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Chapter 2

A Simple Model of Money

Example 2.1 The answer is summarized in the following table

Period	Young Alive	Old Alive	Total Population
1	120	100	220
2	144	120	264

Calculating from the table, we can see that the total population also grows at a net rate of 20% [= (264-220)/220]. In general, we can prove that the total population grows at the rate n:

$$(Total\ Population)_t = N_t + N_{t-1}$$

$$= nN_{t-1} + nN_{t-2}$$

$$= (N_{t-1} + N_{t-2})$$

$$= n(Total\ Population)_{t-1}$$

Exercise 2.1 (a) Feasible set: $100c_{1,t} + 100c_{2,t} \leq 100y = 100(20) \Rightarrow c_{1,t} + c_{2,t} \leq 20$

The graph is easy. The horizontal and vertical intercepts equal 20. Note that until you know more about the preferences, you cannot find exact values for c_1 and c_2 but you can draw a general graph. An example with properly drawn indifference curves is Figure 2A.

(b) First period: $c_{1,t} + v_t m_t \leq y$

Second Period: $c_{2,t+1} \leq v_{t+1} m_t$

Lifetime: $c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq y \Rightarrow c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq 20$

(c) The money market clearing condition is:

$$v_t M_t = N_t\left(y - c_{1,t}\right) \Rightarrow 400v_t = 100\left(y - c_{1,t}\right) \Rightarrow v_t = \frac{100\left(y - c_{1,t}\right)}{400}$$

You could substitute for y here, but this form is good enough for our purposes.

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We want to find $\left[\frac{v_{t+1}}{v_t}\right]$.

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$$\left[\frac{v_{t+1}}{v_t}\right] = \frac{\frac{100(y - c_{1,t+1})}{400}}{\frac{100(y - c_{1,t})}{400}} = 1$$

where the last equality follows from the cancellation and imposing stationarity. ($c_{1,t} = c_{1,t+1}$ *for all* t).

(d) Since the rate of return on fiat money is 1, we find that the real demand for fiat money is:

$$v_t m_t = \frac{y}{1 + \frac{v_t}{v_{t+1}}} = \frac{20}{1+1} = 10$$

Note from the first-period budget constraint that $v_t m_t$ is equal to $(y - c_1)$, so that $(y - c_1) = 10$. Since $v_t m_t = 10, c_1 = y - y - v_t m_t = 20 - 10 = 10$. Half the endowment is consumed and half is sold for real money balances. Using $(y - c_1)$ for the expression derived in part c,

$$v_t = \frac{100(10)}{400} = 2.5 \Rightarrow p_t = \frac{1}{v_t} = 0.4$$

(Question to answer on your own: What will c_2 be?)

(e) We saw in this chapter that the rate of return on fiat money is n in an economy with a constant fiat money stock and a changing population. So an increase in n will cause an increase in the rate of return on fiat money.

An increase in the rate of return on fiat money will increase the real money balances. This should make intuitive sense and is easy to see by plugging a few numbers into the money demand function (*e.g.* suppose that $\frac{v_{t+1}}{v_t}$ increases to 2, re-solve for real money demand).

Given that the real demand for money increases, the money market-clearing condition tells us that the value of money will increase in the initial period. This also should be intuitive—an increase in the demand for apples increases the value of apples. Since the value of money increases in the initial period, the initial old (the initial holders of money) are made better off. (The initial old are better off whenever the real value of money in period 1 increases.)

(f) Following part c, we get, $v_t = \frac{100(10)}{800} = 1.25$. The value of money is cut in half (the price level doubles to $1/v_t = 0.8$. However, the rate of return on fiat money is still one if the population is held constant. Notice that the total real value of the fiat money stock, which is initially held by the initial old, does not change (v_t is cut in half whereas M doubles). This implies that the welfare of the

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initial old does not change. Their holdings of money will not buy any more (or less) of the consumption good.

Exercise 2.2 (a) Since N and M are constant, the rate of return on fiat money in a stationary equilibrium will be one in each country. Intuitively, the economies are identical in the sense of how they change over time. They do not change. Even if each country has a diff erent value of money, v_t, that value does not change over time; therefore, the rate of return on fiat money, $\frac{v_{t+1}}{v_t}$, equals 1.

(b) The value of money in economies A *and* B are, respectively

$$\frac{N(y-c_1^A)}{M} \text{ and } \frac{N(y-c_1^B)}{M}$$

Where c_1^A, c_1^B are first period consumption in economies A and B respectively. The assumption on preferences implies that $c_1^A > c_1^B$ so that $(y - c_1^A) < (y - c_1^B)$.

This, in turn, implies that the value of money in economy A will be lower than the value of money in economy B. Intuitively, the demand for money will be larger in economy B than in economy A. This is because individuals in economy B want to hold relatively more money to finance their higher second-period consumption. Since all else is equal between the two economies (importantly, the supply of money and population), money will have a higher value in economy B than in economy A.

Exercise 2.3 (a) The total amount of the consumption good in period t is $N_t y_1 + N_{t-1} y_2$. This is the total endowment of the economy at time t (young and old). Hence, the feasible set with a stationary allocation is

$$N_t c_1 + N_{t-1} c_2 \leq N_t y_1 + N_{t-1} y_2$$

Dividing through both sides of this equation by N_t,

$$c_1 + \frac{N_{t-1}}{N_t} c_2 \leq y_1 + \frac{N_{t-1}}{N_t} y_2$$

Noting that $N_t = nN_{t-1}$,the feasible set is:

$$c_1 + \left[\frac{1}{n}\right] c_2 \leq y_1 + \left[\frac{1}{n}\right] y_2$$

(b) Plotting the feasible set and superimposing arbitrary indifference curves as in Figure 2B, we find the consumption allocation that maximizes the

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welfare of future generations, (c_1^*, c_2^*).

(c) The constraints facing the individuals are:

$$c_{1,t} + v_t m_t \leq y \text{ and } c_{2,t+1} \leq v_{t+1} m_t + y_2$$

From the first-period constraint, we see that individual real demand for money is $(y_1 - c_{1,t})$. So, aggregate real money demand is $N_t(y_1 - c_{1,t})$. Setting this equal to the total real supply of money,

$$v_t m_t = N_t(y_1 - c_{1,t})$$

(d) From part c, we find that:

$$v_t = \frac{N_t\,(y_1 - c_{1,t}\,)}{M_t} \quad \text{and} \quad v_{t+1} = \frac{N_{t+1}\,(y_1 - c_{1,t+1}\,)}{M_{t+1}}$$

so that the rate of return on fiat money is

$$\left[\frac{v_{t+1}}{v_t}\right] = \frac{\dfrac{N_{t+1}\,(y_1 - c_{1,t+1}\,)}{M_{t+1}}}{\dfrac{N_t\,(y_1 - c_{1,t}\,)}{M_t}} = n$$

due to stationarity and a constant money supply. Work the algebra out completely so you can see this.

(e) To draw the individual's budget set, we need to construct the lifetime budget constraint. Use the first- and second-period constraints found at the beginning of part c. Also impose stationarity. From the second-period constraint at equality, we see that

$$m_2 = \frac{(c_2 - y_2)}{v_{t+1}}$$

Substituting this into the first-period budget constraint, we get

$$c_1 + \frac{v_t}{v_{t+1}}(c_2 - y_2) \leq y_1$$

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$$c_1 + \frac{v_t}{v_{t+1}}(c_2) \leq y_1 + \frac{v_t}{v_{t+1}}(y_2)$$

$$c_1 + \frac{1}{n}(c_2) \leq y_1 + \frac{1}{n}(y_2) \quad \text{(from part d).}$$

Reference to part a shows that this individual budget constraint is the same as the feasible set (the constraint faced by a central planner). This implies that individuals in the monetary equilibrium will choose the same (c_1^*, c_2^*) combination as the one which maximizes the utility of all future generations. This shows that the basic results of Chapter 2 do not change with this alteration in the environment. The monetary equilibrium here can attain the golden rule allocation.

Exercise 2.4 (a) First-period constraint: $c_{1,t} + v_t m_t \leq y_t$ or $c_{1,t} + \frac{y_t}{2} \leq y_t \Rightarrow c_{1,t} \leq \frac{y_t}{2}$.

Second-period constraint: $c_{2,t+1} \leq v_{t+1} m_t$.

Lifetime constraint: $c_{1,t} + \frac{v_t}{v_{t+1}}\left(c_{2,t+1}\right) \leq y_t$

(b) By assumption, individual real money holdings are

$$(y_1 - c_{1,t}) = v_t m_t = \frac{y_t}{2}$$

In such a case, the money market clearing condition becomes:

$$v_t M = N\left(y - c_{1,t}\right) \Rightarrow v_t M = \frac{N y_t}{2} \Rightarrow v_t = \frac{N y_t}{2M}$$

The rate of return of fiat money will be:

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N y_{t+1}}{2M}}{\frac{N y_t}{2M}} = \frac{y_{t+1}}{y_t} = \frac{\alpha y_t}{y_t} = \alpha$$

Note the similarity of this case to that found in Chapter 2. In that chapter, we modeled growth in the economy by growth in the number of young people born each period $N_t = nN_{t-1}$ We found that in that case, the rate of return of fiat money equal

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to, the growth rate of the economy. In this example, α is the growth rate of the economy (it is the gross rate of change of the total endowment). We discover that even in this more complicated setup, the rate of return of fiat money is equal to the growth rate of the economy when the money supply is fixed.

Exercise 2.5

N_ty: Total amount of goods available for allocation at time 't'.

Stationary allocation implies: $c_{1,t} = c_1$ for all t and $c_{2,t} = c_2$ for all t

Therefore the feasible set with a stationary allocation may be written as:

$$N_t c_1 + N_{t-1} c_2 \leq N_t y$$

Dividing this above equation throughout by N_t,

$$c_1 + \frac{N_{t-1}}{N_t} c_2 \leq y$$

With population growing at the rate of 'n', $N_t = nN_{t-1}$

Plugging in this value of N_t in the feasibility constraint, we obtain the feasibility constraint for a stationary allocation with a growing population as follows:

$$c_1 + \frac{N_{t-1}}{nN_{t-1}} c_2 \leq y$$

$$=> c_1 + \frac{c_2}{n} \leq y$$

To find the horizontal intercept, set $c_2 = 0$ in the above equation. This yields:

$$c_1 = y$$

To find the vertical intercept, set $c_1 = 0$ in the above equation. This yields:

$$c_2 = ny$$

Exercise 2.6 (a) $v_t m_t$: This is the aggregate real value of the number of dollars acquired by an individual measured in terms of goods. So, the unit of measure is a good.

(b) M_t: This is the nominal stock of money. This measure is in terms of the number of units of fiat money i.e. number of dollars, so the unit of measure is a dollar.

(c) v_t: This is the value of a unit of fiat money in terms of the good. (The inverse of this measure is the number of dollars that one needs to pay for each unit of the good). So the unit of measure is a good.

(d) p_t: This is the dollar price of a consumption good. (The inverse of this measure is the price in terms of the number of goods that need to be given up to get one dollar). So this measure is in terms of dollars.

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Appendix Exercise 2.1 (a) Follow equations (1.33) to (1.35) of the text. Using the notation of the appendix of Chapter 1, utility is

$$\ln(y - q_t) + \beta \ln(\frac{v_{t+1}}{v_t}[q_t])$$

Differentiating this expression with respect to q_t and setting the result equal to zero:

$$\frac{-1}{y - q_t^*} + \beta \frac{\frac{v_{t+1}}{v_t}}{\frac{v_{t+1}}{v_t}[q_t^*]} = 0$$

Canceling terms and rearranging, we get

$$\frac{\beta}{q_t^*} = \frac{1}{y - q_t^*} \Rightarrow \beta(y - q_t^*) = q_t^* \Rightarrow q_t^* = \frac{\beta y}{1 + \beta}$$

(b) $$c_{1,t}^* = y - q_t^* = \frac{y}{1+\beta}, c_{2,t+1}^* = \frac{v_{t+1}}{v_t}[q_t^*] = \frac{v_{t+1}}{v_t}\left[\frac{\beta y}{1+\beta}\right]$$

(c) The analytical way to solve this is to differentiate the expressions for $q_t^*, c_{1,t}^* \text{ and } c_{2,t+1}^*$ with respect to β and sign the results. (Try this on your own, taking money rate of return as given)

However a little thought can save us this hassle. It is easy to see that as β increases, $c_{1,t}^*$ falls. A smaller value for first period consumption more of the endowment to trade for money so money holdings rise. Furthermore, an increase in q_t^* will then imply larger second period consumption.

An increase in β increases the "weight" put on second-period consumption in the expression for utility. Intuitively, an individual with a large β places a larger importance on second-period consumption than someone with a small β. And, as we see, a "large" β will cause first-period consumption to be reduced. Money holdings will be "large" to finance a "large" among of second period consumption.

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