R_4$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 17 & -1 & -3 \\ 0 & 32 & -5 & -7 \end{bmatrix}$ | $\left(-\frac{1}{3}\right)R_2 \rightarrow R_2$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 16 & 48 \\ 0 & 32 & -5 & -7 \end{bmatrix}$ | $R_3 + (-17)R_2 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -17 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 16 & 48 \\ 0 & 0 & 27 & 89 \end{bmatrix}$ | $R_4 + (-32)R_2 \rightarrow R_2$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -32 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 27 & 89 \end{bmatrix}$ | $\left(\frac{1}{16}\right)R_3 \rightarrow R_3$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 8 \end{bmatrix}$ | $R_4 + (-27)R_3 \rightarrow R_4$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -27 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\left(\frac{1}{8}\right)R_4 \rightarrow R_4$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{8} \end{bmatrix}$ |

So,
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{8} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -27 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{16} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -32 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -17 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & -3 & 3 & 9 \\ 2 & 5 & -1 & 1 \\ 4 & 8 & -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -6 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

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**20.** To obtain the inverse matrix, reverse the elementary row operation that produced it. So, multiply the first row by $\frac{1}{25}$ to obtain

$$E^{-1} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix}.$$

**22.** To obtain the inverse matrix, reverse the elementary row operation that produced it. So, add 3 times the second row to the third row to obtain

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}.$$

**24.** To obtain the inverse matrix, reverse the elementary row operation that produced it. So, add $-k$ times the third row to the second row to obtain

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

**26.** Find a sequence of elementary row operations that can be used to rewrite $A$ in reduced row-echelon form.

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \left(\tfrac{1}{2}\right)R_1 \rightarrow R_1 \qquad E_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} R_2 - R_1 \rightarrow R_2 \qquad E_2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

Use the elementary matrices to find the inverse.

$$A^{-1} = E_2E_1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

**28.** Find a sequence of elementary row operations that can be used to rewrite $A$ in reduced row-echelon form.

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \left(\tfrac{1}{2}\right)R_2 \rightarrow R_2 \qquad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} R_1 + 2R_3 \rightarrow R_1 \qquad E_2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} R_2 - \left(\tfrac{1}{2}\right)R_3 \rightarrow R_2 \qquad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Use the elementary matrices to find the inverse.

$$A^{-1} = E_3E_2E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

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**For Exercises 30–36, answers will vary. Sample answers are shown below.**

**30.** The matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is itself an elementary matrix, so the factorization is

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

**32.** Reduce the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ as follows.

| Matrix | Elementary Row Operation | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ | Add $-2$ times row one to row two. | $E_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ | Multiply row two by $-1$. | $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | Add $-1$ times row two to row one. | $E_3 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ |

So, one way to factor $A$ is

$$A = E_1^{-1}E_2^{-1}E_3^{-1} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

**34.** Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$ as follows.

| Matrix | Elementary Row Operation | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$ | Add $-2$ times row one to row two. | $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ | Add $-1$ times row one to row three. | $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | Add $-1$ times row two to row three. | $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | Add $-3$ times row three to row one. | $E_4 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | Add $-2$ times row two to row one. | $E_5 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |

So, one way to factor $A$ is

$$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}E_5^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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**36.** Find a sequence of elementary row operations that can be used to rewrite $A$ in reduced row-echelon form.

$$\begin{bmatrix}1&0&0&\frac{1}{2}\\0&1&0&1\\0&0&-1&2\\1&0&0&-2\end{bmatrix} \quad \left(\tfrac{1}{4}\right)R_1 \rightarrow R_1 \qquad E_1 = \begin{bmatrix}\frac{1}{4}&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&\frac{1}{2}\\0&1&0&1\\0&0&-1&2\\0&0&0&-\frac{5}{2}\end{bmatrix} \quad R_4 - R_1 \rightarrow R_4 \qquad E_2 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\-1&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&\frac{1}{2}\\0&1&0&1\\0&0&-1&2\\0&0&0&1\end{bmatrix} \quad \left(-\tfrac{2}{5}\right)R_4 \rightarrow R_4 \qquad E_3 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-\frac{2}{5}\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&\frac{1}{2}\\0&1&0&1\\0&0&1&-2\\0&0&0&1\end{bmatrix} \quad -R_3 \rightarrow R_3 \qquad E_4 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&-2\\0&0&0&1\end{bmatrix} \quad R_1 - \left(\tfrac{1}{2}\right)R_4 \rightarrow R_1 \qquad E_5 = \begin{bmatrix}1&0&0&-\frac{1}{2}\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&-2\\0&0&0&1\end{bmatrix} \quad R_2 - R_4 \rightarrow R_2 \qquad E_6 = \begin{bmatrix}1&0&0&0\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \quad R_3 + 2R_4 \rightarrow R_3 \qquad E_7 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&2\\0&0&0&1\end{bmatrix}$$

So, one way to factor $A$ is

$$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}E_5^{-1}E_6^{-1}E_7^{-1}$$

$$= \begin{bmatrix}4&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\1&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-\frac{5}{2}\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&\frac{1}{2}\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&-2\\0&0&0&1\end{bmatrix}.$$

**38.** (a) $EA$ has the same rows as $A$ except the two rows that are interchanged in $E$ will be interchanged in $EA$.

(b) Multiplying a matrix on the left by $E$ interchanges the same two rows that are interchanged from $I_n$ in $E$. So, multiplying $E$ by itself interchanges the rows twice and $E^2 = I_n$.

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**40.** $$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -a & 0 \\ -b & 1+ab & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}.$$

**42.** (a) False. It is impossible to obtain the zero matrix by applying any elementary row operation to the identity matrix.

(b) True. If $A = E_1E_2 \ldots E_k$, where each $E_i$ is an elementary matrix, then $A$ is invertible (because every elementary matrix is) and $A^{-1} = E_k^{-1} \ldots E_2^{-1}E_1^{-1}$.

(c) True. See equivalent conditions (2) and (3) of Theorem 2.15.

**44.** Matrix Elementary Matrix

$$\begin{bmatrix} -2 & 1 \\ -6 & 4 \end{bmatrix} = A$$

$$\begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} = U \qquad E_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$E_1A = U \Rightarrow A = E_1^{-1}U = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} = LU$$

**46.** Matrix Elementary Matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 10 & 12 & 3 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 12 & 3 \end{bmatrix} \qquad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 7 \end{bmatrix} = U \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

$$E_2E_1A = U \Rightarrow A = E_1^{-1}E_2^{-1}U$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= LU$$

**48.** Matrix Elementary Matrix

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ -2 & 1 & -1 & 0 \\ 6 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 6 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = U \qquad E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_3E_2E_1A = U \Rightarrow A = E_1^{-1}E_2^{-1}E_3^{-1}U$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$= LU$$

$L\mathbf{y} = \mathbf{b}$:
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ 15 \\ -1 \end{bmatrix}$$

$y_1 = 4, -y_1 + y_2 = -4 \Rightarrow y_2 = 0,$
$3y_1 + 2y_2 + y_3 = 15 \Rightarrow y_3 = 3,$ and $y_4 = -1.$

$U\mathbf{x} = \mathbf{y}$:
$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \\ -1 \end{bmatrix}$$

$x_4 = 1,\ x_3 = 1,\ x_2 - x_3 = 0 \Rightarrow x_2 = 1,$ and $x_1 = 2.$

So, the solution to the system $A\mathbf{x} = \mathbf{b}$ is: $x_1 = 2,$ $x_2 = x_3 = x_4 = 1.$

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**50.** $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \neq A.$

Because $A^2 \neq A$, $A$ is *not* idempotent.

**52.** $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

Because $A^2 \neq A$, $A$ is *not* idempotent.

**54.** Assume $A$ is idempotent. Then

$$A^2 = A$$
$$\left(A^2\right)^T = A^T$$
$$\left(A^T A^T\right) = A^T$$

which means that $A^T$ is idempotent.

Now assume $A^T$ is idempotent. Then

$$A^T A^T = A^T$$
$$\left(A^T A^T\right)^T = \left(A^T\right)^T$$
$$AA = A$$

which means that $A$ is idempotent.

**56.** $$\begin{aligned} (AB)^2 &= (AB)(AB) \\ &= A(BA)B \\ &= A(AB)B \\ &= (AA)(BB) \\ &= AB \end{aligned}$$

So, $(AB)^2 = AB$, and $AB$ is idempotent.

**58.** If $A$ is row-equivalent to $B$, then

$A = E_k \cdots E_2 E_1 B,$

where $E_1, \ldots, E_k$ are elementary matrices.

So,

$B = E_1^{-1} E_2^{-1} \cdots E_k^{-1} A,$

which shows that $B$ is row equivalent to $A$.

**60.** (a) When an elementary row operation is performed on a matrix $A$, perform the same operation on $I$ to obtain the matrix $E$.

(b) Keep track of the row operations used to reduce $A$ to an upper triangular matrix $U$. If $A$ row reduces to $U$ using only the row operation of adding a multiple of one row to another row below it, then the inverse of the product of the elementary matrices is the matrix $L$, and $A = LU$.

(c) For the system $A\mathbf{x} = \mathbf{b}$, find an $LU$ factorization of $A$. Then solve the system $L\mathbf{y} = \mathbf{b}$ for $\mathbf{y}$ and $U\mathbf{x} = \mathbf{y}$ for $\mathbf{x}$.

## Section 2.5 Markov Chains

**2.** The matrix is *not* stochastic because every entry of a stochastic matrix satisfies the inequality $0 \leq a_{ij} \leq 1$.

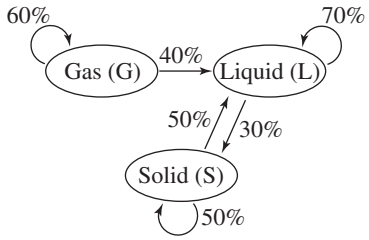
**4.** The matrix is *not* stochastic because the sum of entries in a column of a stochastic matrix is 1.

**6.** The matrix is stochastic because each entry is between 0 and 1, and each column adds up to 1.

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**8.**




The matrix of transition probabilities is shown.

$$P = \begin{array}{c} \overbrace{\phantom{000}}^{\text{From}} \\ \text{G}\quad\text{L}\quad\text{S} \end{array} \begin{bmatrix} 0.60 & 0 & 0 \\ 0.40 & 0.70 & 0.50 \\ 0 & 0.30 & 0.50 \end{bmatrix} \begin{matrix} \text{G} \\ \text{L} \\ \text{S} \end{matrix} \Big\} \text{To}$$

The initial state matrix represents the amounts of the physical states is shown.

$$X_0 = \begin{bmatrix} 0.20(10{,}000) \\ 0.60(10{,}000) \\ 0.20(10{,}000) \end{bmatrix} = \begin{bmatrix} 2000 \\ 6000 \\ 2000 \end{bmatrix}$$

To represent the amount of each physical state after the catalyst is added, multiply $P$ by $X_0$ to obtain

$$PX_0 = \begin{bmatrix} 0.60 & 0 & 0 \\ 0.40 & 0.70 & 0.50 \\ 0 & 0.30 & 0.50 \end{bmatrix} \begin{bmatrix} 2000 \\ 6000 \\ 2000 \end{bmatrix} = \begin{bmatrix} 1200 \\ 6000 \\ 2800 \end{bmatrix}.$$

So, after the catalyst is added there are 1200 molecules in a gas state, 6000 molecules in a liquid state, and 2800 molecules in a solid state.

**10.**

$$X_1 = PX_0 = \begin{bmatrix} 0.6 & 0.2 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{15} \\ \frac{1}{3} \\ \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 0.2\overline{6} \\ 0.\overline{3} \\ 0.4 \end{bmatrix}$$

$$X_2 = PX_1 = \begin{bmatrix} 0.6 & 0.2 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} \frac{4}{15} \\ \frac{1}{3} \\ \frac{3}{5} \end{bmatrix} = \begin{bmatrix} \frac{17}{75} \\ \frac{49}{150} \\ \frac{67}{150} \end{bmatrix} = \begin{bmatrix} 0.226 \\ 0.32\overline{6} \\ 0.44\overline{6} \end{bmatrix}$$

$$X_3 = PX_2 = \begin{bmatrix} 0.6 & 0.2 & 0 \\ 0.2 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} \frac{17}{75} \\ \frac{49}{150} \\ \frac{67}{150} \end{bmatrix} = \begin{bmatrix} \frac{151}{750} \\ \frac{239}{750} \\ \frac{12}{25} \end{bmatrix} = \begin{bmatrix} 0.201\overline{3} \\ 0.318\overline{6} \\ 0.48 \end{bmatrix}$$

**12.** Form the matrix representing the given transition probabilities. $A$ represents infected mice and $B$ noninfected.

$$P = \begin{array}{c} \overbrace{\phantom{00}}^{\text{From}} \\ A\quad B \end{array} \begin{bmatrix} 0.2 & 0.1 \\ 0.8 & 0.9 \end{bmatrix} \begin{matrix} A \\ B \end{matrix} \Big\} \text{To}$$

The state matrix representing the current population is

$$X_0 = \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}.$$

(a) The state matrix for next week is

$$X_1 = PX_0 = \begin{bmatrix} 0.2 & 0.1 \\ 0.8 & 0.9 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix} = \begin{bmatrix} 0.13 \\ 0.87 \end{bmatrix}.$$

So, next week $0.13(1000) = 130$ mice will be infected.

(b) $$X_2 = PX_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.8 & 0.9 \end{bmatrix} \begin{bmatrix} 0.13 \\ 0.87 \end{bmatrix} = \begin{bmatrix} 0.113 \\ 0.887 \end{bmatrix}$$

$$X_3 = PX_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.8 & 0.9 \end{bmatrix} \begin{bmatrix} 0.113 \\ 0.887 \end{bmatrix} = \begin{bmatrix} 0.1113 \\ 0.8887 \end{bmatrix}$$

In 3 weeks, $0.1113(1000) \approx 111$ mice will be infected.

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**14.** Form the matrix representing the given transition probabilities. Let $S$ represent those who swim and $B$ represent those who play basketball.

$$P = \begin{matrix} \overbrace{\phantom{00}}^{\text{From}} \\ S \quad B \end{matrix} \begin{matrix} \\ \begin{bmatrix} 0.30 & 0.40 \\ 0.70 & 0.60 \end{bmatrix} \end{matrix} \begin{matrix} S \\ B \end{matrix} \Big\} \text{To}$$

The state matrix representing the students is

$$X_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} \begin{matrix} S \\ B \end{matrix}.$$

(a) The state matrix for tomorrow is

$$X_1 = PX_0 = \begin{bmatrix} 0.30 & 0.40 \\ 0.70 & 0.60 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.36 \\ 0.64 \end{bmatrix}.$$

So, tomorrow $0.36(250) = 90$ students will swim and $0.64(250) = 160$ students will play basketball.

(b) The state matrix for two days from now is

$$X_2 = P^2X_0 = \begin{bmatrix} 0.37 & 0.36 \\ 0.63 & 0.64 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.364 \\ 0.636 \end{bmatrix}.$$

So, two days from now $0.364(250) = 91$ students will swim and $0.636(250) = 159$ students will play basketball.

(c) The state matrix for four days from now is

$$X_4 = P^4X_0 = \begin{bmatrix} 0.363637 & 0.363637 \\ 0.636363 & 0.636363 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.36364 \\ 0.63636 \end{bmatrix}.$$

So, four days from now, $0.36364(250) \approx 91$ students will swim and $0.63636(250) \approx 159$ students will play basketball.

**16.** Form the matrix representing the given transition probabilities. Let $A$ represent users of Brand A, $B$ users of Brand B, and $N$ users of neither brands.

$$P = \begin{matrix} \overbrace{\phantom{000}}^{\text{From}} \\ A \quad B \quad N \end{matrix} \begin{matrix} \\ \begin{bmatrix} 0.75 & 0.15 & 0.10 \\ 0.20 & 0.75 & 0.15 \\ 0.05 & 0.10 & 0.75 \end{bmatrix} \end{matrix} \begin{matrix} A \\ B \\ N \end{matrix} \Big\} \text{To}$$

The state matrix representing the current product usage is

$$X_0 = \begin{bmatrix} \frac{2}{11} \\ \frac{3}{11} \\ \frac{5}{11} \end{bmatrix} \begin{matrix} A \\ B \\ N \end{matrix}$$

(a) The state matrix for next month is

$$X_1 = P^1X_0 = \begin{bmatrix} 0.75 & 0.15 & 0.10 \\ 0.20 & 0.75 & 0.15 \\ 0.05 & 0.10 & 0.75 \end{bmatrix} \begin{bmatrix} \frac{2}{11} \\ \frac{3}{11} \\ \frac{5}{11} \end{bmatrix} = \begin{bmatrix} 0.222\overline{7} \\ 0.30\overline{9} \\ 0.37\overline{2} \end{bmatrix}.$$

So, next month the distribution of users will be

$0.222\overline{7} \cdot 110{,}000 = 24{,}500$ for Brand A,

$0.30\overline{9} \cdot 110{,}000 = 34{,}000$ for Brand B, and

$0.37\overline{2} \cdot 110{,}000 = 41{,}500$ for neither.

(b) $X_2 = P^2X_0 \approx \begin{bmatrix} 0.2511 \\ 0.3330 \\ 0.325 \end{bmatrix}$

In 2 months, the distribution of users will be

$0.2511 \cdot 110{,}000 = 27{,}625$ for Brand A,

$0.3330 \cdot 110{,}000 = 36{,}625$ for Brand B, and

$0.325 \cdot 110{,}000 = 35{,}750$ for neither.

(c) $X_{18} = P^{18}X_0 \approx \begin{bmatrix} 0.3139 \\ 0.3801 \\ 0.2151 \end{bmatrix}$

In 18 months, the distribution of users will be

$0.3139 \cdot 110{,}000 \approx 34{,}530$ for Brand A,

$0.3801 \cdot 110{,}000 \approx 41{,}808$ for Brand B, and

$0.2151 \cdot 110{,}000 \approx 23{,}662$ for neither.

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**18.** The stochastic matrix

$$P = \begin{bmatrix} 0 & 0.3 \\ 1 & 0.7 \end{bmatrix}$$

is regular because $P^2$ has only positive entries.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} 0 & 0.3 \\ 1 & 0.7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 0.3x_2 &= x_1 \\ x_1 + 0.7x_2 &= x_2 \end{aligned}$$

Because $x_1 + x_2 = 1$, the system of linear equations is as follows.

$$\begin{aligned} -x_1 + 0.3x_2 &= 0 \\ x_1 - 0.3x_2 &= 0 \\ x_1 + x_2 &= 1 \end{aligned}$$

The solution to the system is $x_2 = \frac{10}{13}$ and $x_1 = 1 - \frac{10}{13} = \frac{3}{13}$.

So, $\bar{X} = \begin{bmatrix} \frac{3}{13} \\ \frac{10}{13} \end{bmatrix}$.

**20.** The stochastic matrix

$$P = \begin{bmatrix} 0.2 & 0 \\ 0.8 & 1 \end{bmatrix}$$

is not regular because every power of $P$ has a zero in the second column.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} 0.2 & 0 \\ 0.8 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 0.2x_1 &= x_1 \\ 0.8x_1 + x_2 &= x_2 \end{aligned}$$

Because $x_1 + x_2 = 1$, the system of linear equations is as follows.

$$\begin{aligned} -0.8x_1 &= 0 \\ 0.8x_1 &= 0 \\ x_1 + x_2 &= 1 \end{aligned}$$

The solution of the system is $x_1 = 0$ and $x_2 = 1$.

So, $\bar{X} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

**22.** The stochastic matrix

$$P = \begin{bmatrix} \frac{2}{5} & \frac{7}{10} \\ \frac{3}{5} & \frac{3}{10} \end{bmatrix}$$

is regular because $P^1$ has only positive entries.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} \frac{2}{5} & \frac{7}{10} \\ \frac{3}{5} & \frac{3}{10} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} \tfrac{2}{5}x_1 + \tfrac{7}{10}x_2 &= x_1 \\ \tfrac{3}{5}x_1 + \tfrac{3}{10}x_2 &= x_2 \end{aligned}$$

Because $x_1 + x_2 = 1$, the system of linear equations is as follows.

$$\begin{aligned} -\tfrac{3}{5}x_1 + \tfrac{7}{10}x_2 &= 0 \\ \tfrac{3}{5}x_1 - \tfrac{7}{10}x_2 &= 0 \\ x_1 + x_2 &= 1 \end{aligned}$$

The solution of the system is $x_2 = \frac{6}{13}$ and $x_1 = 1 - \frac{6}{13} = \frac{7}{13}$.

So, $\bar{X} = \begin{bmatrix} \frac{7}{13} \\ \frac{6}{13} \end{bmatrix}$.

**24.** The stochastic matrix

$$P = \begin{bmatrix} \frac{2}{9} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{4}{9} & \frac{1}{4} & \frac{1}{3} \end{bmatrix}$$

is regular because $P^1$ has only positive entries.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} \frac{2}{9} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{4}{9} & \frac{1}{4} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{aligned} \tfrac{2}{9}x_1 + \tfrac{1}{4}x_2 + \tfrac{1}{3}x_3 &= x_1 \\ \tfrac{1}{3}x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{3}x_3 &= x_2 \\ \tfrac{4}{9}x_1 + \tfrac{1}{4}x_2 + \tfrac{1}{3}x_3 &= x_3 \end{aligned}$$

Because $x_1 + x_2 + x_3 = 1$, the system of linear equations is as follows.

$$\begin{aligned} -\tfrac{7}{9}x_1 + \tfrac{1}{4}x_2 + \tfrac{1}{3}x_3 &= 0 \\ \tfrac{1}{3}x_1 - \tfrac{1}{2}x_2 + \tfrac{1}{3}x_3 &= 0 \\ \tfrac{4}{9}x_1 + \tfrac{1}{4}x_2 + \tfrac{2}{3}x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

The solution of the system is $x_3 = 0.33$, $x_2 = 0.4$, and $x_1 = 1 - 0.4 - 0.33 = 0.27$.

So, $\bar{X} = \begin{bmatrix} 0.27 \\ 0.4 \\ 0.33 \end{bmatrix}$.

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**26.** The stochastic matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{5} & 1 \\ \frac{1}{3} & \frac{1}{5} & 0 \\ \frac{1}{6} & \frac{3}{5} & 0 \end{bmatrix}$$

is regular because $P^2$ has only positive entries.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} \frac{1}{2} & \frac{1}{5} & 1 \\ \frac{1}{3} & \frac{1}{5} & 0 \\ \frac{1}{6} & \frac{3}{5} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{aligned} \tfrac{1}{2}x_1 + \tfrac{1}{5}x_2 + x_3 &= x_1 \\ \tfrac{1}{3}x_1 + \tfrac{1}{5}x_2 &= x_2 \\ \tfrac{1}{6}x_1 + \tfrac{3}{5}x_2 &= x_3 \end{aligned}$$

Because $x_1 + x_2 + x_3 = 1$, the system of linear equations is as follows.

$$\begin{aligned} -\tfrac{1}{2}x_1 + \tfrac{1}{5}x_2 + x_3 &= 0 \\ \tfrac{1}{3}x_1 - \tfrac{4}{5}x_2 &= 0 \\ \tfrac{1}{6}x_1 + \tfrac{3}{5}x_2 - x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

The solution of the system is

$x_3 = \frac{5}{22}$, $x_2 = \frac{5}{17} - \frac{5}{17}\left(\frac{5}{22}\right) = \frac{5}{22}$, and

$x_1 = 1 - \frac{5}{22} - \frac{5}{22} = \frac{6}{11}$.

So, $\bar{X} = \begin{bmatrix} \frac{6}{11} \\ \frac{5}{22} \\ \frac{5}{22} \end{bmatrix} \approx \begin{bmatrix} 0.54 \\ 0.22\overline{7} \\ 0.22\overline{7} \end{bmatrix}$.

**28.** The stochastic matrix

$$P = \begin{bmatrix} 0.1 & 0 & 0.3 \\ 0.7 & 1 & 0.3 \\ 0.2 & 0 & 0.4 \end{bmatrix}$$

is not regular because every power of $P$ has two zeros in the second column.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} 0.1 & 0 & 0.3 \\ 0.7 & 1 & 0.3 \\ 0.2 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{aligned} 0.1x_1 + 0.3x_3 &= x_1 \\ 0.7x_1 + x_2 + 0.3x_3 &= x_2 \\ 0.2x_1 + 0.4x_3 &= x_3 \end{aligned}$$

Because $x_1 + x_2 + x_3 = 1$, the system of linear equations is as follows.

$$\begin{aligned} -0.9x_1 + 0.3x_3 &= 0 \\ 0.7x_1 + 0.3x_3 &= 0 \\ 0.2x_1 - 0.6x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

The solution of the system is $x_3 = 0$, $x_2 = 1 - 0 = 1$, and $x_1 = 1 - 1 - 0 = 0$.

So, $\bar{X} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

**30.** The stochastic matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is not regular because every power of $P$ has three zeros in the first column.

$$P\bar{X} = \bar{X} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_1 \\ x_3 &= x_2 \\ x_2 &= x_3 \\ x_4 &= x_4 \end{aligned}$$

Because $x_1 + x_2 + x_3 + x_4 = 1$, the system of linear equations is as follows.

$$\begin{aligned} 0 &= 0 \\ -x_2 + x_3 &= 0 \\ x_2 - x_3 &= 0 \\ 0 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 1 \end{aligned}$$

Let $x_3 = s$ and $x_4 = t$. The solution of the system is $x_4 = t$, $x_3 = s$, $x_2 = s$, and $x_1 = 1 - 2s - t$, where $0 \le s \le 1$, $0 \le t \le 1$, and $2s + t \le 1$.

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**32.** Exercise 3: To find $\bar{X}$, let $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Then use the matrix equation $P\bar{X} = \bar{X}$ to obtain

$$\begin{bmatrix} 0.\overline{3} & 0.1\overline{6} & 0.25 \\ 0.\overline{3} & 0.\overline{6} & 0.25 \\ 0.\overline{3} & 0.1\overline{6} & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

or

$$\begin{aligned} 0.\overline{3}x_1 + 0.1\overline{6}x_2 + 0.25x_3 &= x_1 \\ 0.\overline{3}x_1 + 0.\overline{6}x_2 + 0.25x_3 &= x_2 \\ 0.\overline{3}x_1 + 0.1\overline{6}x_2 + 0.5x_3 &= x_3 \end{aligned}$$

Use these equations and the fact that $x_1 + x_2 + x_3 = 1$ to write the system of linear equations shown.

$$\begin{aligned} -0.\overline{6}x_1 + 0.1\overline{6}x_2 + 0.25x_3 &= 0 \\ 0.\overline{3}x_1 - 0.\overline{3}x_2 + 0.25x_3 &= 0 \\ 0.\overline{3}x_1 + 0.1\overline{6}x_2 + 0.5x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

The solution of the system is

$x_1 = \frac{3}{13}$, $x_2 = \frac{6}{13}$, and $x_3 = \frac{4}{13}$.

So, the steady state matrix is

$$\bar{X} = \begin{bmatrix} \frac{3}{13} \\ \frac{6}{13} \\ \frac{4}{13} \end{bmatrix}.$$

Exercise 5: To find $\bar{X}$, let $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$. Then use the matrix equation $P\bar{X} = \bar{X}$ to obtain

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

or

$$\begin{aligned} x_1 &= x_1 \\ x_2 &= x_2 \\ x_3 &= x_3 \\ x_4 &= x_4 \end{aligned}$$

Use these equations and the fact that $x_1 + x_2 + x_3 + x_4 = 1$ to write the system of linear equations shown.

$x_1 + x_2 + x_3 + x_4 = 1$

Let $x_2 = r$, $x_3 = s$, and $x_4 = t$, where $r$, $s$, and $t$ are real numbers between 0 and 1.

The solution of the system is

$x_1 = 1 - r - s - t$, $x_2 = r$, $x_3 = s$, and $x_4 = t$, where $r$, $s$, and $t$ are real numbers such that $0 \le r \le 1, 0 \le s \le 1, 0 \le t \le 1$, and $r + s + t \le 1$.

So, the steady state matrix is

$$\bar{X} = \begin{bmatrix} 1 - r - s - t \\ r \\ s \\ t \end{bmatrix}.$$

Exercise 6: To find $\bar{X}$, let $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$. Then use the matrix equation $P\bar{X} = \bar{X}$ to obtain

$$\begin{bmatrix} \frac{1}{2} & \frac{2}{9} & \frac{1}{4} & \frac{4}{15} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{4} & \frac{4}{15} \\ \frac{1}{6} & \frac{2}{9} & \frac{1}{4} & \frac{4}{15} \\ \frac{1}{6} & \frac{2}{9} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

or

$$\begin{aligned} \tfrac{1}{2}x_1 + \tfrac{2}{9}x_2 + \tfrac{1}{4}x_3 + \tfrac{4}{15}x_4 &= x_1 \\ \tfrac{1}{6}x_1 + \tfrac{1}{3}x_2 + \tfrac{1}{4}x_3 + \tfrac{4}{15}x_4 &= x_2 \\ \tfrac{1}{6}x_1 + \tfrac{2}{9}x_2 + \tfrac{1}{4}x_3 + \tfrac{4}{15}x_4 &= x_3 \\ \tfrac{1}{6}x_1 + \tfrac{2}{9}x_2 + \tfrac{1}{4}x_3 + \tfrac{1}{5}x_4 &= x_4 \end{aligned}$$

Use these equations and the fact that $x_1 + x_2 + x_3 + x_4 = 1$ to write the system of equations shown.

$$\begin{aligned} -\tfrac{1}{2}x_1 + \tfrac{2}{9}x_2 + \tfrac{1}{4}x_3 + \tfrac{4}{15}x_4 &= 0 \\ \tfrac{1}{6}x_1 - \tfrac{2}{3}x_2 + \tfrac{1}{4}x_3 + \tfrac{4}{15}x_4 &= 0 \\ \tfrac{1}{6}x_1 - \tfrac{2}{9}x_2 - \tfrac{3}{4}x_3 + \tfrac{4}{15}x_4 &= 0 \\ \tfrac{1}{6}x_1 + \tfrac{2}{9}x_2 + \tfrac{1}{4}x_3 - \tfrac{4}{5}x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 1 \end{aligned}$$

The solution of the system is

$x_1 = \frac{24}{73}$, $x_2 = \frac{18}{73}$, $x_3 = \frac{16}{73}$, and $x_4 = \frac{15}{73}$.

So, the steady state matrix is

$$\bar{X} = \begin{bmatrix} \frac{24}{73} \\ \frac{18}{73} \\ \frac{16}{73} \\ \frac{15}{73} \end{bmatrix} \approx \begin{bmatrix} 0.3288 \\ 0.2466 \\ 0.2192 \\ 0.2055 \end{bmatrix}.$$

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**34.** Form the matrix representing the given transition probabilities. Let $A$ represent those who received an "A" and let $N$ represent those who did not.

$$P = \begin{matrix} \overbrace{\phantom{00}}^{\text{From}} \\ A \quad N \end{matrix} \begin{matrix} \\ \begin{bmatrix} 0.70 & 0.10 \\ 0.30 & 0.90 \end{bmatrix} \end{matrix} \begin{matrix} A \\ N \end{matrix} \Big\} \text{To}$$

To find the steady state matrix, solve the equation $P\bar{X} = \bar{X}$, where $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and use the fact that $x_1 + x_2 = 1$ to write a system of equations.

$$\begin{aligned} 0.70x_1 + 0.10x_2 &= x_1 \\ 0.30x_1 + 0.90x_2 &= x_2 \\ x_1 + x_2 &= 1 \end{aligned} \Rightarrow \begin{aligned} -0.3x_1 + 0.1x_2 &= 0 \\ 0.3x_1 - 0.1x_2 &= 0 \\ x_1 + x_2 &= 1 \end{aligned}$$

The solution of the system is $x_1 = \frac{1}{4}$ and $x_2 = \frac{3}{4}$. So, the steady state matrix is $\bar{X} = \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$. This indicates that eventually $\frac{1}{4}$ of the students will receive assignment grades of "A" and $\frac{3}{4}$ of the students will not.

**36.** Form the matrix representing transition probabilities. Let $A$ represent Theatre A, let $B$ represent Theatre B, and let $N$ represent neither theatre.

$$P = \begin{matrix} \overbrace{\phantom{000}}^{\text{From}} \\ A \quad B \quad N \end{matrix} \begin{matrix} \\ \begin{bmatrix} 0.10 & 0.06 & 0.03 \\ 0.05 & 0.08 & 0.04 \\ 0.85 & 0.86 & 0.97 \end{bmatrix} \end{matrix} \begin{matrix} A \\ B \\ B \end{matrix} \Big\} \text{To}$$

To find the steady state matrix, solve the equation $P\bar{X} = \bar{X}$ where $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and use the fact that $x_1 + x_2 + x_3 = 1$ to write a system of equations.

$$\begin{aligned} 0.10x_1 + 0.06x_2 + 0.03x_3 &= x_1 \\ 0.05x_1 + 0.08x_2 + 0.04x_3 &= x_2 \\ 0.85x_1 + 0.86x_2 + 0.97x_3 &= x_3 \\ x_1 + x_2 + x_3 &= 1 \end{aligned} \Rightarrow \begin{aligned} -0.90x_1 + 0.06x_2 + 0.03x_3 &= 0 \\ 0.05x_1 - 0.92x_2 + 0.04x_3 &= 0 \\ 0.85x_1 + 0.86x_2 - 0.03x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

The solution of the system is $x_1 = \frac{4}{119}$, $x_2 = \frac{5}{119}$, and $x_3 = \frac{110}{119}$. So, the steady state matrix is $\bar{X} = \begin{bmatrix} \frac{4}{119} \\ \frac{5}{119} \\ \frac{110}{119} \end{bmatrix}$. This indicates that eventually $\frac{4}{119} \approx 3.4\%$ of the people will attend Theatre A, $\frac{5}{119} \approx 4.2\%$ of the people will attend Theatre B, and $\frac{110}{119} \approx 92.4\%$ of the people will attend neither theatre on any given night.

**38.** The matrix is not absorbing; The first state $S_1$ is absorbing, however the corresponding Markov chain is not absorbing because there is no way to move from $S_2$ or $S_3$ to $S_1$.

**40.** The matrix is absorbing; The fourth state $S_4$ is absorbing and it is possible to move from any of the states to $S_4$ in one transition.

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**42.** Use the matrix equation $P\bar{X} = \bar{X}$, or

$$\begin{bmatrix} 0.1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0.7 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

along with the equation $x_1 + x_2 + x_3 = 1$ to write the linear system

$$\begin{aligned} -0.9x_1 &= 0 \\ 0.2x_1 &= 0 \\ 0.7x_1 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}.$$

The solution of this system is $x_1 = 0$, $x_2 = 1 - t$, and $x_3 = t$, where $t$ is a real number such that $0 \le t \le 1$.

So, the steady state matrix is $\bar{X} = \begin{bmatrix} 0 \\ 1 - t \\ t \end{bmatrix}$, where $0 \le t \le 1$.

**44.** Use the matrix equation $P\bar{X} = \bar{X}$ or

$$\begin{bmatrix} 0.7 & 0 & 0.2 & 0.1 \\ 0.1 & 1 & 0.5 & 0.6 \\ 0 & 0 & 0.2 & 0.2 \\ 0.2 & 0 & 0.1 & 0.1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

along with the equation $x_1 + x_2 + x_3 + x_4 = 1$ to write the linear system

$$\begin{aligned} -0.3x_1 + 0.2x_3 + 0.1x_4 &= 0 \\ 0.1x_1 + 0.5x_3 + 0.6x_4 &= 0 \\ -0.8x_3 + 0.2x_4 &= 0 \\ 0.2x_1 + 0.1x_3 - 0.9x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 1 \end{aligned}.$$

The solution of this system is $x_1 = 0$, $x_2 = 1$, $x_3 = 0$, and $x_4 = 0$. So, the steady state matrix is $\bar{X} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

**46.** Let $S_n$ be the state that Player 1 has $n$ chips.

$$P = \begin{array}{c} \overbrace{\phantom{00000}}^{\text{From}} \\ \begin{matrix} S_0 & S_1 & S_2 & S_3 & S_4 \end{matrix} \\ \begin{bmatrix} 1 & 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0 & 0 \\ 0 & 0.3 & 0 & 0.7 & 0 \\ 0 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 1 \end{bmatrix} \begin{matrix} S_0 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{matrix} \end{array} \text{ To and } X_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

So,

$$P^n X_0 \to \bar{P}X_0 = \begin{bmatrix} \frac{49}{58} \\ 0 \\ 0 \\ 0 \\ \frac{9}{58} \end{bmatrix}.$$

So, the probability that Player 1 reaches $S_4$ and wins the tournament is $\frac{9}{58} \approx 0.155$.

**48.** (a) To find the $n$th state matrix of a Markov chain, compute $X_n = P^n X_0$, where $X_0$ is the initial state matrix.

(b) To find the steady state matrix of a Markov chain, determine the limit of $P^n X_0$, as $n \to \infty$, where $X_0$ is the initial state matrix.

(c) The regular Markov chain is $PX_0, P^2X_0, P^3X_0, \ldots$, where $P$ is a regular stochastic matrix and $X_0$ is the initial state matrix.

(d) An absorbing Markov chain is a Markov chain with at least one absorbing state and it is possible for a member of the population to move from any nonabsorbing state to an absorbing state in a finite number of transitions.

(e) An absorbing Markov chain is concerned with having an entry of 1 and the rest 0 in a column, whereas a regular Markov chain is concerned with the repeated multiplication of the regular stochastic matrix.

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**50.** (a) When the chain reaches $S_1$ or $S_4$, it is certain in the next step to transition to an adjacent state, $S_2$ or $S_3$, respectively, so $S_1$ and $S_4$ reflect to $S_2$ or $S_3$.

(b) $$P = \begin{bmatrix} 0 & 0.4 & 0 & 0 \\ 1 & 0 & 0.3 & 0 \\ 0 & 0.6 & 0 & 1 \\ 0 & 0 & 0.7 & 0 \end{bmatrix}$$

(c) $$P^{30} \approx \begin{bmatrix} \frac{1}{6} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{5}{12} & 0 & \frac{5}{12} \\ \frac{5}{6} & 0 & \frac{5}{6} & 0 \\ 0 & \frac{7}{12} & 0 & \frac{7}{12} \end{bmatrix}$$

$$P^{30} \approx \begin{bmatrix} 0 & \frac{1}{6} & 0 & \frac{1}{6} \\ \frac{5}{6} & 0 & \frac{5}{12} & 0 \\ 0 & \frac{5}{6} & 0 & \frac{5}{6} \\ \frac{7}{12} & 0 & \frac{7}{12} & 0 \end{bmatrix}$$

Other high even or odd powers of $P$ give similar results where the columns alternate.

(d) $$\bar{X} = \begin{bmatrix} \frac{1}{12} \\ \frac{5}{24} \\ \frac{5}{12} \\ \frac{7}{24} \end{bmatrix}$$

Half the sum entries in the corresponding columns of $P^n$ and $P^{n+1}$ approach the corresponding entries in $\bar{X}$.

**52.** (a) Yes, it is possible.

(b) Yes, it is possible.

Both matrices $X$ satisfy $P^1X = X$. The steady state matrix depends on the initial state matrix. In general, the steady state matrix is $\bar{X} = \begin{bmatrix} \frac{6}{11} - t \\ \frac{5}{11} - \frac{5}{6}t \\ \frac{5}{6}t \\ t \end{bmatrix}$, where $t$ is any real number such that $0 \le t \le \frac{6}{11}$. In part (a) $t = 0$ and in part (b), $t = \frac{6}{11}$.

**54.** Let

$$P = \begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix}$$

be a $2 \times 2$ stochastic matrix, and consider the system of equations $PX = X$.

$$\begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

You have

$$\begin{aligned} ax_1 + bx_2 &= x_1 \\ (1-a)x_1 + (1-b)x_2 &= x_2 \end{aligned}$$

or

$$\begin{aligned} (a-1)x_1 + bx_2 &= 0 \\ (1-a)x_1 - bx_2 &= 0. \end{aligned}$$

Letting $x_1 = b$ and $x_2 = 1 - a$, you have the $2 \times 1$ state matrix $X$ satisfying $PX = X$

$$X = \begin{bmatrix} b \\ 1-a \end{bmatrix}.$$

**56.** Let $P$ be a regular stochastic matrix and $X_0$ be the initial state matrix.

$$\begin{aligned} \lim_{n\to\infty} P^nX_0 &= \lim_{n\to\infty} P^n(x_1 + x_2 + \cdots + x_k) \\ &= \lim_{n\to\infty} P^n \cdot x_1 + \lim_{n\to\infty} P^n \cdot x_2 + \cdots + \lim_{n\to\infty} P^n \cdot x_k \\ &= \bar{P}x_1 + \bar{P}x_2 + \cdots + \bar{P}x_k \\ &= \bar{P}(x_1 + x_2 + \cdots + x_k) \\ &= \bar{P}X_0 \\ &= \bar{X}, \text{ where } \bar{X} \text{ is a unique steady state matrix.} \end{aligned}$$

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## Section 2.6 More Applications of Matrix Operations

**2.** Divide the message into groups of four and form the uncoded matrices.

H E L P _ I S _ C O M I N G _ _

$[8 \quad 5 \quad 12 \quad 16] \quad [0 \quad 9 \quad 19 \quad 0] \quad [3 \quad 15 \quad 13 \quad 9] \quad [14 \quad 7 \quad 0 \quad 0]$

Multiplying each uncoded row matrix on the right by $A$ yields the coded row matrices

$$[8 \quad 5 \quad 12 \quad 16]A = [8 \quad 5 \quad 12 \quad 16]\begin{bmatrix} -2 & 3 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 2 \\ 3 & 1 & -2 & -4 \end{bmatrix}$$

$$= [15 \quad 33 \quad -23 \quad -43]$$

$[0 \quad 9 \quad 19 \quad 0]A = [-28 \quad -10 \quad 28 \quad 47]$

$[3 \quad 15 \quad 13 \quad 9]A = [-7 \quad 20 \quad 7 \quad 2]$

$[14 \quad 7 \quad 0 \quad 0]A = [-35 \quad 49 \quad -7 \quad -7].$

So, the coded message is $15, 33, -23, -43, -28, -10, 28, 47, -7, 20, 7, 2, -35, 49, -7, -7$.

**4.** Find $A^{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$, and multiply each coded row matrix on the right by $A^{-1}$ to find the associated uncoded row matrix.

$$[85 \quad 120]\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = [20 \quad 15] \Rightarrow \text{T, } O$$

$[6 \quad 8]A^{-1} = [0 \quad 2] \Rightarrow$ _, B

$[10 \quad 15]A^{-1} = [5 \quad 0] \Rightarrow$ E, _

$[84 \quad 117]A^{-1} = [15 \quad 18] \Rightarrow$ O, R

$[42 \quad 56]A^{-1} = [0 \quad 14] \Rightarrow$ _, N

$[90 \quad 125]A^{-1} = [15 \quad 20] \Rightarrow$ O, T

$[60 \quad 80]A^{-1} = [0 \quad 20] \Rightarrow$ _, T

$[30 \quad 45]A^{-1} = [15 \quad 0] \Rightarrow$ O, _

$[19 \quad 26]A^{-1} = [2 \quad 5] \Rightarrow$ B, E

So, the message is TO_BE_OR_NOT_TO_BE.

**6.** Find $A^{-1} = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix}$, and multiply each coded row matrix on the right by $A^{-1}$ to find the associated uncoded row matrix.

$$[112 \quad -140 \quad 83]A^{-1} = [112 \quad -140 \quad 83]\begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} = [8 \quad 1 \quad 22] \Rightarrow \text{H, A, V}$$

$[19 \quad -25 \quad 13]A^{-1} = [5 \quad 0 \quad 1] \Rightarrow$ E, _, A

$[72 \quad -76 \quad 61]A^{-1} = [0 \quad 7 \quad 18] \Rightarrow$ _, G, R

$[95 \quad -118 \quad 71]A^{-1} = [5 \quad 1 \quad 20] \Rightarrow$ E, A, T

$[20 \quad 21 \quad 38]A^{-1} = [0 \quad 23 \quad 5] \Rightarrow$ _, W, E

$[35 \quad -23 \quad 36]A^{-1} = [5 \quad 11 \quad 5] \Rightarrow$ E, K, E

$[42 \quad -48 \quad 32]A^{-1} = [14 \quad 4 \quad 0] \Rightarrow$ N, D, _

The message is HAVE_A_GREAT_WEEKEND_.

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**8.** Let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and find that

$$\begin{matrix} & & _ & \text{S} \\ [-19 & -19]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = [0 & 19] \\ & & \text{U} & \text{E} \\ [37 & 16]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = [21 & 5]. \end{matrix}$$

This produces a system of 4 equations.

$$\begin{aligned} -19a - 19c &= 0 \\ -19b - 19d &= 19 \\ 37a + 16c &= 21 \\ 37b + 16d &= 5. \end{aligned}$$

Solving this system, you find $a = 1$, $b = 1$, $c = -1$, and $d = -2$. So,

$$A^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}.$$

Multiply each coded row matrix on the right by $A^{-1}$ to yield the uncoded row matrices.

$[3 \quad 1], [14 \quad 3], [5 \quad 12], [0 \quad 15], [18 \quad 4],$

$[5 \quad 18], [19 \quad 0], [0 \quad 19], [21 \quad 5].$

This corresponds to the message

CANCEL_ORDERS_SUE.

**10.** You have

$$[45 \quad -35]\begin{bmatrix} w & x \\ y & z \end{bmatrix} = [10 \quad 15] \text{ and}$$

$$[38 \quad -30]\begin{bmatrix} w & x \\ y & z \end{bmatrix} = [8 \quad 14].$$

So, $45w - 35y = 10$ and $45x - 35z = 15$

$38w - 30y = 8$ $\quad 38x - 30z = 14.$

Solving these two systems gives $w = y = 1$, $x = -2$, and $z = -3$. So,

$$A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & -3 \end{bmatrix}.$$

(b) Decoding, you have:

$$\begin{aligned} [45 \quad -35]A^{-1} &= [10 \quad 15] \Rightarrow \text{J, O} \\ [38 \quad -30]A^{-1} &= [8 \quad 14] \Rightarrow \text{H, N} \\ [18 \quad -18]A^{-1} &= [0 \quad 18] \Rightarrow _\text{, R} \\ [35 \quad -30]A^{-1} &= [5 \quad 20] \Rightarrow \text{E, T} \\ [81 \quad -60]A^{-1} &= [21 \quad 18] \Rightarrow \text{U, R} \\ [42 \quad -28]A^{-1} &= [14 \quad 0] \Rightarrow \text{N, } _ \\ [75 \quad -55]A^{-1} &= [20 \quad 15] \Rightarrow \text{T, O} \\ [2 \quad -2]A^{-1} &= [0 \quad 2] \Rightarrow _\text{, B} \\ [22 \quad -21]A^{-1} &= [1 \quad 19] \Rightarrow \text{A, S} \\ [15 \quad -10]A^{-1} &= [5 \quad 0] \Rightarrow \text{E, } _ \end{aligned}$$

The message is JOHN_RETURN_TO_BASE_.

**12.** Use the given information to find $D$.

$$D = \begin{matrix} \overbrace{\text{A} \quad \text{B}}^{\text{User}} \\ \begin{bmatrix} 0.30 & 0.20 \\ 0.40 & 0.40 \end{bmatrix} \begin{matrix} \text{A} \\ \text{B} \end{matrix} \end{matrix} \Big\} \text{Supplier}$$

The equation $X = DX + E$ may be rewritten in the form $(I - D)X = E$, that is

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.4 & 0.6 \end{bmatrix} X = \begin{bmatrix} 10{,}000 \\ 20{,}000 \end{bmatrix}.$$

Solve this system by using Gauss-Jordan elimination to obtain

$$x \approx \begin{bmatrix} 29{,}412 \\ 52{,}941 \end{bmatrix}.$$

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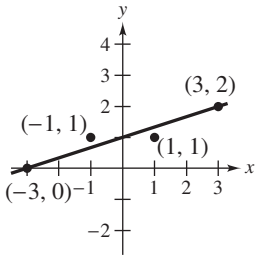
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**14.** From the given matrix $D$, form the linear system $X = DX + E$, which can be written as $(I - D)X = E$, that is

$$\begin{bmatrix} 0.8 & -0.4 & -0.4 \\ -0.4 & 0.8 & -0.2 \\ 0 & -0.2 & 0.8 \end{bmatrix} X = \begin{bmatrix} 5000 \\ 2000 \\ 8000 \end{bmatrix}.$$

Solving this system, $X = \begin{bmatrix} 21{,}875 \\ 17{,}000 \\ 14{,}250 \end{bmatrix}$.

**16.** (a) The line that best fits the given points is shown in the graph.




(b) Using the matrices

$$X = \begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix},$$

you have $X^TX = \begin{bmatrix} 4 & 0 \\ 0 & 20 \end{bmatrix}$, $X^TY = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, and

$$A = (X^TX)^{-1}X^TY = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{20} \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{10} \end{bmatrix}.$$

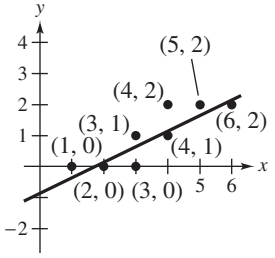
So, the least squares regression line is $y = \frac{3}{10}x + 1$.

(c) Solving $Y = XA + E$ for $E$, you have

$$E = Y - XA = \begin{bmatrix} -0.1 \\ 0.3 \\ -0.3 \\ 0.1 \end{bmatrix}.$$

So, the sum of the squares error is $E^TE = 0.2$.

**18.** (a) The line that best fits the given points is shown in the graph.




(b) Using the matrices

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 8 & 28 \\ 28 & 116 \end{bmatrix}, X^TY = \begin{bmatrix} 8 \\ 37 \end{bmatrix}, \text{ and}$$

$$A = (X^TX)^{-1}(X^TY) = \begin{bmatrix} -\frac{3}{4} \\ \frac{1}{2} \end{bmatrix}.$$

So, the least squares regression line is $y = \frac{1}{2}x - \frac{3}{4}$.

(c) Solving $Y = XA + E$ for $E$, you have

$$E = Y - XA = \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \end{bmatrix}^T$$

and the sum of the squares error is $E^TE = 1.5$.

**20.** Using the matrices

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 9 & 35 \end{bmatrix},$$

$$X^TY = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 9 \\ 39 \end{bmatrix}, \text{ and}$$

$$A = (X^TX)^{-1}(X^TY) = \begin{bmatrix} -\frac{3}{2} \\ \frac{3}{2} \end{bmatrix}.$$

So, the least squares regression line is $y = \frac{3}{2}x - \frac{3}{2}$.

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**22.** Using matrices

$$X = \begin{bmatrix} 1 & -4 \\ 1 & -2 \\ 1 & 2 \\ 1 & 4 \end{bmatrix} \text{ and } Y = \begin{bmatrix} -1 \\ 0 \\ 4 \\ 5 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 4 & 0 \\ 0 & 40 \end{bmatrix}, \quad X^TY = \begin{bmatrix} 8 \\ 32 \end{bmatrix}, \text{ and}$$

$$A = \left(X^TX\right)^{-1}\left(X^TY\right) = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{40} \end{bmatrix}\begin{bmatrix} 8 \\ 32 \end{bmatrix} = \begin{bmatrix} 2 \\ 0.8 \end{bmatrix}.$$

So, the least squares regression line is $y = 0.8x + 2$.

**24.** Using matrices

$$X = \begin{bmatrix} 1 & -3 \\ 1 & -1 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 0 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 4 & 0 \\ 0 & 20 \end{bmatrix}, \quad X^TY = \begin{bmatrix} 7 \\ -13 \end{bmatrix}, \text{ and}$$

$$A = \left(X^TX\right)^{-1}\left(X^TY\right) = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{20} \end{bmatrix}\begin{bmatrix} 7 \\ -13 \end{bmatrix} = \begin{bmatrix} \frac{7}{4} \\ -\frac{13}{20} \end{bmatrix}.$$

So, the least squares regression line is $y = -0.65x + 1.75$.

**26.** Using matrices

$$X = \begin{bmatrix} 1 & 0 \\ 1 & 4 \\ 1 & 5 \\ 1 & 8 \\ 1 & 10 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 6 \\ 3 \\ 0 \\ -4 \\ -5 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 4 & 5 & 8 & 10 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 4 \\ 1 & 5 \\ 1 & 8 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 5 & 27 \\ 27 & 205 \end{bmatrix},$$

$$X^TY = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 4 & 5 & 8 & 10 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 0 \\ -4 \\ -5 \end{bmatrix} = \begin{bmatrix} 0 \\ -70 \end{bmatrix}, \text{ and}$$

$$A = \left(X^TX\right)^{-1}\left(X^TY\right) = \frac{1}{296}\begin{bmatrix} 205 & -27 \\ -27 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ -70 \end{bmatrix}$$

$$= \frac{1}{296}\begin{bmatrix} 1890 \\ -350 \end{bmatrix}.$$

So, the least squares regression line is $y = -\frac{175}{148}x + \frac{945}{148}$.

**28.** Using matrices

$$X = \begin{bmatrix} 1 & 9 \\ 1 & 10 \\ 1 & 11 \\ 1 & 12 \\ 1 & 13 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 0.72 \\ 0.92 \\ 1.17 \\ 1.34 \\ 1.60 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 5 & 55 \\ 55 & 615 \end{bmatrix} \text{ and } X^TY = \begin{bmatrix} 5.75 \\ 65.43 \end{bmatrix}.$$

$$A = \left(X^TX\right)^{-1}X^TY = \begin{bmatrix} -1.248 \\ 0.218 \end{bmatrix}$$

So, the least squares regression line is $y = 0.218x - 1.248$.

**30.** (a) To encode a message, convert the message to numbers and partition it into uncoded row matrices of size $1 \times n$. Then multiply on the right by an invertible $n \times n$ matrix $A$ to obtain coded row matrices. To decode a message, multiply the coded row matrices on the right by $A^{-1}$ and convert the numbers back to letters.

(b) A Leontief input-output model uses an $n \times n$ matrix to represent the input needs of an economic system, and an $n \times 1$ matrix to represent any external demands on the system.

(c) The coefficients of the least squares regression line are given by $A = \left(X^TX\right)^{-1}X^TY$.

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## Review Exercises for Chapter 2

**2.** $$-2\begin{bmatrix}1 & 2\\ 5 & -4\\ 6 & 0\end{bmatrix}+8\begin{bmatrix}7 & 1\\ 1 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}-2 & -4\\ -10 & 8\\ -12 & 0\end{bmatrix}+\begin{bmatrix}56 & 8\\ 8 & 16\\ 8 & 32\end{bmatrix}=\begin{bmatrix}54 & 4\\ -2 & 24\\ -4 & 32\end{bmatrix}$$

**4.** $$\begin{bmatrix}1 & 5\\ 2 & -4\end{bmatrix}\begin{bmatrix}6 & -2 & 8\\ 4 & 0 & 0\end{bmatrix}=\begin{bmatrix}1(6)+5(4) & 1(-2)+5(0) & 1(8)+5(0)\\ 2(6)-4(4) & 2(-2)-4(0) & 2(8)-4(0)\end{bmatrix}=\begin{bmatrix}26 & -2 & 8\\ -4 & -4 & 16\end{bmatrix}$$

**6.** $$\begin{bmatrix}2 & 1\\ 6 & 0\end{bmatrix}\begin{bmatrix}4 & 2\\ -3 & 1\end{bmatrix}+\begin{bmatrix}-2 & 4\\ 0 & 4\end{bmatrix}=\begin{bmatrix}5 & 5\\ 24 & 12\end{bmatrix}+\begin{bmatrix}-2 & 4\\ 0 & 4\end{bmatrix}=\begin{bmatrix}3 & 9\\ 24 & 16\end{bmatrix}$$

**8.** Letting $A=\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$, $\mathbf{x}=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, and $\mathbf{b}=\begin{bmatrix}5\\ -4\end{bmatrix}$, the system can be written as

$$A\mathbf{x}=\mathbf{b}$$

$$\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}5\\ -4\end{bmatrix}.$$

Using Gaussian elimination, the solution of the system is

$$\mathbf{x}=\begin{bmatrix}\frac{6}{7}\\ -\frac{23}{7}\end{bmatrix}.$$

**10.** Letting $A=\begin{bmatrix}2 & 3 & 1\\ 2 & -3 & -3\\ 4 & -2 & 3\end{bmatrix}$, $\mathbf{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$, and $\mathbf{b}=\begin{bmatrix}10\\ 22\\ -2\end{bmatrix}$, the system can be written as

$$A\mathbf{x}=\mathbf{b}$$

$$\begin{bmatrix}2 & 3 & 1\\ 2 & -3 & -3\\ 4 & -2 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}10\\ 22\\ -2\end{bmatrix}.$$

Using Gaussian elimination, the solution of the system is

$$\mathbf{x}=\begin{bmatrix}5\\ 2\\ -6\end{bmatrix}.$$

**12.** $$A^T=\begin{bmatrix}3 & 2\\ -1 & 0\end{bmatrix}$$

$$A^TA=\begin{bmatrix}3 & 2\\ -1 & 0\end{bmatrix}\begin{bmatrix}3 & -1\\ 2 & 0\end{bmatrix}=\begin{bmatrix}13 & -3\\ -3 & 1\end{bmatrix}$$

$$AA^T=\begin{bmatrix}3 & -1\\ 2 & 0\end{bmatrix}\begin{bmatrix}3 & 2\\ -1 & 0\end{bmatrix}=\begin{bmatrix}10 & 6\\ 6 & 4\end{bmatrix}$$

**14.** $$A^T=\begin{bmatrix}1\\ -2\\ -3\end{bmatrix}$$

$$A^TA=\begin{bmatrix}1\\ -2\\ -3\end{bmatrix}\begin{bmatrix}1 & -2 & -3\end{bmatrix}=\begin{bmatrix}1 & -2 & -3\\ -2 & 4 & 6\\ -3 & 6 & 9\end{bmatrix}$$

$$AA^T=\begin{bmatrix}1 & -2 & -3\end{bmatrix}\begin{bmatrix}1\\ -2\\ -3\end{bmatrix}=\begin{bmatrix}14\end{bmatrix}$$

**16.** From the formula

$$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix},$$

you see that $ad-bc=4(2)-(-1)(-8)=0$, and so the matrix has no inverse.

**18.** Begin by adjoining the identity matrix to the given matrix.

$$\begin{bmatrix}A & I\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}$$

This matrix reduces to

$$\begin{bmatrix}I & A^{-1}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 1 & -1 & 0\\ 0 & 1 & 0 & 0 & 1 & -1\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}.$$

So, the inverse matrix is

$$A^{-1}=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}.$$

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**20.** $A \quad \mathbf{x} \quad \mathbf{b}$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Because $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}$, solve the equation $A\mathbf{x} = \mathbf{b}$ as follows.

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

**22.** $A \quad \mathbf{x} \quad \mathbf{b}$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

Using Gauss-Jordan elimination, you find that

$$A^{-1} = \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{3}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}.$$

Solve the equation $A\mathbf{x} = \mathbf{b}$ as follows.

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{3}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

**24.** $A \quad \mathbf{x} \quad \mathbf{b}$

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Because $A^{-1} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}$, solve the equation $A\mathbf{x} = \mathbf{b}$ as follows.

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} \frac{4}{11} & \frac{1}{11} \\ -\frac{3}{11} & \frac{2}{11} \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{18}{11} \\ -\frac{19}{11} \end{bmatrix}$$

**26.** $A \quad \mathbf{x} \quad \mathbf{b}$

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 1 \\ 4 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -7 \end{bmatrix}$$

Using Gauss-Jordan elimination, you find that

$$A^{-1} = \begin{bmatrix} \frac{5}{18} & \frac{1}{9} & \frac{1}{6} \\ -\frac{8}{9} & \frac{4}{9} & -\frac{1}{3} \\ \frac{17}{18} & -\frac{2}{9} & \frac{1}{6} \end{bmatrix}$$

Solve the equation $A\mathbf{x} = \mathbf{b}$ as follows.

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} \frac{5}{18} & \frac{1}{9} & \frac{1}{6} \\ -\frac{8}{9} & \frac{4}{9} & -\frac{1}{3} \\ \frac{17}{18} & -\frac{2}{9} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} 0 \\ -1 \\ -7 \end{bmatrix} = \begin{bmatrix} -\frac{23}{18} \\ \frac{17}{9} \\ -\frac{17}{18} \end{bmatrix}$$

**28.** Because $(2A)^{-1} = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$, you can use the formula for the inverse of a $2 \times 2$ matrix to obtain

$$2A = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}^{-1} = \frac{1}{2-0}\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}.$$

So, $A = \frac{1}{4}\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -1 \\ 0 & \frac{1}{2} \end{bmatrix}$.

**30.** The matrix $\begin{bmatrix} 2 & x \\ 1 & 4 \end{bmatrix}$ will be nonsingular if $ad - bc = (2)(4) - (1)(x) \neq 0$, which implies that $x \neq 8$.

**32.** Because the given matrix represents 6 times the second row, the inverse will be $\frac{1}{6}$ times the second row.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**For Exercises 34 and 36, answers will vary. Sample answers are shown below.**

**34.** Begin by finding a sequence of elementary row operations to write $A$ in reduced row-echelon form.

| Matrix | Elementary Row Operation | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & -4 \\ -3 & 13 \end{bmatrix}$ | Interchange the rows. | $E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$ | Add 3 times row 1 to row 2. | $E_2 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | Add 4 times row 2 to row 1. | $E_3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$ |

Then, you can factor $A$ as follows.

$$A = E_1^{-1}E_2^{-1}E_3^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

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**36.** Begin by finding a sequence of elementary row operations to write $A$ in reduced row-echelon form.

| Matrix | Elementary Row Operation | Elementary Matrix |
|---|---|---|
| $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$ | Multiply row one by $\frac{1}{3}$. | $E_1 = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | Add $-1$ times row one to row three. | $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | Add $-2$ times row three to row one. | $E_3 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | Multiply row two by $\frac{1}{2}$. | $E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |

So, you can factor $A$ as follows.

$$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**38.** Letting $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, you have

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & cb + d^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

So, many answers are possible.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, etc.

**40.** There are many possible answers.

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

But, $BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \neq 0.$

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**42.** Because $(A^{-1} + B^{-1})(A^{-1} + B^{-1}) = I$, if $(A^{-1} + B^{-1})^{-1}$ exists, it is sufficient to show that $(A^{-1} + B^{-1})(A(A + B)^{-1}B) = I$ for equality of the second factors in each equation.

$$\begin{aligned}
(A^{-1} + B^{-1})(A(A + B)^{-1}B) &= A^{-1}(A(A + B)^{-1}B) + B^{-1}(A(A + B)^{-1}B) \\
&= A^{-1}A(A + B)^{-1}B + B^{-1}A(A + B)^{-1}B \\
&= I(A + B)^{-1}B + B^{-1}A(A + B)^{-1}B \\
&= (I + B^{-1}A)((A + B)^{-1}B) \\
&= (B^{-1}B + B^{-1}A)((A + B)^{-1}B) \\
&= B^{-1}(B + A)(A + B)^{-1}B \\
&= B^{-1}(A + B)(A + B)^{-1}B \\
&= B^{-1}IB \\
&= B^{-1}B \\
&= I
\end{aligned}$$

Therefore, $(A^{-1} + B^{-1})^{-1} = A(A + B)^{-1}B$.

**44.** Answers will vary. Sample answer:

| Matrix | Elementary Matrix |
|---|---|
| $\begin{bmatrix} -3 & 1 \\ 12 & 0 \end{bmatrix} = A$ | |
| $\begin{bmatrix} -3 & 1 \\ 0 & 4 \end{bmatrix} = U$ | $E = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}$ |

$EA = U$

$$A = E^{-1}U = \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & 4 \end{bmatrix} = LU$$

**46.**

| Matrix | Elementary Matrix |
|---|---|
| $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = A$ | |
| $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ | $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ | $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ |
| $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = U$ | $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ |

$$E_3E_2E_1A = U \Rightarrow A = E_1^{-1}E_2^{-1}E_3^{-1}U = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = LU$$

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**48.** Matrix Elementary Matrix

$$\begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & -2 & 0 \\ 2 & 1 & 1 & -2 \end{bmatrix} = A$$

$$\begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = U \qquad E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$EA = U \Rightarrow A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = LU$$

$$L\mathbf{y} = \mathbf{b}\text{:} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 2 \\ 8 \end{bmatrix} \Rightarrow \mathbf{y} = \begin{bmatrix} 7 \\ -3 \\ 2 \\ 1 \end{bmatrix}$$

$$U\mathbf{x} = \mathbf{y}\text{:} \begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \mathbf{x} = \begin{bmatrix} 4 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

**50.** $1.1\begin{bmatrix} 100 & 90 & 70 & 30 \\ 40 & 20 & 60 & 60 \end{bmatrix} = \begin{bmatrix} 110 & 99 & 77 & 33 \\ 44 & 22 & 66 & 66 \end{bmatrix}$

**52.** (a) In matrix $B$, grading system 1 counts each midterm as 25% of the grade and the final exam as 50% of the grade.
Grading system 2 counts each midterm as 20% of the grade and the final exam as 60% of the grade.

(b) $$AB = \begin{bmatrix} 78 & 82 & 80 \\ 84 & 88 & 85 \\ 92 & 93 & 90 \\ 88 & 86 & 90 \\ 74 & 78 & 80 \\ 96 & 95 & 98 \end{bmatrix}\begin{bmatrix} 0.25 & 0.20 \\ 0.25 & 0.20 \\ 0.50 & 0.60 \end{bmatrix} = \begin{bmatrix} 80 & 80 \\ 85.5 & 85.4 \\ 91.25 & 91 \\ 88.5 & 88.8 \\ 78 & 78.4 \\ 96.75 & 97 \end{bmatrix}$$

(c) Two students received an "A" in each grading system.

**54.** $$f(A) = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}^3 - 3\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} - \begin{bmatrix} 6 & 3 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**56.** The matrix is not stochastic because the sum of entries in columns 1 and 2 do not add up to 1.

**58.** This matrix is stochastic because each entry is between 0 and 1, and each column adds up to 1.

**60.** $$X_1 = PX_0 = \begin{bmatrix} 0.307 \\ 0.693 \end{bmatrix}$$
$$X_2 = PX_1 = \begin{bmatrix} 0.38246 \\ 0.61754 \end{bmatrix}$$
$$X_3 = PX_2 = \begin{bmatrix} 0.3659 \\ 0.6341 \end{bmatrix}$$

**62.** $$X_1 = PX_0 = \begin{bmatrix} \frac{4}{9} \\ \frac{5}{27} \\ \frac{10}{27} \end{bmatrix} \approx \begin{bmatrix} 0.\overline{4} \\ 0.\overline{185} \\ 0.\overline{370} \end{bmatrix}$$
$$X_2 = PX_1 = \begin{bmatrix} \frac{37}{81} \\ \frac{22}{81} \\ \frac{22}{81} \end{bmatrix} \approx \begin{bmatrix} 0.4568 \\ 0.2716 \\ 0.2716 \end{bmatrix}$$
$$X_3 = PX_2 = \begin{bmatrix} \frac{103}{243} \\ \frac{59}{243} \\ \frac{1}{3} \end{bmatrix} \approx \begin{bmatrix} 0.4239 \\ 0.2428 \\ 0.\overline{3} \end{bmatrix}$$

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**64.** Begin by forming the matrix of transition probabilities.

$$P = \begin{matrix} \overbrace{\phantom{000}}^{\text{From Region}} \\ 1 \quad 2 \quad 3 \end{matrix} \begin{matrix} \left[\begin{matrix} 0.85 & 0.15 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.05 & 0.05 & 0.80 \end{matrix}\right] \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \left.\vphantom{\begin{matrix} 1 \\ 2 \\ 3 \end{matrix}}\right\} \text{To Region}$$

(a) The population in each region after 1 year is given by

$$PX = \begin{bmatrix} 0.85 & 0.15 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.05 & 0.05 & 0.80 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 0.3\overline{6} \\ 0.\overline{3} \\ 0.3 \end{bmatrix}.$$

$$\text{So, } 300{,}000 \begin{bmatrix} 0.3\overline{6} \\ 0.\overline{3} \\ 0.3 \end{bmatrix} = \begin{bmatrix} 110{,}000 \\ 100{,}000 \\ 90{,}000 \end{bmatrix} \begin{matrix} \text{Region 1} \\ \text{Region 2.} \\ \text{Region 3} \end{matrix}$$

(b) The population in each region after 3 years is given by

$$P^3X = \begin{bmatrix} 0.665375 & 0.322375 & 0.2435 \\ 0.219 & 0.562 & 0.219 \\ 0.115625 & 0.115625 & 0.5375 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 0.4104 \\ 0.\overline{3} \\ 0.25625 \end{bmatrix}.$$

$$\text{So, } 300{,}000 \begin{bmatrix} 0.4104 \\ 0.\overline{3} \\ 0.25625 \end{bmatrix} = \begin{bmatrix} 123{,}125 \\ 100{,}000 \\ 76{,}875 \end{bmatrix} \begin{matrix} \text{Region 1} \\ \text{Region 2.} \\ \text{Region 3} \end{matrix}$$

**66.** The stochastic matrix

$$P = \begin{bmatrix} 1 & \frac{4}{7} \\ 0 & \frac{3}{7} \end{bmatrix}$$

is not regular because $P^n$ has a zero in the first column for all powers.

To find $\overline{X}$, begin by letting $\overline{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Then use the matrix equation $P\overline{X} = \overline{X}$ to obtain

$$\begin{bmatrix} 1 & \frac{4}{7} \\ 0 & \frac{3}{7} \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Use these matrices and the fact that $x_1 + x_2 = 1$ to write the system of linear equations shown.

$$\begin{aligned} \tfrac{4}{7}x_2 &= 0 \\ -\tfrac{4}{7}x_2 &= 0 \\ x_1 + x_2 &= 1 \end{aligned}$$

The solution of the system is $x_1 = 1$ and $x_2 = 0$

So, the steady state matrix is $\overline{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

**68.** The stochastic matrix

$$P = \begin{bmatrix} 0 & 0 & 0.2 \\ 0.5 & 0.9 & 0 \\ 0.5 & 0.1 & 0.8 \end{bmatrix}$$

is regular because $P^2$ has only positive entries.

To find $\overline{X}$, let $\overline{X} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$. Then use the matrix equation $P\overline{X} = \overline{X}$ to obtain.

$$\begin{bmatrix} 0 & 0 & 0.2 \\ 0.5 & 0.9 & 0 \\ 0.5 & 0.1 & 0.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Use these matrices and the fact that $x_1 + x_2 + x_3 = 1$ to write the system of linear equations shown.

$$\begin{aligned} -x_1 \qquad + 0.2x_3 &= 0 \\ 0.5x_1 - 0.1x_2 \qquad &= 0 \\ 0.5x_1 + 0.1x_2 - 0.2x_3 &= 0 \\ x_1 + x_2 + x_3 &= 1 \end{aligned}$$

The solution of the system is $x_1 = \frac{1}{11}$, $x_2 = \frac{5}{11}$, and $x_3 = \frac{5}{11}$.

So, the steady state matrix is $\overline{X} = \begin{bmatrix} \frac{1}{11} \\ \frac{5}{11} \\ \frac{5}{11} \end{bmatrix}$.

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**70.** Form the matrix representing the given probabilities. Let $C$ represent the classified documents, $D$ represent the declassified documents, and $S$ represent the shredded documents.

$$P = \begin{matrix} \overbrace{\begin{matrix} C & D & S \end{matrix}}^{\text{From}} \\ \begin{bmatrix} 0.70 & 0.20 & 0 \\ 0.10 & 0.75 & 0 \\ 0.20 & 0.05 & 1 \end{bmatrix} \end{matrix} \begin{matrix} C \\ D \\ S \end{matrix} \Big\} \text{To}$$

Solve the equation $P\bar{X} = \bar{X}$, where $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and use the fact that $x_1 + x_2 + x_3 = 1$ to write a system of equations.

$$\begin{array}{rcl} 0.70x_1 + 0.20x_2 = x_1 \\ 0.10x_1 + 0.75x_2 = x_2 \\ 0.20x_1 + 0.05x_2 + x_3 = x_3 \\ x_1 + x_2 + x_3 = 1 \end{array} \Rightarrow \begin{array}{rcl} -0.3x_1 + 0.2x_2 = 0 \\ 0.1x_1 - 0.25x_2 = 0 \\ 0.2x_1 + 0.05x_2 = 0 \\ x_1 + x_2 + x_3 = 0 \end{array}$$

So, the steady state matrix is $\bar{X} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

This indicates that eventually all of the documents will be shredded.

**72.** The matrix

$$P = \begin{bmatrix} 1 & 0 & 0.38 \\ 0 & 0.30 & 0 \\ 0 & 0.70 & 0.62 \end{bmatrix}$$

is absorbing. The first state $S_1$ is absorbing and it is possible to move from $S_2$ to $S_1$ in two transitions and to move from $S_3$ to $S_1$ in one transition.

**74.** (a) False. See Exercise 65, page 61.

(b) False. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

Then $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

$A + B$ is a *singular* matrix, while both $A$ and $B$ are *nonsingular* matrices.

**76.** (a) True. See Section 2.5, Example 4(b).

(b) False. See Section 2.5, Example 7(a).

**78.** The uncoded row matrices are

| B | E | A | M | _ | M | E | _ | U | P | _ | S | C | O | T | T | Y | _ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [2 | 5 | 1] | [13 | 0 | 13] | [5 | 0 | 21] | [16 | 0 | 19] | [3 | 15 | 20] | [20 | 25 | 0] |

Multiplying each $1 \times 3$ matrix on the right by $A$ yields the coded row matrices.

$$[17 \quad 6 \quad 20] \quad [0 \quad 0 \quad 13] \quad [-32 \quad -16 \quad -43] \quad [-6 \quad -3 \quad 7] \quad [11 \quad -2 \quad -3] \quad [115 \quad 45 \quad 155]$$

So, the coded message is

17, 6, 20, 0, 0, 13, −32, −16, −43, −6, −3, 7, 11, −2, −3, 115, 45, 155.

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**80.** Find $A^{-1}$ to be

$$A^{-1} = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

and the coded row matrices are

$$[11 \quad 52],\ [-8 \quad -9],\ [-13 \quad -39],\ [5 \quad 20],\ [12 \quad 56],\ [5 \quad 20],\ [-2 \quad 7],\ [9 \quad 41],\ [25 \quad 100].$$

Multiplying each coded row matrix on the right by $A^{-1}$ yields the uncoded row matrices.

| S | H | O | W | _ | M | E | _ | T | H | E | _ | M | O | N | E | Y | _ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [19 | 8] | [15 | 23] | [0 | 13] | [5 | 0] | [20 | 8] | [5 | 0] | [13 | 15] | [14 | 5] | [25 | 0] |

So, the message is SHOW_ME_THE_MONEY_.

**82.** Find $A^{-1}$ to be

$$A^{-1} = \begin{bmatrix} \frac{4}{13} & \frac{2}{13} & \frac{1}{13} \\ \frac{8}{13} & -\frac{9}{13} & \frac{2}{13} \\ \frac{5}{13} & -\frac{4}{13} & -\frac{2}{13} \end{bmatrix},$$

and multiply each coded row matrix on the right by $A^{-1}$ to find the associated uncoded row matrix.

$$[66 \quad 27 \quad -31]A^{-1} = [66 \quad 27 \quad -31]\begin{bmatrix} \frac{4}{13} & \frac{2}{13} & \frac{1}{13} \\ \frac{8}{13} & -\frac{9}{13} & \frac{2}{13} \\ \frac{5}{13} & -\frac{4}{13} & -\frac{2}{13} \end{bmatrix} = [25 \quad 1 \quad 14] \Rightarrow \text{Y, A, N}$$

$$[37 \quad 5 \quad -9]A^{-1} = [11 \quad 5 \quad 5] \Rightarrow \text{K, E, E}$$
$$[61 \quad 46 \quad -73]A^{-1} = [19 \quad 0 \quad 23] \Rightarrow \text{S, _, W}$$
$$[46 \quad -14 \quad 9]A^{-1} = [9 \quad 14 \quad 0] \Rightarrow \text{I, N, _}$$
$$[94 \quad 21 \quad -49]A^{-1} = [23 \quad 15 \quad 18] \Rightarrow \text{W, O, R}$$
$$[32 \quad -4 \quad 12]A^{-1} = [12 \quad 4 \quad 0] \Rightarrow \text{L, D, _}$$
$$[66 \quad 31 \quad -53]A^{-1} = [19 \quad 5 \quad 18] \Rightarrow \text{S, E, R}$$
$$[47 \quad 33 \quad -67]A^{-1} = [9 \quad 5 \quad 19] \Rightarrow \text{I, E, S}$$

$$[32 \quad 19 \quad -56]A^{-1} = [0 \quad 9 \quad 14] \Rightarrow \text{_, I, N}$$
$$[43 \quad -9 \quad -20]A^{-1} = [0 \quad 19 \quad 5] \Rightarrow \text{_, S, E}$$
$$[68 \quad 23 \quad -34]A^{-1} = [22 \quad 5 \quad 14] \Rightarrow \text{V, E, N}$$

The message is YANKEES_WIN_WORLD_SERIES_IN_SEVEN.

**84.** Solve the equation $X = DX + E$ for $X$ to obtain $(I - D)X = E$, which corresponds to solving the augmented matrix.

$$\begin{bmatrix} 0.9 & -0.3 & -0.2 & 3000 \\ 0 & 0.8 & -0.3 & 3500 \\ -0.4 & -0.1 & 0.9 & 8500 \end{bmatrix}$$

The solution to this system is

$$X = \begin{bmatrix} 10{,}000 \\ 10{,}000 \\ 15{,}000 \end{bmatrix}.$$

**86.** Using the matrices

$$X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \\ 1 & 6 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \\ 4 \end{bmatrix},$$

you have

$$X^TX = \begin{bmatrix} 5 & 20 \\ 20 & 90 \end{bmatrix},\ X^TY = \begin{bmatrix} 14 \\ 63 \end{bmatrix}, \text{ and}$$

$$A = (X^TX)^{-1}X^TY = \begin{bmatrix} 1.8 & -0.4 \\ -0.4 & 0.1 \end{bmatrix}\begin{bmatrix} 14 \\ 63 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7 \end{bmatrix}.$$

So, the least squares regression line is $y = 0.7x$.

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**88.** Using the matrices

$$X = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 4 \\ 2 \\ 1 \\ -2 \\ -3 \end{bmatrix}, \text{ you have}$$

$$X^TX = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}, \quad X^TY = \begin{bmatrix} 2 \\ -18 \end{bmatrix}, \text{ and } A = \left(X^TX\right)^{-1}X^TY = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{10} \end{bmatrix}\begin{bmatrix} 2 \\ -18 \end{bmatrix} = \begin{bmatrix} 0.4 \\ -1.8 \end{bmatrix}.$$

So, the least squares regression line is $y = -1.8x + 0.4$, or $y = -\frac{9}{5}x + \frac{2}{5}$.

**90.** (a) Using the matrices $X = \begin{bmatrix} 1 & 8 \\ 1 & 9 \\ 1 & 10 \\ 1 & 11 \\ 1 & 12 \\ 1 & 13 \end{bmatrix}$ and $Y = \begin{bmatrix} 2.93 \\ 3.00 \\ 3.01 \\ 3.10 \\ 3.21 \\ 3.39 \end{bmatrix}$, you have

$$X^TX = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 8 & 9 & 10 & 11 & 12 & 13 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 1 & 9 \\ 1 & 10 \\ 1 & 11 \\ 1 & 12 \\ 1 & 13 \end{bmatrix} = \begin{bmatrix} 6 & 63 \\ 63 & 679 \end{bmatrix}$$

and

$$X^TY = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 8 & 9 & 10 & 11 & 12 & 13 \end{bmatrix}\begin{bmatrix} 2.93 \\ 3.00 \\ 3.01 \\ 3.10 \\ 3.21 \\ 3.39 \end{bmatrix} = \begin{bmatrix} 18.64 \\ 197.23 \end{bmatrix}.$$

Now, using $\left(X^TX\right)^{-1}$ to find the coefficient matrix $A$, you have

$$A = \left(X^TX\right)^{-1}X^TY = \begin{bmatrix} \frac{97}{15} & \frac{-3}{5} \\ \frac{-3}{5} & \frac{2}{35} \end{bmatrix}\begin{bmatrix} 18.64 \\ 197.23 \end{bmatrix} \approx \begin{bmatrix} 2.2007 \\ 0.0863 \end{bmatrix}.$$

So, the least squares regression line is $y = 0.0863x + 2.2007$.

(b) Using a graphing utility, the regression line is $y = 0.0863x + 2.2007$.

(c)

| Year | 2008 | 2009 | 2009 | 2010 | 2011 | 2012 | 2013 |
|---|---|---|---|---|---|---|---|
| Actual | 2.93 | 2.93 | 3.00 | 3.01 | 3.10 | 3.21 | 3.39 |
| Estimated | 2.89 | 2.89 | 2.98 | 3.06 | 3.15 | 3.24 | 3.32 |

The estimated values are close to the actual values.

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# Project Solutions for Chapter 2

## 1 Exploring Matrix Multiplication

**1.** Test 1 seems to be the more difficult test. The averages were:

Test 1 average $= 75$

Test 2 average $= 85.5$

**2.** Anna, David, Chris, Bruce

**3.** Answers will vary. Sample answer:

$M\begin{bmatrix}1\\0\end{bmatrix}$ represents scores on the first test.

$M\begin{bmatrix}0\\1\end{bmatrix}$ represents scores on the second test.

**4.** Answers will vary. Sample answer:

$\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}M$ represents Anna's scores.

$\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}M$ represents Chris's scores.

**5.** Answers will vary. Sample answer:

$M\begin{bmatrix}1\\1\end{bmatrix}$ represents the sum of the test scores for each student, and $\frac{1}{2}M\begin{bmatrix}1\\1\end{bmatrix}$ represents each students' average.

**6.** $\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}M$ represents the sum of scores on each test; $\frac{1}{4}\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}M$ represents the average on each test.

**7.** $\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}M\begin{bmatrix}1\\1\end{bmatrix}$ represents the overall points total for all students on all tests.

**8.** $\frac{1}{8}\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}M\begin{bmatrix}1\\1\end{bmatrix} = 80.25$

**9.** $M\begin{bmatrix}1.1\\1.0\end{bmatrix}$

## 2 Nilpotent Matrices

**1.** $A^2 \neq 0$ and $A^3 = 0$, so the index is 3.

**2.** (a) Nilpotent of index 2
(b) Not nilpotent
(c) Nilpotent of index 2
(d) Not nilpotent
(e) Nilpotent of index 2
(f) Nilpotent of index 3

**3.** $\begin{bmatrix}0 & 0 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$ index 2; $\begin{bmatrix}0 & 1 & 1\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}$ index 3

**4.** $\begin{bmatrix}0 & 0 & 0 & 1\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$ index 2; $\begin{bmatrix}0 & 0 & 1 & 1\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$ index 3;

$\begin{bmatrix}0 & 1 & 1 & 1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}$ index 4

**5.** $\begin{bmatrix}0 & 1 & 1 & 1 & 1\\0 & 0 & 1 & 1 & 1\\0 & 0 & 0 & 1 & 1\\0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 0\end{bmatrix}$

**6.** No. If $A$ is nilpotent and invertible, then $A^k = O$ for some $k$ and $A^{k-1} \neq O$. So,

$$A^{-1}A = I \Rightarrow O = A^{-1}A^k = (A^{-1}A)A^{k-1} = IA^{k-1} \neq O,$$

which is impossible.

**7.** If $A$ is nilpotent, then $(A^k)^T = (A^T)^k = O$. But $(A^T)^{k-1} = (A^{k-1})^T \neq O$, which shows that $A^T$ is nilpotent with the same index.

**8.** Let $A$ be nilpotent of index $k$. Then

$$(I - A)(A^{k-1} + A^{k-2} + \cdots + A^2 + A + I) = I - A^k = I,$$

which shows that

$$(A^{k-1} + A^{k-2} + \cdots + A^2 + A + I)$$

is the inverse of $I - A$.

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