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1 □ FUNCTIONS AND LIMITS

1.1 Four Ways to Represent a Function

1. The functions $f(x) = x + \sqrt{2 - x}$ and $g(u) = u + \sqrt{2 - u}$ give exactly the same output values for every input value, so f and g are equal.

2. $f(x) = \dfrac{x^2 - x}{x - 1} = \dfrac{x(x-1)}{x-1} = x$ for $x - 1 \neq 0$, so f and g [where $g(x) = x$] are not equal because $f(1)$ is undefined and $g(1) = 1$.

3. (a) The point $(-2, 2)$ lies on the graph of g, so $g(-2) = 2$. Similarly, $g(0) = -2$, $g(2) = 1$, and $g(3) \approx 2.5$.

(b) Only the point $(-4, 3)$ on the graph has a y-value of 3, so the only value of x for which $g(x) = 3$ is -4.

(c) The function outputs $g(x)$ are never greater than 3, so $g(x) \leq 3$ for the entire domain of the function. Thus, $g(x) \leq 3$ for $-4 \leq x \leq 4$ (or, equivalently, on the interval $[-4, 4]$).

(d) The domain consists of all x-values on the graph of g: $\{x \mid -4 \leq x \leq 4\} = [-4, 4]$. The range of g consists of all the y-values on the graph of g: $\{y \mid -2 \leq y \leq 3\} = [-2, 3]$.

(e) For any $x_1 < x_2$ in the interval $[0, 2]$, we have $g(x_1) < g(x_2)$. [The graph rises from $(0, -2)$ to $(2, 1)$.] Thus, $g(x)$ is increasing on $[0, 2]$.

4. (a) From the graph, we have $f(-4) = -2$ and $g(3) = 4$.

(b) Since $f(-3) = -1$ and $g(-3) = 2$, or by observing that the graph of g is above the graph of f at $x = -3$, $g(-3)$ is larger than $f(-3)$.

(c) The graphs of f and g intersect at $x = -2$ and $x = 2$, so $f(x) = g(x)$ at these two values of x.

(d) The graph of f lies below or on the graph of g for $-4 \leq x \leq -2$ and for $2 \leq x \leq 3$. Thus, the intervals on which $f(x) \leq g(x)$ are $[-4, -2]$ and $[2, 3]$.

(e) $f(x) = -1$ is equivalent to $y = -1$, and the points on the graph of f with y-values of -1 are $(-3, -1)$ and $(4, -1)$, so the solution of the equation $f(x) = -1$ is $x = -3$ or $x = 4$.

(f) For any $x_1 < x_2$ in the interval $[-4, 0]$, we have $g(x_1) > g(x_2)$. Thus, $g(x)$ is decreasing on $[-4, 0]$.

(g) The domain of f is $\{x \mid -4 \leq x \leq 4\} = [-4, 4]$. The range of f is $\{y \mid -2 \leq y \leq 3\} = [-2, 3]$.

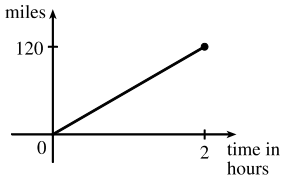
(h) The domain of g is $\{x \mid -4 \leq x \leq 3\} = [-4, 3]$. Estimating the lowest point of the graph of g as having coordinates $(0, 0.5)$, the range of g is approximately $\{y \mid 0.5 \leq y \leq 4\} = [0.5, 4]$.

5. From Figure 1 in the text, the lowest point occurs at about $(t, a) = (12, -85)$. The highest point occurs at about $(17, 115)$. Thus, the range of the vertical ground acceleration is $-85 \leq a \leq 115$. Written in interval notation, the range is $[-85, 115]$.

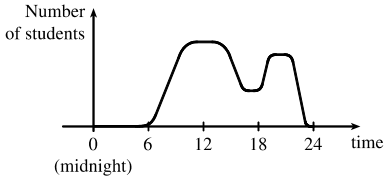
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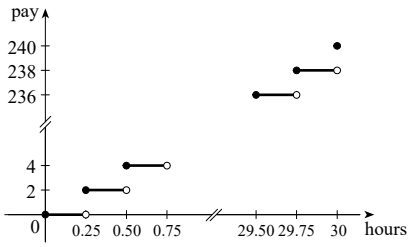
6. *Example 1:* A car is driven at 60 mi/h for 2 hours. The distance d traveled by the car is a function of the time t. The domain of the function is $\{t \mid 0 \le t \le 2\}$, where t is measured in hours. The range of the function is $\{d \mid 0 \le d \le 120\}$, where d is measured in miles.



Example 2: At a certain university, the number of students N on campus at any time on a particular day is a function of the time t after midnight. The domain of the function is $\{t \mid 0 \le t \le 24\}$, where t is measured in hours. The range of the function is $\{N \mid 0 \le N \le k\}$, where N is an integer and k is the largest number of students on campus at once.



Example 3: A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay P is a function of the number of hours worked h. The domain of the function is $[0, 30]$ and the range of the function is $\{0, 2.00, 4.00, \ldots, 238.00, 240.00\}$.



7. We solve $3x - 5y = 7$ for y: $3x - 5y = 7 \quad \Leftrightarrow \quad -5y = -3x + 7 \quad \Leftrightarrow \quad y = \frac{3}{5}x - \frac{7}{5}$. Since the equation determines exactly one value of y for each value of x, the equation defines y as a function of x.

8. We solve $3x^2 - 2y = 5$ for y: $3x^2 - 2y = 5 \quad \Leftrightarrow \quad -2y = -3x^2 + 5 \quad \Leftrightarrow \quad y = \frac{3}{2}x^2 - \frac{5}{2}$. Since the equation determines exactly one value of y for each value of x, the equation defines y as a function of x.

9. We solve $x^2 + (y-3)^2 = 5$ for y: $x^2 + (y-3)^2 = 5 \quad \Leftrightarrow \quad (y-3)^2 = 5 - x^2 \quad \Leftrightarrow \quad y - 3 = \pm\sqrt{5 - x^2} \quad \Leftrightarrow$ $y = 3 \pm \sqrt{5 - x^2}$. Some input values x correspond to more than one output y. (For instance, $x = 1$ corresponds to $y = 1$ and to $y = 5$.) Thus, the equation does *not* define y as a function of x.

10. We solve $2xy + 5y^2 = 4$ for y: $2xy + 5y^2 = 4 \quad \Leftrightarrow \quad 5y^2 + (2x)\,y - 4 = 0 \quad \Leftrightarrow$

$y = \dfrac{-2x \pm \sqrt{(2x)^2 - 4(5)(-4)}}{2(5)} = \dfrac{-2x \pm \sqrt{4x^2 + 80}}{10} = \dfrac{-x \pm \sqrt{x^2 + 20}}{5}$ (using the quadratic formula). Some input values x correspond to more than one output y. (For instance, $x = 4$ corresponds to $y = -2$ and to $y = 2/5$.) Thus, the equation does *not* define y as a function of x.

11. We solve $(y+3)^3 + 1 = 2x$ for y: $(y+3)^3 + 1 = 2x \quad \Leftrightarrow \quad (y+3)^3 = 2x - 1 \quad \Leftrightarrow \quad y + 3 = \sqrt[3]{2x - 1} \quad \Leftrightarrow$ $y = -3 + \sqrt[3]{2x - 1}$. Since the equation determines exactly one value of y for each value of x, the equation defines y as a function of x.

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12. We solve $2x - |y| = 0$ for y: $2x - |y| = 0 \quad \Leftrightarrow \quad |y| = 2x \quad \Leftrightarrow \quad y = \pm 2x$. Some input values x correspond to more than one output y. (For instance, $x = 1$ corresponds to $y = -2$ and to $y = 2$.) Thus, the equation does *not* define y as a function of x.

13. The height 60 in $(x = 60)$ corresponds to shoe sizes 7 and 8 ($y = 7$ and $y = 8$). Since an input value x corresponds to more than output value y, the table does *not* define y as a function of x.

14. Each year x corresponds to exactly one tuition cost y. Thus, the table defines y as a function of x.

15. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.

16. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-2, 2]$ and the range is $[-1, 2]$.

17. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-3, -2) \cup [-1, 3]$.

18. No, the curve is not the graph of a function since for $x = 0, \pm 1$, and ± 2, there are infinitely many points on the curve.

19. (a) When $t = 1950$, $T \approx 13.8^\circ\text{C}$, so the global average temperature in 1950 was about 13.8°C.

(b) When $T = 14.2^\circ\text{C}$, $t \approx 1990$.

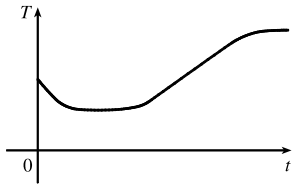
(c) The global average temperature was smallest in 1910 (the year corresponding to the lowest point on the graph) and largest in 2000 (the year corresponding to the highest point on the graph).

(d) When $t = 1910$, $T \approx 13.5^\circ\text{C}$, and when $t = 2000$, $T \approx 14.4^\circ\text{C}$. Thus, the range of T is about $[13.5, 14.4]$.

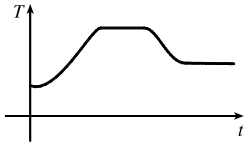
20. (a) The ring width varies from near 0 mm to about 1.6 mm, so the range of the ring width function is approximately $[0, 1.6]$.

(b) According to the graph, the earth gradually cooled from 1550 to 1700, warmed into the late 1700s, cooled again into the late 1800s, and has been steadily warming since then. In the mid-19th century, there was variation that could have been associated with volcanic eruptions.

21. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



22. The temperature of the pie would increase rapidly, level off to oven temperature, decrease rapidly, and then level off to room temperature.



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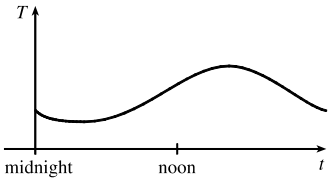
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23. (a) The power consumption at 6 AM is 500 MW, which is obtained by reading the value of power P when $t = 6$ from the graph. At 6 PM we read the value of P when $t = 18$, obtaining approximately 730 MW.

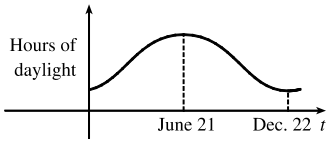
(b) The minimum power consumption is determined by finding the time for the lowest point on the graph, $t = 4$, or 4 AM. The maximum power consumption corresponds to the highest point on the graph, which occurs just before $t = 12$, or right before noon. These times are reasonable, considering the power consumption schedules of most individuals and businesses.

24. Runner A won the race, reaching the finish line at 100 meters in about 15 seconds, followed by runner B with a time of about 19 seconds, and then by runner C who finished in around 23 seconds. B initially led the race, followed by C, and then A. C then passed B to lead for a while. Then A passed first B, and then passed C to take the lead and finish first. Finally, B passed C to finish in second place. All three runners completed the race.

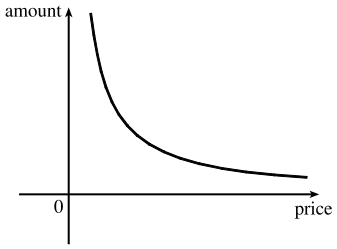
25. Of course, this graph depends strongly on the geographical location!



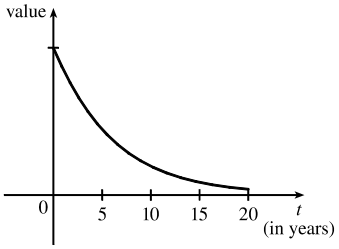
26. The summer solstice (the longest day of the year) is around June 21, and the winter solstice (the shortest day) is around December 22. (Exchange the dates for the southern hemisphere.)



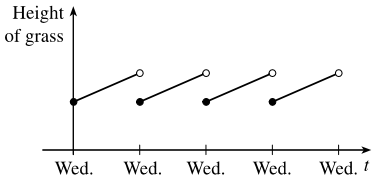
27. As the price increases, the amount sold decreases.



28. The value of the car decreases fairly rapidly initially, then somewhat less rapidly.



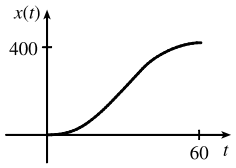
29.



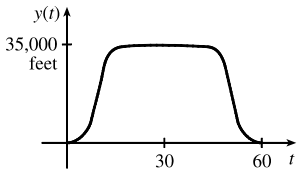
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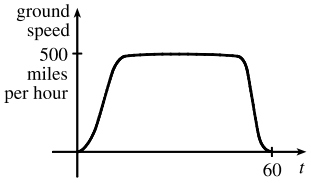
30. (a)



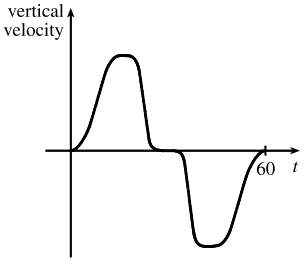
(b)



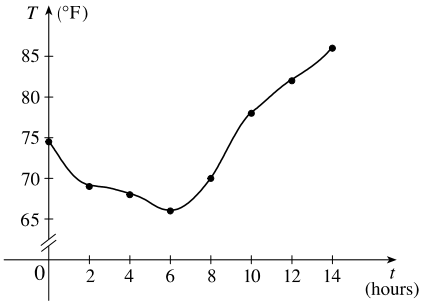
(c)



(d)

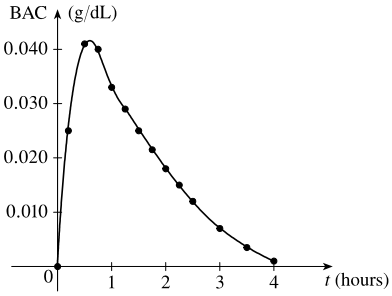


31. (a)



(b) 9:00 AM corresponds to $t = 9$. When $t = 9$, the temperature T is about 74°F.

32. (a)



(b) The blood alcohol concentration rises rapidly, then slowly decreases to near zero.

33. $f(x) = 3x^2 - x + 2$.

$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12$.

$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16$.

$f(a) = 3a^2 - a + 2$.

$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2$.

$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 = 3a^2 + 5a + 4$.

$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4$.

$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2$.

[continued]

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$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2.$

$[f(a)]^2 = \left[3a^2 - a + 2\right]^2 = \left(3a^2 - a + 2\right)\left(3a^2 - a + 2\right)$
$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4.$

$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2.$

34. $g(x) = \dfrac{x}{\sqrt{x+1}}$.

$g(0) = \dfrac{0}{\sqrt{0+1}} = 0.$

$g(3) = \dfrac{3}{\sqrt{3+1}} = \dfrac{3}{2}.$

$5g(a) = 5 \cdot \dfrac{a}{\sqrt{a+1}} = \dfrac{5a}{\sqrt{a+1}}.$

$\dfrac{1}{2}g(4a) = \dfrac{1}{2} \cdot g(4a) = \dfrac{1}{2} \cdot \dfrac{4a}{\sqrt{4a+1}} = \dfrac{2a}{\sqrt{4a+1}}.$

$g(a^2) = \dfrac{a^2}{\sqrt{a^2+1}}$; $[g(a)]^2 = \left(\dfrac{a}{\sqrt{a+1}}\right)^2 = \dfrac{a^2}{a+1}.$

$g(a+h) = \dfrac{(a+h)}{\sqrt{(a+h)+1}} = \dfrac{a+h}{\sqrt{a+h+1}}.$

$g(x-a) = \dfrac{(x-a)}{\sqrt{(x-a)+1}} = \dfrac{x-a}{\sqrt{x-a+1}}.$

35. $f(x) = 4 + 3x - x^2$, so $f(3+h) = 4 + 3(3+h) - (3+h)^2 = 4 + 9 + 3h - (9 + 6h + h^2) = 4 - 3h - h^2$,
and $\dfrac{f(3+h) - f(3)}{h} = \dfrac{(4 - 3h - h^2) - 4}{h} = \dfrac{h(-3-h)}{h} = -3 - h.$

36. $f(x) = x^3$, so $f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$,
and $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(a^3 + 3a^2h + 3ah^2 + h^3) - a^3}{h} = \dfrac{h(3a^2 + 3ah + h^2)}{h} = 3a^2 + 3ah + h^2.$

37. $f(x) = \dfrac{1}{x}$, so $\dfrac{f(x) - f(a)}{x - a} = \dfrac{\frac{1}{x} - \frac{1}{a}}{x - a} = \dfrac{\frac{a-x}{xa}}{x-a} = \dfrac{a-x}{xa(x-a)} = \dfrac{-1(x-a)}{xa(x-a)} = -\dfrac{1}{ax}.$

38. $f(x) = \sqrt{x+2}$, so $\dfrac{f(x) - f(1)}{x-1} = \dfrac{\sqrt{x+2} - \sqrt{3}}{x-1}$. Depending upon the context, this may be considered simplified.

Note: We may also rationalize the numerator:

$$\frac{\sqrt{x+2} - \sqrt{3}}{x-1} = \frac{\sqrt{x+2} - \sqrt{3}}{x-1} \cdot \frac{\sqrt{x+2} + \sqrt{3}}{\sqrt{x+2} + \sqrt{3}} = \frac{(x+2) - 3}{(x-1)\left(\sqrt{x-2} + \sqrt{3}\right)}$$
$$= \frac{x-1}{(x-1)\left(\sqrt{x-2} + \sqrt{3}\right)} = \frac{1}{\sqrt{x+2} + \sqrt{3}}$$

39. $f(x) = (x+4)/(x^2 - 9)$ is defined for all x except when $0 = x^2 - 9 \quad \Leftrightarrow \quad 0 = (x+3)(x-3) \quad \Leftrightarrow \quad x = -3$ or 3, so the domain is $\{x \in \mathbb{R} \mid x \neq -3, 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

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40. The function $f(x) = \dfrac{x^2+1}{x^2+4x-21}$ is defined for all values of x except those for which $x^2+4x-21=0 \quad \Leftrightarrow$ $(x+7)(x-3)=0 \quad \Leftrightarrow \quad x=-7$ or $x=3$. Thus, the domain is $\{x \in \mathbb{R} \mid x \neq -7, 3\} = (-\infty,-7) \cup (-7,3) \cup (3,\infty)$.

41. $f(t) = \sqrt[3]{2t-1}$ is defined for all real numbers. In fact $\sqrt[3]{p(t)}$, where $p(t)$ is a polynomial, is defined for all real numbers. Thus, the domain is $\mathbb{R}$, or $(-\infty,\infty)$.

42. $g(t) = \sqrt{3-t} - \sqrt{2+t}$ is defined when $3-t \geq 0 \quad \Leftrightarrow \quad t \leq 3$ and $2+t \geq 0 \quad \Leftrightarrow \quad t \geq -2$. Thus, the domain is $-2 \leq t \leq 3$, or $[-2,3]$.

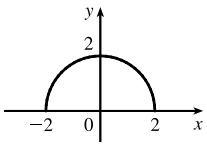
43. $h(x) = 1 \,/\, \sqrt[4]{x^2-5x}$ is defined when $x^2-5x>0 \quad \Leftrightarrow \quad x(x-5)>0$. Note that $x^2-5x \neq 0$ since that would result in division by zero. The expression $x(x-5)$ is positive if $x<0$ or $x>5$. (See Appendix A for methods for solving inequalities.) Thus, the domain is $(-\infty,0) \cup (5,\infty)$.

44. $f(u) = \dfrac{u+1}{1+\dfrac{1}{u+1}}$ is defined when $u+1 \neq 0 \;\; [u \neq -1]$ and $1+\dfrac{1}{u+1} \neq 0$. Since $1+\dfrac{1}{u+1}=0 \quad \Leftrightarrow$ $\dfrac{1}{u+1}=-1 \quad \Leftrightarrow \quad 1=-u-1 \quad \Leftrightarrow \quad u=-2$, the domain is $\{u \mid u \neq -2, u \neq -1\} = (-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$.

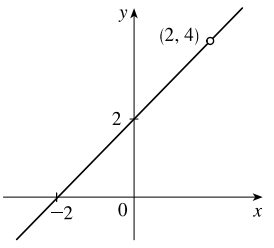
45. $F(p) = \sqrt{2-\sqrt{p}}$ is defined when $p \geq 0$ and $2-\sqrt{p} \geq 0$. Since $2-\sqrt{p} \geq 0 \quad \Leftrightarrow \quad 2 \geq \sqrt{p} \quad \Leftrightarrow \quad \sqrt{p} \leq 2 \quad \Leftrightarrow$ $0 \leq p \leq 4$, the domain is $[0,4]$.

46. The function $h(x) = \sqrt{x^2-4x-5}$ is defined when $x^2-4x-5 \geq 0 \quad \Leftrightarrow \quad (x+1)(x-5) \geq 0$. The polynomial $p(x) = x^2-4x-5$ may change signs only at its zeros, so we test values of x on the intervals separated by $x=-1$ and $x=5$: $p(-2)=7>0$, $p(0)=-5<0$, and $p(6)=7>0$. Thus, the domain of h, equivalent to the solution intervals of $p(x) \geq 0$, is $\{x \mid x \leq -1 \text{ or } x \geq 5\} = (-\infty,-1] \cup [5,\infty)$.

47. $h(x) = \sqrt{4-x^2}$. Now $y = \sqrt{4-x^2} \quad \Rightarrow \quad y^2 = 4-x^2 \quad \Leftrightarrow \quad x^2+y^2=4$, so the graph is the top half of a circle of radius 2 with center at the origin. The domain is $\{x \mid 4-x^2 \geq 0\} = \{x \mid 4 \geq x^2\} = \{x \mid 2 \geq |x|\} = [-2,2]$. From the graph, the range is $0 \leq y \leq 2$, or $[0,2]$.



48. The function $f(x) = \dfrac{x^2-4}{x-2}$ is defined when $x-2 \neq 0 \Leftrightarrow x \neq 2$, so the domain is $\{x \mid x \neq 2\} = (-\infty,2) \cup (2,\infty)$. On its domain, $f(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x-2)(x+2)}{x-2} = x+2$. Thus, the graph of f is the line $y=x+2$ with a hole at $(2,4)$.

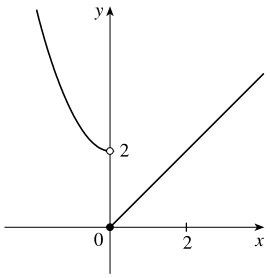


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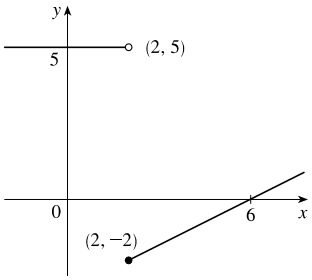
49. $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

$f(-3) = (-3)^2 + 2 = 11$, $f(0) = 0$, and $f(2) = 2$.



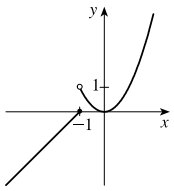
50. $f(x) = \begin{cases} 5 & \text{if } x < 2 \\ \frac{1}{2}x - 3 & \text{if } x \geq 2 \end{cases}$

$f(-3) = 5$, $f(0) = 5$, and $f(2) = \frac{1}{2}(2) - 3 = -2$.



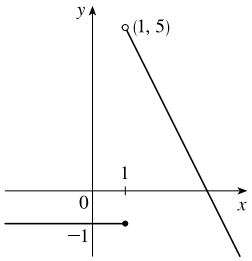
51. $f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

$f(-3) = -3 + 1 = -2$, $f(0) = 0^2 = 0$, and $f(2) = 2^2 = 4$.



52. $f(x) = \begin{cases} -1 & \text{if } x \leq 1 \\ 7 - 2x & \text{if } x > 1 \end{cases}$

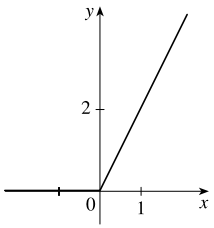
$f(-3) = -1$, $f(0) = -1$, and $f(2) = 7 - 2(2) = 3$.



53. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

so $f(x) = x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

Graph the line $y = 2x$ for $x \geq 0$ and graph $y = 0$ (the x-axis) for $x < 0$.

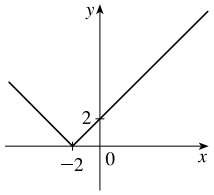


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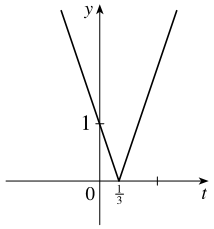
54. $f(x) = |x+2| = \begin{cases} x+2 & \text{if } x+2 \geq 0 \\ -(x+2) & \text{if } x+2 < 0 \end{cases}$

$= \begin{cases} x+2 & \text{if } x \geq -2 \\ -x-2 & \text{if } x < -2 \end{cases}$



55. $g(t) = |1-3t| = \begin{cases} 1-3t & \text{if } 1-3t \geq 0 \\ -(1-3t) & \text{if } 1-3t < 0 \end{cases}$

$= \begin{cases} 1-3t & \text{if } t \leq \frac{1}{3} \\ 3t-1 & \text{if } t > \frac{1}{3} \end{cases}$

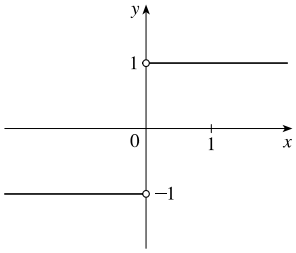


56. $f(x) = \dfrac{|x|}{x}$

The domain of f is $\{x \mid x \neq 0\}$ and $|x| = x$ if $x > 0$, $|x| = -x$ if $x < 0$.

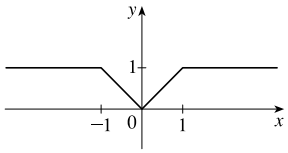
So we can write

$$f(x) = \begin{cases} \dfrac{-x}{x} = -1 & \text{if } x < 0 \\ \dfrac{x}{x} = 1 & \text{if } x > 0 \end{cases}$$



57. To graph $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 1 & \text{if } |x| > 1 \end{cases}$, graph $y = |x|$ [Figure 16]

for $-1 \leq x \leq 1$ and graph $y = 1$ for $x > 1$ and for $x < -1$.

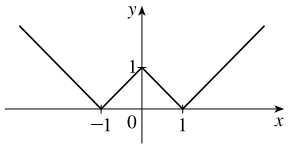


We could rewrite f as $f(x) = \begin{cases} 1 & \text{if } x < -1 \\ -x & \text{if } -1 \leq x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$.

58. $g(x) = \big||x|-1\big| = \begin{cases} |x|-1 & \text{if } |x|-1 \geq 0 \\ -(|x|-1) & \text{if } |x|-1 < 0 \end{cases}$

$= \begin{cases} |x|-1 & \text{if } |x| \geq 1 \\ -|x|+1 & \text{if } |x| < 1 \end{cases}$

$= \begin{cases} x-1 & \text{if } |x| \geq 1 \text{ and } x \geq 0 \\ -x-1 & \text{if } |x| \geq 1 \text{ and } x < 0 \\ -x+1 & \text{if } |x| < 1 \text{ and } x \geq 0 \\ -(-x)+1 & \text{if } |x| < 1 \text{ and } x < 0 \end{cases} = \begin{cases} x-1 & \text{if } x \geq 1 \\ -x-1 & \text{if } x \leq -1 \\ -x+1 & \text{if } 0 \leq x < 1 \\ x+1 & \text{if } -1 < x < 0 \end{cases}$



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59. Recall that the slope m of a line between the two points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and an equation of the line connecting those two points is $y - y_1 = m(x - x_1)$. The slope of the line segment joining the points $(1, -3)$ and $(5, 7)$ is $\dfrac{7 - (-3)}{5 - 1} = \dfrac{5}{2}$, so an equation is $y - (-3) = \frac{5}{2}(x - 1)$. The function is $f(x) = \frac{5}{2}x - \frac{11}{2}$, $1 \le x \le 5$.

60. The slope of the line segment joining the points $(-5, 10)$ and $(7, -10)$ is $\dfrac{-10 - 10}{7 - (-5)} = -\dfrac{5}{3}$, so an equation is $y - 10 = -\frac{5}{3}[x - (-5)]$. The function is $f(x) = -\frac{5}{3}x + \frac{5}{3}$, $-5 \le x \le 7$.

61. We need to solve the given equation for y. $x + (y-1)^2 = 0 \;\Leftrightarrow\; (y-1)^2 = -x \;\Leftrightarrow\; y - 1 = \pm\sqrt{-x} \;\Leftrightarrow\; y = 1 \pm \sqrt{-x}$. The expression with the positive radical represents the top half of the parabola, and the one with the negative radical represents the bottom half. Hence, we want $f(x) = 1 - \sqrt{-x}$. Note that the domain is $x \le 0$.

62. $x^2 + (y-2)^2 = 4 \;\Leftrightarrow\; (y-2)^2 = 4 - x^2 \;\Leftrightarrow\; y - 2 = \pm\sqrt{4 - x^2} \;\Leftrightarrow\; y = 2 \pm \sqrt{4 - x^2}$. The top half is given by the function $f(x) = 2 + \sqrt{4 - x^2}$, $-2 \le x \le 2$.

63. For $0 \le x \le 3$, the graph is the line with slope -1 and y-intercept 3, that is, $y = -x + 3$. For $3 < x \le 5$, the graph is the line with slope 2 passing through $(3, 0)$; that is, $y - 0 = 2(x - 3)$, or $y = 2x - 6$. So the function is

$$f(x) = \begin{cases} -x + 3 & \text{if } 0 \le x \le 3 \\ 2x - 6 & \text{if } 3 < x \le 5 \end{cases}$$

64. For $-4 \le x \le -2$, the graph is the line with slope $-\frac{3}{2}$ passing through $(-2, 0)$; that is, $y - 0 = -\frac{3}{2}[x - (-2)]$, or $y = -\frac{3}{2}x - 3$. For $-2 < x < 2$, the graph is the top half of the circle with center $(0, 0)$ and radius 2. An equation of the circle is $x^2 + y^2 = 4$, so an equation of the top half is $y = \sqrt{4 - x^2}$. For $2 \le x \le 4$, the graph is the line with slope $\frac{3}{2}$ passing through $(2, 0)$; that is, $y - 0 = \frac{3}{2}(x - 2)$, or $y = \frac{3}{2}x - 3$. So the function is

$$f(x) = \begin{cases} -\frac{3}{2}x - 3 & \text{if } -4 \le x \le -2 \\ \sqrt{4 - x^2} & \text{if } -2 < x < 2 \\ \frac{3}{2}x - 3 & \text{if } \quad 2 \le x \le 4 \end{cases}$$

65. Let the length and width of the rectangle be L and W. Then the perimeter is $2L + 2W = 20$ and the area is $A = LW$. Solving the first equation for W in terms of L gives $W = \dfrac{20 - 2L}{2} = 10 - L$. Thus, $A(L) = L(10 - L) = 10L - L^2$. Since lengths are positive, the domain of A is $0 < L < 10$. If we further restrict L to be larger than W, then $5 < L < 10$ would be the domain.

66. Let the length and width of the rectangle be L and W. Then the area is $LW = 16$, so that $W = 16/L$. The perimeter is $P = 2L + 2W$, so $P(L) = 2L + 2(16/L) = 2L + 32/L$, and the domain of P is $L > 0$, since lengths must be positive quantities. If we further restrict L to be larger than W, then $L > 4$ would be the domain.

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67. Let the length of a side of the equilateral triangle be x. Then by the Pythagorean Theorem, the height y of the triangle satisfies $y^2 + \left(\frac{1}{2}x\right)^2 = x^2$, so that $y^2 = x^2 - \frac{1}{4}x^2 = \frac{3}{4}x^2$ and $y = \frac{\sqrt{3}}{2}x$. Using the formula for the area A of a triangle, $A = \frac{1}{2}(\text{base})(\text{height})$, we obtain $A(x) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$, with domain $x > 0$.

68. Let the length, width, and height of the closed rectangular box be denoted by L, W, and H, respectively. The length is twice the width, so $L = 2W$. The volume V of the box is given by $V = LWH$. Since $V = 8$, we have $8 = (2W)WH \quad \Rightarrow$ $8 = 2W^2H \quad \Rightarrow \quad H = \dfrac{8}{2W^2} = \dfrac{4}{W^2}$, and so $H = f(W) = \dfrac{4}{W^2}$.

69. Let each side of the base of the box have length x, and let the height of the box be h. Since the volume is 2, we know that $2 = hx^2$, so that $h = 2/x^2$, and the surface area is $S = x^2 + 4xh$. Thus, $S(x) = x^2 + 4x(2/x^2) = x^2 + (8/x)$, with domain $x > 0$.

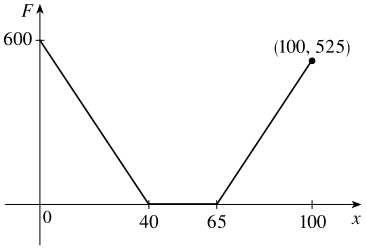
70. Let r and h denote the radius and the height of the right circular cylinder, respectively. Then the volume V is given by $V = \pi r^2 h$, and for this particular cylinder we have $\pi r^2 h = 25 \quad \Leftrightarrow \quad r^2 = \dfrac{25}{\pi h}$. Solving for r and rejecting the negative solution gives $r = \dfrac{5}{\sqrt{\pi h}}$, so $r = f(h) = \dfrac{5}{\sqrt{\pi h}}$ in.

71. The height of the box is x and the length and width are $L = 20 - 2x$, $W = 12 - 2x$. Then $V = LWx$ and so $V(x) = (20 - 2x)(12 - 2x)(x) = 4(10 - x)(6 - x)(x) = 4x(60 - 16x + x^2) = 4x^3 - 64x^2 + 240x$.
The sides L, W, and x must be positive. Thus, $L > 0 \quad \Leftrightarrow \quad 20 - 2x > 0 \quad \Leftrightarrow \quad x < 10$;
$W > 0 \quad \Leftrightarrow \quad 12 - 2x > 0 \quad \Leftrightarrow \quad x < 6$; and $x > 0$. Combining these restrictions gives us the domain $0 < x < 6$.

72. The area of the window is $A = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = xh + \dfrac{\pi x^2}{8}$, where h is the height of the rectangular portion of the window. The perimeter is $P = 2h + x + \frac{1}{2}\pi x = 30 \quad \Leftrightarrow \quad 2h = 30 - x - \frac{1}{2}\pi x \quad \Leftrightarrow \quad h = \frac{1}{4}(60 - 2x - \pi x)$. Thus,
$A(x) = x\,\dfrac{60 - 2x - \pi x}{4} + \dfrac{\pi x^2}{8} = 15x - \dfrac{1}{2}x^2 - \dfrac{\pi}{4}x^2 + \dfrac{\pi}{8}x^2 = 15x - \dfrac{4}{8}x^2 - \dfrac{\pi}{8}x^2 = 15x - x^2\left(\dfrac{\pi + 4}{8}\right)$.
Since the lengths x and h must be positive quantities, we have $x > 0$ and $h > 0$. For $h > 0$, we have $2h > 0 \quad \Leftrightarrow$ $30 - x - \frac{1}{2}\pi x > 0 \quad \Leftrightarrow \quad 60 > 2x + \pi x \quad \Leftrightarrow \quad x < \dfrac{60}{2 + \pi}$. Hence, the domain of A is $0 < x < \dfrac{60}{2 + \pi}$.

73. We can summarize the amount of the fine with a piecewise defined function.

$$F(x) = \begin{cases} 15(40 - x) & \text{if } 0 \le x < 40 \\ 0 & \text{if } 40 \le x \le 65 \\ 15(x - 65) & \text{if } x > 65 \end{cases}$$



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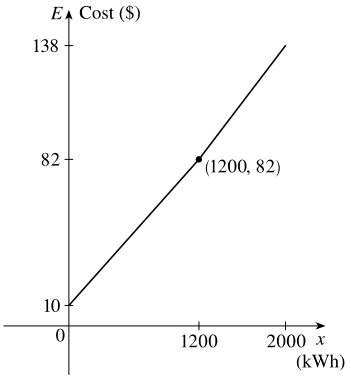
74. For the first 1200 kWh, $E(x) = 10 + 0.06x$.

For usage over 1200 kWh, the cost is

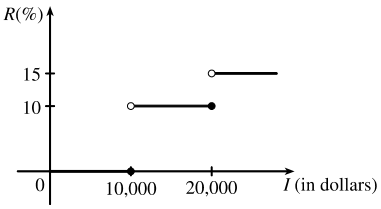
$E(x) = 10 + 0.06(1200) + 0.07(x - 1200) = 82 + 0.07(x - 1200)$.

Thus,

$$E(x) = \begin{cases} 10 + 0.06x & \text{if } 0 \le x \le 1200 \\ 82 + 0.07(x - 1200) & \text{if } x > 1200 \end{cases}$$



75. (a)

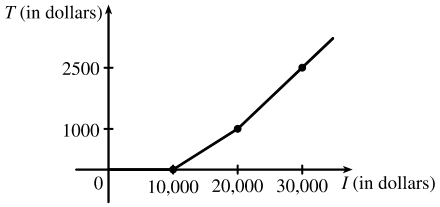


(b) On \$14,000, tax is assessed on \$4000, and $10\%(\$4000) = \$400$.

On \$26,000, tax is assessed on \$16,000, and

$10\%(\$10{,}000) + 15\%(\$6000) = \$1000 + \$900 = \$1900$.

(c) As in part (b), there is \$1000 tax assessed on \$20,000 of income, so the graph of T is a line segment from $(10{,}000, 0)$ to $(20{,}000, 1000)$. The tax on \$30,000 is \$2500, so the graph of T for $x > 20{,}000$ is the ray with initial point $(20{,}000, 1000)$ that passes through $(30{,}000, 2500)$.



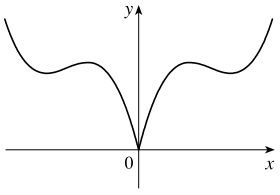
76. (a) Because an even function is symmetric with respect to the y-axis, and the point $(5, 3)$ is on the graph of this even function, the point $(-5, 3)$ must also be on its graph.

(b) Because an odd function is symmetric with respect to the origin, and the point $(5, 3)$ is on the graph of this odd function, the point $(-5, -3)$ must also be on its graph.

77. f is an odd function because its graph is symmetric about the origin. g is an even function because its graph is symmetric with respect to the y-axis.

78. f is not an even function since it is not symmetric with respect to the y-axis. f is not an odd function since it is not symmetric about the origin. Hence, f is *neither* even nor odd. g is an even function because its graph is symmetric with respect to the y-axis.

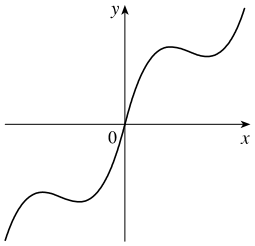
79. (a) The graph of an even function is symmetric about the y-axis. We reflect the given portion of the graph of f about the y-axis in order to complete it.



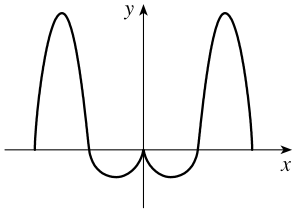
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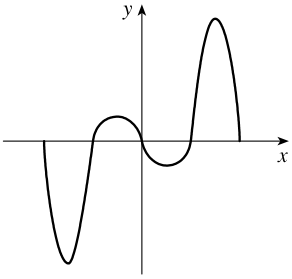
(b) For an odd function, $f(-x) = -f(x)$. The graph of an odd function is symmetric about the origin. We rotate the given portion of the graph of f through $180°$ about the origin in order to complete it.



80. (a) The graph of an even function is symmetric about the y-axis. We reflect the given portion of the graph of f about the y-axis in order to complete it.



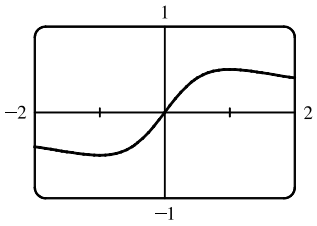
(b) The graph of an odd function is symmetric about the origin. We rotate the given portion of the graph of f through $180°$ about the origin in order to complete it.



81. $f(x) = \dfrac{x}{x^2+1}$.

$$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -\frac{x}{x^2+1} = -f(x).$$

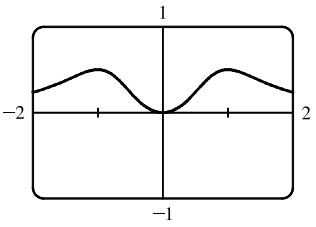
Since $f(-x) = -f(x)$, f is an odd function.



82. $f(x) = \dfrac{x^2}{x^4+1}$.

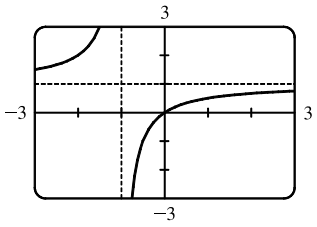
$$f(-x) = \frac{(-x)^2}{(-x)^4+1} = \frac{x^2}{x^4+1} = f(x).$$

Since $f(-x) = f(x)$, f is an even function.



83. $f(x) = \dfrac{x}{x+1}$, so $f(-x) = \dfrac{-x}{-x+1} = \dfrac{x}{x-1}$.

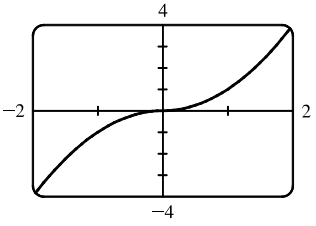
Since this is neither $f(x)$ nor $-f(x)$, the function f is neither even nor odd.



84. $f(x) = x\,|x|$.

$$f(-x) = (-x)\,|-x| = (-x)\,|x| = -(x\,|x|)$$
$$= -f(x)$$

Since $f(-x) = -f(x)$, f is an odd function.



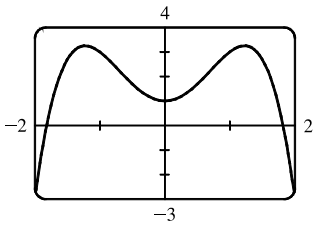
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85. $f(x) = 1 + 3x^2 - x^4$.

$f(-x) = 1+3(-x)^2-(-x)^4 = 1+3x^2-x^4 = f(x)$.

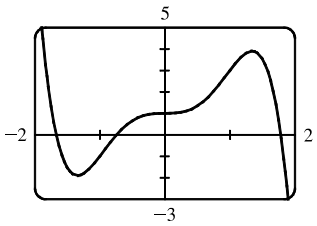
Since $f(-x) = f(x)$, f is an even function.



86. $f(x) = 1 + 3x^3 - x^5$, so

$$f(-x) = 1 + 3(-x)^3 - (-x)^5 = 1 + 3(-x^3) - (-x^5)$$
$$= 1 - 3x^3 + x^5$$

Since this is neither $f(x)$ nor $-f(x)$, the function f is neither even nor odd.



87. (i) If f and g are both even functions, then $f(-x) = f(x)$ and $g(-x) = g(x)$. Now $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$, so $f+g$ is an *even* function.

(ii) If f and g are both odd functions, then $f(-x) = -f(x)$ and $g(-x) = -g(x)$. Now $(f+g)(-x) = f(-x) + g(-x) = -f(x) + [-g(x)] = -[f(x) + g(x)] = -(f+g)(x)$, so $f+g$ is an *odd* function.

(iii) If f is an even function and g is an odd function, then $(f+g)(-x) = f(-x) + g(-x) = f(x) + [-g(x)] = f(x) - g(x)$, which is not $(f+g)(x)$ nor $-(f+g)(x)$, so $f+g$ is *neither* even nor odd. (Exception: if f is the zero function, then $f+g$ will be *odd*. If g is the zero function, then $f+g$ will be *even*.)

88. (i) If f and g are both even functions, then $f(-x) = f(x)$ and $g(-x) = g(x)$. Now $(fg)(-x) = f(-x)g(-x) = f(x)g(x) = (fg)(x)$, so fg is an *even* function.

(ii) If f and g are both odd functions, then $f(-x) = -f(x)$ and $g(-x) = -g(x)$. Now $(fg)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (fg)(x)$, so fg is an *even* function.

(iii) If f is an even function and g is an odd function, then $(fg)(-x) = f(-x)g(-x) = f(x)[-g(x)] = -[f(x)g(x)] = -(fg)(x)$, so fg is an *odd* function.

1.2 Mathematical Models: A Catalog of Essential Functions

1. (a) $f(x) = x^3 + 3x^2$ is a polynomial function of degree 3. (This function is also an algebraic function.)

(b) $g(t) = \cos^2 t - \sin t$ is a trigonometric function.

(c) $r(t) = t^{\sqrt{3}}$ is a power function.

(d) $v(t) = 8^t$ is an exponential function.

(e) $y = \dfrac{\sqrt{x}}{x^2+1}$ is an algebraic function. It is the quotient of a root of a polynomial and a polynomial of degree 2.

(f) $g(u) = \log_{10} u$ is a logarithmic function.

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2. (a) $f(t) = \dfrac{3t^2 + 2}{t}$ is a rational function. (This function is also an algebraic function.)

(b) $h(r) = 2.3^r$ is an exponential function.

(c) $s(t) = \sqrt{t+4}$ is an algebraic function. It is a root of a polynomial.

(d) $y = x^4 + 5$ is a polynomial function of degree 4.

(e) $g(x) = \sqrt[3]{x}$ is a root function. Rewriting $g(x)$ as $x^{1/3}$, we recognize the function also as a power function. (This function is, further, an algebraic function because it is a root of a polynomial.)

(f) $y = \dfrac{1}{x^2}$ is a rational function. Rewriting y as x^{-2}, we recognize the function also as a power function. (This function is, further, an algebraic function because it is the quotient of two polynomials.)

3. We notice from the figure that g and h are even functions (symmetric with respect to the y-axis) and that f is an odd function (symmetric with respect to the origin). So (b) $\left[y = x^5\right]$ must be f. Since g is flatter than h near the origin, we must have (c) $\left[y = x^8\right]$ matched with g and (a) $\left[y = x^2\right]$ matched with h.

4. (a) The graph of $y = 3x$ is a line (choice G).

(b) $y = 3^x$ is an exponential function (choice f).

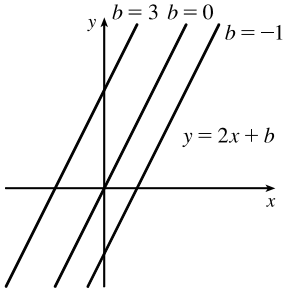
(c) $y = x^3$ is an odd polynomial function or power function (choice F).

(d) $y = \sqrt[3]{x} = x^{1/3}$ is a root function (choice g).

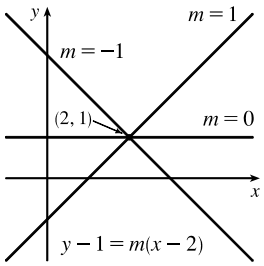
5. The denominator cannot equal 0, so $1 - \sin x \neq 0 \quad \Leftrightarrow \quad \sin x \neq 1 \quad \Leftrightarrow \quad x \neq \frac{\pi}{2} + 2n\pi$. Thus, the domain of $f(x) = \dfrac{\cos x}{1 - \sin x}$ is $\{x \mid x \neq \frac{\pi}{2} + 2n\pi,\ n \text{ an integer}\}$.

6. The denominator cannot equal 0, so $1 - \tan x \neq 0 \quad \Leftrightarrow \quad \tan x \neq 1 \quad \Leftrightarrow \quad x \neq \frac{\pi}{4} + n\pi$. The tangent function is not defined if $x \neq \frac{\pi}{2} + n\pi$. Thus, the domain of $g(x) = \dfrac{1}{1 - \tan x}$ is $\{x \mid x \neq \frac{\pi}{4} + n\pi,\ x \neq \frac{\pi}{2} + n\pi,\ n \text{ an integer}\}$.

7. (a) An equation for the family of linear functions with slope 2 is $y = f(x) = 2x + b$, where b is the y-intercept.



(b) $f(2) = 1$ means that the point $(2, 1)$ is on the graph of f. We can use the point-slope form of a line to obtain an equation for the family of linear functions through the point $(2, 1)$. $y - 1 = m(x - 2)$, which is equivalent to $y = mx + (1 - 2m)$ in slope-intercept form.

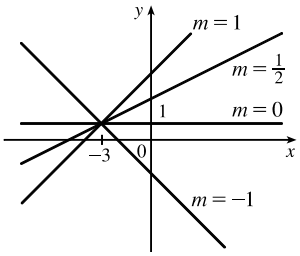


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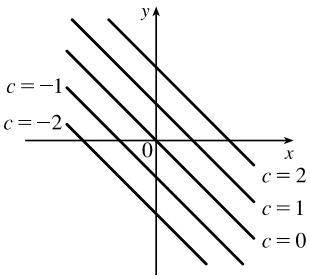
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(c) To belong to both families, an equation must have slope $m = 2$, so the equation in part (b), $y = mx + (1 - 2m)$, becomes $y = 2x - 3$. It is the *only* function that belongs to both families.

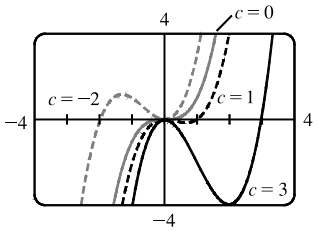
8. All members of the family of linear functions $f(x) = 1 + m(x + 3)$ have graphs that are lines passing through the point $(-3, 1)$.



9. All members of the family of linear functions $f(x) = c - x$ have graphs that are lines with slope -1. The y-intercept is c.



10. We graph $P(x) = x^3 - cx^2$ for $c = -2$, 0, 1, and 3. For $c \neq 0$, $P(x) = x^3 - cx^2 = x^2(x - c)$ has two x-intercepts, 0 and c. The curve has one decreasing portion that begins or ends at the origin and increases in length as $|c|$ increases; the decreasing portion is in quadrant II for $c < 0$ and in quadrant IV for $c > 0$.



11. Because f is a quadratic function, we know it is of the form $f(x) = ax^2 + bx + c$. The y-intercept is 18, so $f(0) = 18 \Rightarrow c = 18$ and $f(x) = ax^2 + bx + 18$. Since the points $(3, 0)$ and $(4, 2)$ lie on the graph of f, we have

$$f(3) = 0 \Rightarrow 9a + 3b + 18 = 0 \Rightarrow 3a + b = -6 \quad \textbf{(1)}$$
$$f(4) = 2 \Rightarrow 16a + 4b + 18 = 2 \Rightarrow 4a + b = -4 \quad \textbf{(2)}$$

This is a system of two equations in the unknowns a and b, and subtracting **(1)** from **(2)** gives $a = 2$. From **(1)**, $3(2) + b = -6 \Rightarrow b = -12$, so a formula for f is $f(x) = 2x^2 - 12x + 18$.

12. g is a quadratic function so $g(x) = ax^2 + bx + c$. The y-intercept is 1, so $g(0) = 1 \Rightarrow c = 1$ and $g(x) = ax^2 + bx + 1$. Since the points $(-2, 2)$ and $(1, -2.5)$ lie on the graph of g, we have

$$g(-2) = 2 \Rightarrow 4a - 2b + 1 = 2 \Rightarrow 4a - 2b = 1 \quad \textbf{(1)}$$
$$g(1) = -2.5 \Rightarrow a + b + 1 = -2.5 \Rightarrow a + b = -3.5 \quad \textbf{(2)}$$

Then **(1)** $+ 2 \cdot$ **(2)** gives us $6a = -6 \Rightarrow a = -1$ and from **(2)**, we have $-1 + b = -3.5 \Rightarrow b = -2.5$, so a formula for g is $g(x) = -x^2 - 2.5x + 1$.

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13. Since $f(-1) = f(0) = f(2) = 0$, f has zeros of -1, 0, and 2, so an equation for f is $f(x) = a[x-(-1)](x-0)(x-2)$, or $f(x) = ax(x+1)(x-2)$. Because $f(1) = 6$, we'll substitute 1 for x and 6 for $f(x)$.

$6 = a(1)(2)(-1) \quad \Rightarrow \quad -2a = 6 \quad \Rightarrow \quad a = -3$, so an equation for f is $f(x) = -3x(x+1)(x-2)$.

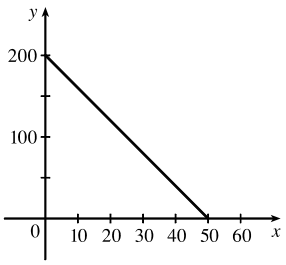
14. (a) For $T = 0.02t + 8.50$, the slope is 0.02, which means that the average surface temperature of the world is increasing at a rate of $0.02\ ^\circ\text{C}$ per year. The T-intercept is 8.50, which represents the average surface temperature in $^\circ\text{C}$ in the year 1900.

(b) $t = 2100 - 1900 = 200 \quad \Rightarrow \quad T = 0.02(200) + 8.50 = 12.50\ ^\circ\text{C}$

15. (a) $D = 200$, so $c = 0.0417D(a+1) = 0.0417(200)(a+1) = 8.34a + 8.34$. The slope is 8.34, which represents the change in mg of the dosage for a child for each change of 1 year in age.

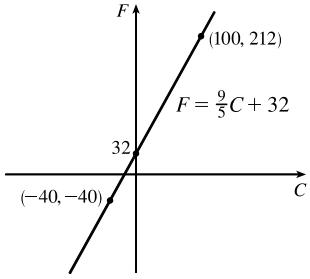
(b) For a newborn, $a = 0$, so $c = 8.34$ mg.

16. (a)



(b) The slope of -4 means that for each increase of 1 dollar for a rental space, the number of spaces rented *decreases* by 4. The y-intercept of 200 is the number of spaces that would be occupied if there were no charge for each space. The x-intercept of 50 is the smallest rental fee that results in no spaces rented.

17. (a)



(b) The slope of $\frac{9}{5}$ means that F increases $\frac{9}{5}$ degrees for each increase of 1°C. (Equivalently, F increases by 9 when C increases by 5 and F decreases by 9 when C decreases by 5.) The F-intercept of 32 is the Fahrenheit temperature corresponding to a Celsius temperature of 0.

18. (a) Jari is traveling faster since the line representing her distance versus time is steeper than the corresponding line for Jade.

(b) At $t = 0$, Jade has traveled 10 miles. At $t = 6$, Jade has traveled 16 miles. Thus, Jade's speed is

$$\frac{16 \text{ miles} - 10 \text{ miles}}{6 \text{ minutes} - 0 \text{ minutes}} = 1 \text{ mi/min. This is } \frac{1 \text{ mile}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 60 \text{ mi/h}$$

At $t = 0$, Jari has traveled 0 miles. At $t = 6$, Jari has traveled 7 miles. Thus, Jari's speed is

$$\frac{7 \text{ miles} - 0 \text{ miles}}{6 \text{ minutes} - 0 \text{ minutes}} = \frac{7}{6} \text{ mi/min or } \frac{7 \text{ miles}}{6 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 70 \text{ mi/h}$$

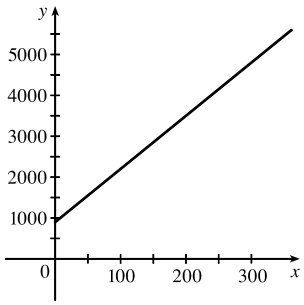
(c) From part (b), we have a slope of 1 (mile/minute) for the linear function f modeling the distance traveled by Jade and from the graph the y-intercept is 10. Thus, $f(t) = 1t + 10 = t + 10$. Similarly, we have a slope of $\frac{7}{6}$ miles/minute for

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Jari and a y-intercept of 0. Thus, the distance traveled by Jari as a function of time t (in minutes) is modeled by $g(t) = \frac{7}{6}t + 0 = \frac{7}{6}t$.

19. (a) Let x denote the number of chairs produced in one day and y the associated cost. Using the points $(100, 2200)$ and $(300, 4800)$, we get the slope $\frac{4800-2200}{300-100} = \frac{2600}{200} = 13$. So $y - 2200 = 13(x - 100) \quad \Leftrightarrow$ $y = 13x + 900$.



(b) The slope of the line in part (a) is 13 and it represents the cost (in dollars) of producing each additional chair.

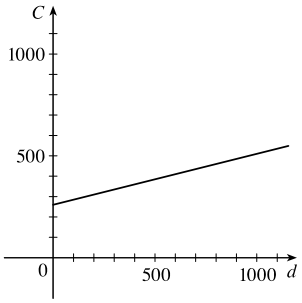
(c) The y-intercept is 900 and it represents the fixed daily costs of operating the factory.

20. (a) Using d in place of x and C in place of y, we find the slope to be $\dfrac{C_2 - C_1}{d_2 - d_1} = \dfrac{460 - 380}{800 - 480} = \dfrac{80}{320} = \dfrac{1}{4}$.

So a linear equation is $C - 460 = \frac{1}{4}(d - 800) \quad \Leftrightarrow \quad C - 460 = \frac{1}{4}d - 200 \quad \Leftrightarrow \quad C = \frac{1}{4}d + 260$.

(b) Letting $d = 1500$ we get $C = \frac{1}{4}(1500) + 260 = 635$.
The cost of driving 1500 miles is \$635.

(c)



The slope of the line represents the cost per mile, \$0.25.

(d) The C-intercept represents the fixed cost, \$260.

(e) A linear function gives a suitable model in this situation because you have fixed monthly costs such as insurance and car payments, as well as costs that increase as you drive, such as gasoline, oil, and tires, and the cost of these for each additional mile driven is a constant.

21. (a) We are given $\dfrac{\text{change in pressure}}{\text{10 feet change in depth}} = \dfrac{4.34}{10} = 0.434$. Using P for pressure and d for depth with the point $(d, P) = (0, 15)$, we have the slope-intercept form of the line, $P = 0.434d + 15$.

(b) When $P = 100$, then $100 = 0.434d + 15 \quad \Leftrightarrow \quad 0.434d = 85 \quad \Leftrightarrow \quad d = \frac{85}{0.434} \approx 195.85$ feet. Thus, the pressure is 100 lb/in^2 at a depth of approximately 196 feet.

22. (a) $R(x) = kx^{-2}$ and $R(0.005) = 140$, so $140 = k(0.005)^{-2} \quad \Leftrightarrow \quad k = 140(0.005)^2 = 0.0035$.

(b) $R(x) = 0.0035x^{-2}$, so for a diameter of 0.008 m the resistance is $R(0.008) = 0.0035(0.008)^{-2} \approx 54.7$ ohms.

23. If x is the original distance from the source, then the illumination is $f(x) = kx^{-2} = k/x^2$. Moving halfway to the lamp gives an illumination of $f\left(\frac{1}{2}x\right) = k\left(\frac{1}{2}x\right)^{-2} = k(2/x)^2 = 4(k/x^2)$, so the light is four times as bright.

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24. (a) $P = k/V$ and $P = 39$ kPa when $V = 0.671\ \text{m}^3$, so $39 = k/0.671 \quad \Leftrightarrow \quad k = 39(0.671) = 26.169$.

(b) When $V = 0.916$, $P = 26.169/V = 26.169/0.916 \approx 28.6$, so the pressure is reduced to approximately 28.6 kPa.

25. (a) $P = kAv^3$ so doubling the windspeed v gives $P = kA(2v)^3 = 8(kAv^3)$. Thus, the power output is increased by a factor of eight.

(b) The area swept out by the blades is given by $A = \pi l^2$, where l is the blade length, so the power output is $P = kAv^3 = k\pi l^2 v^3$. Doubling the blade length gives $P = k\pi(2l)^2 v^3 = 4(k\pi l^2 v^3)$. Thus, the power output is increased by a factor of four.

(c) From part (b) we have $P = k\pi l^2 v^3$, and $k = 0.214\ \text{kg/m}^3$, $l = 30$ m gives

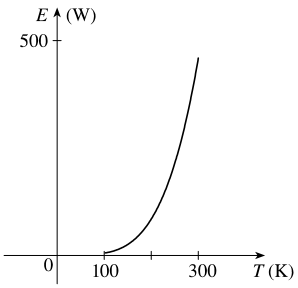
$$P = 0.214\ \frac{\text{kg}}{\text{m}^3} \cdot 900\pi\ \text{m}^2 \cdot v^3 = 192.6\pi v^3\ \frac{\text{kg}}{\text{m}}$$

For $v = 10$ m/s, we have

$$P = 192.6\pi\left(10\ \frac{\text{m}}{\text{s}}\right)^3\ \frac{\text{kg}}{\text{m}} = 192{,}600\pi\ \frac{\text{m}^2 \cdot \text{kg}}{\text{s}^3} \approx 605{,}000\ \text{W}$$

Similarly, $v = 15$ m/s gives $P = 650{,}025\pi \approx 2{,}042{,}000$ W and $v = 25$ m/s gives $P = 3{,}009{,}375\pi \approx 9{,}454{,}000$ W.

26. (a) We graph $E(T) = (5.67 \times 10^{-8})T^4$ for $100 \le T \le 300$:



(b) From the graph, we see that as temperature increases, energy increases—slowly at first, but then at an increasing rate.

27. (a) The data appear to be periodic and a sine or cosine function would make the best model. A model of the form $f(x) = a\cos(bx) + c$ seems appropriate.

(b) The data appear to be decreasing in a linear fashion. A model of the form $f(x) = mx + b$ seems appropriate.

28. (a) The data appear to be increasing exponentially. A model of the form $f(x) = a \cdot b^x$ or $f(x) = a \cdot b^x + c$ seems appropriate.

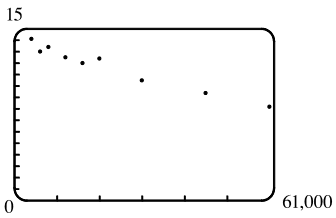
(b) The data appear to be decreasing similarly to the values of the reciprocal function. A model of the form $f(x) = a/x$ seems appropriate.

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Exercises 29–33: Some values are given to many decimal places. The results may depend on the technology used—rounding is left to the reader.

29. (a)

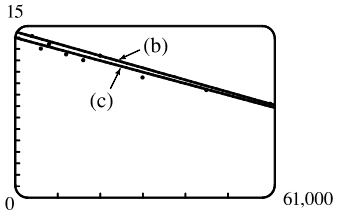


A linear model does seem appropriate.

(b) Using the points $(4000, 14.1)$ and $(60{,}000, 8.2)$, we obtain

$y - 14.1 = \dfrac{8.2 - 14.1}{60{,}000 - 4000}(x - 4000)$ or, equivalently,

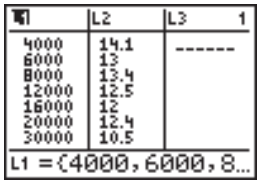
$y \approx -0.000105357x + 14.521429$.

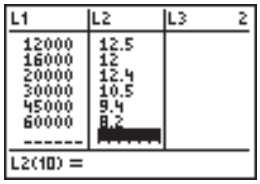


(c) Using a computing device, we obtain the regression line $y = -0.0000997855x + 13.950764$.

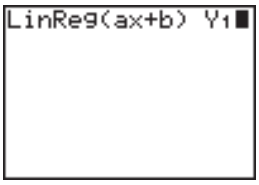
The following commands and screens illustrate how to find the regression line on a TI-84 Plus calculator.

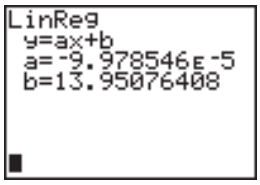
Enter the data into list one (L1) and list two (L2). Press STAT 1 to enter the editor.

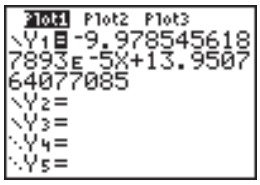




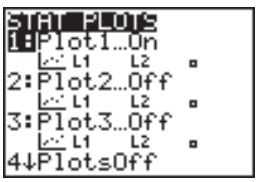
Find the regession line and store it in Y_1. Press 2nd QUIT STAT ▸ 4 VARS ▸ 1 1 ENTER.

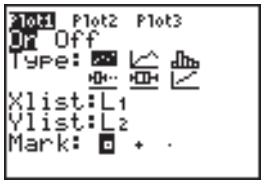




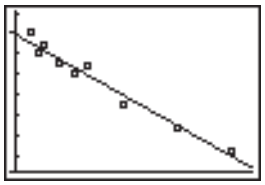


Note from the last figure that the regression line has been stored in Y_1 and that Plot1 has been turned on (Plot1 is highlighted). You can turn on Plot1 from the Y= menu by placing the cursor on Plot1 and pressing ENTER or by pressing 2nd STAT PLOT 1 ENTER.





Now press ZOOM 9 to produce a graph of the data and the regression line. Note that choice 9 of the ZOOM menu automatically selects a window that displays all of the data.



(d) When $x = 25{,}000$, $y \approx 11.456$; or about 11.5 per 100 population.

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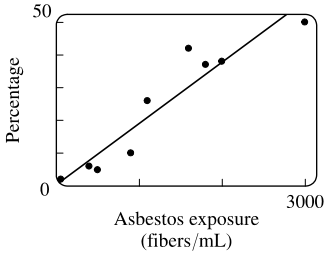
(e) When $x = 80{,}000$, $y \approx 5.968$; or about a 6% chance.

(f) When $x = 200{,}000$, y is negative, so the model does not apply.

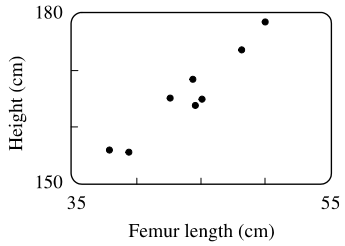
30. (a) Using a computing device, we obtain the regression line $y = 0.01879x + 0.30480$.

(b) The regression line appears to be a suitable model for the data.

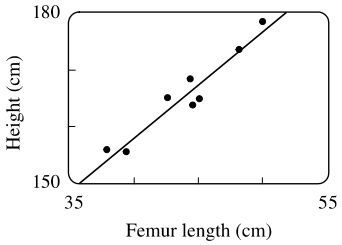
(c) The y-intercept represents the percentage of laboratory rats that develop lung tumors when *not* exposed to asbestos fibers.



31. (a)



(b) Using a computing device, we obtain the regression line $y = 1.88074x + 82.64974$.

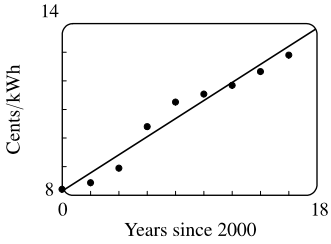


(c) When $x = 53$ cm, $y \approx 182.3$ cm.

32. (a) See the scatter plot in part (b). A linear model seems appropriate.

(b) Using a computing device, we obtain the regression line $y = 0.31567x + 8.15578$.

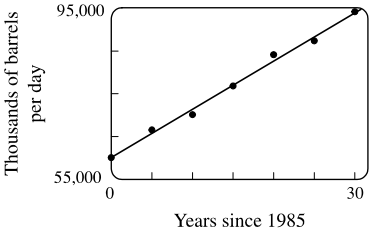
(c) For 2005, $x = 5$ and $y \approx 9.73$ cents/kWh. For 2017, $x = 17$ and $y \approx 13.52$ cents/kWh.



33. (a) See the scatter plot in part (b). A linear model seems appropriate.

(b) Using a computing device, we obtain the regression line $y = 1124.86x + 60{,}119.86$.

(c) For 2002, $x = 17$ and $y \approx 79{,}242$ thousands of barrels per day. For 2017, $x = 32$ and $y \approx 96{,}115$ thousands of barrels per day.



34. (a) $T = 1.000\,431\,227 d^{\,1.499\,528\,750}$

(b) The power model in part (a) is approximately $T = d^{1.5}$. Squaring both sides gives us $T^2 = d^3$, so the model matches Kepler's Third Law, $T^2 = kd^3$.

35. (a) If $A = 60$, then $S = 0.7A^{0.3} \approx 2.39$, so you would expect to find 2 species of bats in that cave.

(b) $S = 4 \;\Rightarrow\; 4 = 0.7A^{0.3} \;\Rightarrow\; \frac{40}{7} = A^{3/10} \;\Rightarrow\; A = \left(\frac{40}{7}\right)^{10/3} \approx 333.6$, so we estimate the surface area of the cave to be $334\ \text{m}^2$.

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36. (a) Using a computing device, we obtain a power function $N = cA^b$, where $c \approx 3.1046$ and $b \approx 0.308$.

(b) If $A = 291$, then $N = cA^b \approx 17.8$, so you would expect to find 18 species of reptiles and amphibians on Dominica.

37. We have $I = \dfrac{S}{4\pi r^2} = \left(\dfrac{S}{4\pi}\right)\left(\dfrac{1}{r^2}\right) = \dfrac{S/(4\pi)}{r^2}$. Thus, $I = \dfrac{k}{r^2}$ with $k = \dfrac{S}{4\pi}$.

1.3 New Functions from Old Functions

1. (a) If the graph of f is shifted 3 units upward, its equation becomes $y = f(x) + 3$.

(b) If the graph of f is shifted 3 units downward, its equation becomes $y = f(x) - 3$.

(c) If the graph of f is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.

(d) If the graph of f is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.

(e) If the graph of f is reflected about the x-axis, its equation becomes $y = -f(x)$.

(f) If the graph of f is reflected about the y-axis, its equation becomes $y = f(-x)$.

(g) If the graph of f is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.

(h) If the graph of f is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3}f(x)$.

2. (a) To obtain the graph of $y = f(x) + 8$ from the graph of $y = f(x)$, shift the graph 8 units upward.

(b) To obtain the graph of $y = f(x + 8)$ from the graph of $y = f(x)$, shift the graph 8 units to the left.

(c) To obtain the graph of $y = 8f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 8.

(d) To obtain the graph of $y = f(8x)$ from the graph of $y = f(x)$, shrink the graph horizontally by a factor of 8.

(e) To obtain the graph of $y = -f(x) - 1$ from the graph of $y = f(x)$, first reflect the graph about the x-axis, and then shift it 1 unit downward.

(f) To obtain the graph of $y = 8f(\frac{1}{8}x)$ from the graph of $y = f(x)$, stretch the graph horizontally and vertically by a factor of 8.

3. (a) *Graph 3:* The graph of f is shifted 4 units to the right and has equation $y = f(x - 4)$.

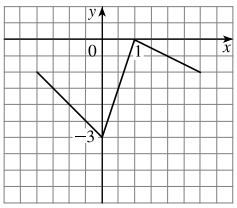
(b) *Graph 1:* The graph of f is shifted 3 units upward and has equation $y = f(x) + 3$.

(c) *Graph 4:* The graph of f is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.

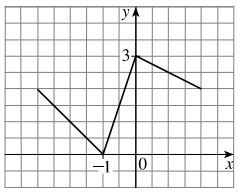
(d) *Graph 5:* The graph of f is shifted 4 units to the left and reflected about the x-axis. Its equation is $y = -f(x + 4)$.

(e) *Graph 2:* The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x + 6)$.

4. (a) $y = f(x) - 3$: Shift the graph of f 3 units down.



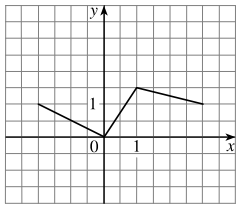
(b) $y = f(x + 1)$: Shift the graph of f 1 unit to the left.



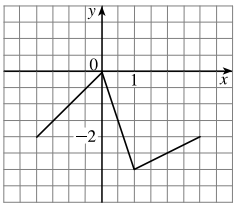
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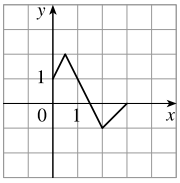
(c) $y = \frac{1}{2}f(x)$: Shrink the graph of f vertically by a factor of 2.



(d) $y = -f(x)$: Reflect the graph of f about the x-axis.

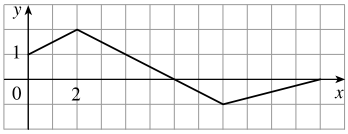


5. (a) To graph $y = f(2x)$ we shrink the graph of f horizontally by a factor of 2.



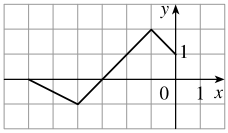
The point $(4, -1)$ on the graph of f corresponds to the point $\left(\frac{1}{2} \cdot 4, -1\right) = (2, -1)$.

(b) To graph $y = f\left(\frac{1}{2}x\right)$ we stretch the graph of f horizontally by a factor of 2.



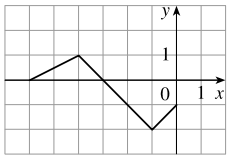
The point $(4, -1)$ on the graph of f corresponds to the point $(2 \cdot 4, -1) = (8, -1)$.

(c) To graph $y = f(-x)$ we reflect the graph of f about the y-axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1) = (-4, -1)$.

(d) To graph $y = -f(-x)$ we reflect the graph of f about the y-axis, then about the x-axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1 \cdot -1) = (-4, 1)$.

6. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 2 units to the right and stretched vertically by a factor of 2. Thus, a function describing the graph is

$$y = 2f(x-2) = 2\sqrt{3(x-2) - (x-2)^2} = 2\sqrt{3x - 6 - (x^2 - 4x + 4)} = 2\sqrt{-x^2 + 7x - 10}$$

7. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the x-axis, and shifted downward 1 unit. Thus, a function describing the graph is

$$y = \underbrace{-1 \cdot}_{\substack{\text{reflect} \\ \text{about } x\text{-axis}}} \quad f\ \underbrace{(x+4)}_{\substack{\text{shift} \\ \text{4 units left}}} \quad \underbrace{-\ 1}_{\substack{\text{shift} \\ \text{1 unit left}}}$$

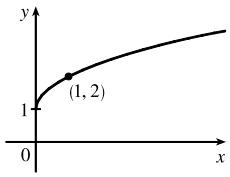
This function can be written as

$$y = -f(x+4) - 1 = -\sqrt{3(x+4) - (x+4)^2} - 1$$
$$= -\sqrt{3x + 12 - (x^2 + 8x + 16)} - 1 = -\sqrt{-x^2 - 5x - 4} - 1$$

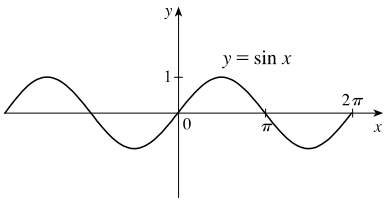
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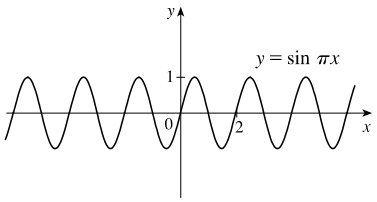
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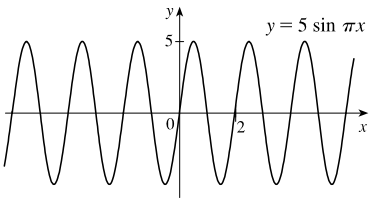
8. (a) The graph of $y = 1 + \sqrt{x}$ can be obtained from the graph of $y = \sqrt{x}$ by shifting it upward 1 unit.



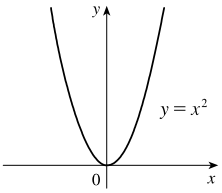
(b) The graph of $y = \sin \pi x$ can be obtained from the graph of $y = \sin x$ by compressing horizontally by a factor of π, giving a period of $2\pi/\pi = 2$. The graph of $y = 5 \sin \pi x$ is then obtained by stretching vertically by a factor of 5.

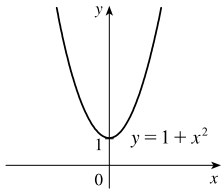




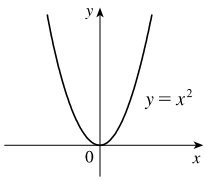


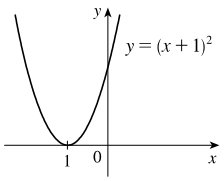
9. $y = 1 + x^2$. Start with the graph of $y = x^2$ and shift 1 unit upward



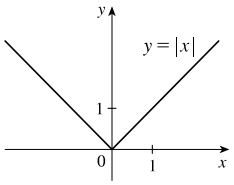


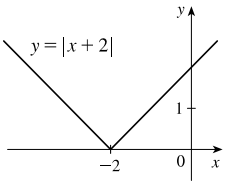
10. $y = (x + 1)^2$. Start with the graph of $y = x^2$ and shift 1 unit to the left.



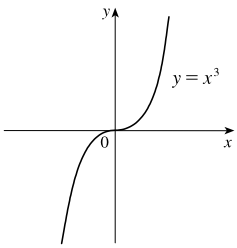


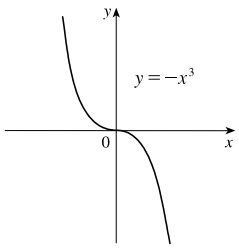
11. $y = |x + 2|$. Start with the graph of $y = |x|$ and shift 2 units to the left.

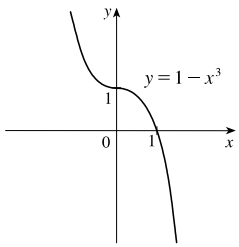




12. $y = 1 - x^3$. Start with the graph of $y = x^3$, reflect about the x-axis, and then shift 1 unit upward.



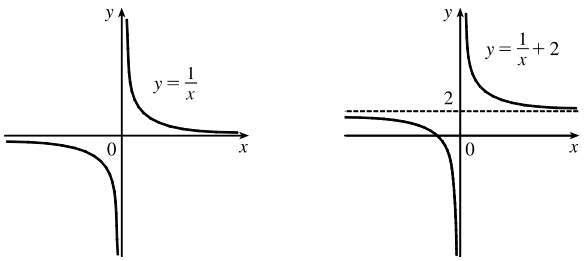




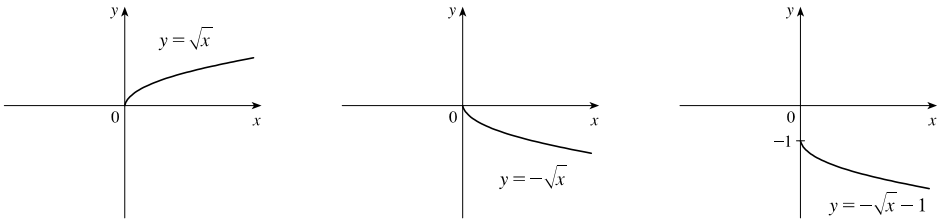
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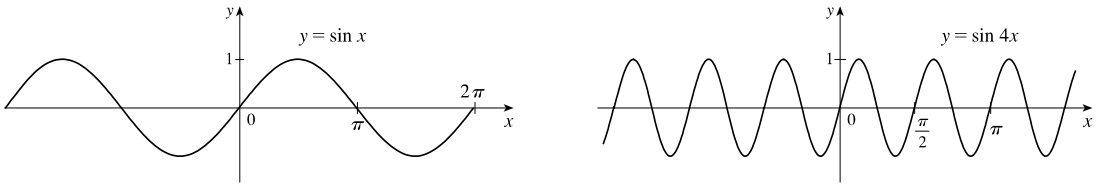
13. $y = \dfrac{1}{x} + 2$. Start with the graph of $y = \dfrac{1}{x}$ and shift 2 units upward.



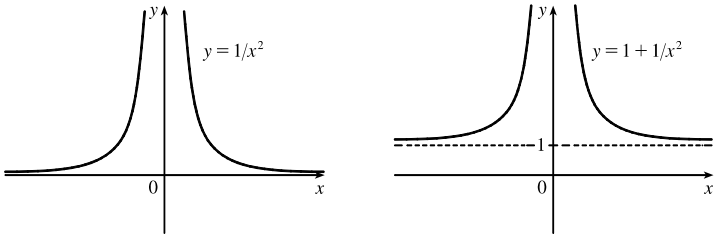
14. $y = -\sqrt{x} - 1$. Start with the graph of $y = \sqrt{x}$, reflect about the x-axis, and then shift 1 unit downward.



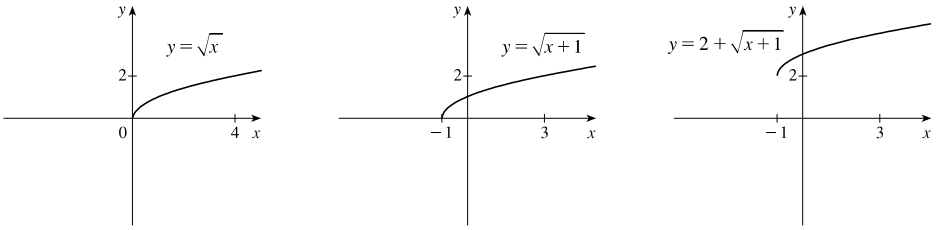
15. $y = \sin 4x$. Start with the graph of $y = \sin x$ and compress horizontally by a factor of 4. The period becomes $2\pi/4 = \pi/2$.



16. $y = 1 + \dfrac{1}{x^2}$. Start with the graph of $y = \dfrac{1}{x^2}$ and shift 1 unit upward.



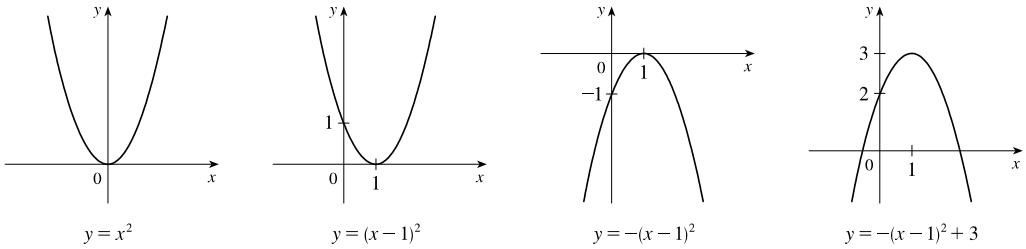
17. $y = 2 + \sqrt{x+1}$. Start with the graph of $y = \sqrt{x}$, shift 1 unit to the left, and then shift 2 units upward.



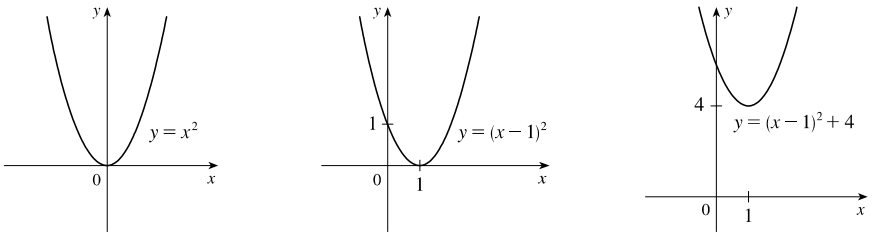
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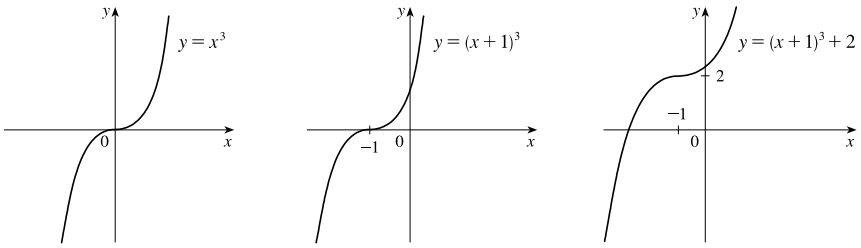
18. $y = -(x-1)^2 + 3$. Start with the graph of $y = x^2$, shift 1 unit to the right, reflect about the x-axis, and then shift 3 units upward.



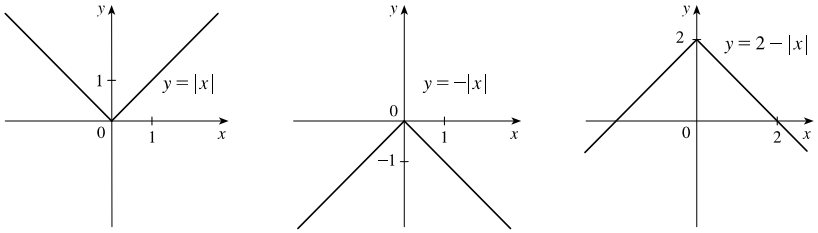
19. $y = x^2 - 2x + 5 = (x^2 - 2x + 1) + 4 = (x-1)^2 + 4$. Start with the graph of $y = x^2$, shift 1 unit to the right, and then shift 4 units upward.



20. $y = (x+1)^3 + 2$. Start with the graph of $y = x^3$, shift 1 unit to the left, and then shift 2 units upward.



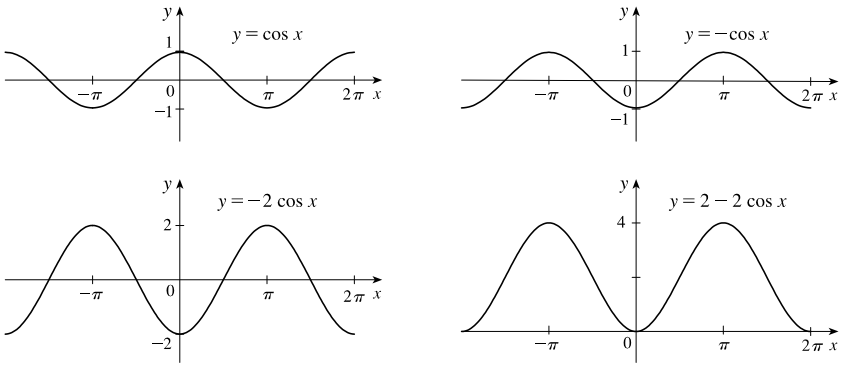
21. $y = 2 - |x|$. Start with the graph of $y = |x|$, reflect about the x-axis, and then shift 2 units upward.



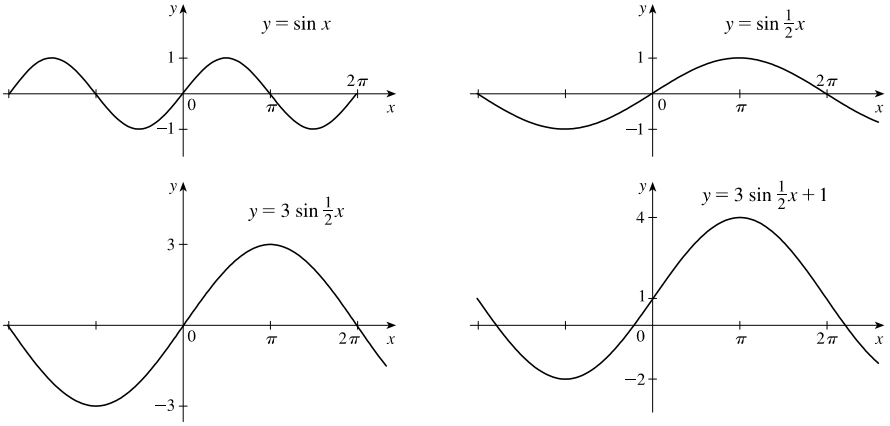
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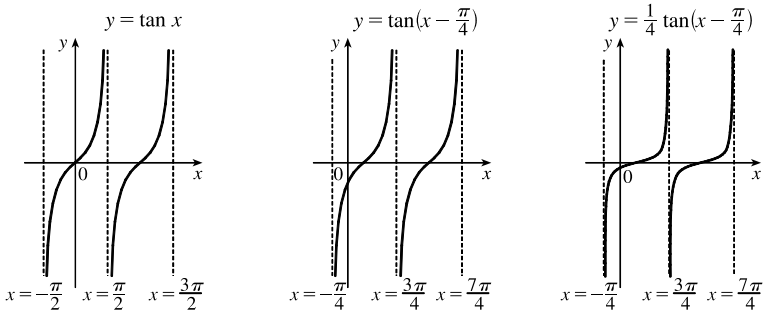
22. $y = 2 - 2\cos x$. Start with the graph of $y = \cos x$, reflect about the x-axis, stretch vertically by a factor of 2, and then shift 2 units upward.



23. $y = 3\sin\frac{1}{2}x + 1$. Start with the graph of $y = \sin x$, stretch horizontally by a factor of 2, stretch vertically by a factor of 3, and then shift 1 unit upward.



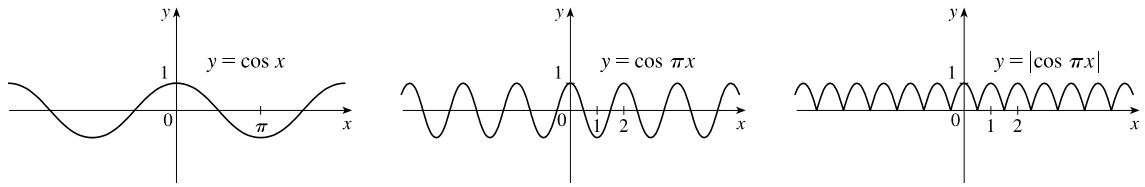
24. $y = \frac{1}{4}\tan\left(x - \frac{\pi}{4}\right)$. Start with the graph of $y = \tan x$, shift $\frac{\pi}{4}$ units to the right, and then compress vertically by a factor of 4.



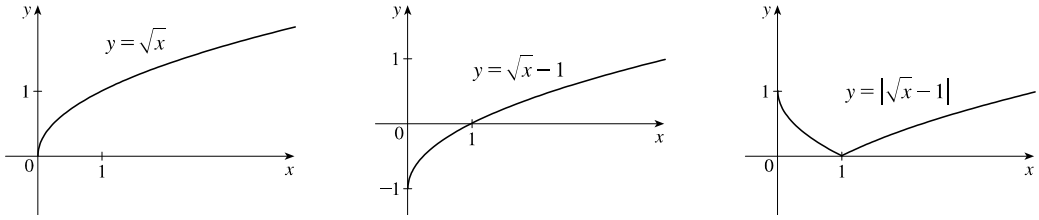
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25. $y = |\cos \pi x|$. Start with the graph of $y = \cos x$, shrink horizontally by a factor of π, and reflect all the parts of the graph below the x-axis about the x-axis.



26. $y = \left|\sqrt{x} - 1\right|$. Start with the graph of $y = \sqrt{x}$, shift 1 unit downward, and then reflect the portion of the graph below the x-axis about the x-axis.



27. This is just like the solution to Example 4 except the amplitude of the curve (the 30°N curve in Figure 9 on June 21) is $14 - 12 = 2$. So the function is $L(t) = 12 + 2\sin\left[\frac{2\pi}{365}(t - 80)\right]$. March 31 is the 90th day of the year, so the model gives $L(90) \approx 12.34$ h. The daylight time (5:51 AM to 6:18 PM) is 12 hours and 27 minutes, or 12.45 h. The model value differs from the actual value by $\frac{12.45 - 12.34}{12.45} \approx 0.009$, less than 1%.

28. Using a sine function to model the brightness of Delta Cephei as a function of time, we take its period to be 5.4 days, its amplitude to be 0.35 (on the scale of magnitude), and its average magnitude to be 4.0. If we take $t = 0$ at a time of average brightness, then the magnitude (brightness) as a function of time t in days can be modeled by the formula $M(t) = 4.0 + 0.35\sin\left(\frac{2\pi}{5.4}t\right)$.

29. The water depth $D(t)$ can be modeled by a cosine function with amplitude $\dfrac{12 - 2}{2} = 5$ m, average magnitude $\dfrac{12 + 2}{2} = 7$ m, and period 12 hours. High tide occurred at time 6:45 AM ($t = 6.75$ h), so the curve begins a cycle at time $t = 6.75$ h (shift 6.75 units to the right). Thus, $D(t) = 5\cos\left[\frac{2\pi}{12}(t - 6.75)\right] + 7 = 5\cos\left[\frac{\pi}{6}(t - 6.75)\right] + 7$, where D is in meters and t is the number of hours after midnight.

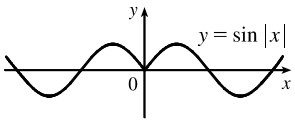
30. The total volume of air $V(t)$ in the lungs can be modeled by a sine function with amplitude $\dfrac{2500 - 2000}{2} = 250$ mL, average volume $\dfrac{2500 + 2000}{2} = 2250$ mL, and period 4 seconds. Thus, $V(t) = 250\sin\frac{2\pi}{4}t + 2250 = 250\sin\frac{\pi}{2}t + 2250$, where V is in mL and t is in seconds.

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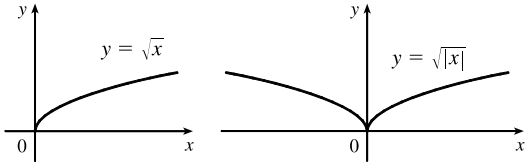
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31. (a) To obtain $y = f(|x|)$, the portion of the graph of $y = f(x)$ to the right of the y-axis is reflected about the y-axis.

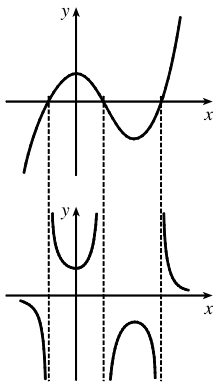
(b) $y = \sin|x|$



(c) $y = \sqrt{|x|}$



32. The most important features of the given graph are the x-intercepts and the maximum and minimum points. The graph of $y = 1/f(x)$ has vertical asymptotes at the x-values where there are x-intercepts on the graph of $y = f(x)$. The maximum of 1 on the graph of $y = f(x)$ corresponds to a minimum of $1/1 = 1$ on $y = 1/f(x)$. Similarly, the minimum on the graph of $y = f(x)$ corresponds to a maximum on the graph of $y = 1/f(x)$. As the values of y get large (positively or negatively) on the graph of $y = f(x)$, the values of y get close to zero on the graph of $y = 1/f(x)$.



33. $f(x) = \sqrt{25 - x^2}$ is defined only when $25 - x^2 \geq 0 \quad \Leftrightarrow \quad x^2 \leq 25 \quad \Leftrightarrow \quad -5 \leq x \leq 5$, so the domain of f is $[-5, 5]$. For $g(x) = \sqrt{x+1}$, we must have $x + 1 \geq 0 \quad \Leftrightarrow \quad x \geq -1$, so the domain of g is $[-1, \infty)$.

(a) $(f+g)(x) = \sqrt{25 - x^2} + \sqrt{x+1}$. The domain of $f + g$ is found by intersecting the domains of f and g: $[-1, 5]$.

(b) $(f-g)(x) = \sqrt{25 - x^2} - \sqrt{x+1}$. The domain of $f - g$ is found by intersecting the domains of f and g: $[-1, 5]$.

(c) $(fg)(x) = \sqrt{25 - x^2} \cdot \sqrt{x+1} = \sqrt{-x^3 - x^2 + 25x + 25}$. The domain of fg is found by intersecting the domains of f and g: $[-1, 5]$.

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{25 - x^2}}{\sqrt{x+1}} = \sqrt{\dfrac{25 - x^2}{x+1}}$. Notice that we must have $x + 1 \neq 0$ in addition to any previous restrictions. Thus, the domain of f/g is $(-1, 5]$.

34. For $f(x) = \dfrac{1}{x-1}$, we must have $x - 1 \neq 0 \quad \Leftrightarrow \quad x \neq 1$. For $g(x) = \dfrac{1}{x} - 2$, we must have $x \neq 0$.

(a) $(f+g)(x) = \dfrac{1}{x-1} + \dfrac{1}{x} - 2 = \dfrac{x + x - 1 - 2x(x-1)}{x(x-1)} = \dfrac{2x - 1 - 2x^2 + 2x}{x^2 - x} = -\dfrac{2x^2 - 4x + 1}{x^2 - x}, \quad \{x \mid x \neq 0, 1\}$

(b) $(f-g)(x) = \dfrac{1}{x-1} - \left(\dfrac{1}{x} - 2\right) = \dfrac{x - (x-1) + 2x(x-1)}{x(x-1)} = \dfrac{1 + 2x^2 - 2x}{x^2 - x} = \dfrac{2x^2 - 2x + 1}{x^2 - x}, \quad \{x \mid x \neq 0, 1\}$

(c) $(fg)(x) = \dfrac{1}{x-1}\left(\dfrac{1}{x} - 2\right) = \dfrac{1}{x^2 - x} - \dfrac{2}{x-1} = \dfrac{1 - 2x}{x^2 - x}, \quad \{x \mid x \neq 0, 1\}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\dfrac{1}{x-1}}{\dfrac{1}{x} - 2} = \dfrac{\dfrac{1}{x-1}}{\dfrac{1 - 2x}{x}} = \dfrac{1}{x-1} \cdot \dfrac{x}{1 - 2x} = \dfrac{x}{(x-1)(1-2x)} = -\dfrac{x}{(x-1)(2x-1)}$

$= -\dfrac{x}{2x^2 - 3x + 1}, \quad \{x \mid x \neq 0, \frac{1}{2}, 1\}$

[Note the additional domain restriction $g(x) \neq 0 \quad \Rightarrow \quad x \neq \frac{1}{2}$.]

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35. $f(x) = x^3 + 5$ and $g(x) = \sqrt[3]{x}$. The domain of each function is $(-\infty, \infty)$.

(a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 + 5 = x + 5$. The domain is $(-\infty, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g(x^3 + 5) = \sqrt[3]{x^3 + 5}$. The domain is $(-\infty, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f(x^3 + 5) = (x^3 + 5)^3 + 5$. The domain is $(-\infty, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{x}\right) = \sqrt[3]{\sqrt[3]{x}} = \sqrt[9]{x}$. The domain is $(-\infty, \infty)$.

36. $f(x) = 1/x$ and $g(x) = 2x + 1$. The domain of f is $(-\infty, 0) \cup (0, \infty)$. The domain of g is $(-\infty, \infty)$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x + 1) = \dfrac{1}{2x + 1}$. The domain is
$\{x \mid 2x + 1 \neq 0\} = \left\{x \mid x \neq -\frac{1}{2}\right\} = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = 2\left(\dfrac{1}{x}\right) + 1 = \dfrac{2}{x} + 1$. We must have $x \neq 0$, so the domain is $(-\infty, 0) \cup (0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$. Since f requires $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x + 1) = 2(2x + 1) + 1 = 4x + 3$. The domain is $(-\infty, \infty)$.

37. $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = x + 1$. The domain of f is $(0, \infty)$. The domain of g is $(-\infty, \infty)$.

(a) $(f \circ g)(x) = f(g(x)) = f(x + 1) = \dfrac{1}{\sqrt{x + 1}}$. We must have $x + 1 > 0$, or $x > -1$, so the domain is $(-1, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{\sqrt{x}} + 1$. We must have $x > 0$, so the domain is $(0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{\sqrt{1/\sqrt{x}}} = \dfrac{1}{1/\sqrt{\sqrt{x}}} = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. We must have $x > 0$, so the domain is $(0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g(x + 1) = (x + 1) + 1 = x + 2$. The domain is $(-\infty, \infty)$.

38. $f(x) = \dfrac{x}{x + 1}$ and $g(x) = 2x - 1$. The domain of f is $(-\infty, -1) \cup (-1, \infty)$. The domain of g is $(-\infty, \infty)$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x - 1) = \dfrac{2x - 1}{(2x - 1) + 1} = \dfrac{2x - 1}{2x}$. We must have $2x \neq 0 \quad \Leftrightarrow \quad x \neq 0$. Thus, the domain is $(-\infty, 0) \cup (0, \infty)$.

(b) $(g \circ f)(x) = g(f(x)) = 2\left(\dfrac{x}{x + 1}\right) - 1 = \dfrac{2x}{x + 1} - 1 = \dfrac{2x - 1(x + 1)}{x + 1} = \dfrac{x - 1}{x + 1}$. We must have $x + 1 \neq 0 \quad \Leftrightarrow$ $x \neq -1$. Thus, the domain is $(-\infty, -1) \cup (-1, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = \dfrac{\dfrac{x}{x + 1}}{\dfrac{x}{x + 1} + 1} = \dfrac{\dfrac{x}{x + 1}}{\dfrac{x}{x + 1} + 1} \cdot \dfrac{x + 1}{x + 1} = \dfrac{x}{x + (x + 1)} = \dfrac{x}{2x + 1}$. We must have both $x + 1 \neq 0$ and $2x + 1 \neq 0$, so the domain excludes both -1 and $-\frac{1}{2}$. Thus, the domain is $(-\infty, -1) \cup \left(-1, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x - 1) = 2(2x - 1) - 1 = 4x - 3$. The domain is $(-\infty, \infty)$.

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39. $f(x)=\dfrac{2}{x}$ and $g(x)=\sin x$. The domain of f is $(-\infty,0)\cup(0,\infty)$. The domain of g is $(-\infty,\infty)$.

(a) $(f\circ g)(x)=f(g(x))=f(\sin x)=\dfrac{2}{\sin x}=2\csc x$. We must have $\sin x\neq 0$, so the domain is $\{x \mid x\neq k\pi,\ k \text{ an integer}\}$.

(b) $(g\circ f)(x)=g(f(x))=g\left(\dfrac{2}{x}\right)=\sin\left(\dfrac{2}{x}\right)$. We must have $x\neq 0$, so the domain is $(-\infty,0)\cup(0,\infty)$.

(c) $(f\circ f)(x)=f(f(x))=f\left(\dfrac{2}{x}\right)=\dfrac{2}{\frac{2}{x}}=x$. Since f requires $x\neq 0$, the domain is $(-\infty,0)\cup(0,\infty)$.

(d) $(g\circ g)(x)=g(g(x))=g(\sin x)=\sin(\sin x)$. The domain is $(-\infty,\infty)$.

40. $f(x)=\sqrt{5-x}$ and $g(x)=\sqrt{x-1}$. The domain of f is $(-\infty,5]$ and the domain of g is $[1,\infty)$.

(a) $(f\circ g)(x)=f(g(x))=f\left(\sqrt{x-1}\right)=\sqrt{5-\sqrt{x-1}}$. We must have $x-1\geq 0 \quad\Leftrightarrow\quad x\geq 1$ and $5-\sqrt{x-1}\geq 0 \quad\Leftrightarrow\quad \sqrt{x-1}\leq 5 \quad\Leftrightarrow\quad 0\leq x-1\leq 25 \quad\Leftrightarrow\quad 1\leq x\leq 26$. Thus, the domain is $[1,26]$.

(b) $(g\circ f)(x)=g(f(x))=g\left(\sqrt{5-x}\right)=\sqrt{\sqrt{5-x}-1}$. We must have $5-x\geq 0 \quad\Leftrightarrow\quad x\leq 5$ and $\sqrt{5-x}-1\geq 0 \quad\Leftrightarrow\quad \sqrt{5-x}\geq 1 \quad\Leftrightarrow\quad 5-x\geq 1 \quad\Leftrightarrow\quad x\leq 4$. Intersecting the restrictions on x gives a domain of $(-\infty,4]$.

(c) $(f\circ f)(x)=f(f(x))=f\left(\sqrt{5-x}\right)=\sqrt{5-\sqrt{5-x}}$. We must have $5-x\geq 0 \quad\Leftrightarrow\quad x\leq 5$ and $5-\sqrt{5-x}\geq 0 \quad\Leftrightarrow\quad \sqrt{5-x}\leq 5 \quad\Leftrightarrow\quad 0\leq 5-x\leq 25 \quad\Leftrightarrow\quad -5\leq -x\leq 20 \quad\Leftrightarrow\quad -20\leq x\leq 5$. Intersecting the restrictions on x gives a domain of $[-20,5]$.

(d) $(g\circ g)(x)=g(g(x))=g\left(\sqrt{x-1}\right)=\sqrt{\sqrt{x-1}-1}$. We must have $x-1\geq 0 \quad\Leftrightarrow\quad x\geq 1$ and $\sqrt{x-1}-1\geq 0 \quad\Leftrightarrow\quad \sqrt{x-1}\geq 1 \quad\Leftrightarrow\quad x-1\geq 1 \quad\Leftrightarrow\quad x\geq 2$. Intersecting the restrictons on x gives a domain of $[2,\infty)$.

41. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x^2))=f(\sin(x^2))=3\sin(x^2)-2$

42. $(f\circ g\circ h)(x)=f(g(h(x)))=f\left(g\left(\sqrt{x}\right)\right)=f\left(2^{\sqrt{x}}\right)=\left|2^{\sqrt{x}}-4\right|$

43. $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x^3+2))=f[(x^3+2)^2]=f(x^6+4x^3+4)$
$=\sqrt{(x^6+4x^3+4)-3}=\sqrt{x^6+4x^3+1}$

44. $(f\circ g\circ h)(x)=f(g(h(x)))=f\left(g\left(\sqrt[3]{x}\right)\right)=f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)=\tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

45. Let $g(x)=2x+x^2$ and $f(x)=x^4$. Then $(f\circ g)(x)=f(g(x))=f(2x+x^2)=(2x+x^2)^4=F(x)$.

46. Let $g(x)=\cos x$ and $f(x)=x^2$. Then $(f\circ g)(x)=f(g(x))=f(\cos x)=(\cos x)^2=\cos^2 x=F(x)$.

47. Let $g(x)=\sqrt[3]{x}$ and $f(x)=\dfrac{x}{1+x}$. Then $(f\circ g)(x)=f(g(x))=f\left(\sqrt[3]{x}\right)=\dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}=F(x)$.

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48. Let $g(x) = \dfrac{x}{1+x}$ and $f(x) = \sqrt[3]{x}$. Then $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x}{1+x}\right) = \sqrt[3]{\dfrac{x}{1+x}} = G(x)$.

49. Let $g(t) = t^2$ and $f(t) = \sec t \tan t$. Then $(f \circ g)(t) = f(g(t)) = f(t^2) = \sec(t^2)\ \tan(t^2) = v(t)$.

50. Let $g(x) = \sqrt{x}$ and $f(x) = \sqrt{1+x}$. Then $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right) = \sqrt{1+\sqrt{x}} = H(x)$.

51. Let $h(x) = \sqrt{x}$, $g(x) = x - 1$, and $f(x) = \sqrt{x}$. Then

$(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{x}\right)\right) = f\left(\sqrt{x} - 1\right) = \sqrt{\sqrt{x} - 1} = R(x)$.

52. Let $h(x) = |x|$, $g(x) = 2 + x$, and $f(x) = \sqrt[8]{x}$. Then

$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f\left(2 + |x|\right) = \sqrt[8]{2 + |x|} = H(x)$.

53. Let $h(t) = \cos t$, $g(t) = \sin t$, and $f(t) = t^2$. Then

$(f \circ g \circ h)(t) = f(g(h(t))) = f(g(\cos t)) = f(\sin(\cos t)) = [\sin(\cos t)]^2 = \sin^2(\cos t) = S(t)$.

54. Let $h(t) = \tan t$, $g(t) = \sqrt{t} + 1$, and $f(t) = \cos t$. Then

$(f \circ g \circ h)(t) = f\left(g(h(t))\right) = f\left(g(\tan t)\right) = f\left(\sqrt{\tan t} + 1\right) = \cos\left(\sqrt{\tan t} + 1\right) = H(t)$.

55. (a) $f(g(3)) = f(4) = 6$.

(b) $g(f(2)) = g(1) = 5$.

(c) $(f \circ g)(5) = f(g(5)) = f(3) = 5$.

(d) $(g \circ f)(5) = g(f(5)) = g(2) = 3$.

56. (a) $g(g(g(2))) = g(g(3)) = g(4) = 1$.

(b) $(f \circ f \circ f)(1) = f(f(f(1))) = f(f(3)) = f(5) = 2$.

(c) $(f \circ f \circ g)(1) = f(f(g(1))) = f(f(5)) = f(2) = 1$.

(d) $(g \circ f \circ g)(3) = g(f(g(3))) = g(f(4)) = g(6) = 2$.

57. (a) $g(2) = 5$, because the point $(2, 5)$ is on the graph of g. Thus, $f(g(2)) = f(5) = 4$, because the point $(5, 4)$ is on the graph of f.

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$. This value is not defined, because there is no point on the graph of g that has x-coordinate 6.

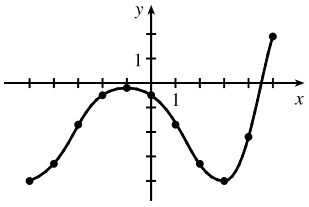
(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

58. To find a particular value of $f(g(x))$, say for $x = 0$, we note from the graph that $g(0) \approx 2.8$ and $f(2.8) \approx -0.5$. Thus, $f(g\,(0)) \approx f(2.8) \approx -0.5$. The other values listed in the table were obtained in a similar fashion.

x	$g(x)$	$f(g(x))$
-5	-0.2	-4
-4	1.2	-3.3
-3	2.2	-1.7
-2	2.8	-0.5
-1	3	-0.2

x	$g(x)$	$f(g(x))$
0	2.8	-0.5
1	2.2	-1.7
2	1.2	-3.3
3	-0.2	-4
4	-1.9	-2.2
5	-4.1	1.9



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59. (a) Using the relationship *distance* = *rate* · *time* with the radius r as the distance, we have $r(t) = 60t$.

(b) $A = \pi r^2 \quad \Rightarrow \quad (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$. This formula gives us the extent of the rippled area (in cm^2) at any time t.

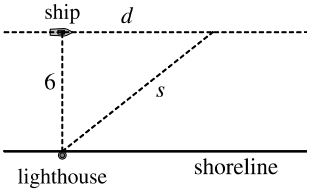
60. (a) The radius r of the balloon is increasing at a rate of 2 cm/s, so $r(t) = (2 \text{ cm/s})(t \text{ s}) = 2t$ (in cm).

(b) Using $V = \frac{4}{3}\pi r^3$, we get $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$.

The result, $V = \frac{32}{3}\pi t^3$, gives the volume of the balloon (in cm^3) as a function of time (in s).

61. (a) From the figure, we have a right triangle with legs 6 and d, and hypotenuse s.

By the Pythagorean Theorem, $d^2 + 6^2 = s^2 \quad \Rightarrow \quad s = f(d) = \sqrt{d^2 + 36}$.



(b) Using $d = rt$, we get $d = (30 \text{ km/h})(t \text{ hours}) = 30t$ (in km). Thus, $d = g(t) = 30t$.

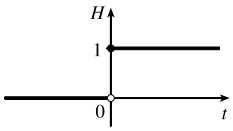
(c) $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$. This function represents the distance between the lighthouse and the ship as a function of the time elapsed since noon.

62. (a) $d = rt \quad \Rightarrow \quad d(t) = 350t$

(b) There is a Pythagorean relationship involving the legs with lengths d and 1 and the hypotenuse with length s: $d^2 + 1^2 = s^2$. Thus, $s(d) = \sqrt{d^2 + 1}$.

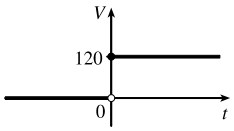
(c) $(s \circ d)(t) = s(d(t)) = s(350t) = \sqrt{(350t)^2 + 1}$

63. (a)



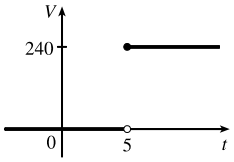
$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

(b)



$$V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 120 & \text{if } t \geq 0 \end{cases} \quad \text{so } V(t) = 120H(t).$$

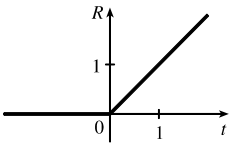
(c)



Starting with the formula in part (b), we replace 120 with 240 to reflect the different voltage. Also, because we are starting 5 units to the right of $t = 0$, we replace t with $t - 5$. Thus, the formula is $V(t) = 240H(t - 5)$.

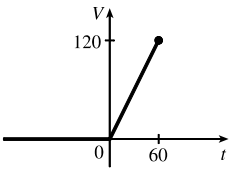
64. (a) $R(t) = tH(t)$

$$= \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}$$



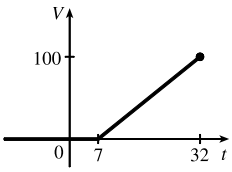
(b) $V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 2t & \text{if } 0 \leq t \leq 60 \end{cases}$

so $V(t) = 2tH(t), t \leq 60$.



(c) $V(t) = \begin{cases} 0 & \text{if } t < 7 \\ 4(t-7) & \text{if } 7 \leq t \leq 32 \end{cases}$

so $V(t) = 4(t - 7)H(t - 7), t \leq 32$.



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65. If $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$, then

$(f \circ g)(x) = f(g(x)) = f(m_2x + b_2) = m_1(m_2x + b_2) + b_1 = m_1m_2x + m_1b_2 + b_1$.

So $f \circ g$ is a linear function with slope m_1m_2.

66. If $A(x) = 1.04x$, then

$(A \circ A)(x) = A(A(x)) = A(1.04x) = 1.04(1.04x) = (1.04)^2x$,

$(A \circ A \circ A)(x) = A((A \circ A)(x)) = A((1.04)^2x) = 1.04(1.04)^2x = (1.04)^3x$, and

$(A \circ A \circ A \circ A)(x) = A((A \circ A \circ A)(x)) = A((1.04)^3x) = 1.04(1.04)^3x, = (1.04)^4x$.

These compositions represent the amount of the investment after 2, 3, and 4 years.

Based on this pattern, when we compose n copies of A, we get the formula $\underbrace{(A \circ A \circ \cdots \circ A)}_{n\ A\text{'s}}(x) = (1.04)^n x$.

67. (a) By examining the variable terms in g and h, we deduce that we must square g to get the terms $4x^2$ and $4x$ in h. If we let $f(x) = x^2 + c$, then $(f \circ g)(x) = f(g(x)) = f(2x + 1) = (2x + 1)^2 + c = 4x^2 + 4x + (1 + c)$. Since $h(x) = 4x^2 + 4x + 7$, we must have $1 + c = 7$. So $c = 6$ and $f(x) = x^2 + 6$.

(b) We need a function g so that $f(g(x)) = 3(g(x)) + 5 = h(x)$. But $h(x) = 3x^2 + 3x + 2 = 3(x^2 + x) + 2 = 3(x^2 + x - 1) + 5$, so we see that $g(x) = x^2 + x - 1$.

68. We need a function g so that $g(f(x)) = g(x + 4) = h(x) = 4x - 1 = 4(x + 4) - 17$. So we see that the function g must be $g(x) = 4x - 17$.

69. We need to examine $h(-x)$.

$$h(-x) = (f \circ g)(-x) = f(g(-x)) = f(g(x)) \quad \text{[because } g \text{ is even]} \quad = h(x)$$

Because $h(-x) = h(x)$, h is an even function.

70. $h(-x) = f(g(-x)) = f(-g(x))$. At this point, we can't simplify the expression, so we might try to find a counterexample to show that h is not an odd function. Let $g(x) = x$, an odd function, and $f(x) = x^2 + x$. Then $h(x) = x^2 + x$, which is neither even nor odd.

Now suppose f is an odd function. Then $f(-g(x)) = -f(g(x)) = -h(x)$. Hence, $h(-x) = -h(x)$, and so h is odd if both f and g are odd.

Now suppose f is an even function. Then $f(-g(x)) = f(g(x)) = h(x)$. Hence, $h(-x) = h(x)$, and so h is even if g is odd and f is even.

71. (a) $E(x) = f(x) + f(-x) \quad \Rightarrow \quad E(-x) = f(-x) + f(-(-x)) = f(-x) + f(x) = E(x)$. Since $E(-x) = E(x)$, E is an even function.

(b) $O(x) = f(x) - f(-x) \quad \Rightarrow \quad O(-x) = f(-x) - f(-(-x)) = f(-x) - f(x) = -[f(x) - f(-x)] = -O(x)$.

Since $O(-x) = -O(x)$, O is an odd function.

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(c) For any function f with domain $\mathbb{R}$, define functions E and O as in parts (a) and (b). Then $\frac{1}{2}E$ is even, $\frac{1}{2}O$ is odd, and we show that $f(x) = \frac{1}{2}E(x) + \frac{1}{2}O(x)$:

$$\begin{aligned}\tfrac{1}{2}E(x) + \tfrac{1}{2}O(x) &= \tfrac{1}{2}\left[f(x) + f(-x)\right] + \tfrac{1}{2}\left[f(x) - f(-x)\right] \\ &= \tfrac{1}{2}[f(x) + f(-x) + f(x) - f(-x)] \\ &= \tfrac{1}{2}[2f(x)] = f(x)\end{aligned}$$

as desired.

(d) $f(x) = 2^x + (x-3)^2$ has domain $\mathbb{R}$, so we know from part (c) that $f(x) = \frac{1}{2}E(x) + \frac{1}{2}O(x)$, where

$$\begin{aligned}E(x) = f(x) + f(-x) &= 2^x + (x-3)^2 + 2^{-x} + (-x-3)^2 \\ &= 2^x + 2^{-x} + (x-3)^2 + (x+3)^2\end{aligned}$$

and

$$\begin{aligned}O(x) = f(x) - f(-x) &= 2^x + (x-3)^2 - [2^{-x} + (-x-3)^2] \\ &= 2^x - 2^{-x} + (x-3)^2 - (x+3)^2\end{aligned}$$

1.4 The Tangent and Velocity Problems

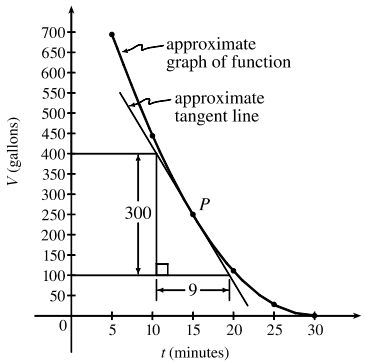
1. (a) Using $P(15, 250)$, we construct the following table:

t	Q	slope $= m_{PQ}$
5	$(5, 694)$	$\frac{694-250}{5-15} = -\frac{444}{10} = -44.4$
10	$(10, 444)$	$\frac{444-250}{10-15} = -\frac{194}{5} = -38.8$
20	$(20, 111)$	$\frac{111-250}{20-15} = -\frac{139}{5} = -27.8$
25	$(25, 28)$	$\frac{28-250}{25-15} = -\frac{222}{10} = -22.2$
30	$(30, 0)$	$\frac{0-250}{30-15} = -\frac{250}{15} = -16.\overline{6}$

(b) Using the values of t that correspond to the points closest to P ($t = 10$ and $t = 20$), we have

$$\frac{-38.8 + (-27.8)}{2} = -33.3$$

(c) From the graph, we can estimate the slope of the tangent line at P to be $\frac{-300}{9} = -33.\overline{3}$.



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2. (a) (i) On the interval $[0, 40]$, slope $= \dfrac{7398 - 3438}{40 - 0} = 99.$

(ii) On the interval $[10, 20]$, slope $= \dfrac{5622 - 4559}{20 - 10} = 106.3.$

(iii) On the interval $[20, 30]$, slope $= \dfrac{6536 - 5622}{30 - 20} = 91.4.$

The slopes represent the average number of steps per minute the student walked during the respective time intervals.

(b) Averaging the slopes of the secant lines corresponding to the intervals immediately before and after $t = 20$, we have

$$\frac{106.3 + 91.4}{2} = 98.85$$

The student's walking pace is approximately 99 steps per minute at 3:20 PM.

3. (a) $y = \dfrac{1}{1 - x}$, $P(2, -1)$

	x	$Q(x, 1/(1-x))$	m_{PQ}
(i)	1.5	$(1.5, -2)$	2
(ii)	1.9	$(1.9, -1.111\,111)$	1.111 111
(iii)	1.99	$(1.99, -1.010\,101)$	1.010 101
(iv)	1.999	$(1.999, -1.001\,001)$	1.001 001
(v)	2.5	$(2.5, -0.666\,667)$	0.666 667
(vi)	2.1	$(2.1, -0.909\,091)$	0.909 091
(vii)	2.01	$(2.01, -0.990\,099)$	0.990 099
(viii)	2.001	$(2.001, -0.999\,001)$	0.999 001

(b) The slope appears to be 1.

(c) Using $m = 1$, an equation of the tangent line to the curve at $P(2, -1)$ is $y - (-1) = 1(x - 2)$, or $y = x - 3$.

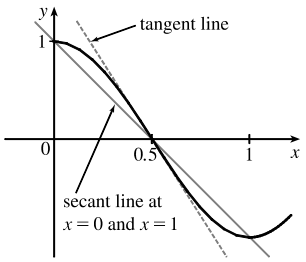
4. (a) $y = \cos \pi x$, $P(0.5, 0)$

	x	Q	m_{PQ}
(i)	0	$(0, 1)$	-2
(ii)	0.4	$(0.4, 0.309017)$	-3.090170
(iii)	0.49	$(0.49, 0.031411)$	-3.141076
(iv)	0.499	$(0.499, 0.003142)$	-3.141587
(v)	1	$(1, -1)$	-2
(vi)	0.6	$(0.6, -0.309017)$	-3.090170
(vii)	0.51	$(0.51, -0.031411)$	-3.141076
(viii)	0.501	$(0.501, -0.003142)$	-3.141587

(b) The slope appears to be $-\pi$.

(c) $y - 0 = -\pi(x - 0.5)$ or $y = -\pi x + \frac{1}{2}\pi$.

(d)



5. (a) $y = y(t) = 275 - 16t^2$. At $t = 4$, $y = 275 - 16(4)^2 = 19$. The average velocity between times 4 and $4 + h$ is

$$v_{\text{avg}} = \frac{y(4+h) - y(4)}{(4+h) - 4} = \frac{\left[275 - 16(4+h)^2\right] - 19}{h} = \frac{-128h - 16h^2}{h} = -128 - 16h \quad \text{if } h \neq 0$$

(i) 0.1 seconds: $h = 0.1$, $v_{\text{avg}} = -129.6$ ft/s

(ii) 0.05 seconds: $h = 0.05$, $v_{\text{avg}} = -128.8$ ft/s

(iii) 0.01 seconds: $h = 0.01$, $v_{\text{avg}} = -128.16$ ft/s

(b) The instantaneous velocity when $t = 4$ (h approaches 0) is -128 ft/s.

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6. (a) $y = y(t) = 10t - 1.86t^2$. At $t = 1$, $y = 10(1) - 1.86(1)^2 = 8.14$. The average velocity between times 1 and $1 + h$ is

$$v_{\text{avg}} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{\left[10(1+h) - 1.86(1+h)^2\right] - 8.14}{h} = \frac{6.28h - 1.86h^2}{h} = 6.28 - 1.86h, \text{ if } h \neq 0.$$

(i) $[1, 2]$: $h = 1$, $v_{\text{avg}} = 4.42$ m/s

(ii) $[1, 1.5]$: $h = 0.5$, $v_{\text{avg}} = 5.35$ m/s

(iii) $[1, 1.1]$: $h = 0.1$, $v_{\text{avg}} = 6.094$ m/s

(iv) $[1, 1.01]$: $h = 0.01$, $v_{\text{avg}} = 6.2614$ m/s

(v) $[1, 1.001]$: $h = 0.001$, $v_{\text{avg}} = 6.27814$ m/s

(b) The instantaneous velocity when $t = 1$ (h approaches 0) is 6.28 m/s.

7. (a) (i) On the interval $[2, 4]$, $v_{\text{avg}} = \dfrac{s(4) - s(2)}{4 - 2} = \dfrac{79.2 - 20.6}{2} = 29.3$ ft/s.

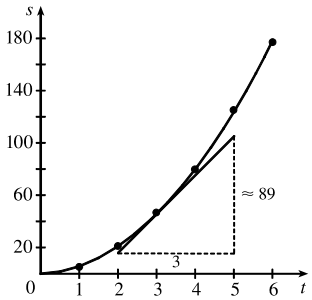
(ii) On the interval $[3, 4]$, $v_{\text{avg}} = \dfrac{s(4) - s(3)}{4 - 3} = \dfrac{79.2 - 46.5}{1} = 32.7$ ft/s.

(iii) On the interval $[4, 5]$, $v_{\text{avg}} = \dfrac{s(5) - s(4)}{5 - 4} = \dfrac{124.8 - 79.2}{1} = 45.6$ ft/s.

(iv) On the interval $[4, 6]$, $v_{\text{avg}} = \dfrac{s(6) - s(4)}{6 - 4} = \dfrac{176.7 - 79.2}{2} = 48.75$ ft/s.

(b) Using the points $(2, 16)$ and $(5, 105)$ from the approximate tangent line, the instantaneous velocity at $t = 3$ is about

$$\frac{105 - 16}{5 - 2} = \frac{89}{3} \approx 29.7 \text{ ft/s}.$$



8. (a) (i) $s = s(t) = 2\sin \pi t + 3\cos \pi t$. On the interval $[1, 2]$, $v_{\text{avg}} = \dfrac{s(2) - s(1)}{2 - 1} = \dfrac{3 - (-3)}{1} = 6$ cm/s.

(ii) On the interval $[1, 1.1]$, $v_{\text{avg}} = \dfrac{s(1.1) - s(1)}{1.1 - 1} \approx \dfrac{-3.471 - (-3)}{0.1} = -4.71$ cm/s.

(iii) On the interval $[1, 1.01]$, $v_{\text{avg}} = \dfrac{s(1.01) - s(1)}{1.01 - 1} \approx \dfrac{-3.0613 - (-3)}{0.01} = -6.13$ cm/s.

(iv) On the interval $[1, 1.001]$, $v_{\text{avg}} = \dfrac{s(1.001) - s(1)}{1.001 - 1} \approx \dfrac{-3.00627 - (-3)}{0.001} = -6.27$ cm/s.

(b) The instantaneous velocity of the particle when $t = 1$ appears to be about -6.3 cm/s.

9. (a) For the curve $y = \sin(10\pi/x)$ and the point $P(1, 0)$:

x	Q	m_{PQ}
2	$(2, 0)$	0
1.5	$(1.5, 0.8660)$	1.7321
1.4	$(1.4, -0.4339)$	-1.0847
1.3	$(1.3, -0.8230)$	-2.7433
1.2	$(1.2, 0.8660)$	4.3301
1.1	$(1.1, -0.2817)$	-2.8173

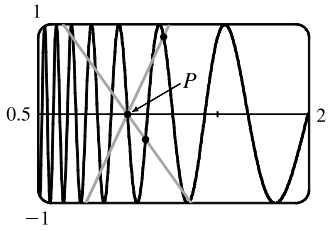
x	Q	m_{PQ}
0.5	$(0.5, 0)$	0
0.6	$(0.6, 0.8660)$	-2.1651
0.7	$(0.7, 0.7818)$	-2.6061
0.8	$(0.8, 1)$	-5
0.9	$(0.9, -0.3420)$	3.4202

As x approaches 1, the slopes do not appear to be approaching any particular value.

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(b)



We see that problems with estimation are caused by the frequent oscillations of the graph. The tangent is so steep at P that we need to take x-values much closer to 1 in order to get accurate estimates of its slope.

(c) If we choose $x = 1.001$, then the point Q is $(1.001, -0.0314)$ and $m_{PQ} \approx -31.3794$. If $x = 0.999$, then Q is $(0.999, 0.0314)$ and $m_{PQ} = -31.4422$. The average of these slopes is -31.4108. So we estimate that the slope of the tangent line at P is about -31.4.

1.5 The Limit of a Function

1. As x approaches 2, $f(x)$ approaches 5. [Or, the values of $f(x)$ can be made as close to 5 as we like by taking x sufficiently close to 2 (but $x \neq 2$).] Yes, the graph could have a hole at $(2, 5)$ and be defined such that $f(2) = 3$.

2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 7. No, the limit does not exist because the left- and right-hand limits are different.

3. (a) $\lim_{x \to -3} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to -3 (but not equal to -3).

(b) $\lim_{x \to 4^+} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to 4 through values larger than 4.

4. (a) As x approaches 2 from the left, the values of $f(x)$ approach 3, so $\lim_{x \to 2^-} f(x) = 3$.

(b) As x approaches 2 from the right, the values of $f(x)$ approach 1, so $\lim_{x \to 2^+} f(x) = 1$.

(c) $\lim_{x \to 2} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

(d) When $x = 2$, $y = 3$, so $f(2) = 3$.

(e) As x approaches 4, the values of $f(x)$ approach 4, so $\lim_{x \to 4} f(x) = 4$.

(f) There is no value of $f(x)$ when $x = 4$, so $f(4)$ does not exist.

5. (a) As x approaches 1, the values of $f(x)$ approach 2, so $\lim_{x \to 1} f(x) = 2$.

(b) As x approaches 3 from the left, the values of $f(x)$ approach 1, so $\lim_{x \to 3^-} f(x) = 1$.

(c) As x approaches 3 from the right, the values of $f(x)$ approach 4, so $\lim_{x \to 3^+} f(x) = 4$.

(d) $\lim_{x \to 3} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

(e) When $x = 3$, $y = 3$, so $f(3) = 3$.

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6. (a) $h(x)$ approaches 4 as x approaches -3 from the left, so $\lim_{x \to -3^-} h(x) = 4$.

(b) $h(x)$ approaches 4 as x approaches -3 from the right, so $\lim_{x \to -3^+} h(x) = 4$.

(c) $\lim_{x \to -3} h(x) = 4$ because the limits in part (a) and part (b) are equal.

(d) $h(-3)$ is not defined, so it doesn't exist.

(e) $h(x)$ approaches 1 as x approaches 0 from the left, so $\lim_{x \to 0^-} h(x) = 1$.

(f) $h(x)$ approaches -1 as x approaches 0 from the right, so $\lim_{x \to 0^+} h(x) = -1$.

(g) $\lim_{x \to 0} h(x)$ does not exist because the limits in part (e) and part (f) are not equal.

(h) $h(0) = 1$ since the point $(0, 1)$ is on the graph of h.

(i) Since $\lim_{x \to 2^-} h(x) = 2$ and $\lim_{x \to 2^+} h(x) = 2$, we have $\lim_{x \to 2} h(x) = 2$.

(j) $h(2)$ is not defined, so it doesn't exist.

(k) $h(x)$ approaches 3 as x approaches 5 from the right, so $\lim_{x \to 5^+} h(x) = 3$.

(l) $h(x)$ does not approach any one number as x approaches 5 from the left, so $\lim_{x \to 5^-} h(x)$ does not exist.

7. (a) $\lim_{x \to 4^-} g(x) \neq \lim_{x \to 4^+} g(x)$, so $\lim_{x \to 4} g(x)$ does not exist. However, there is a point on the graph representing $g(4)$. Thus, $a = 4$ satisfies the given description.

(b) $\lim_{x \to 5^-} g(x) = \lim_{x \to 5^+} g(x)$, so $\lim_{x \to 5} g(x)$ exists. However, $g(5)$ is not defined. Thus, $a = 5$ satisfies the given description.

(c) From part (a), $a = 4$ satisfies the given description. Also, $\lim_{x \to 2^-} g(x)$ and $\lim_{x \to 2^+} g(x)$ exist, but $\lim_{x \to 2^-} g(x) \neq \lim_{x \to 2^+} g(x)$. Thus, $\lim_{x \to 2} g(x)$ does not exist, and $a = 2$ also satisfies the given description.

(d) $\lim_{x \to 4^+} g(x) = g(4)$, but $\lim_{x \to 4^-} g(x) \neq g(4)$. Thus, $a = 4$ satisfies the given description.

8. (a) $\lim_{x \to -3} A(x) = \infty$ (b) $\lim_{x \to 2^-} A(x) = -\infty$

(c) $\lim_{x \to 2^+} A(x) = \infty$ (d) $\lim_{x \to -1} A(x) = -\infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = -1$ and $x = 2$.

9. (a) $\lim_{x \to -7} f(x) = -\infty$ (b) $\lim_{x \to -3} f(x) = \infty$ (c) $\lim_{x \to 0} f(x) = \infty$

(d) $\lim_{x \to 6^-} f(x) = -\infty$ (e) $\lim_{x \to 6^+} f(x) = \infty$

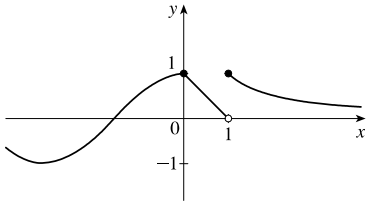
(f) The equations of the vertical asymptotes are $x = -7$, $x = -3$, $x = 0$, and $x = 6$.

10. $\lim_{t \to 12^-} f(t) = 150$ mg and $\lim_{t \to 12^+} f(t) = 300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t = 12$ h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

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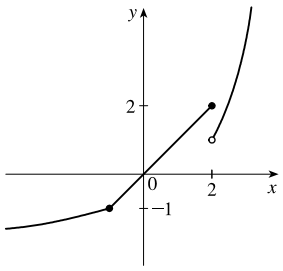
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11.



From the grapn of f we see that $\lim_{x\to 1^-} f(x) = 0$, but $\lim_{x\to 1^+} f(x) = 1$, so $\lim_{x\to a} f(x)$ does not exist for $a = 1$. However, $\lim_{x\to a} f(x)$ exists for all other values of a. Thus, $\lim_{x\to a} f(x)$ exists for all a in $(-\infty, 1) \cup (1, \infty)$.

12.

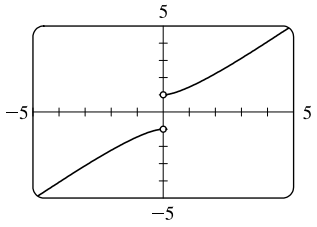


From the graph of f we see that $\lim_{x\to 2^-} f(x) = 2$, but $\lim_{x\to 2^+} f(x) = 1$, so $\lim_{x\to a} f(x)$ does not exist for $a = 2$. However, $\lim_{x\to a} f(x)$ exists for all other values of a. Thus, $\lim_{x\to a} f(x)$ exists for all a in $(-\infty, 2) \cup (2, \infty)$.

13. (a) From the graph, $\lim_{x\to 0^-} f(x) = -1$.

(b) From the graph, $\lim_{x\to 0^+} f(x) = 1$.

(c) Since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x)$ does not exist.

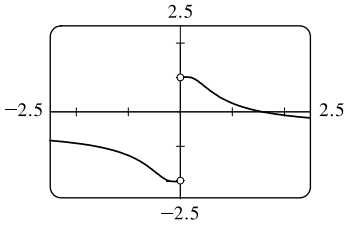


$f(x) = x\sqrt{1 + x^{-2}}$

14. (a) From the graph, $\lim_{x\to 0^-} f(x) = -2$.

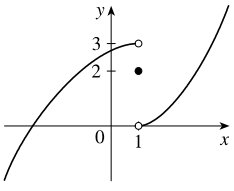
(b) From the graph, $\lim_{x\to 0^+} f(x) = 1$.

(c) Since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$, $\lim_{x\to 0} f(x)$ does not exist.

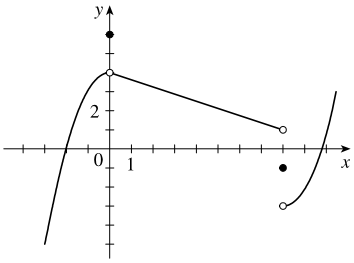


$f(x) = \dfrac{3^{1/x} - 2}{3^{1/x} + 1}$

15. $\lim_{x\to 1^-} f(x) = 3$, $\lim_{x\to 1^+} f(x) = 0$, $f(1) = 2$



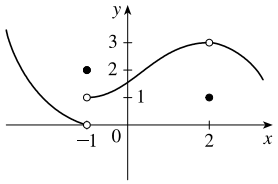
16. $\lim_{x\to 0} f(x) = 4$, $\lim_{x\to 8^-} f(x) = 1$, $\lim_{x\to 8^+} f(x) = -3$, $f(0) = 6$, $f(8) = -1$



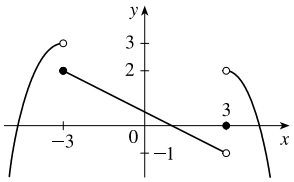
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17. $\lim_{x\to -1^-} f(x) = 0$, $\lim_{x\to -1^+} f(x) = 1$, $\lim_{x\to 2} f(x) = 3$, $f(-1) = 2$, $f(2) = 1$



18. $\lim_{x\to -3^-} f(x) = 3$, $\lim_{x\to -3^+} f(x) = 2$, $\lim_{x\to 3^-} f(x) = -1$, $\lim_{x\to 3^+} f(x) = 2$, $f(-3) = 2$, $f(3) = 0$



19. For $f(x) = \dfrac{x^2 - 3x}{x^2 - 9}$:

x	$f(x)$
3.1	0.508 197
3.05	0.504 132
3.01	0.500 832
3.001	0.500 083
3.0001	0.500 008

x	$f(x)$
2.9	0.491 525
2.95	0.495 798
2.99	0.499 165
2.999	0.499 917
2.9999	0.499 992

It appears that $\lim_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 9} = \dfrac{1}{2}$.

20. For $f(x) = \dfrac{x^2 - 3x}{x^2 - 9}$:

x	$f(x)$
−2.5	−5
−2.9	−29
−2.95	−59
−2.99	−299
−2.999	−2999
−2.9999	−29,999

x	$f(x)$
−3.5	7
−3.1	31
−3.05	61
−3.01	301
−3.001	3001
−3.0001	30,001

It appears that $\lim_{x\to -3^+} f(x) = -\infty$ and that $\lim_{x\to -3^-} f(x) = \infty$, so $\lim_{x\to -3} \dfrac{x^2 - 3x}{x^2 - 9}$ does not exist.

21. For $f(x) = \dfrac{\sin x}{x + \tan x}$:

x	$f(x)$
±1	0.329 033
±0.5	0.458 209
±0.2	0.493 331
±0.1	0.498 333
±0.05	0.499 583
±0.01	0.499 983

It appears that $\lim_{x\to 0} \dfrac{\sin x}{x + \tan x} = 0.5 = \dfrac{1}{2}$.

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22. For $f(h) = \dfrac{(2+h)^5 - 32}{h}$:

h	$f(h)$
0.5	131.312 500
0.1	88.410 100
0.01	80.804 010
0.001	80.080 040
0.0001	80.008 000

h	$f(h)$
−0.5	48.812 500
−0.1	72.390 100
−0.01	79.203 990
−0.001	79.920 040
−0.0001	79.992 000

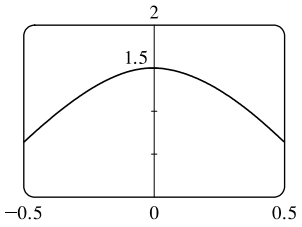
It appears that $\lim\limits_{h\to 0} \dfrac{(2+h)^5 - 32}{h} = 80$.

23. For $f(\theta) = \dfrac{\sin 3\theta}{\tan 2\theta}$:

θ	$f(\theta)$
±0.1	1.457 847
±0.01	1.499 575
±0.001	1.499 996
±0.0001	1.500 000

It appears that $\lim\limits_{\theta\to 0} \dfrac{\sin 3\theta}{\tan 2\theta} = 1.5$.

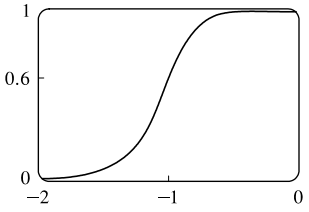
The graph confirms that result.



24. For $f(p) = \dfrac{1+p^9}{1+p^{15}}$:

p	$f(p)$
−1.1	0.427 397
−1.01	0.582 008
−1.001	0.598 200
−1.0001	0.599 820

p	$f(p)$
−0.9	0.771 405
−0.99	0.617 992
−0.999	0.601 800
−0.9999	0.600 180



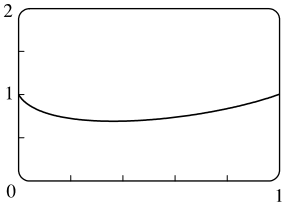
It appears that $\lim\limits_{p\to -1} f(p) = 0.6$. The graph confirms that result.

25. For $f(x) = x^x$:

x	$f(x)$
0.1	0.794 328
0.01	0.954 993
0.001	0.993 116
0.0001	0.999 079

It appears that $\lim\limits_{x\to 0^+} f(x) = 1$.

The graph confirms that result.



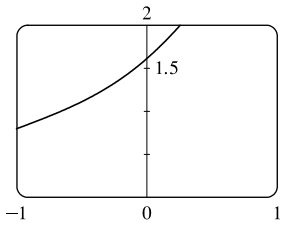
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26. For $f(t) = \dfrac{5^t - 1}{t}$:

t	$f(t)$
0.1	1.746 189
0.01	1.622 459
0.001	1.610 734
0.0001	1.609 567

t	$f(t)$
-0.1	1.486 601
-0.01	1.596 556
-0.001	1.608 143
-0.0001	1.609 308



It appears that $\lim_{t \to 0} f(t) \approx 1.6094$. The graph confirms that result.

27. $\lim_{x \to 5^+} \dfrac{x+1}{x-5} = \infty$ since the numerator is positive and the denominator approaches 0 from the positive side as $x \to 5^+$.

28. $\lim_{x \to 5^-} \dfrac{x+1}{x-5} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 5^-$.

29. $\lim_{x \to 2} \dfrac{x^2}{(x-2)^2} = \infty$ since the numerator is positive and the denominator approaches 0 through positive values as $x \to 2$.

30. $\lim_{x \to 3^-} \dfrac{\sqrt{x}}{(x-3)^5} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 3^-$.

31. $\lim_{x \to -2^+} \dfrac{x-1}{x^2(x+2)} = -\infty$ since $(x+2) \to 0$ as $x \to -2^+$ and $\dfrac{x-1}{x^2(x+2)} < 0$ for $-2 < x < 0$.

32. $\lim_{x \to 0} \dfrac{x-1}{x^2(x+2)} = -\infty$ since $x^2 \to 0$ as $x \to 0$ and $\dfrac{x-1}{x^2(x+2)} < 0$ for $0 < x < 1$ and for $-2 < x < 0$.

33. $\lim_{x \to (\pi/2)^+} \dfrac{1}{x} \sec x = -\infty$ since $\dfrac{1}{x}$ is positive and $\sec x \to -\infty$ as $x \to (\pi/2)^+$.

34. $\lim_{x \to \pi^-} x \cot x = -\infty$ since x is positive and $\cot x \to -\infty$ as $x \to \pi^-$.

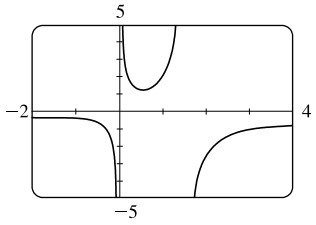
35. $\lim_{x \to 1} \dfrac{x^2+2x}{x^2-2x+1} = \lim_{x \to 1} \dfrac{x^2+2x}{(x-1)^2} = \infty$ since the numerator is positive and the denominator approaches 0 through positive values as $x \to 1$.

36. $\lim_{x \to 3^-} \dfrac{x^2+4x}{x^2-2x-3} = \lim_{x \to 3^-} \dfrac{x^2+4x}{(x-3)(x+1)} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \to 3^-$.

37. The denominator of $f(x) = \dfrac{x-1}{2x+4}$ is equal to 0 when $x = -2$ (and the numerator is not), so $x = -2$ is the vertical asymptote of the function.

38. (a) The denominator of $y = \dfrac{x^2+1}{3x-2x^2} = \dfrac{x^2+1}{x(3-2x)}$ is equal to zero when $x = 0$ and $x = \frac{3}{2}$ (and the numerator is not), so $x = 0$ and $x = 1.5$ are vertical asymptotes of the function.

(b)



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39. (a) $f(x) = \dfrac{1}{x^3 - 1}$.

x	$f(x)$
0.5	−1.14
0.9	−3.69
0.99	−33.7
0.999	−333.7
0.9999	−3333.7
0.99999	−33,333.7

x	$f(x)$
1.5	0.42
1.1	3.02
1.01	33.0
1.001	333.0
1.0001	3333.0
1.00001	33,333.3

From these calculations, it seems that

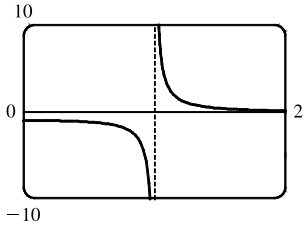
$\lim_{x \to 1^-} f(x) = -\infty$ and $\lim_{x \to 1^+} f(x) = \infty$.

(b) If x is slightly smaller than 1, then $x^3 - 1$ will be a negative number close to 0, and the reciprocal of $x^3 - 1$, that is, $f(x)$, will be a negative number with large absolute value. So $\lim_{x \to 1^-} f(x) = -\infty$.

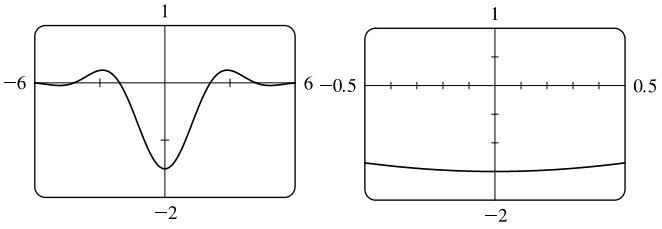
If x is slightly larger than 1, then $x^3 - 1$ will be a small positive number, and its reciprocal, $f(x)$, will be a large positive number. So $\lim_{x \to 1^+} f(x) = \infty$.

(c) It appears from the graph of f that

$\lim_{x \to 1^-} f(x) = -\infty$ and $\lim_{x \to 1^+} f(x) = \infty$.



40. (a) From the graphs, it seems that $\lim_{x \to 0} \dfrac{\cos 2x - \cos x}{x^2} = -1.5$.



(b)

x	$f(x)$
±0.1	−1.493 759
±0.01	−1.499 938
±0.001	−1.499 999
±0.0001	−1.500 000

41. For $f(x) = x^2 - (2^x/1000)$:

(a)

x	$f(x)$
1	0.998 000
0.8	0.638 259
0.6	0.358 484
0.4	0.158 680
0.2	0.038 851
0.1	0.008 928
0.05	0.001 465

It appears that $\lim_{x \to 0} f(x) = 0$.

(b)

x	$f(x)$
0.04	0.000 572
0.02	−0.000 614
0.01	−0.000 907
0.005	−0.000 978
0.003	−0.000 993
0.001	−0.001 000

It appears that $\lim_{x \to 0} f(x) = -0.001$.

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42. For $h(x) = \dfrac{\tan x - x}{x^3}$:

(a)

x	$h(x)$
1.0	0.557 407 73
0.5	0.370 419 92
0.1	0.334 672 09
0.05	0.333 667 00
0.01	0.333 346 67
0.005	0.333 336 67

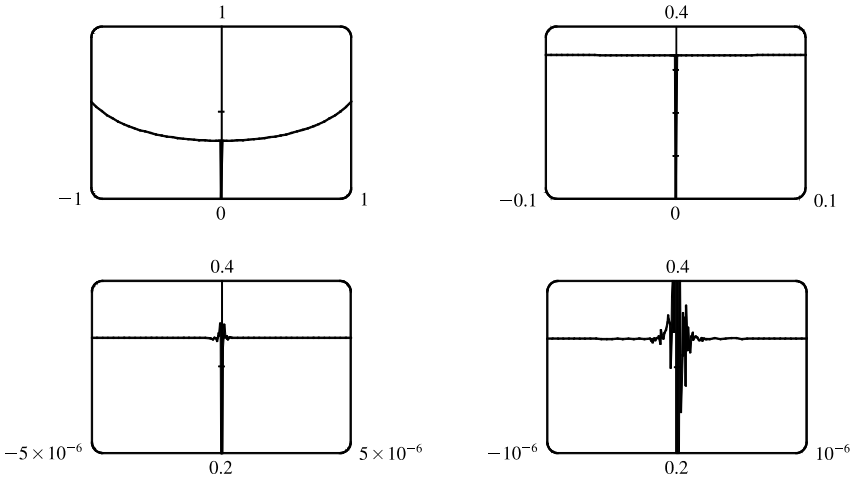
(b) It seems that $\lim_{x \to 0} h(x) = \frac{1}{3}$.

(c)

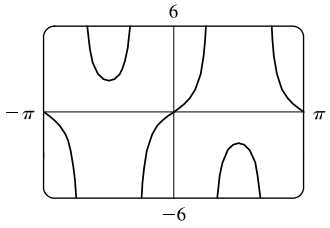
x	$h(x)$
0.001	0.333 333 50
0.0005	0.333 333 44
0.0001	0.333 330 00
0.00005	0.333 336 00
0.00001	0.333 000 00
0.000001	0.000 000 00

Here the values will vary from one calculator to another. Every calculator will eventually give *false values*.

(d) As in part (c), when we take a small enough viewing rectangle we get incorrect output.



43.



There appear to be vertical asymptotes of the curve $y = \tan(2 \sin x)$ at $x \approx \pm 0.90$ and $x \approx \pm 2.24$. To find the exact equations of these asymptotes, we note that the graph of the tangent function has vertical asymptotes at $x = \frac{\pi}{2} + \pi n$. Thus, we must have $2 \sin x = \frac{\pi}{2} + \pi n$, or equivalently, $\sin x = \frac{\pi}{4} + \frac{\pi}{2} n$. Since $-1 \le \sin x \le 1$, we must have $\sin x = \pm \frac{\pi}{4}$ and so $x = \pm \sin^{-1} \frac{\pi}{4}$ (corresponding to $x \approx \pm 0.90$). Just as $150°$ is the reference angle for $30°$, $\pi - \sin^{-1} \frac{\pi}{4}$ is the reference angle for $\sin^{-1} \frac{\pi}{4}$. So $x = \pm\left(\pi - \sin^{-1} \frac{\pi}{4}\right)$ are also equations of vertical asymptotes (corresponding to $x \approx \pm 2.24$).

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44. (a) For any positive integer n, if $x = \dfrac{1}{n\pi}$, then $f(x) = \tan\dfrac{1}{x} = \tan(n\pi) = 0$. (Remember that the tangent function has period π.)

(b) For any nonnegative number n, if $x = \dfrac{4}{(4n+1)\pi}$, then

$$f(x) = \tan\frac{1}{x} = \tan\frac{(4n+1)\pi}{4} = \tan\left(\frac{4n\pi}{4} + \frac{\pi}{4}\right) = \tan\left(n\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

(c) From part (a), $f(x) = 0$ infinitely often as $x \to 0$. From part (b), $f(x) = 1$ infinitely often as $x \to 0$. Thus, $\lim\limits_{x\to 0} \tan\dfrac{1}{x}$ does not exist since $f(x)$ does not get close to a fixed number as $x \to 0$.

45. $\lim\limits_{v\to c^-} m = \lim\limits_{v\to c^-} \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. As $v \to c^-$, $\sqrt{1 - v^2/c^2} \to 0^+$, and $m \to \infty$.

1.6 Calculating Limits Using the Limit Laws

1. (a) $\lim\limits_{x\to 2}[f(x) + 5g(x)] = \lim\limits_{x\to 2} f(x) + \lim\limits_{x\to 2}[5g(x)]$ [Limit Law 1]

$= \lim\limits_{x\to 2} f(x) + 5\lim\limits_{x\to 2} g(x)$ [Limit Law 3]

$= 4 + 5(-2) = -6$

(b) $\lim\limits_{x\to 2}[g(x)]^3 = \left[\lim\limits_{x\to 2} g(x)\right]^3$ [Limit Law 6]

$= (-2)^3 = -8$

(c) $\lim\limits_{x\to 2}\sqrt{f(x)} = \sqrt{\lim\limits_{x\to 2} f(x)}$ [Limit Law 7]

$= \sqrt{4} = 2$

(d) $\lim\limits_{x\to 2}\dfrac{3f(x)}{g(x)} = \dfrac{\lim\limits_{x\to 2}[3f(x)]}{\lim\limits_{x\to 2} g(x)}$ [Limit Law 5]

$= \dfrac{3\lim\limits_{x\to 2} f(x)}{\lim\limits_{x\to 2} g(x)}$ [Limit Law 3]

$= \dfrac{3(4)}{-2} = -6$

(e) Because the limit of the denominator is 0, we can't use Limit Law 5. The given limit, $\lim\limits_{x\to 2}\dfrac{g(x)}{h(x)}$, does not exist because the denominator approaches 0 while the numerator approaches a nonzero number.

(f) $\lim\limits_{x\to 2}\dfrac{g(x)\,h(x)}{f(x)} = \dfrac{\lim\limits_{x\to 2}[g(x)\,h(x)]}{\lim\limits_{x\to 2} f(x)}$ [Limit Law 5]

$= \dfrac{\lim\limits_{x\to 2} g(x)\cdot\lim\limits_{x\to 2} h(x)}{\lim\limits_{x\to 2} f(x)}$ [Limit Law 4]

$= \dfrac{-2\cdot 0}{4} = 0$

2. (a) $\lim\limits_{x\to 2}[f(x) + g(x)] = \lim\limits_{x\to 2} f(x) + \lim\limits_{x\to 2} g(x)$ [Limit Law 1]

$= -1 + 2$

$= 1$

(b) $\lim\limits_{x\to 0} f(x)$ exists, but $\lim\limits_{x\to 0} g(x)$ does not exist, so we cannot apply Limit Law 2 to $\lim\limits_{x\to 0}[f(x) - g(x)]$. The limit does not exist.

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(c) $\lim_{x\to -1}[f(x)\,g(x)] = \lim_{x\to -1} f(x)\cdot \lim_{x\to -1} g(x)$ [Limit Law 4]
$= 1\cdot 2$
$= 2$

(d) $\lim_{x\to 3} f(x) = 1$, but $\lim_{x\to 3} g(x) = 0$, so we cannot apply Limit Law 5 to $\lim_{x\to 3} \dfrac{f(x)}{g(x)}$. The limit does not exist.

Note: $\lim_{x\to 3^-} \dfrac{f(x)}{g(x)} = \infty$ since $g(x)\to 0^+$ as $x\to 3^-$ and $\lim_{x\to 3^+} \dfrac{f(x)}{g(x)} = -\infty$ since $g(x)\to 0^-$ as $x\to 3^+$.

Therefore, the limit does not exist, even as an infinite limit.

(e) $\lim_{x\to 2}\left[x^2 f(x)\right] = \lim_{x\to 2} x^2 \cdot \lim_{x\to 2} f(x)$ [Limit Law 4]
$= 2^2\cdot(-1)$
$= -4$

(f) $f(-1) + \lim_{x\to -1} g(x) = 3 + 2$
$= 5$

3. $\lim_{x\to 5}(4x^2 - 5x) = \lim_{x\to 5}(4x^2) - \lim_{x\to 5}(5x)$ [Limit Law 2]
$= 4\lim_{x\to 5} x^2 - 5\lim_{x\to 5} x$ [3]
$= 4(5^2) - 5(5)$ [10, 9]
$= 75$

4. $\lim_{x\to -3}(2x^3 + 6x^2 - 9) = \lim_{x\to -3}(2x^3) + \lim_{x\to -3}(6x^2) - \lim_{x\to -3} 9$ [Limits Laws 1 and 2]
$= 2\lim_{x\to -3} x^3 + 6\lim_{x\to -3} x^2 - \lim_{x\to -3} 9$ [3]
$= 2(-3)^3 + 6(-3)^2 - 9$ [10, 8]
$= -9$

5. $\lim_{v\to 2}(v^2 + 2v)(2v^3 - 5) = \lim_{v\to 2}(v^2 + 2v)\cdot \lim_{v\to 2}(2v^3 - 5)$ [Limit Law 4]
$= \left(\lim_{v\to 2} v^2 + \lim_{v\to 2} 2v\right)\left(\lim_{v\to 2} 2v^3 - \lim_{v\to 2} 5\right)$ [1 and 2]
$= \left(\lim_{v\to 2} v^2 + 2\lim_{v\to 2} v\right)\left(2\lim_{v\to 2} v^3 - \lim_{v\to 2} 5\right)$ [3]
$= \left[2^2 + 2(2)\right]\left[2(2)^3 - 5\right]$ [10, 9, and 8]
$= (8)(11) = 88$

6. $\lim_{t\to 7} \dfrac{3t^2+1}{t^2-5t+2} = \dfrac{\lim_{t\to 7}(3t^2+1)}{\lim_{t\to 7}(t^2-5t+2)}$ [Limit Law 5]
$= \dfrac{\lim_{t\to 7} 3t^2 + \lim_{t\to 7} 1}{\lim_{t\to 7} t^2 - \lim_{t\to 7} 5t + \lim_{t\to 7} 2}$ [1 and 2]
$= \dfrac{3\lim_{t\to 7} t^2 + \lim_{t\to 7} 1}{\lim_{t\to 7} t^2 - 5\lim_{t\to 7} t + \lim_{t\to 7} 2}$ [3]
$= \dfrac{3(7^2)+1}{7^2 - 5(7) + 2}$ [10, 9, and 8]
$= \dfrac{148}{16} = \dfrac{37}{4}$

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7. $\lim\limits_{u\to-2}\sqrt{9-u^3+2u^2}=\sqrt{\lim\limits_{u\to-2}(9-u^3+2u^2)}$ [Limit Law 7]

$=\sqrt{\lim\limits_{u\to-2}9-\lim\limits_{u\to-2}u^3+\lim\limits_{u\to-2}2u^2}$ [2 and 1]

$=\sqrt{\lim\limits_{u\to-2}9-\lim\limits_{u\to-2}u^3+2\lim\limits_{u\to-2}u^2}$ [3]

$=\sqrt{9-(-2)^3+2(-2)^2}$ [8 and 10]

$=\sqrt{25}=5$

8. $\lim\limits_{x\to3}\sqrt[3]{x+5}\,(2x^2-3x)=\lim\limits_{x\to3}\sqrt[3]{x+5}\cdot\lim\limits_{x\to3}(2x^2-3x)$ [Limit Law 4]

$=\sqrt[3]{\lim\limits_{x\to3}(x+5)}\cdot\lim\limits_{x\to3}(2x^2-3x)$ [7]

$=\sqrt[3]{\lim\limits_{x\to3}x+\lim\limits_{x\to3}5}\cdot\left(\lim\limits_{x\to3}2x^2-\lim\limits_{x\to3}3x\right)$ [1 and 2]

$=\sqrt[3]{\lim\limits_{x\to3}x+\lim\limits_{x\to3}5}\cdot\left(2\lim\limits_{x\to3}x^2-3\lim\limits_{x\to3}x\right)$ [3]

$=\sqrt[3]{3+5}\cdot\left[2(3^2)-3(3)\right]$ [9, 8, and 10]

$=2\cdot(18-9)=18$

9. $\lim\limits_{t\to-1}\left(\dfrac{2t^5-t^4}{5t^2+4}\right)^3=\left(\lim\limits_{t\to-1}\dfrac{2t^5-t^4}{5t^2+4}\right)^3$ [Limit Law 6]

$=\left(\dfrac{\lim\limits_{t\to-1}(2t^5-t^4)}{\lim\limits_{t\to-1}(5t^2+4)}\right)^3$ [5]

$=\left(\dfrac{2\lim\limits_{t\to-1}t^5-\lim\limits_{t\to-1}t^4}{5\lim\limits_{t\to-1}t^2+\lim\limits_{t\to-1}4}\right)^3$ [3, 2, and 1]

$=\left(\dfrac{2(-1)^5-(-1)^4}{5(-1)^2+4}\right)^3$ [10 and 8]

$=\left(-\dfrac{3}{9}\right)^3=-\dfrac{1}{27}$

10. (a) The left-hand side of the equation is not defined for $x=2$, but the right-hand side is.

(b) Since the equation holds for all $x\neq2$, it follows that both sides of the equation approach the same limit as $x\to2$, just as in Example 3. Remember that in finding $\lim\limits_{x\to a}f(x)$, we never consider $x=a$.

11. $\lim\limits_{x\to-2}(3x-7)=3(-2)-7=-13$

12. $\lim\limits_{x\to6}\left(8-\frac{1}{2}x\right)=8-\frac{1}{2}(6)=5$

13. $\lim\limits_{t\to4}\dfrac{t^2-2t-8}{t-4}=\lim\limits_{t\to4}\dfrac{(t-4)(t+2)}{t-4}=\lim\limits_{t\to4}(t+2)=4+2=6$

14. $\lim\limits_{x\to-3}\dfrac{x^2+3x}{x^2-x-12}=\lim\limits_{x\to-3}\dfrac{x(x+3)}{(x-4)(x+3)}=\lim\limits_{x\to-3}\dfrac{x}{x-4}=\dfrac{-3}{-3-4}=\dfrac{3}{7}$

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15. $\lim\limits_{x\to 2}\dfrac{x^2+5x+4}{x-2}$ does not exist since $x-2\to 0$, but $x^2+5x+4\to 18$ as $x\to 2$.

16. $\lim\limits_{x\to 4}\dfrac{x^2+3x}{x^2-x-12}=\lim\limits_{x\to 4}\dfrac{x(x+3)}{(x-4)(x+3)}=\lim\limits_{x\to 4}\dfrac{x}{x-4}$. The last limit does not exist since $\lim\limits_{x\to 4^-}\dfrac{x}{x-4}=-\infty$ and $\lim\limits_{x\to 4^+}\dfrac{x}{x-4}=\infty$.

17. $\lim\limits_{x\to -2}\dfrac{x^2-x-6}{3x^2+5x-2}=\lim\limits_{x\to -2}\dfrac{(x-3)(x+2)}{(3x-1)(x+2)}=\lim\limits_{x\to -2}\dfrac{x-3}{3x-1}=\dfrac{-2-3}{3(-2)-1}=\dfrac{-5}{-7}=\dfrac{5}{7}$

18. $\lim\limits_{x\to -5}\dfrac{2x^2+9x-5}{x^2-25}=\lim\limits_{x\to -5}\dfrac{(2x-1)(x+5)}{(x-5)(x+5)}=\lim\limits_{x\to -5}\dfrac{2x-1}{x-5}=\dfrac{2(-5)-1}{-5-5}=\dfrac{-11}{-10}=\dfrac{11}{10}$

19. Factoring t^3-27 as the difference of two cubes, we have

$$\lim_{t\to 3}\frac{t^3-27}{t^2-9}=\lim_{t\to 3}\frac{(t-3)(t^2+3t+9)}{(t-3)(t+3)}=\lim_{t\to 3}\frac{t^2+3t+9}{t+3}=\frac{3^2+3(3)+9}{3+3}=\frac{27}{6}=\frac{9}{2}.$$

20. Factoring u^3+1 as the sum of two cubes, we have

$$\lim_{u\to -1}\frac{u+1}{u^3+1}=\lim_{u\to -1}\frac{u+1}{(u+1)(u^2-u+1)}=\lim_{u\to -1}\frac{1}{u^2-u+1}=\frac{1}{(-1)^2-(-1)+1}=\frac{1}{3}.$$

21. $\lim\limits_{h\to 0}\dfrac{(h-3)^2-9}{h}=\lim\limits_{h\to 0}\dfrac{h^2-6h+9-9}{h}=\lim\limits_{h\to 0}\dfrac{h^2-6h}{h}=\lim\limits_{h\to 0}\dfrac{h(h-6)}{h}=\lim\limits_{h\to 0}(h-6)=0-6=-6$

22. $\lim\limits_{x\to 9}\dfrac{9-x}{3-\sqrt{x}}=\lim\limits_{x\to 9}\dfrac{9-x}{3-\sqrt{x}}\cdot\dfrac{3+\sqrt{x}}{3+\sqrt{x}}=\lim\limits_{x\to 9}\dfrac{(9-x)(3+\sqrt{x}\,)}{9-x}=\lim\limits_{x\to 9}(3+\sqrt{x}\,)=3+\sqrt{9}=6$

23.
$$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}=\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}\cdot\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}=\lim_{h\to 0}\frac{\left(\sqrt{9+h}\,\right)^2-3^2}{h\left(\sqrt{9+h}+3\right)}=\lim_{h\to 0}\frac{(9+h)-9}{h\left(\sqrt{9+h}+3\right)}$$
$$=\lim_{h\to 0}\frac{h}{h\left(\sqrt{9+h}+3\right)}=\lim_{h\to 0}\frac{1}{\sqrt{9+h}+3}=\frac{1}{3+3}=\frac{1}{6}$$

24.
$$\lim_{x\to 2}\frac{2-x}{\sqrt{x+2}-2}=\lim_{x\to 2}\frac{2-x}{\sqrt{x+2}-2}\cdot\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}=\lim_{x\to 2}\frac{(2-x)\left(\sqrt{x+2}+2\right)}{\left(\sqrt{x+2}\right)^2-4}=\lim_{x\to 2}\frac{-(x-2)\left(\sqrt{x+2}+2\right)}{x-2}$$
$$=\lim_{x\to 2}\left[-\left(\sqrt{x+2}+2\right)\right]=-\left(\sqrt{4}+2\right)=-4$$

25. $\lim\limits_{x\to 3}\dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}=\lim\limits_{x\to 3}\dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}\cdot\dfrac{3x}{3x}=\lim\limits_{x\to 3}\dfrac{3-x}{3x(x-3)}=\lim\limits_{x\to 3}\dfrac{-1}{3x}=-\dfrac{1}{9}$

26.
$$\lim_{h\to 0}\frac{(-2+h)^{-1}+2^{-1}}{h}=\lim_{h\to 0}\frac{\dfrac{1}{h-2}+\dfrac{1}{2}}{h}=\lim_{h\to 0}\frac{\dfrac{2+(h-2)}{2(h-2)}}{h}=\lim_{h\to 0}\frac{2+(h-2)}{2h(h-2)}$$
$$=\lim_{h\to 0}\frac{h}{2h(h-2)}=\lim_{h\to 0}\frac{1}{2(h-2)}=\frac{1}{2(0-2)}=-\frac{1}{4}$$

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27. $\lim_{t\to 0}\dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}=\lim_{t\to 0}\dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}\cdot\dfrac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}=\lim_{t\to 0}\dfrac{\left(\sqrt{1+t}\right)^2-\left(\sqrt{1-t}\right)^2}{t\left(\sqrt{1+t}+\sqrt{1-t}\right)}$

$=\lim_{t\to 0}\dfrac{(1+t)-(1-t)}{t\left(\sqrt{1+t}+\sqrt{1-t}\right)}=\lim_{t\to 0}\dfrac{2t}{t\left(\sqrt{1+t}+\sqrt{1-t}\right)}=\lim_{t\to 0}\dfrac{2}{\sqrt{1+t}+\sqrt{1-t}}$

$=\dfrac{2}{\sqrt{1}+\sqrt{1}}=\dfrac{2}{2}=1$

28. $\lim_{t\to 0}\left(\dfrac{1}{t}-\dfrac{1}{t^2+t}\right)=\lim_{t\to 0}\left(\dfrac{1}{t}-\dfrac{1}{t(t+1)}\right)=\lim_{t\to 0}\dfrac{t+1-1}{t(t+1)}=\lim_{t\to 0}\dfrac{1}{t+1}=\dfrac{1}{0+1}=1$

29. $\lim_{x\to 16}\dfrac{4-\sqrt{x}}{16x-x^2}=\lim_{x\to 16}\dfrac{(4-\sqrt{x})(4+\sqrt{x})}{(16x-x^2)(4+\sqrt{x})}=\lim_{x\to 16}\dfrac{16-x}{x(16-x)(4+\sqrt{x})}$

$=\lim_{x\to 16}\dfrac{1}{x(4+\sqrt{x})}=\dfrac{1}{16\left(4+\sqrt{16}\right)}=\dfrac{1}{16(8)}=\dfrac{1}{128}$

30. $\lim_{x\to 2}\dfrac{x^2-4x+4}{x^4-3x^2-4}=\lim_{x\to 2}\dfrac{(x-2)^2}{(x^2-4)(x^2+1)}=\lim_{x\to 2}\dfrac{(x-2)^2}{(x+2)(x-2)(x^2+1)}$

$=\lim_{x\to 2}\dfrac{x-2}{(x+2)(x^2+1)}=\dfrac{0}{4\cdot 5}=0$

31. $\lim_{t\to 0}\left(\dfrac{1}{t\sqrt{1+t}}-\dfrac{1}{t}\right)=\lim_{t\to 0}\dfrac{1-\sqrt{1+t}}{t\sqrt{1+t}}=\lim_{t\to 0}\dfrac{\left(1-\sqrt{1+t}\right)\left(1+\sqrt{1+t}\right)}{t\sqrt{t+1}\left(1+\sqrt{1+t}\right)}=\lim_{t\to 0}\dfrac{-t}{t\sqrt{1+t}\left(1+\sqrt{1+t}\right)}$

$=\lim_{t\to 0}\dfrac{-1}{\sqrt{1+t}\left(1+\sqrt{1+t}\right)}=\dfrac{-1}{\sqrt{1+0}\left(1+\sqrt{1+0}\right)}=-\dfrac{1}{2}$

32. $\lim_{x\to -4}\dfrac{\sqrt{x^2+9}-5}{x+4}=\lim_{x\to -4}\dfrac{\left(\sqrt{x^2+9}-5\right)\left(\sqrt{x^2+9}+5\right)}{(x+4)\left(\sqrt{x^2+9}+5\right)}=\lim_{x\to -4}\dfrac{(x^2+9)-25}{(x+4)\left(\sqrt{x^2+9}+5\right)}$

$=\lim_{x\to -4}\dfrac{x^2-16}{(x+4)\left(\sqrt{x^2+9}+5\right)}=\lim_{x\to -4}\dfrac{(x+4)(x-4)}{(x+4)\left(\sqrt{x^2+9}+5\right)}$

$=\lim_{x\to -4}\dfrac{x-4}{\sqrt{x^2+9}+5}=\dfrac{-4-4}{\sqrt{16+9}+5}=\dfrac{-8}{5+5}=-\dfrac{4}{5}$

33. $\lim_{h\to 0}\dfrac{(x+h)^3-x^3}{h}=\lim_{h\to 0}\dfrac{(x^3+3x^2h+3xh^2+h^3)-x^3}{h}=\lim_{h\to 0}\dfrac{3x^2h+3xh^2+h^3}{h}$

$=\lim_{h\to 0}\dfrac{h(3x^2+3xh+h^2)}{h}=\lim_{h\to 0}(3x^2+3xh+h^2)=3x^2$

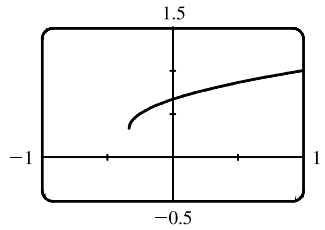
34. $\lim_{h\to 0}\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}=\lim_{h\to 0}\dfrac{\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}}{h}=\lim_{h\to 0}\dfrac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}=\lim_{h\to 0}\dfrac{-h(2x+h)}{hx^2(x+h)^2}$

$=\lim_{h\to 0}\dfrac{-(2x+h)}{x^2(x+h)^2}=\dfrac{-2x}{x^2\cdot x^2}=-\dfrac{2}{x^3}$

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35. (a)



$$\lim_{x\to 0}\frac{x}{\sqrt{1+3x}-1}\approx\frac{2}{3}$$

(b)

x	$f(x)$
−0.001	0.666 166 3
−0.000 1	0.666 616 7
−0.000 01	0.666 661 7
−0.000 001	0.666 666 2
0.000 001	0.666 667 2
0.000 01	0.666 671 7
0.000 1	0.666 716 7
0.001	0.667 166 3

The limit appears to be $\frac{2}{3}$.

(c) $\lim_{x\to 0}\left(\dfrac{x}{\sqrt{1+3x}-1}\cdot\dfrac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1}\right)=\lim_{x\to 0}\dfrac{x\left(\sqrt{1+3x}+1\right)}{(1+3x)-1}=\lim_{x\to 0}\dfrac{x\left(\sqrt{1+3x}+1\right)}{3x}$

$$=\frac{1}{3}\lim_{x\to 0}\left(\sqrt{1+3x}+1\right) \qquad \text{[Limit Law 3]}$$

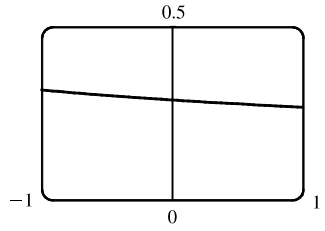
$$=\frac{1}{3}\left[\sqrt{\lim_{x\to 0}(1+3x)}+\lim_{x\to 0}1\right] \qquad \text{[1 and 7]}$$

$$=\frac{1}{3}\left(\sqrt{\lim_{x\to 0}1+3\lim_{x\to 0}x}+1\right) \qquad \text{[1, 3, and 8]}$$

$$=\frac{1}{3}\left(\sqrt{1+3\cdot 0}+1\right) \qquad \text{[8 and 9]}$$

$$=\frac{1}{3}(1+1)=\frac{2}{3}$$

36. (a)



$$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3}}{x}\approx 0.29$$

(b)

x	$f(x)$
−0.001	0.288 699 2
−0.000 1	0.288 677 5
−0.000 01	0.288 675 4
−0.000 001	0.288 675 2
0.000 001	0.288 675 1
0.000 01	0.288 674 9
0.000 1	0.288 672 7
0.001	0.288 651 1

The limit appears to be approximately 0.2887.

(c) $\lim_{x\to 0}\left(\dfrac{\sqrt{3+x}-\sqrt{3}}{x}\cdot\dfrac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}\right)=\lim_{x\to 0}\dfrac{(3+x)-3}{x\left(\sqrt{3+x}+\sqrt{3}\right)}=\lim_{x\to 0}\dfrac{1}{\sqrt{3+x}+\sqrt{3}}$

$$=\frac{\lim_{x\to 0}1}{\lim_{x\to 0}\sqrt{3+x}+\lim_{x\to 0}\sqrt{3}} \qquad \text{[Limit Laws 5 and 1]}$$

$$=\frac{1}{\sqrt{\lim_{x\to 0}(3+x)}+\sqrt{3}} \qquad \text{[7 and 8]}$$

$$=\frac{1}{\sqrt{3+0}+\sqrt{3}} \qquad \text{[1, 8, and 9]}$$

$$=\frac{1}{2\sqrt{3}}$$

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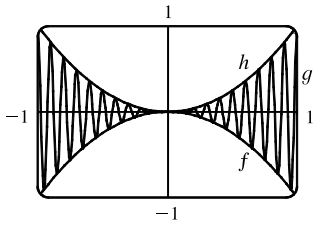
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37. Let $f(x) = -x^2$, $g(x) = x^2 \cos 20\pi x$ and $h(x) = x^2$. Then

$-1 \le \cos 20\pi x \le 1 \quad \Rightarrow \quad -x^2 \le x^2 \cos 20\pi x \le x^2 \quad \Rightarrow \quad f(x) \le g(x) \le h(x)$.

So since $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, by the Squeeze Theorem we have

$\lim_{x\to 0} g(x) = 0$.

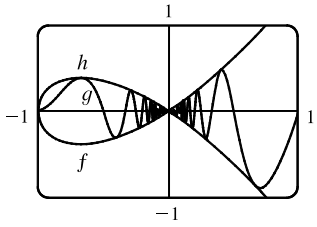


38. Let $f(x) = -\sqrt{x^3+x^2}$, $g(x) = \sqrt{x^3+x^2}\,\sin(\pi/x)$, and $h(x) = \sqrt{x^3+x^2}$. Then

$-1 \le \sin(\pi/x) \le 1 \quad \Rightarrow \quad -\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\sin(\pi/x) \le \sqrt{x^3+x^2} \quad \Rightarrow$

$f(x) \le g(x) \le h(x)$. So since $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, by the Squeeze Theorem

we have $\lim_{x\to 0} g(x) = 0$.



39. We have $\lim_{x\to 4}(4x-9) = 4(4)-9 = 7$ and $\lim_{x\to 4}\left(x^2-4x+7\right) = 4^2-4(4)+7 = 7$. Since $4x-9 \le f(x) \le x^2-4x+7$ for $x \ge 0$, $\lim_{x\to 4} f(x) = 7$ by the Squeeze Theorem.

40. We have $\lim_{x\to 1}(2x) = 2(1) = 2$ and $\lim_{x\to 1}(x^4-x^2+2) = 1^4-1^2+2 = 2$. Since $2x \le g(x) \le x^4-x^2+2$ for all x,

$\lim_{x\to 1} g(x) = 2$ by the Squeeze Theorem.

41. $-1 \le \cos(2/x) \le 1 \quad \Rightarrow \quad -x^4 \le x^4\cos(2/x) \le x^4$. Since $\lim_{x\to 0}\left(-x^4\right) = 0$ and $\lim_{x\to 0} x^4 = 0$, we have

$\lim_{x\to 0}\left[x^4\cos(2/x)\right] = 0$ by the Squeeze Theorem.

42. $-1 \le \sin(2\pi/x) \le 1 \quad \Rightarrow \quad 0 \le \sin^2(2\pi/x) \le 1 \quad \Rightarrow \quad 1 \le 1+\sin^2(2\pi/x) \le 2 \quad \Rightarrow$

$\sqrt{x} \le \sqrt{x}\left[1+\sin^2(2\pi/x)\right] \le 2\sqrt{x}$. Since $\lim_{x\to 0^+}\sqrt{x} = 0$ and $\lim_{x\to 0^+} 2\sqrt{x} = 0$, we have

$\lim_{x\to 0^+}\left[\sqrt{x}\,(1+\sin^2(2\pi/x))\right] = 0$ by the Squeeze Theorem.

43. $|x+4| = \begin{cases} x+4 & \text{if } x+4 \ge 0 \\ -(x+4) & \text{if } x+4 < 0 \end{cases} = \begin{cases} x+4 & \text{if } x \ge -4 \\ -(x+4) & \text{if } x < -4 \end{cases}$

Thus, $\lim_{x\to -4^+}(|x+4|-2x) = \lim_{x\to -4^+}(x+4-2x) = \lim_{x\to -4^+}(-x+4) = 4+4 = 8$ and

$\lim_{x\to -4^-}(|x+4|-2x) = \lim_{x\to -4^-}(-(x+4)-2x) = \lim_{x\to -4^-}(-3x-4) = 12-4 = 8$.

The left and right limits are equal, so $\lim_{x\to -4}(|x+4|-2x) = 8$.

44. $|x+4| = \begin{cases} x+4 & \text{if } x+4 \ge 0 \\ -(x+4) & \text{if } x+4 < 0 \end{cases} = \begin{cases} x+4 & \text{if } x \ge -4 \\ -(x+4) & \text{if } x < -4 \end{cases}$

Thus, $\lim_{x\to -4^+}\dfrac{|x+4|}{2x+8} = \lim_{x\to -4^+}\dfrac{x+4}{2x+8} = \lim_{x\to -4^+}\dfrac{x+4}{2(x+4)} = \lim_{x\to -4^+}\dfrac{1}{2} = \dfrac{1}{2}$ and

$\lim_{x\to -4^-}\dfrac{|x+4|}{2x+8} = \lim_{x\to -4^-}\dfrac{-(x+4)}{2x+8} = \lim_{x\to -4^-}\dfrac{-(x+4)}{2(x+4)} = \lim_{x\to -4^-}\dfrac{-1}{2} = -\dfrac{1}{2}$.

The left and right limits are different, so $\lim_{x\to -4}\dfrac{|x+4|}{2x+8}$ does not exist.

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45. $|2x^3 - x^2| = |x^2(2x-1)| = |x^2| \cdot |2x-1| = x^2\,|2x-1|$

$$|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \geq 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1 & \text{if } x \geq 0.5 \\ -(2x-1) & \text{if } x < 0.5 \end{cases}$$

So $|2x^3 - x^2| = x^2[-(2x-1)]$ for $x < 0.5$.

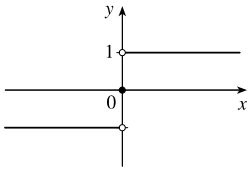
Thus, $\lim\limits_{x \to 0.5^-} \dfrac{2x-1}{|2x^3 - x^2|} = \lim\limits_{x \to 0.5^-} \dfrac{2x-1}{x^2[-(2x-1)]} = \lim\limits_{x \to 0.5^-} \dfrac{-1}{x^2} = \dfrac{-1}{(0.5)^2} = \dfrac{-1}{0.25} = -4.$

46. Since $|x| = -x$ for $x < 0$, we have $\lim\limits_{x \to -2} \dfrac{2-|x|}{2+x} = \lim\limits_{x \to -2} \dfrac{2-(-x)}{2+x} = \lim\limits_{x \to -2} \dfrac{2+x}{2+x} = \lim\limits_{x \to -2} 1 = 1.$

47. Since $|x| = -x$ for $x < 0$, we have $\lim\limits_{x \to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right) = \lim\limits_{x \to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{-x}\right) = \lim\limits_{x \to 0^-} \dfrac{2}{x}$, which does not exist since the denominator approaches 0 and the numerator does not.

48. Since $|x| = x$ for $x > 0$, we have $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right) = \lim\limits_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{x}\right) = \lim\limits_{x \to 0^+} 0 = 0.$

49. (a)



(b) (i) Since $\operatorname{sgn} x = 1$ for $x > 0$, $\lim\limits_{x \to 0^+} \operatorname{sgn} x = \lim\limits_{x \to 0^+} 1 = 1$.

(ii) Since $\operatorname{sgn} x = -1$ for $x < 0$, $\lim\limits_{x \to 0^-} \operatorname{sgn} x = \lim\limits_{x \to 0^-} -1 = -1$.

(iii) Since $\lim\limits_{x \to 0^-} \operatorname{sgn} x \neq \lim\limits_{x \to 0^+} \operatorname{sgn} x$, $\lim\limits_{x \to 0} \operatorname{sgn} x$ does not exist.

(iv) Since $|\operatorname{sgn} x| = 1$ for $x \neq 0$, $\lim\limits_{x \to 0} |\operatorname{sgn} x| = \lim\limits_{x \to 0} 1 = 1$.

50. (a) $g(x) = \operatorname{sgn}(\sin x) = \begin{cases} -1 & \text{if } \sin x < 0 \\ 0 & \text{if } \sin x = 0 \\ 1 & \text{if } \sin x > 0 \end{cases}$

(i) $\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} \operatorname{sgn}(\sin x) = 1$ since $\sin x$ is positive for small positive values of x.

(ii) $\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} \operatorname{sgn}(\sin x) = -1$ since $\sin x$ is negative for small negative values of x.

(iii) $\lim\limits_{x \to 0} g(x)$ does not exist since $\lim\limits_{x \to 0^+} g(x) \neq \lim\limits_{x \to 0^-} g(x)$.

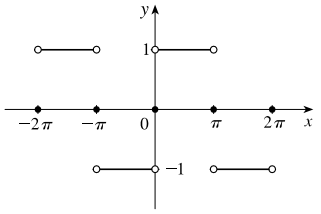
(iv) $\lim\limits_{x \to \pi^+} g(x) = \lim\limits_{x \to \pi^+} \operatorname{sgn}(\sin x) = -1$ since $\sin x$ is negative for values of x slightly greater than π.

(v) $\lim\limits_{x \to \pi^-} g(x) = \lim\limits_{x \to \pi^-} \operatorname{sgn}(\sin x) = 1$ since $\sin x$ is positive for values of x slightly less than π.

(vi) $\lim\limits_{x \to \pi} g(x)$ does not exist since $\lim\limits_{x \to \pi^+} g(x) \neq \lim\limits_{x \to \pi^-} g(x)$.

(b) The sine function changes sign at every integer multiple of π, so the signum function equals 1 on one side and -1 on the other side of $n\pi$, n an integer. Thus, $\lim\limits_{x \to a} g(x)$ does not exist for $a = n\pi$, n an integer.

(c)



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51. (a) (i) $\lim\limits_{x\to 2^+} g(x) = \lim\limits_{x\to 2^+} \dfrac{x^2+x-6}{|x-2|} = \lim\limits_{x\to 2^+} \dfrac{(x+3)(x-2)}{|x-2|}$

$= \lim\limits_{x\to 2^+} \dfrac{(x+3)(x-2)}{x-2}$ [since $x-2>0$ if $x\to 2^+$]

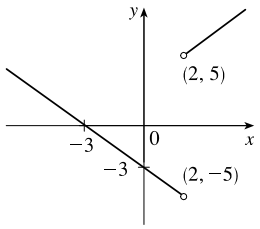
$= \lim\limits_{x\to 2^+} (x+3) = 5$

(ii) The solution is similar to the solution in part (i), but now $|x-2| = 2-x$ since $x-2<0$ if $x\to 2^-$.

Thus, $\lim\limits_{x\to 2^-} g(x) = \lim\limits_{x\to 2^-} -(x+3) = -5$.

(b) Since the right-hand and left-hand limits of g at $x=2$ are not equal, $\lim\limits_{x\to 2} g(x)$ does not exist.

(c)

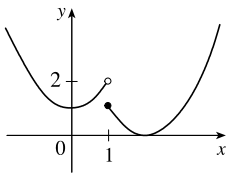


52. (a) $f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ (x-2)^2 & \text{if } x\geq 1 \end{cases}$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (x^2+1) = 1^2+1 = 2, \quad \lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1+} (x-2)^2 = (-1)^2 = 1$

(b) Since the right-hand and left-hand limits of f at $x=1$ are not equal, $\lim\limits_{x\to 1} f(x)$ does not exist.

(c)



53. For the $\lim\limits_{t\to 2} B(t)$ to exist, the one-sided limits at $t=2$ must be equal. $\lim\limits_{t\to 2^-} B(t) = \lim\limits_{t\to 2^-} \left(4-\frac{1}{2}t\right) = 4-1 = 3$ and

$\lim\limits_{t\to 2^+} B(t) = \lim\limits_{t\to 2^+} \sqrt{t+c} = \sqrt{2+c}$. Now $3 = \sqrt{2+c} \;\Rightarrow\; 9 = 2+c \;\Leftrightarrow\; c = 7$.

54. (a) (i) $\lim\limits_{x\to 1^-} g(x) = \lim\limits_{x\to 1^-} x = 1$

(ii) $\lim\limits_{x\to 1^+} g(x) = \lim\limits_{x\to 1^+} (2-x^2) = 2-1^2 = 1$. Since $\lim\limits_{x\to 1^-} g(x) = 1$ and $\lim\limits_{x\to 1^+} g(x) = 1$, we have $\lim\limits_{x\to 1} g(x) = 1$.

Note that the fact $g(1) = 3$ does not affect the value of the limit.

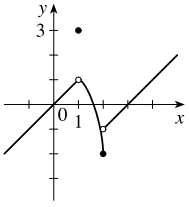
(iii) When $x=1$, $g(x) = 3$, so $g(1) = 3$.

(iv) $\lim\limits_{x\to 2^-} g(x) = \lim\limits_{x\to 2^-} (2-x^2) = 2-2^2 = 2-4 = -2$

(v) $\lim\limits_{x\to 2^+} g(x) = \lim\limits_{x\to 2^+} (x-3) = 2-3 = -1$

(vi) $\lim\limits_{x\to 2} g(x)$ does not exist since $\lim\limits_{x\to 2^-} g(x) \neq \lim\limits_{x\to 2^+} g(x)$.

(b) $g(x) = \begin{cases} x & \text{if } x<1 \\ 3 & \text{if } x=1 \\ 2-x^2 & \text{if } 1<x\leq 2 \\ x-3 & \text{if } x>2 \end{cases}$



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55. (a) (i) $[\![x]\!] = -2$ for $-2 \le x < -1$, so $\lim_{x \to -2^+} [\![x]\!] = \lim_{x \to -2^+} (-2) = -2$

(ii) $[\![x]\!] = -3$ for $-3 \le x < -2$, so $\lim_{x \to -2^-} [\![x]\!] = \lim_{x \to -2^-} (-3) = -3$.

The right and left limits are different, so $\lim_{x \to -2} [\![x]\!]$ does not exist.

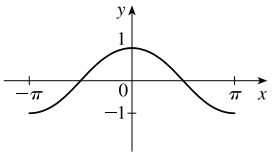
(iii) $[\![x]\!] = -3$ for $-3 \le x < -2$, so $\lim_{x \to -2.4} [\![x]\!] = \lim_{x \to -2.4} (-3) = -3$.

(b) (i) $[\![x]\!] = n - 1$ for $n - 1 \le x < n$, so $\lim_{x \to n^-} [\![x]\!] = \lim_{x \to n^-} (n - 1) = n - 1$.

(ii) $[\![x]\!] = n$ for $n \le x < n + 1$, so $\lim_{x \to n^+} [\![x]\!] = \lim_{x \to n^+} n = n$.

(c) $\lim_{x \to a} [\![x]\!]$ exists $\Leftrightarrow$ a is not an integer.

56. (a) See the graph of $y = \cos x$.

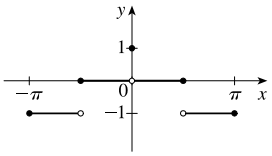


Since $-1 \le \cos x < 0$ on $[-\pi, -\pi/2)$, we have $y = f(x) = [\![\cos x]\!] = -1$ on $[-\pi, -\pi/2)$.

Since $0 \le \cos x < 1$ on $[-\pi/2, 0) \cup (0, \pi/2]$, we have $f(x) = 0$ on $[-\pi/2, 0) \cup (0, \pi/2]$.

Since $-1 \le \cos x < 0$ on $(\pi/2, \pi]$, we have $f(x) = -1$ on $(\pi/2, \pi]$.

Note that $f(0) = 1$.



(b) (i) $\lim_{x \to 0^-} f(x) = 0$ and $\lim_{x \to 0^+} f(x) = 0$, so $\lim_{x \to 0} f(x) = 0$.

(ii) As $x \to (\pi/2)^-$, $f(x) \to 0$, so $\lim_{x \to (\pi/2)^-} f(x) = 0$.

(iii) As $x \to (\pi/2)^+$, $f(x) \to -1$, so $\lim_{x \to (\pi/2)^+} f(x) = -1$.

(iv) Since the answers in parts (ii) and (iii) are not equal, $\lim_{x \to \pi/2} f(x)$ does not exist.

(c) $\lim_{x \to a} f(x)$ exists for all a in the open interval $(-\pi, \pi)$ except $a = -\pi/2$ and $a = \pi/2$.

57. The graph of $f(x) = [\![x]\!] + [\![-x]\!]$ is the same as the graph of $g(x) = -1$ with holes at each integer, since $f(a) = 0$ for any integer a. Thus, $\lim_{x \to 2^-} f(x) = -1$ and $\lim_{x \to 2^+} f(x) = -1$, so $\lim_{x \to 2} f(x) = -1$. However, $f(2) = [\![2]\!] + [\![-2]\!] = 2 + (-2) = 0$, so $\lim_{x \to 2} f(x) \ne f(2)$.

58. $\lim_{v \to c^-} \left(L_0 \sqrt{1 - \dfrac{v^2}{c^2}} \right) = L_0 \sqrt{1 - 1} = 0$. As the velocity approaches the speed of light, the length approaches 0.

A left-hand limit is necessary since L is not defined for $v > c$.

59. Since $p(x)$ is a polynomial, $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$. Thus, by the Limit Laws,

$$\lim_{x \to a} p(x) = \lim_{x \to a} \left(a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \right) = a_0 + a_1 \lim_{x \to a} x + a_2 \lim_{x \to a} x^2 + \cdots + a_n \lim_{x \to a} x^n$$
$$= a_0 + a_1 a + a_2 a^2 + \cdots + a_n a^n = p(a)$$

Thus, for any polynomial p, $\lim_{x \to a} p(x) = p(a)$.

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60. Let $r(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are any polynomials, and suppose that $q(a) \neq 0$. Then

$$\lim_{x\to a} r(x) = \lim_{x\to a} \frac{p(x)}{q(x)} = \frac{\lim\limits_{x\to a} p(x)}{\lim\limits_{x\to a} q(x)} \quad \text{[Limit Law 5]} \quad = \frac{p(a)}{q(a)} \quad \text{[Exercise 59]} \quad = r(a).$$

61. $\lim\limits_{x\to 1}[f(x)-8] = \lim\limits_{x\to 1}\left[\dfrac{f(x)-8}{x-1}\cdot(x-1)\right] = \lim\limits_{x\to 1}\dfrac{f(x)-8}{x-1}\cdot\lim\limits_{x\to 1}(x-1) = 10\cdot 0 = 0.$

Thus, $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1}\{[f(x)-8]+8\} = \lim\limits_{x\to 1}[f(x)-8] + \lim\limits_{x\to 1} 8 = 0+8 = 8.$

Note: The value of $\lim\limits_{x\to 1}\dfrac{f(x)-8}{x-1}$ does not affect the answer since it's multiplied by 0. What's important is that

$\lim\limits_{x\to 1}\dfrac{f(x)-8}{x-1}$ exists.

62. (a) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0}\left[\dfrac{f(x)}{x^2}\cdot x^2\right] = \lim\limits_{x\to 0}\dfrac{f(x)}{x^2}\cdot\lim\limits_{x\to 0} x^2 = 5\cdot 0 = 0$

(b) $\lim\limits_{x\to 0}\dfrac{f(x)}{x} = \lim\limits_{x\to 0}\left[\dfrac{f(x)}{x^2}\cdot x\right] = \lim\limits_{x\to 0}\dfrac{f(x)}{x^2}\cdot\lim\limits_{x\to 0} x = 5\cdot 0 = 0$

63. Observe that $0 \le f(x) \le x^2$ for all x, and $\lim\limits_{x\to 0} 0 = 0 = \lim\limits_{x\to 0} x^2$. So, by the Squeeze Theorem, $\lim\limits_{x\to 0} f(x) = 0$.

64. Let $f(x) = [\![x]\!]$ and $g(x) = -[\![x]\!]$. Then $\lim\limits_{x\to 3} f(x)$ and $\lim\limits_{x\to 3} g(x)$ do not exist [Example 10]

but $\lim\limits_{x\to 3}[f(x)+g(x)] = \lim\limits_{x\to 3}([\![x]\!]-[\![x]\!]) = \lim\limits_{x\to 3} 0 = 0.$

65. Let $f(x) = H(x)$ and $g(x) = 1 - H(x)$, where H is the Heaviside function defined in Exercise 1.3.63.

Thus, either f or g is 0 for any value of x. Then $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 0} g(x)$ do not exist, but $\lim\limits_{x\to 0}[f(x)g(x)] = \lim\limits_{x\to 0} 0 = 0.$

66. $\lim\limits_{x\to 2}\dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1} = \lim\limits_{x\to 2}\left(\dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}\cdot\dfrac{\sqrt{6-x}+2}{\sqrt{6-x}+2}\cdot\dfrac{\sqrt{3-x}+1}{\sqrt{3-x}+1}\right)$

$$= \lim_{x\to 2}\left[\frac{\left(\sqrt{6-x}\right)^2-2^2}{\left(\sqrt{3-x}\right)^2-1^2}\cdot\frac{\sqrt{3-x}+1}{\sqrt{6-x}+2}\right] = \lim_{x\to 2}\left(\frac{6-x-4}{3-x-1}\cdot\frac{\sqrt{3-x}+1}{\sqrt{6-x}+2}\right)$$

$$= \lim_{x\to 2}\frac{(2-x)\left(\sqrt{3-x}+1\right)}{(2-x)\left(\sqrt{6-x}+2\right)} = \lim_{x\to 2}\frac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \frac{1}{2}$$

67. Since the denominator approaches 0 as $x \to -2$, the limit will exist only if the numerator also approaches

0 as $x \to -2$. In order for this to happen, we need $\lim\limits_{x\to -2}\left(3x^2+ax+a+3\right) = 0 \quad \Leftrightarrow$

$3(-2)^2 + a(-2) + a + 3 = 0 \quad \Leftrightarrow \quad 12 - 2a + a + 3 = 0 \quad \Leftrightarrow \quad a = 15$. With $a = 15$, the limit becomes

$$\lim_{x\to -2}\frac{3x^2+15x+18}{x^2+x-2} = \lim_{x\to -2}\frac{3(x+2)(x+3)}{(x-1)(x+2)} = \lim_{x\to -2}\frac{3(x+3)}{x-1} = \frac{3(-2+3)}{-2-1} = \frac{3}{-3} = -1.$$

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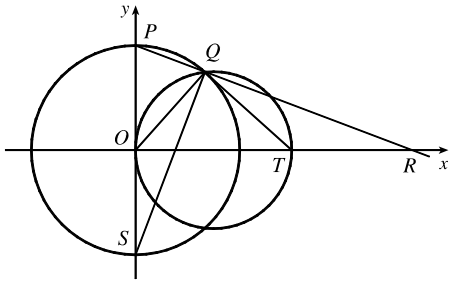
68. *Solution 1:* First, we find the coordinates of P and Q as functions of r. Then we can find the equation of the line determined by these two points, and thus find the x-intercept (the point R), and take the limit as $r \to 0$. The coordinates of P are $(0, r)$. The point Q is the point of intersection of the two circles $x^2 + y^2 = r^2$ and $(x-1)^2 + y^2 = 1$. Eliminating y from these equations, we get $r^2 - x^2 = 1 - (x-1)^2 \quad \Leftrightarrow \quad r^2 = 1 + 2x - 1 \quad \Leftrightarrow \quad x = \frac{1}{2}r^2$. Substituting back into the equation of the shrinking circle to find the y-coordinate, we get $\left(\frac{1}{2}r^2\right)^2 + y^2 = r^2 \quad \Leftrightarrow \quad y^2 = r^2\left(1 - \frac{1}{4}r^2\right) \quad \Leftrightarrow \quad y = r\sqrt{1 - \frac{1}{4}r^2}$ (the positive y-value). So the coordinates of Q are $\left(\frac{1}{2}r^2, r\sqrt{1 - \frac{1}{4}r^2}\right)$. The equation of the line joining P and Q is thus $y - r = \dfrac{r\sqrt{1 - \frac{1}{4}r^2} - r}{\frac{1}{2}r^2 - 0}(x - 0)$. We set $y = 0$ in order to find the x-intercept, and get

$$x = -r\frac{\frac{1}{2}r^2}{r\left(\sqrt{1 - \frac{1}{4}r^2} - 1\right)} = \frac{-\frac{1}{2}r^2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right)}{1 - \frac{1}{4}r^2 - 1} = 2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right)$$

Now we take the limit as $r \to 0^+$: $\lim\limits_{r\to 0^+} x = \lim\limits_{r\to 0^+} 2\left(\sqrt{1 - \frac{1}{4}r^2} + 1\right) = \lim\limits_{r\to 0^+} 2\left(\sqrt{1} + 1\right) = 4$.

So the limiting position of R is the point $(4, 0)$.

Solution 2: We add a few lines to the diagram, as shown. Note that $\angle PQS = 90°$ (subtended by diameter PS). So $\angle SQR = 90° = \angle OQT$ (subtended by diameter OT). It follows that $\angle OQS = \angle TQR$. Also $\angle PSQ = 90° - \angle SPQ = \angle ORP$. Since $\triangle QOS$ is isosceles, so is $\triangle QTR$, implying that $QT = TR$. As the circle C_2 shrinks, the point Q plainly approaches the origin, so the point R must approach a point twice as far from the origin as T, that is, the point $(4, 0)$, as above.



1.7 The Precise Definition of a Limit

1. If $|f(x) - 1| < 0.2$, then $-0.2 < f(x) - 1 < 0.2 \quad \Rightarrow \quad 0.8 < f(x) < 1.2$. From the graph, we see that the last inequality is true if $0.7 < x < 1.1$, so we can choose $\delta = \min\{1 - 0.7, 1.1 - 1\} = \min\{0.3, 0.1\} = 0.1$ (or any smaller positive number).

2. If $|f(x) - 2| < 0.5$, then $-0.5 < f(x) - 2 < 0.5 \quad \Rightarrow \quad 1.5 < f(x) < 2.5$. From the graph, we see that the last inequality is true if $2.6 < x < 3.8$, so we can take $\delta = \min\{3 - 2.6, 3.8 - 3\} = \min\{0.4, 0.8\} = 0.4$ (or any smaller positive number). Note that $x \neq 3$.

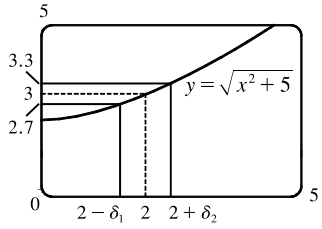
3. On the given graph, the leftmost question mark is the solution of $\sqrt{x} = 1.6$ and the rightmost, $\sqrt{x} = 2.4$. So the values are $1.6^2 = 2.56$ and $2.4^2 = 5.76$. On the left side, we need $|x - 4| < |2.56 - 4| = 1.44$. On the right side, we need $|x - 4| < |5.76 - 4| = 1.76$. To satisfy both conditions, we need the more restrictive condition to hold—namely, $|x - 4| < 1.44$. Thus, we can choose $\delta = 1.44$, or any smaller positive number.

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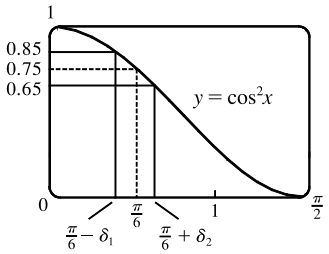
4. On the given graph, the leftmost question mark is the positive solution of $x^2 = \frac{1}{2}$, that is, $x = \frac{1}{\sqrt{2}}$, and the rightmost question mark is the positive solution of $x^2 = \frac{3}{2}$, that is, $x = \sqrt{\frac{3}{2}}$. On the left side, we need $|x - 1| < \left|\frac{1}{\sqrt{2}} - 1\right| \approx 0.292$ (rounding down to be safe). On the right side, we need $|x - 1| < \left|\sqrt{\frac{3}{2}} - 1\right| \approx 0.224$. The more restrictive of these two conditions must apply, so we choose $\delta = 0.224$ (or any smaller positive number).

5.



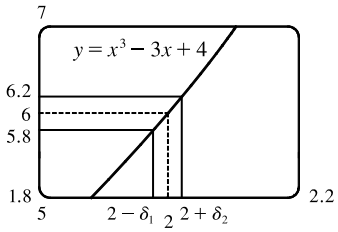
From the graph, we find that $y = \sqrt{x^2 + 5} = 2.7\ [3 - 0.3]$ when $x \approx 1.513$, so $2 - \delta_1 \approx 1.513 \quad \Rightarrow \quad \delta_1 \approx 2 - 1.513 = 0.487$. Also, $y = \sqrt{x^2 + 5} = 3.3\ [3 + 0.3]$ when $x \approx 2.426$, so $2 + \delta_2 \approx 2.426 \quad \Rightarrow$ $\delta_2 \approx 2.426 - 2 = 0.426$. Thus, we choose $\delta = 0.426$ (or any smaller positive number) since this is the smaller of δ_1 and δ_2.

6.



From the graph, we find that $y = \cos^2 x = 0.85\ [0.75 + 0.10]$ when $x \approx 0.398$, so $\frac{\pi}{6} - \delta_1 \approx 0.398 \quad \Rightarrow \quad \delta_1 \approx \frac{\pi}{6} - 0.398 \approx 0.126$. Also, $y = \cos^2 x = 0.65\ [0.75 - 0.10]$ when $x \approx 0.633$, so $\frac{\pi}{6} + \delta_2 \approx 0.633 \quad \Rightarrow$ $\delta_2 \approx 0.633 - \frac{\pi}{6} \approx 0.109$. Thus, we choose $\delta = 0.109$ (or any smaller positive number) since this is the smaller of δ_1 and δ_2.

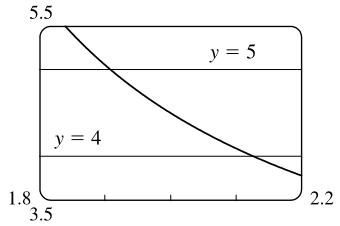
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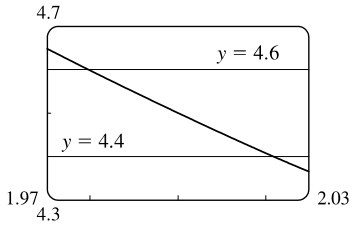


From the graph with $\varepsilon = 0.2$, we find that $y = x^3 - 3x + 4 = 5.8\ [6 - \varepsilon]$ when $x \approx 1.9774$, so $2 - \delta_1 \approx 1.9774 \quad \Rightarrow \quad \delta_1 \approx 0.0226$. Also, $y = x^3 - 3x + 4 = 6.2\ [6 + \varepsilon]$ when $x \approx 2.022$, so $2 + \delta_2 \approx 2.0219 \quad \Rightarrow$ $\delta_2 \approx 0.0219$. Thus, we choose $\delta = 0.0219$ (or any smaller positive number) since this is the smaller of δ_1 and δ_2.

For $\varepsilon = 0.1$, we get $\delta_1 \approx 0.0112$ and $\delta_2 \approx 0.0110$, so we choose $\delta = 0.011$ (or any smaller positive number).

8. For $y = (4x + 1)/(3x - 4)$ and $\varepsilon = 0.5$, we need $1.91 \le x \le 2.125$. So since $|2 - 1.91| = 0.09$ and $|2 - 2.125| = 0.125$, we can take $0 < \delta \le 0.09$. For $\varepsilon = 0.1$, we need $1.980 \le 2.021$. So since $|2 - 1.980| = 0.02$ and $|2 - 2.021| = 0.021$, we can take $\delta = 0.02$ (or any smaller positive number).

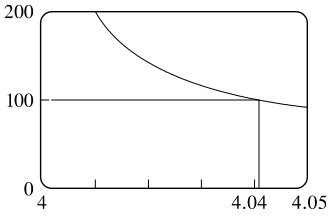




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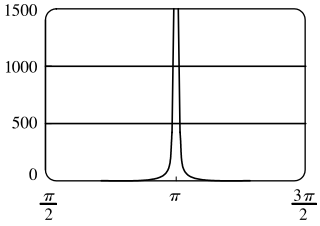
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9. (a) The graph of $y = \dfrac{x^2+4}{\sqrt{x-4}}$ shows that $y = 100$ when $x \approx 4.04$ (more accurately, 4.04134). Thus, we choose $\delta = 0.04$ (or any smaller positive number).



(b) From part (a), we see that as x gets closer to 4 from the right, y increases without bound. In symbols, $\lim\limits_{x \to 4^+} \dfrac{x^2+4}{\sqrt{x-4}} = \infty$.

10. We graph $y = \csc^2 x$ and $y = 500$. The graphs intersect at $x \approx 3.186$, so we choose $\delta = 3.186 - \pi \approx 0.044$. Thus, if $0 < |x - \pi| < 0.044$, then $\csc^2 x > 500$. Similarly, for $M = 1000$, we get $\delta = 3.173 - \pi \approx 0.031$.

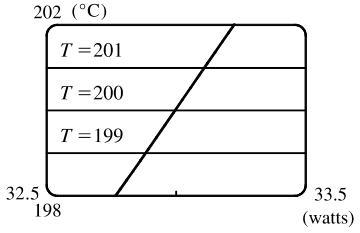


11. (a) $A = \pi r^2$ and $A = 1000$ cm^2 $\Rightarrow$ $\pi r^2 = 1000$ $\Rightarrow$ $r^2 = \frac{1000}{\pi}$ $\Rightarrow$ $r = \sqrt{\frac{1000}{\pi}}$ $(r > 0)$ ≈ 17.8412 cm.

(b) $|A - 1000| \le 5$ $\Rightarrow$ $-5 \le \pi r^2 - 1000 \le 5$ $\Rightarrow$ $1000 - 5 \le \pi r^2 \le 1000 + 5$ $\Rightarrow$ $\sqrt{\frac{995}{\pi}} \le r \le \sqrt{\frac{1005}{\pi}}$ $\Rightarrow$ $17.7966 \le r \le 17.8858$. $\sqrt{\frac{1000}{\pi}} - \sqrt{\frac{995}{\pi}} \approx 0.04466$ and $\sqrt{\frac{1005}{\pi}} - \sqrt{\frac{1000}{\pi}} \approx 0.04455$. So if the machinist gets the radius within 0.0445 cm of 17.8412, the area will be within 5 cm^2 of 1000.

(c) x is the radius, $f(x)$ is the area, a is the target radius given in part (a), L is the target area (1000 cm^2), ε is the magnitude of the error tolerance in the area (5 cm^2), and δ is the tolerance in the radius given in part (b).

12. (a) $T = 0.1w^2 + 2.155w + 20$ and $T = 200$ $\Rightarrow$ $0.1w^2 + 2.155w + 20 = 200$ $\Rightarrow$ [by the quadratic formula or from the graph] $w \approx 33.0$ watts $(w > 0)$



(b) From the graph, $199 \le T \le 201$ $\Rightarrow$ $32.89 < w < 33.11$.

(c) x is the input power, $f(x)$ is the temperature, a is the target input power given in part (a), L is the target temperature (200), ε is the tolerance in the temperature (1), and δ is the tolerance in the power input in watts indicated in part (b) (0.11 watts).

13. (a) $|4x - 8| = 4\,|x - 2| < 0.1$ $\Leftrightarrow$ $|x - 2| < \dfrac{0.1}{4}$, so $\delta = \dfrac{0.1}{4} = 0.025$.

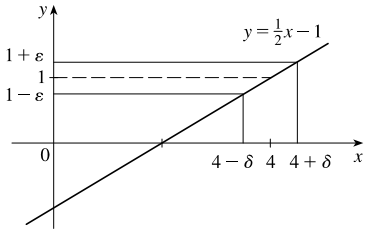
(b) $|4x - 8| = 4\,|x - 2| < 0.01$ $\Leftrightarrow$ $|x - 2| < \dfrac{0.01}{4}$, so $\delta = \dfrac{0.01}{4} = 0.0025$.

14. $|(5x - 7) - 3| = |5x - 10| = |5(x - 2)| = 5\,|x - 2|$. We must have $|f(x) - L| < \varepsilon$, so $5\,|x - 2| < \varepsilon$ $\Leftrightarrow$ $|x - 2| < \varepsilon/5$. Thus, choose $\delta = \varepsilon/5$. For $\varepsilon = 0.1$, $\delta = 0.02$; for $\varepsilon = 0.05$, $\delta = 0.01$; for $\varepsilon = 0.01$, $\delta = 0.002$.

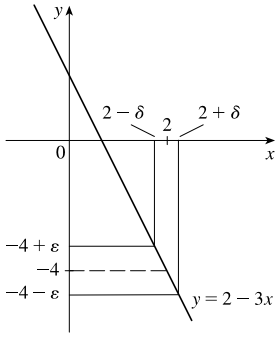
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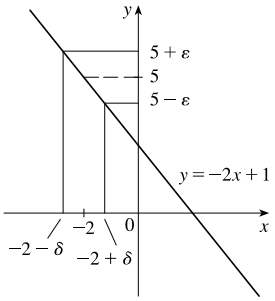
15. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-4| < \delta$, then $\left|(\frac{1}{2}x-1)-1\right| < \varepsilon$. But $\left|(\frac{1}{2}x-1)-1\right| < \varepsilon \quad \Leftrightarrow \quad \left|\frac{1}{2}x-2\right| < \varepsilon \quad \Leftrightarrow \quad \left|\frac{1}{2}\right||x-4| < \varepsilon \quad \Leftrightarrow \quad |x-4| < 2\varepsilon$. So if we choose $\delta = 2\varepsilon$, then $0 < |x-4| < \delta \quad \Rightarrow \quad \left|(\frac{1}{2}x-1)-1\right| < \varepsilon$. Thus, $\lim_{x\to 4}\left(\frac{1}{2}x-1\right) = 1$ by the definition of a limit.



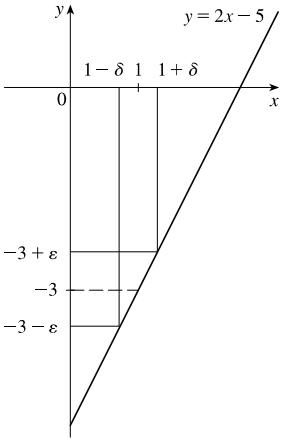
16. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-2| < \delta$, then $|(2-3x)-(-4)| < \varepsilon$. But $|(2-3x)-(-4)| < \varepsilon \quad \Leftrightarrow \quad |6-3x| < \varepsilon \quad \Leftrightarrow \quad |-3|\,|x-2| < \varepsilon \quad \Leftrightarrow \quad |x-2| < \frac{1}{3}\varepsilon$. So if we choose $\delta = \frac{1}{3}\varepsilon$, then $0 < |x-2| < \delta \quad \Rightarrow \quad |(2-3x)-(-4)| < \varepsilon$. Thus, $\lim_{x\to 2}(2-3x) = -4$ by the definition of a limit.



17. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-(-2)| < \delta$, then $|(-2x+1)-5| < \varepsilon$. But $|(-2x+1)-5| < \varepsilon \quad \Leftrightarrow \quad |-2x-4| < \varepsilon \quad \Leftrightarrow \quad |-2|\,|x-(-2)| < \varepsilon \quad \Leftrightarrow \quad |x-(-2)| < \frac{1}{2}\varepsilon$. So if we choose $\delta = \frac{1}{2}\varepsilon$, then $0 < |x-(-2)| < \delta \quad \Rightarrow \quad |(-2x+1)-5| < \varepsilon$. Thus, $\lim_{x\to -2}(-2x+1) = 5$ by the definition of a limit.



18. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-1| < \delta$, then $|(2x-5)-(-3)| < \varepsilon$. But $|(2x-5)-(-3)| < \varepsilon \quad \Leftrightarrow \quad |2x-2| < \varepsilon \quad \Leftrightarrow \quad |2|\,|x-1| < \varepsilon \quad \Leftrightarrow \quad |x-1| < \frac{1}{2}\varepsilon$. So if we choose $\delta = \frac{1}{2}\varepsilon$, then $0 < |x-1| < \delta \quad \Rightarrow \quad |(2x-5)-(-3)| < \varepsilon$. Thus, $\lim_{x\to 1}(2x-5) = -3$ by the definition of a limit.



19. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-9| < \delta$, then $\left|\left(1-\frac{1}{3}x\right)-(-2)\right| < \varepsilon$. But $\left|\left(1-\frac{1}{3}x\right)-(-2)\right| < \varepsilon \quad \Leftrightarrow \quad \left|3-\frac{1}{3}x\right| < \varepsilon \quad \Leftrightarrow \quad \left|-\frac{1}{3}\right||x-9| < \varepsilon \quad \Leftrightarrow \quad |x-9| < 3\varepsilon$. So if we choose $\delta = 3\varepsilon$, then $0 < |x-9| < \delta \quad \Rightarrow \quad \left|\left(1-\frac{1}{3}x\right)-(-2)\right| < \varepsilon$. Thus, $\lim_{x\to 9}\left(1-\frac{1}{3}x\right) = -2$ by the definition of a limit.

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20. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-5| < \delta$, then $\left|\left(\frac{3}{2}x - \frac{1}{2}\right) - 7\right| < \varepsilon$. But $\left|\left(\frac{3}{2}x - \frac{1}{2}\right) - 7\right| < \varepsilon \quad \Leftrightarrow$ $\left|\frac{3}{2}x - \frac{15}{2}\right| < \varepsilon \quad \Leftrightarrow \quad \left|\frac{3}{2}\right||x-5| < \varepsilon \quad \Leftrightarrow \quad |x-5| < \frac{2}{3}\varepsilon$. So if we choose $\delta = \frac{2}{3}\varepsilon$, then $0 < |x-5| < \delta \quad \Rightarrow$ $\left|\left(\frac{3}{2}x - \frac{1}{2}\right) - 7\right| < \varepsilon$. Thus, $\lim_{x\to 5}\left(\frac{3}{2}x - \frac{1}{2}\right) = 7$ by the definition of a limit.

21. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-4| < \delta$, then $\left|\dfrac{x^2-2x-8}{x-4} - 6\right| < \varepsilon \quad \Leftrightarrow$ $\left|\dfrac{(x-4)(x+2)}{x-4} - 6\right| < \varepsilon \quad \Leftrightarrow \quad |x+2-6| < \varepsilon \quad [x \neq 4] \quad \Leftrightarrow \quad |x-4| < \varepsilon$. So choose $\delta = \varepsilon$. Then $0 < |x-4| < \delta \quad \Rightarrow \quad |x-4| < \varepsilon \quad \Rightarrow \quad |x+2-6| < \varepsilon \quad \Rightarrow \quad \left|\dfrac{(x-4)(x+2)}{x-4} - 6\right| < \varepsilon \quad [x \neq 4] \quad \Rightarrow$ $\left|\dfrac{x^2-2x-8}{x-4} - 6\right| < \varepsilon$. By the definition of a limit, $\lim_{x\to 4}\dfrac{x^2-2x-8}{x-4} = 6$.

22. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x+1.5| < \delta$, then $\left|\dfrac{9-4x^2}{3+2x} - 6\right| < \varepsilon \quad \Leftrightarrow$ $\left|\dfrac{(3+2x)(3-2x)}{3+2x} - 6\right| < \varepsilon \quad \Leftrightarrow \quad |3-2x-6| < \varepsilon \quad [x \neq -1.5] \quad \Leftrightarrow \quad |-2x-3| < \varepsilon \quad \Leftrightarrow \quad |-2|\,|x+1.5| < \varepsilon \quad \Leftrightarrow$ $|x+1.5| < \varepsilon/2$. So choose $\delta = \varepsilon/2$. Then $0 < |x+1.5| < \delta \quad \Rightarrow \quad |x+1.5| < \varepsilon/2 \quad \Rightarrow \quad |-2|\,|x+1.5| < \varepsilon \quad \Rightarrow$ $|-2x-3| < \varepsilon \quad \Rightarrow \quad |3-2x-6| < \varepsilon \quad \Rightarrow \quad \left|\dfrac{(3+2x)(3-2x)}{3+2x} - 6\right| < \varepsilon \quad [x \neq -1.5] \quad \Rightarrow \quad \left|\dfrac{9-4x^2}{3+2x} - 6\right| < \varepsilon.$

By the definition of a limit, $\lim_{x\to -1.5}\dfrac{9-4x^2}{3+2x} = 6$.

23. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-a| < \delta$, then $|x-a| < \varepsilon$. So $\delta = \varepsilon$ will work.

24. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-a| < \delta$, then $|c-c| < \varepsilon$. But $|c-c| = 0$, so this will be true no matter what δ we pick.

25. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-0| < \delta$, then $\left|x^2 - 0\right| < \varepsilon \quad \Leftrightarrow \quad x^2 < \varepsilon \quad \Leftrightarrow \quad |x| < \sqrt{\varepsilon}$. Take $\delta = \sqrt{\varepsilon}$. Then $0 < |x-0| < \delta \quad \Rightarrow \quad \left|x^2 - 0\right| < \varepsilon$. Thus, $\lim_{x\to 0} x^2 = 0$ by the definition of a limit.

26. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-0| < \delta$, then $\left|x^3 - 0\right| < \varepsilon \quad \Leftrightarrow \quad |x|^3 < \varepsilon \quad \Leftrightarrow \quad |x| < \sqrt[3]{\varepsilon}$. Take $\delta = \sqrt[3]{\varepsilon}$. Then $0 < |x-0| < \delta \quad \Rightarrow \quad \left|x^3 - 0\right| < \delta^3 = \varepsilon$. Thus, $\lim_{x\to 0} x^3 = 0$ by the definition of a limit.

27. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-0| < \delta$, then $\left||x| - 0\right| < \varepsilon$. But $\left||x|\right| = |x|$. So this is true if we pick $\delta = \varepsilon$. Thus, $\lim_{x\to 0}|x| = 0$ by the definition of a limit.

28. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < x-(-6) < \delta$, then $\left|\sqrt[8]{6+x} - 0\right| < \varepsilon$. But $\left|\sqrt[8]{6+x} - 0\right| < \varepsilon \quad \Leftrightarrow$ $\sqrt[8]{6+x} < \varepsilon \quad \Leftrightarrow \quad 6+x < \varepsilon^8 \quad \Leftrightarrow \quad x-(-6) < \varepsilon^8$. So if we choose $\delta = \varepsilon^8$, then $0 < x-(-6) < \delta \quad \Rightarrow$ $\left|\sqrt[8]{6+x} - 0\right| < \varepsilon$. Thus, $\lim_{x\to -6^+}\sqrt[8]{6+x} = 0$ by the definition of a right-hand limit.

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29. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-2| < \delta$, then $\left|\left(x^2-4x+5\right)-1\right| < \varepsilon \quad \Leftrightarrow \quad \left|x^2-4x+4\right| < \varepsilon \quad \Leftrightarrow$ $\left|(x-2)^2\right| < \varepsilon$. So take $\delta = \sqrt{\varepsilon}$. Then $0 < |x-2| < \delta \quad \Leftrightarrow \quad |x-2| < \sqrt{\varepsilon} \quad \Leftrightarrow \quad \left|(x-2)^2\right| < \varepsilon$. Thus, $\lim\limits_{x\to 2}\left(x^2-4x+5\right) = 1$ by the definition of a limit.

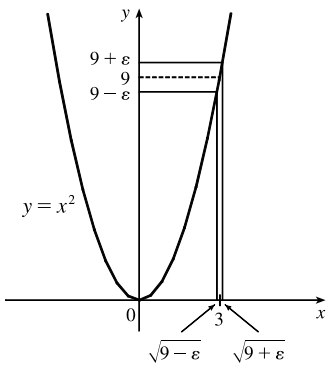
30. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-2| < \delta$, then $\left|\left(x^2+2x-7\right)-1\right| < \varepsilon$. But $\left|\left(x^2+2x-7\right)-1\right| < \varepsilon \quad \Leftrightarrow$ $\left|x^2+2x-8\right| < \varepsilon \quad \Leftrightarrow \quad |x+4|\,|x-2| < \varepsilon$. Thus our goal is to make $|x-2|$ small enough so that its product with $|x+4|$ is less than ε. Suppose we first require that $|x-2| < 1$. Then $-1 < x-2 < 1 \quad \Rightarrow \quad 1 < x < 3 \quad \Rightarrow \quad 5 < x+4 < 7 \quad \Rightarrow$ $|x+4| < 7$, and this gives us $7\,|x-2| < \varepsilon \quad \Rightarrow \quad |x-2| < \varepsilon/7$. Choose $\delta = \min\{1, \varepsilon/7\}$. Then if $0 < |x-2| < \delta$, we have $|x-2| < \varepsilon/7$ and $|x+4| < 7$, so $\left|\left(x^2+2x-7\right)-1\right| = |(x+4)(x-2)| = |x+4|\,|x-2| < 7(\varepsilon/7) = \varepsilon$, as desired. Thus, $\lim\limits_{x\to 2}(x^2+2x-7) = 1$ by the definition of a limit.

31. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-(-2)| < \delta$, then $\left|\left(x^2-1\right)-3\right| < \varepsilon$ or upon simplifying we need $\left|x^2-4\right| < \varepsilon$ whenever $0 < |x+2| < \delta$. Notice that if $|x+2| < 1$, then $-1 < x+2 < 1 \quad \Rightarrow \quad -5 < x-2 < -3 \quad \Rightarrow$ $|x-2| < 5$. So take $\delta = \min\{\varepsilon/5, 1\}$. Then $0 < |x+2| < \delta \quad \Rightarrow \quad |x-2| < 5$ and $|x+2| < \varepsilon/5$, so $\left|\left(x^2-1\right)-3\right| = |(x+2)(x-2)| = |x+2|\,|x-2| < (\varepsilon/5)(5) = \varepsilon$. Thus, by the definition of a limit, $\lim\limits_{x\to -2}(x^2-1) = 3$.

32. Given $\varepsilon > 0$, we need $\delta > 0$ such that if $0 < |x-2| < \delta$, then $\left|x^3-8\right| < \varepsilon$. Now $\left|x^3-8\right| = \left|(x-2)\left(x^2+2x+4\right)\right|$. If $|x-2| < 1$, that is, $1 < x < 3$, then $x^2+2x+4 \;<\; 3^2+2(3)+4 = 19$ and so $\left|x^3-8\right| = |x-2|\left(x^2+2x+4\right) < 19\,|x-2|$. So if we take $\delta = \min\left\{1, \frac{\varepsilon}{19}\right\}$, then $0 < |x-2| < \delta \quad \Rightarrow$ $\left|x^3-8\right| = |x-2|\left(x^2+2x+4\right) < \frac{\varepsilon}{19}\cdot 19 = \varepsilon$. Thus, by the definition of a limit, $\lim\limits_{x\to 2} x^3 = 8$.

33. Given $\varepsilon > 0$, we let $\delta = \min\left\{2, \frac{\varepsilon}{8}\right\}$. If $0 < |x-3| < \delta$, then $|x-3| < 2 \quad \Rightarrow \quad -2 < x-3 < 2 \quad \Rightarrow$ $4 < x+3 < 8 \quad \Rightarrow \quad |x+3| < 8$. Also $|x-3| < \frac{\varepsilon}{8}$, so $\left|x^2-9\right| = |x+3|\,|x-3| < 8\cdot\frac{\varepsilon}{8} = \varepsilon$. Thus, $\lim\limits_{x\to 3} x^2 = 9$.

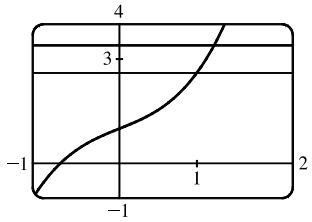
34. From the figure, our choices for δ are $\delta_1 = 3-\sqrt{9-\varepsilon}$ and $\delta_2 = \sqrt{9+\varepsilon}-3$. The *largest* possible choice for δ is the minimum value of $\{\delta_1, \delta_2\}$; that is, $\delta = \min\{\delta_1, \delta_2\} = \delta_2 = \sqrt{9+\varepsilon}-3$.



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35. (a) The points of intersection in the graph are $(x_1, 2.6)$ and $(x_2, 3.4)$ with $x_1 \approx 0.891$ and $x_2 \approx 1.093$. Thus, we can take δ to be the smaller of $1 - x_1$ and $x_2 - 1$. So $\delta = x_2 - 1 \approx 0.093$.



(b) Solving $x^3 + x + 1 = 3 + \varepsilon$ with a CAS gives us two nonreal complex solutions and one real solution, which is $x(\varepsilon) = \dfrac{\left(216 + 108\varepsilon + 12\sqrt{336 + 324\varepsilon + 81\varepsilon^2}\,\right)^{2/3} - 12}{6\left(216 + 108\varepsilon + 12\sqrt{336 + 324\varepsilon + 81\varepsilon^2}\,\right)^{1/3}}$. Thus, $\delta = x(\varepsilon) - 1$.

(c) If $\varepsilon = 0.4$, then $x(\varepsilon) \approx 1.093\,272\,342$ and $\delta = x(\varepsilon) - 1 \approx 0.093$, which agrees with our answer in part (a).

36. *1. Guessing a value for δ* Let $\varepsilon > 0$ be given. We have to find a number $\delta > 0$ such that $\left|\dfrac{1}{x} - \dfrac{1}{2}\right| < \varepsilon$ whenever $0 < |x - 2| < \delta$. But $\left|\dfrac{1}{x} - \dfrac{1}{2}\right| = \left|\dfrac{2 - x}{2x}\right| = \dfrac{|x - 2|}{|2x|} < \varepsilon$. We find a positive constant C such that $\dfrac{1}{|2x|} < C \quad \Rightarrow$ $\dfrac{|x - 2|}{|2x|} < C\,|x - 2|$ and we can make $C\,|x - 2| < \varepsilon$ by taking $|x - 2| < \dfrac{\varepsilon}{C} = \delta$. We restrict x to lie in the interval $|x - 2| < 1 \quad \Rightarrow \quad 1 < x < 3$ so $1 > \dfrac{1}{x} > \dfrac{1}{3} \quad \Rightarrow \quad \dfrac{1}{6} < \dfrac{1}{2x} < \dfrac{1}{2} \quad \Rightarrow \quad \dfrac{1}{|2x|} < \dfrac{1}{2}$. So $C = \dfrac{1}{2}$ is suitable. Thus, we should choose $\delta = \min\{1, 2\varepsilon\}$.

2. Showing that δ works Given $\varepsilon > 0$ we let $\delta = \min\{1, 2\varepsilon\}$. If $0 < |x - 2| < \delta$, then $|x - 2| < 1 \quad \Rightarrow \quad 1 < x < 3 \quad \Rightarrow$ $\dfrac{1}{|2x|} < \dfrac{1}{2}$ (as in part 1). Also $|x - 2| < 2\varepsilon$, so $\left|\dfrac{1}{x} - \dfrac{1}{2}\right| = \dfrac{|x - 2|}{|2x|} < \dfrac{1}{2} \cdot 2\varepsilon = \varepsilon$. This shows that $\lim\limits_{x \to 2}(1/x) = \frac{1}{2}$.

37. *1. Guessing a value for δ* Given $\varepsilon > 0$, we must find $\delta > 0$ such that $|\sqrt{x} - \sqrt{a}| < \varepsilon$ whenever $0 < |x - a| < \delta$. But $|\sqrt{x} - \sqrt{a}| = \dfrac{|x - a|}{\sqrt{x} + \sqrt{a}} < \varepsilon$ (from the hint). Now if we can find a positive constant C such that $\sqrt{x} + \sqrt{a} > C$ then $\dfrac{|x - a|}{\sqrt{x} + \sqrt{a}} < \dfrac{|x - a|}{C} < \varepsilon$, and we take $|x - a| < C\varepsilon$. We can find this number by restricting x to lie in some interval centered at a. If $|x - a| < \frac{1}{2}a$, then $-\frac{1}{2}a < x - a < \frac{1}{2}a \quad \Rightarrow \quad \frac{1}{2}a < x < \frac{3}{2}a \quad \Rightarrow \quad \sqrt{x} + \sqrt{a} > \sqrt{\frac{1}{2}a} + \sqrt{a}$, and so $C = \sqrt{\frac{1}{2}a} + \sqrt{a}$ is a suitable choice for the constant. So $|x - a| < \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon$. This suggests that we let $\delta = \min\left\{\frac{1}{2}a, \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon\right\}$.

2. Showing that δ works Given $\varepsilon > 0$, we let $\delta = \min\left\{\frac{1}{2}a, \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon\right\}$. If $0 < |x - a| < \delta$, then $|x - a| < \frac{1}{2}a \quad \Rightarrow \quad \sqrt{x} + \sqrt{a} > \sqrt{\frac{1}{2}a} + \sqrt{a}$ (as in part 1). Also $|x - a| < \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon$, so $|\sqrt{x} - \sqrt{a}\,| = \dfrac{|x - a|}{\sqrt{x} + \sqrt{a}} < \dfrac{\left(\sqrt{a/2} + \sqrt{a}\right)\varepsilon}{\left(\sqrt{a/2} + \sqrt{a}\right)} = \varepsilon$. Therefore, $\lim\limits_{x \to a} \sqrt{x} = \sqrt{a}$ by the definition of a limit.

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38. Suppose that $\lim_{t\to 0} H(t) = L$. Given $\varepsilon = \frac{1}{2}$, there exists $\delta > 0$ such that $0 < |t| < \delta \;\Rightarrow\; |H(t) - L| < \frac{1}{2} \;\Leftrightarrow$ $L - \frac{1}{2} < H(t) < L + \frac{1}{2}$. For $0 < t < \delta$, $H(t) = 1$, so $1 < L + \frac{1}{2} \;\Rightarrow\; L > \frac{1}{2}$. For $-\delta < t < 0$, $H(t) = 0$, so $L - \frac{1}{2} < 0 \;\Rightarrow\; L < \frac{1}{2}$. This contradicts $L > \frac{1}{2}$. Therefore, $\lim_{t\to 0} H(t)$ does not exist.

39. Suppose that $\lim_{x\to 0} f(x) = L$. Given $\varepsilon = \frac{1}{2}$, there exists $\delta > 0$ such that $0 < |x| < \delta \;\Rightarrow\; |f(x) - L| < \frac{1}{2}$. Take any rational number r with $0 < |r| < \delta$. Then $f(r) = 0$, so $|0 - L| < \frac{1}{2}$, so $L \le |L| < \frac{1}{2}$. Now take any irrational number s with $0 < |s| < \delta$. Then $f(s) = 1$, so $|1 - L| < \frac{1}{2}$. Hence, $1 - L < \frac{1}{2}$, so $L > \frac{1}{2}$. This contradicts $L < \frac{1}{2}$, so $\lim_{x\to 0} f(x)$ does not exist.

40. First suppose that $\lim_{x\to a} f(x) = L$. Then, given $\varepsilon > 0$ there exists $\delta > 0$ so that $0 < |x - a| < \delta \;\Rightarrow\; |f(x) - L| < \varepsilon$. Then $a - \delta < x < a \;\Rightarrow\; 0 < |x - a| < \delta$ so $|f(x) - L| < \varepsilon$. Thus, $\lim_{x\to a^-} f(x) = L$. Also $a < x < a + \delta \;\Rightarrow$ $0 < |x - a| < \delta$ so $|f(x) - L| < \varepsilon$. Hence, $\lim_{x\to a^+} f(x) = L$.

Now suppose $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$. Let $\varepsilon > 0$ be given. Since $\lim_{x\to a^-} f(x) = L$, there exists $\delta_1 > 0$ so that $a - \delta_1 < x < a \;\Rightarrow\; |f(x) - L| < \varepsilon$. Since $\lim_{x\to a^+} f(x) = L$, there exists $\delta_2 > 0$ so that $a < x < a + \delta_2 \;\Rightarrow$ $|f(x) - L| < \varepsilon$. Let δ be the smaller of δ_1 and δ_2. Then $0 < |x - a| < \delta \;\Rightarrow\; a - \delta_1 < x < a$ or $a < x < a + \delta_2$ so $|f(x) - L| < \varepsilon$. Hence, $\lim_{x\to a} f(x) = L$. So we have proved that $\lim_{x\to a} f(x) = L \;\Leftrightarrow\; \lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$.

41. $\dfrac{1}{(x+3)^4} > 10{,}000 \;\Leftrightarrow\; (x+3)^4 < \dfrac{1}{10{,}000} \;\Leftrightarrow\; |x+3| < \dfrac{1}{\sqrt[4]{10{,}000}} \;\Leftrightarrow\; |x-(-3)| < \dfrac{1}{10}$

42. Given $M > 0$, we need $\delta > 0$ such that $0 < |x+3| < \delta \;\Rightarrow\; 1/(x+3)^4 > M$. Now $\dfrac{1}{(x+3)^4} > M \;\Leftrightarrow$ $(x+3)^4 < \dfrac{1}{M} \;\Leftrightarrow\; |x+3| < \dfrac{1}{\sqrt[4]{M}}$. So take $\delta = \dfrac{1}{\sqrt[4]{M}}$. Then $0 < |x+3| < \delta = \dfrac{1}{\sqrt[4]{M}} \;\Rightarrow\; \dfrac{1}{(x+3)^4} > M$, so $\lim_{x\to -3} \dfrac{1}{(x+3)^4} = \infty$.

43. Let $N < 0$ be given. Then, for $x < -1$, we have $\dfrac{5}{(x+1)^3} < N \;\Leftrightarrow\; \dfrac{5}{N} < (x+1)^3 \;\Leftrightarrow\; \sqrt[3]{\dfrac{5}{N}} < x + 1$.

Let $\delta = -\sqrt[3]{\dfrac{5}{N}}$. Then $-1 - \delta < x < -1 \;\Rightarrow\; \sqrt[3]{\dfrac{5}{N}} < x + 1 < 0 \;\Rightarrow\; \dfrac{5}{(x+1)^3} < N$, so $\lim_{x\to -1^-} \dfrac{5}{(x+1)^3} = -\infty$.

44. (a) Let M be given. Since $\lim_{x\to a} f(x) = \infty$, there exists $\delta_1 > 0$ such that $0 < |x - a| < \delta_1 \;\Rightarrow\; f(x) > M + 1 - c$. Since $\lim_{x\to a} g(x) = c$, there exists $\delta_2 > 0$ such that $0 < |x - a| < \delta_2 \;\Rightarrow\; |g(x) - c| < 1 \;\Rightarrow\; g(x) > c - 1$. Let δ be the smaller of δ_1 and δ_2. Then $0 < |x - a| < \delta \;\Rightarrow\; f(x) + g(x) > (M + 1 - c) + (c - 1) = M$. Thus, $\lim_{x\to a} [f(x) + g(x)] = \infty$.

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(b) Let $M > 0$ be given. Since $\lim_{x \to a} g(x) = c > 0$, there exists $\delta_1 > 0$ such that $0 < |x - a| < \delta_1 \quad \Rightarrow$ $|g(x) - c| < c/2 \quad \Rightarrow \quad g(x) > c/2$. Since $\lim_{x \to a} f(x) = \infty$, there exists $\delta_2 > 0$ such that $0 < |x - a| < \delta_2 \quad \Rightarrow$ $f(x) > 2M/c$. Let $\delta = \min\{\delta_1, \delta_2\}$. Then $0 < |x - a| < \delta \quad \Rightarrow \quad f(x)\, g(x) > \dfrac{2M}{c}\,\dfrac{c}{2} = M$, so $\lim_{x \to a} f(x)\, g(x) = \infty$.

(c) Let $N < 0$ be given. Since $\lim_{x \to a} g(x) = c < 0$, there exists $\delta_1 > 0$ such that $0 < |x - a| < \delta_1 \quad \Rightarrow$ $|g(x) - c| < -c/2 \quad \Rightarrow \quad g(x) < c/2$. Since $\lim_{x \to a} f(x) = \infty$, there exists $\delta_2 > 0$ such that $0 < |x - a| < \delta_2 \quad \Rightarrow$ $f(x) > 2N/c$. (Note that $c < 0$ and $N < 0 \quad \Rightarrow \quad 2N/c > 0$.) Let $\delta = \min\{\delta_1, \delta_2\}$. Then $0 < |x - a| < \delta \quad \Rightarrow$ $f(x) > 2N/c \quad \Rightarrow \quad f(x)\, g(x) < \dfrac{2N}{c} \cdot \dfrac{c}{2} = N$, so $\lim_{x \to a} f(x)\, g(x) = -\infty$.

1.8 Continuity

1. From Definition 1, $\lim_{x \to 4} f(x) = f(4)$.

2. The graph of f has no hole, jump, or vertical asymptote.

3. (a) f is discontinuous at -4 since $f(-4)$ is not defined and at -2, 2, and 4 since the limit does not exist (the left and right limits are not the same).

(b) f is continuous from the left at -2 since $\lim_{x \to -2^-} f(x) = f(-2)$. f is continuous from the right at 2 and 4 since $\lim_{x \to 2^+} f(x) = f(2)$ and $\lim_{x \to 4^+} f(x) = f(4)$. The function is not continuous from either side at -4 since $f(-4)$ is undefined.

4. g is not continuous at -2 since $g(-2)$ is not defined. g is not continuous at $a = -1$ since the limit does not exist (the left and right limits are $-\infty$). g is not continuous at $a = 0$ and $a = 1$ since the limit does not exist (the left and right limits are not equal).

5. (a) From the graph we see that $\lim_{x \to a} f(x)$ does not exist at $a = 1$ since the left and right limits are not the same.

(b) f is not continuous at $a = 1$ since $\lim_{x \to 1} f(x)$ does not exist by part (a). Also, f is not continuous at $a = 3$ since $\lim_{x \to 3} f(x) \neq f(3)$.

(c) From the graph we see that $\lim_{x \to 3} f(x) = 3$, but $f(3) = 2$. Since the limit is not equal to $f(3)$, f is not continuous at $a = 3$.

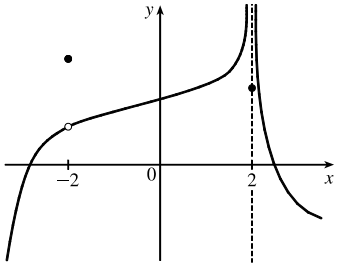
6. (a) From the graph we see that $\lim_{x \to a} f(x)$ does not exist at $a = 1$ since the function increases without bound from the left and from the right. Also, $\lim_{x \to a} f(x)$ does not exist at $a = 5$ since the left and right limits are not the same.

(b) f is not continuous at $a = 1$ and at $a = 5$ since the limits do not exist by part (a). Also, f is not continuous at $a = 3$ since $\lim_{x \to 3} f(x) \neq f(3)$.

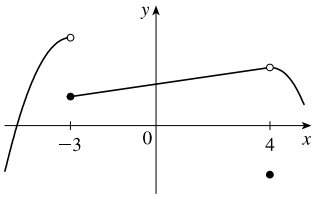
(c) From the graph we see that $\lim_{x \to 3} f(x)$ exists, but the limit is not equal to $f(3)$, so f is not continuous at $a = 3$.

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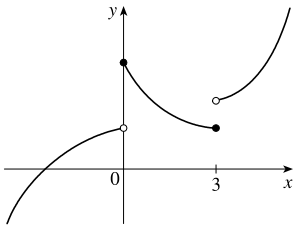
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7. 

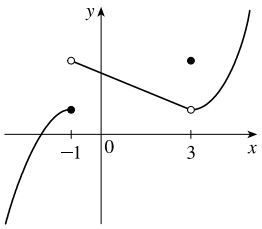
The graph of $y = f(x)$ must have a removable discontinuity (a hole) at $x = -2$ and an infinite discontinuity (a vertical asymptote) at $x = 2$.

8. 

The graph of $y = f(x)$ must have a jump discontinuity at $x = -3$ and a removable discontinuity (a hole) at $x = 4$.

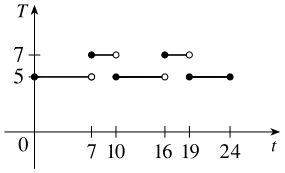
9. 

The graph of $y = f(x)$ must have discontinuities at $x = 0$ and $x = 3$. It must show that $\lim_{x \to 0^+} f(x) = f(0)$ and $\lim_{x \to 3^-} f(x) = f(3)$.

10. 

The graph of $y = f(x)$ must have a discontinuity at $x = -1$ with $\lim_{x \to -1^-} f(x) = f(-1)$ and $\lim_{x \to -1^+} f(x) \neq f(-1)$. The graph must also show that $\lim_{x \to 3^-} f(x) \neq f(3)$ and $\lim_{x \to 3^+} f(x) \neq f(3)$.

11. (a) The toll is \$5 except between 7:00 AM and 10:00 AM and between 4:00 PM and 7:00 PM, when the toll is \$7.



(b) The function T has jump discontinuities at $t = 7$, 10, 16, and 19. Their significance to someone who uses the road is that, because of the sudden jumps in the toll, they may want to avoid the higher rates between $t = 7$ and $t = 10$ and between $t = 16$ and $t = 19$ if feasible.

12. (a) Continuous; at the location in question, the temperature changes smoothly as time passes, without any instantaneous jumps from one temperature to another.

(b) Continuous; the temperature at a specific time changes smoothly as the distance due west from New York City increases, without any instantaneous jumps.

(c) Discontinuous; as the distance due west from New York City increases, the altitude above sea level may jump from one height to another without going through all of the intermediate values — at a cliff, for example.

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(d) Discontinuous; as the distance traveled increases, the cost of the ride jumps in small increments.

(e) Discontinuous; when the lights are switched on (or off), the current suddenly changes between 0 and some nonzero value, without passing through all of the intermediate values. This is debatable, though, depending on your definition of current.

13. $\lim_{x\to -1} f(x) = \lim_{x\to -1}\left[3x^2 + (x+2)^5\right] = \lim_{x\to -1} 3x^2 + \lim_{x\to -1}(x+2)^5 = 3\lim_{x\to -1} x^2 + \lim_{x\to -1}(x+2)^5$
$= 3(-1)^2 + (-1+2)^5 = 4 = f(-1)$

By the definition of continuity, f is continuous at $a = -1$.

14. $\lim_{t\to 2} g(t) = \lim_{t\to 2} \dfrac{t^2+5t}{2t+1} = \dfrac{\lim_{t\to 2}(t^2+5t)}{\lim_{t\to 2}(2t+1)} = \dfrac{\lim_{t\to 2} t^2 + 5\lim_{t\to 2} t}{2\lim_{t\to 2} t + \lim_{t\to 2} 1} = \dfrac{2^2+5(2)}{2(2)+1} = \dfrac{14}{5} = g(2).$

By the definition of continuity, g is continuous at $a = 2$.

15. $\lim_{v\to 1} p(v) = \lim_{v\to 1} 2\sqrt{3v^2+1} = 2\lim_{v\to 1}\sqrt{3v^2+1} = 2\sqrt{\lim_{v\to 1}(3v^2+1)} = 2\sqrt{3\lim_{v\to 1} v^2 + \lim_{v\to 1} 1}$
$= 2\sqrt{3(1)^2+1} = 2\sqrt{4} = 4 = p(1)$

By the definition of continuity, p is continuous at $a = 1$.

16. $\lim_{r\to -2} f(r) = \lim_{r\to -2}\sqrt[3]{4r^2-2r+7} = \sqrt[3]{\lim_{r\to -2}(4r^2-2r+7)} = \sqrt[3]{4(-2)^2-2(-2)+7} = \sqrt[3]{27} = 3 = f(-2)$

By the definition of continuity, f is continuous at $a = -2$.

17. For $a > 4$, we have

$$\begin{aligned}
\lim_{x\to a} f(x) &= \lim_{x\to a}(x+\sqrt{x-4}) = \lim_{x\to a} x + \lim_{x\to a}\sqrt{x-4} && \text{[Limit Law 1]}\\
&= a + \sqrt{\lim_{x\to a} x - \lim_{x\to a} 4} && \text{[8, 11, and 2]}\\
&= a + \sqrt{a-4} && \text{[8 and 7]}\\
&= f(a)
\end{aligned}$$

So f is continuous at $x = a$ for every a in $(4,\infty)$. Also, $\lim_{x\to 4^+} f(x) = 4 = f(4)$, so f is continuous from the right at 4.

Thus, f is continuous on $[4,\infty)$.

18. For $a < -2$, we have

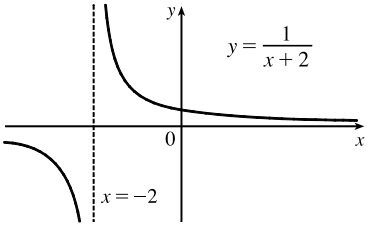
$$\begin{aligned}
\lim_{x\to a} g(x) &= \lim_{x\to a}\frac{x-1}{3x+6} = \frac{\lim_{x\to a}(x-1)}{\lim_{x\to a}(3x+6)} && \text{[Limit Law 5]}\\
&= \frac{\lim_{x\to a} x - \lim_{x\to a} 1}{3\lim_{x\to a} x + \lim_{x\to a} 6} && \text{[2, 1, and 3]}\\
&= \frac{a-1}{3a+6} && \text{[8 and 7]}
\end{aligned}$$

Thus, g is continuous at $x = a$ for every a in $(-\infty,-2)$; that is, g is continuous on $(-\infty,-2)$.

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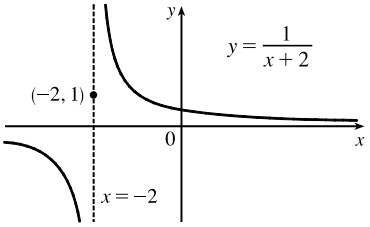
19. $f(x) = \dfrac{1}{x+2}$ is discontinuous at $a = -2$ because $f(-2)$ is undefined.



20. $f(x) = \begin{cases} \dfrac{1}{x+2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$

Here $f(-2) = 1$, but $\lim\limits_{x \to -2^-} f(x) = -\infty$ and $\lim\limits_{x \to -2^+} f(x) = \infty$,

so $\lim\limits_{x \to -2} f(x)$ does not exist and f is discontinuous at -2.



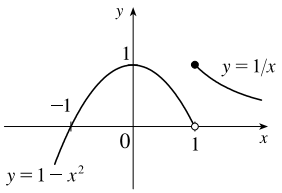
21. $f(x) = \begin{cases} 1 - x^2 & \text{if } x < 1 \\ 1/x & \text{if } x \geq 1 \end{cases}$

The left-hand limit of f at $a = 1$ is

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (1 - x^2) = 0$. The right-hand limit of f at $a = 1$ is

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (1/x) = 1$. Since these limits are not equal, $\lim\limits_{x \to 1} f(x)$

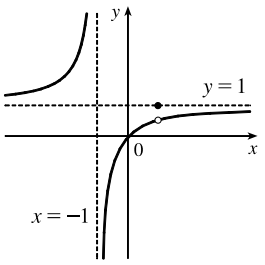
does not exist and f is discontinuous at 1.



22. $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$

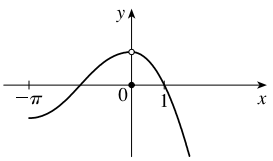
$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \to 1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x \to 1} \frac{x}{x+1} = \frac{1}{2},$$

but $f(1) = 1$, so f is discontinous at 1.



23. $f(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$

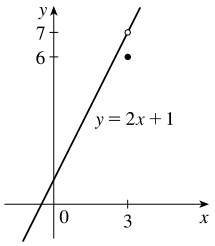
$\lim\limits_{x \to 0} f(x) = 1$, but $f(0) = 0 \neq 1$, so f is discontinuous at 0.



24. $f(x) = \begin{cases} \dfrac{2x^2 - 5x - 3}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{2x^2 - 5x - 3}{x - 3} = \lim_{x \to 3} \frac{(2x+1)(x-3)}{x - 3} = \lim_{x \to 3} (2x + 1) = 7,$$

but $f(3) = 6$, so f is discontinuous at 3.



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25. (a) $f(x) = \dfrac{x-3}{x^2-9} = \dfrac{x-3}{(x-3)(x+3)} = \dfrac{1}{x+3}$ for $x \neq 3$. $f(3)$ is undefined, so f is discontinuous at $x = 3$. Further, $\lim\limits_{x\to 3} f(x) = \dfrac{1}{3+3} = \dfrac{1}{6}$. Since f is discontinuous at $x = 3$, but $\lim\limits_{x\to 3} f(x)$ exists, f has a removable discontinuity at $x = 3$.

(b) If f is redefined to be $\frac{1}{6}$ at $x = 3$, then f will be equivalent to the function $g(x) = \dfrac{1}{x+3}$, which is continuous at $x = 3$.

26. (a) $f(x) = \dfrac{x^2-7x+12}{x-3} = \dfrac{(x-3)(x-4)}{x-3} = x-4$ for $x \neq 3$. $f(3)$ is undefined, so f is discontinuous at $x = 3$. Further, $\lim\limits_{x\to 3} f(x) = 3-4 = -1$. Since f is discontinuous at $x = 3$, but $\lim\limits_{x\to 3} f(x)$ exists, f has a removable discontinuity at $x = 3$.

(b) If f is redefined to be -1 at $x = 3$, then f will be equivalent to the function $g(x) = x - 4$, which is continuous everywhere (and is thus continuous at $x = 3$).

27. The domain of $f(x) = \dfrac{x^2}{\sqrt{x^4+2}}$ is $(-\infty, \infty)$ since the denominator is never 0. By Theorem 5(a), the polynomial x^2 is continuous everywhere. By Theorems 5(a), 7, and 9, $\sqrt{x^4+2}$ is continuous everywhere. Finally, by part 5 of Theorem 4, $f(x)$ is continuous everywhere.

28. $g(v) = \dfrac{3v-1}{v^2+2v-15} = \dfrac{3v-1}{(v+5)(v-3)}$ is a rational function, so it is continuous on its domain, $(-\infty,-5) \cup (-5,3) \cup (3,\infty)$, by Theorem 5(b).

29. $h(t) = \dfrac{\cos(t^2)}{1-t^2} = \dfrac{\cos(t^2)}{(1+t)(1-t)}$. Now t^2 is continuous on $\mathbb{R}$ since it is a polynomial and cosine is continuous everywhere. Thus, $\cos(t^2)$ is continuous on $\mathbb{R}$ by Theorem 9. h is not continuous where its denominator is 0, so h is continuous on $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$ by part 5 of Theorem 4.

30. $g(u) = \dfrac{1}{u} - \dfrac{u^2}{u-2}$ is the difference of two rational functions. $\dfrac{1}{u}$ is continuous on $(-\infty,0) \cup (0,\infty)$ and $\dfrac{u^2}{u-2}$ is continuous on $(-\infty,2) \cup (2,\infty)$. Thus, by part 2 of Theorem 4, g is continuous on $(-\infty,0) \cup (0,2) \cup (2,\infty)$.

31. $L(v) = v\sqrt{9-v^2}$ is defined when $9 - v^2 \geq 0 \quad \Leftrightarrow \quad v^2 \leq 9 \quad \Leftrightarrow \quad |v| \leq 3 \quad \Leftrightarrow \quad -3 \leq v \leq 3$. By Theorems 7 and 9, $\sqrt{9-v^2}$ is continuous on its domain because it is a composite of a root function and a polynomial function. L is the product of a polynomial (continuous everywhere) and $\sqrt{9-v^2}$, so it is continuous at every number in its domain, $[-3,3]$, by part 4 of Theorem 4.

32. $B(u) = \sqrt{3u-2} + \sqrt[3]{2u-3}$ is defined when $3u - 2 \geq 0 \quad \Rightarrow \quad 3u \geq 2 \quad \Rightarrow \quad u \geq \frac{2}{3}$. (Note that $\sqrt[3]{2u-3}$ is defined everywhere.) So B has domain $\left[\frac{2}{3}, \infty\right)$. By Theorems 7 and 9, $\sqrt{3u-2}$ and $\sqrt[3]{2u-3}$ are each continuous on their domain because each is the composite of a root function and a polynomial function. B is the sum of these two functions, so it is continuous at every number in its domain by part 1 of Theorem 4.

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33. $M(x)=\sqrt{1+\dfrac{1}{x}}=\sqrt{\dfrac{x+1}{x}}$ is defined when $\dfrac{x+1}{x}\ge 0 \quad\Rightarrow\quad x+1\ge 0$ and $x>0$ or $x+1\le 0$ and $x<0 \quad\Rightarrow\quad x>0$ or $x\le -1$, so M has domain $(-\infty,-1]\cup(0,\infty)$. M is the composite of a root function and a rational function, so it is continuous at every number in its domain by Theorems 7 and 9.

34. The sine and cosine functions are continuous everywhere by Theorem 7, so $F(x)=\sin(\cos(\sin x))$, which is the composite of sine, cosine, and (once again) sine, is continuous everywhere by Theorem 9.

35. Because x is continuous on $\mathbb{R}$ and $\sqrt{20-x^2}$ is continuous on its domain, $-\sqrt{20}\le x\le\sqrt{20}$, the product $f(x)=x\sqrt{20-x^2}$ is continuous on $-\sqrt{20}\le x\le\sqrt{20}$. The number 2 is in that domain, so f is continuous at 2, and $\lim\limits_{x\to 2} f(x)=f(2)=2\sqrt{16}=8$.

36. The function $f(\theta)=\sin(\tan(\cos\theta))$ is the composite of trigonometric functions, so it is continuous throughout its domain. Now the domain of $\cos\theta$ is $\mathbb{R}$, $-1\le\cos\theta\le 1$, the domain of $\tan\theta$ includes $[-1,1]$, and the domain of $\sin\theta$ is $\mathbb{R}$, so the domain of f is $\mathbb{R}$. Thus f is continuous at $\frac{\pi}{2}$, and $\lim\limits_{0\to\pi/2}\sin(\tan(\cos\theta))=\sin(\tan(\cos\frac{\pi}{2}))=\sin(\tan(0))=\sin(0)=0$.

37. The function $f(x)=x^2\tan x$ is continuous throughout its domain because it is the product of a polynomial and a trigonometric function. The domain of f is the set of all real numbers that are not odd multiples of $\frac{\pi}{2}$; that is, domain $f=\{x\mid x\ne n\pi/2, n \text{ an odd integer}\}$. Thus, $\frac{\pi}{4}$ is in the domain of f and

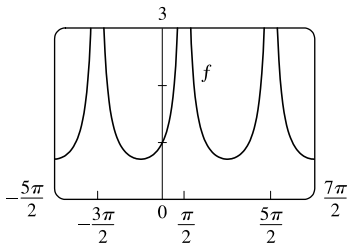
$$\lim_{x\to\pi/4} x^2\tan x=f\left(\frac{\pi}{4}\right)=\left(\frac{\pi}{4}\right)^2\tan\frac{\pi}{4}=\frac{\pi^2}{16}\cdot 1=\frac{\pi^2}{16}$$

38. The function $f(x)=\dfrac{x^3}{\sqrt{x^2+x-2}}$ is continuous throughout its domain because it is the quotient of a polynomial and the square root of a polynomial. The domain of f is

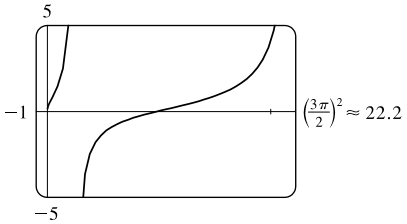
$$\{x\mid x^2+x-2>0\}=\{x\mid (x+2)(x-1)>0\}=\{x\mid x<-2 \text{ or } x>1\}=(-\infty,-2)\cup(1,\infty)$$

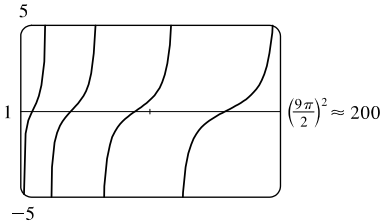
Thus, 2 is in the domain of f and $\lim\limits_{x\to 2}\dfrac{x^3}{\sqrt{x^2+x-2}}=f(2)=\dfrac{2^3}{\sqrt{2^2+2-2}}=\dfrac{8}{\sqrt{4}}=4$.

39. The function $f(x)=\dfrac{1}{\sqrt{1-\sin x}}$ is discontinuous wherever $1-\sin x=0 \quad\Rightarrow\quad \sin x=1 \quad\Rightarrow\quad x=\frac{\pi}{2}+2n\pi$, where n is any integer. The graph shows the discontinuities for $n=-1$, 0, and 1.



40.





The function $y=f(x)=\tan\sqrt{x}$ is continuous throughout its domain because it is the composite of a trigonometric function

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and a root function. The square root function has domain $[0, \infty)$ and the tangent function has domain $\{x \mid x \neq \frac{\pi}{2} + \pi n\}$. So f is discontinuous when $x < 0$ and when $\sqrt{x} = \frac{\pi}{2} + \pi n \quad \Rightarrow \quad x = \left(\frac{\pi}{2} + \pi n\right)^2$, where n is a nonnegative integer. Note that as x increases, the distance between discontinuities increases.

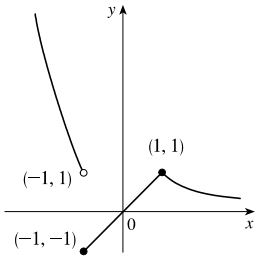
41. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 1 \\ \sqrt{x-1} & \text{if } x > 1 \end{cases}$

By Theorem 5, since $f(x)$ equals the polynomial $1 - x^2$ on $(-\infty, 1]$, f is continuous on $(-\infty, 1]$. By Theorem 7, since $f(x)$ equals the root function $\sqrt{x-1}$ on $(1, \infty)$, f is continuous on $(1, \infty)$. At $x = 1$, $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (1 - x^2) = 1 - 1^2 = 0$ and $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \sqrt{x-1} = \sqrt{1-1} = 0$. Thus, $\lim\limits_{x \to 1} f(x)$ exists and equals 0. Also, $f(1) = 1 - 1^2 = 0$. Therefore, f is continuous at $x = 1$. We conclude that f is continuous on $(-\infty, \infty)$.

42. $f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$

By Theorem 7, the trigonometric functions are continuous. Since $f(x) = \sin x$ on $(-\infty, \pi/4)$ and $f(x) = \cos x$ on $(\pi/4, \infty)$, f is continuous on $(-\infty, \pi/4) \cup (\pi/4, \infty)$. $\lim\limits_{x \to (\pi/4)^-} f(x) = \lim\limits_{x \to (\pi/4)^-} \sin x = \sin \frac{\pi}{4} = 1/\sqrt{2}$ since the sine function is continuous at $\pi/4$. Similarly, $\lim\limits_{x \to (\pi/4)^+} f(x) = \lim\limits_{x \to (\pi/4)^+} \cos x = 1/\sqrt{2}$ by continuity of the cosine function at $\pi/4$. Thus, $\lim\limits_{x \to (\pi/4)} f(x)$ exists and equals $1/\sqrt{2}$, which agrees with the value $f(\pi/4)$. Therefore, f is continuous at $\pi/4$, so f is continuous on $(-\infty, \infty)$.

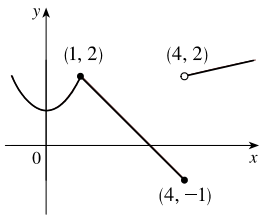
43. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ 1/x & \text{if } x \geq 1 \end{cases}$



f is continuous on $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$, where it is a polynomial, a polynomial, and a rational function, respectively.

Now $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} x^2 = 1$ and $\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} x = -1$, so f is discontinuous at -1. Since $f(-1) = -1$, f is continuous from the right at -1. Also, $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} x = 1$ and $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{1}{x} = 1 = f(1)$, so f is continuous at 1.

44. $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ 3 - x & \text{if } 1 < x \leq 4 \\ \sqrt{x} & \text{if } x > 4 \end{cases}$



f is continuous on $(-\infty, 1)$, $(1, 4)$, and $(4, \infty)$, where it is a polynomial, a polynomial, and a root function, respectively. Now $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (x^2 + 1) = 2$ and $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (3 - x) = 2$.

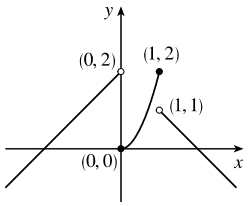
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Since $f(1) = 2$, we have continuity at 1. Also, $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (3 - x) = -1 = f(4)$ and $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \sqrt{x} = 2$, so f is discontinuous at 4, but it is continuous from the left at 4.

45. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 2 - x & \text{if } x > 1 \end{cases}$



f is continuous on $(-\infty, 0)$, $(0, 1)$, and $(1, \infty)$ since on each of these intervals it is a polynomial. Now $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x + 2) = 2$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x^2 = 0$, so f is discontinuous at 0. Since $f(0) = 0$, f is continuous from the right at 0. Also $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x^2 = 2$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2 - x) = 1$, so f is discontinuous at 1. Since $f(1) = 2$, f is continuous from the left at 1.

46. By Theorem 5, each piece of F is continuous on its domain. We need to check for continuity at $r = R$. $\lim_{r\to R^-} F(r) = \lim_{r\to R^-} \dfrac{GMr}{R^3} = \dfrac{GM}{R^2}$ and $\lim_{r\to R^+} F(r) = \lim_{r\to R^+} \dfrac{GM}{r^2} = \dfrac{GM}{R^2}$, so $\lim_{r\to R} F(r) = \dfrac{GM}{R^2}$. Since $F(R) = \dfrac{GM}{R^2}$, F is continuous at R. Therefore, F is a continuous function of r.

47. $f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \ge 2 \end{cases}$

f is continuous on $(-\infty, 2)$ and $(2, \infty)$. Now $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \left(cx^2 + 2x\right) = 4c + 4$ and $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \left(x^3 - cx\right) = 8 - 2c$. So f is continuous $\Leftrightarrow$ $4c + 4 = 8 - 2c$ $\Leftrightarrow$ $6c = 4$ $\Leftrightarrow$ $c = \frac{2}{3}$. Thus, for f to be continuous on $(-\infty, \infty)$, $c = \frac{2}{3}$.

48. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$

At $x = 2$: $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{x^2 - 4}{x - 2} = \lim_{x\to 2^-} \dfrac{(x + 2)(x - 2)}{x - 2} = \lim_{x\to 2^-} (x + 2) = 2 + 2 = 4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (ax^2 - bx + 3) = 4a - 2b + 3$

We must have $4a - 2b + 3 = 4$, or $\mathbf{4a - 2b = 1}$ **(1)**.

At $x = 3$: $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (2x - a + b) = 6 - a + b$

We must have $9a - 3b + 3 = 6 - a + b$, or $\mathbf{10a - 4b = 3}$ **(2)**.

[continued]

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Now solve the system of equations by adding -2 times equation **(1)** to equation **(2)**.

$$\begin{array}{rcl} -8a+4b &=& -2 \\ \underline{10a-4b} &=& \underline{\quad 3} \\ 2a &=& 1 \end{array}$$

So $a=\frac{1}{2}$. Substituting $\frac{1}{2}$ for a in **(1)** gives us $-2b=-1$, so $b=\frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty,\infty)$, $a=b=\frac{1}{2}$.

49. If f and g are continuous and $g(2)=6$, then $\lim_{x\to 2}[3f(x)+f(x)\,g(x)]=36 \;\Rightarrow$

$3\lim_{x\to 2} f(x)+\lim_{x\to 2} f(x)\cdot\lim_{x\to 2} g(x)=36 \;\Rightarrow\; 3f(2)+f(2)\cdot 6=36 \;\Rightarrow\; 9f(2)=36 \;\Rightarrow\; f(2)=4.$

50. (a) $f(x)=\dfrac{1}{x}$ and $g(x)=\dfrac{1}{x^2}$, so $(f\circ g)(x)=f(g(x))=f(1/x^2)=1\,/(1/x^2)=x^2$.

(b) The domain of $f\circ g$ is the set of numbers x in the domain of g (all nonzero reals) such that $g(x)$ is in the domain of f (also all nonzero reals). Thus, the domain is $\left\{x \;\middle|\; x\neq 0 \text{ and } \dfrac{1}{x^2}\neq 0\right\}=\{x \mid x\neq 0\}$ or $(-\infty,0)\cup(0,\infty)$. Since $f\circ g$ is the composite of two rational functions, it is continuous throughout its domain; that is, everywhere except $x=0$.

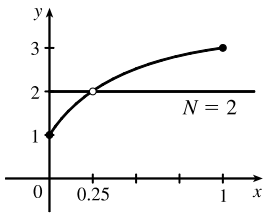
51. (a) $f(x)=\dfrac{x^4-1}{x-1}=\dfrac{(x^2+1)(x^2-1)}{x-1}=\dfrac{(x^2+1)(x+1)(x-1)}{x-1}=(x^2+1)(x+1)$ [or x^3+x^2+x+1]

for $x\neq 1$. The discontinuity is removable and $g(x)=x^3+x^2+x+1$ agrees with f for $x\neq 1$ and is continuous on $\mathbb{R}$.

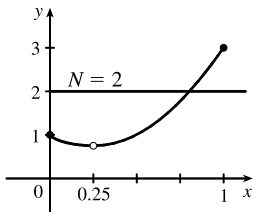
(b) $f(x)=\dfrac{x^3-x^2-2x}{x-2}=\dfrac{x(x^2-x-2)}{x-2}=\dfrac{x(x-2)(x+1)}{x-2}=x(x+1)$ [or x^2+x] for $x\neq 2$. The discontinuity is removable and $g(x)=x^2+x$ agrees with f for $x\neq 2$ and is continuous on $\mathbb{R}$.

(c) $\lim_{x\to\pi^-} f(x)=\lim_{x\to\pi^-}[\![\sin x]\!]=\lim_{x\to\pi^-} 0=0$ and $\lim_{x\to\pi^+} f(x)=\lim_{x\to\pi^+}[\![\sin x]\!]=\lim_{x\to\pi^+}(-1)=-1$, so $\lim_{x\to\pi} f(x)$ does not exist. The discontinuity at $x=\pi$ is a jump discontinuity.

52.



f does not satisfy the conclusion of the Intermediate Value Theorem.



f does satisfy the conclusion of the Intermediate Value Theorem.

53. $f(x)=x^2+10\sin x$ is continuous on the interval $[31,32]$, $f(31)\approx 957$, and $f(32)\approx 1030$. Since $957<1000<1030$, there is a number c in $(31,32)$ such that $f(c)=1000$ by the Intermediate Value Theorem. *Note:* There is also a number c in $(-32,-31)$ such that $f(c)=1000$.

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54. Suppose that $f(3) < 6$. By the Intermediate Value Theorem applied to the continuous function f on the closed interval $[2, 3]$, the fact that $f(2) = 8 > 6$ and $f(3) < 6$ implies that there is a number c in $(2, 3)$ such that $f(c) = 6$. This contradicts the fact that the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. Hence, our supposition that $f(3) < 6$ was incorrect. It follows that $f(3) \geq 6$. But $f(3) \neq 6$ because the only solutions of $f(x) = 6$ are $x = 1$ and $x = 4$. Therefore, $f(3) > 6$.

55. $f(x) = -x^3 + 4x + 1$ is continuous on the interval $[-1, 0]$, $f(-1) = -2$, and $f(0) = 1$. Since $-2 < 0 < 1$, there is a number c in $(-1, 0)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $-x^3 + 4x + 1 = 0$ in the interval $(-1, 0)$.

56. The equation $2/x = x - \sqrt{x}$ is equivalent to the equation $2/x - x + \sqrt{x} = 0$. $f(x) = 2/x - x + \sqrt{x}$ is continuous on the interval $[2, 3]$, $f(2) = 1 - 2 + \sqrt{2} \approx 0.414$, and $f(3) = \frac{2}{3} - 3 + \sqrt{3} \approx -0.601$. Since $f(2) > 0 > f(3)$, there is a number c in $(2, 3)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $2/x - x + \sqrt{x} = 0$, or $2/x = x - \sqrt{x}$, in the interval $(2, 3)$.

57. $f(x) = \cos x - x$ is continuous on the interval $[0, 1]$, $f(0) = 1$, and $f(1) = \cos 1 - 1 \approx -0.46$. Since $-0.46 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $\cos x - x = 0$, or $\cos x = x$, in the interval $(0, 1)$.

58. The equation $\sin x = x^2 - x$ is equivalent to the equation $\sin x - x^2 + x = 0$. $f(x) = \sin x - x^2 + x$ is continuous on the interval $[1, 2]$, $f(1) = \sin 1 \approx 0.84$, and $f(2) = \sin 2 - 2 \approx -1.09$. Since $\sin 1 > 0 > \sin 2 - 2$, there is a number c in $(1, 2)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $\sin x - x^2 + x = 0$, or $\sin x = x^2 - x$, in the interval $(1, 2)$.

59. (a) $f(x) = \cos x - x^3$ is continuous on the interval $[0, 1]$, $f(0) = 1 > 0$, and $f(1) = \cos 1 - 1 \approx -0.46 < 0$. Since $1 > 0 > -0.46$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $\cos x - x^3 = 0$, or $\cos x = x^3$, in the interval $(0, 1)$.

(b) $f(0.86) \approx 0.016 > 0$ and $f(0.87) \approx -0.014 < 0$, so there is a solution between 0.86 and 0.87, that is, in the interval $(0.86, 0.87)$.

60. (a) $f(x) = x^5 - x^2 + 2x + 3$ is continuous on $[-1, 0]$, $f(-1) = -1 < 0$, and $f(0) = 3 > 0$. Since $-1 < 0 < 3$, there is a number c in $(-1, 0)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $x^5 - x^2 + 2x + 3 = 0$ in the interval $(-1, 0)$.

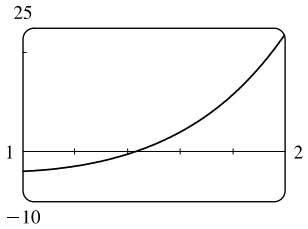
(b) $f(-0.88) \approx -0.062 < 0$ and $f(-0.87) \approx 0.0047 > 0$, so there is a solution between -0.88 and -0.87.

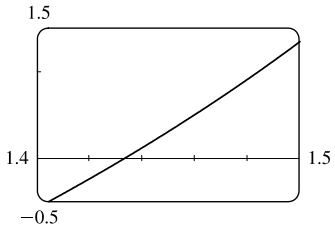
61. (a) Let $f(x) = x^5 - x^2 - 4$. Then $f(1) = 1^5 - 1^2 - 4 = -4 < 0$ and $f(2) = 2^5 - 2^2 - 4 = 24 > 0$. So by the Intermediate Value Theorem, there is a number c in $(1, 2)$ such that $f(c) = c^5 - c^2 - 4 = 0$.

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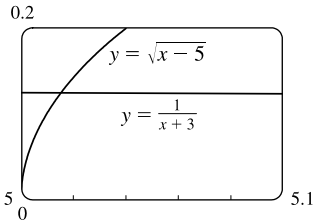
(b) We can see from the graphs that, correct to three decimal places, the solution is $x \approx 1.434$.





62. (a) Let $f(x) = \sqrt{x-5} - \dfrac{1}{x+3}$. Then $f(5) = -\frac{1}{8} < 0$ and $f(6) = \frac{8}{9} > 0$, and f is continuous on $[5, \infty)$. So by the Intermediate Value Theorem, there is a number c in $(5, 6)$ such that $f(c) = 0$. This implies that $\dfrac{1}{c+3} = \sqrt{c-5}$.

(b) Using the intersect feature of the graphing device, we find that the solution of the equation is $x = 5.016$, correct to three decimal places.



63. Let $f(x) = \sin x^3$. Then f is continuous on $[1, 2]$ since f is the composite of the sine function and the cubing function, both of which are continuous on $\mathbb{R}$. The zeros of the sine are at $n\pi$, so we note that $0 < 1 < \pi < \frac{3}{2}\pi < 2\pi < 8 < 3\pi$, and that the pertinent cube roots are related by $1 < \sqrt[3]{\frac{3}{2}\pi}$ [call this value A] < 2. [By observation, we might notice that $x = \sqrt[3]{\pi}$ and $x = \sqrt[3]{2\pi}$ are zeros of f.]

Now $f(1) = \sin 1 > 0$, $f(A) = \sin \frac{3}{2}\pi = -1 < 0$, and $f(2) = \sin 8 > 0$. Applying the Intermediate Value Theorem on $[1, A]$ and then on $[A, 2]$, we see there are numbers c and d in $(1, A)$ and $(A, 2)$ such that $f(c) = f(d) = 0$. Thus, f has at least two x-intercepts in $(1, 2)$.

64. Let $f(x) = x^2 - 3 + 1/x$. Then f is continuous on $(0, 2]$ since f is a rational function whose domain is $(0, \infty)$. By inspection, we see that $f\left(\frac{1}{4}\right) = \frac{17}{16} > 0$, $f(1) = -1 < 0$, and $f(2) = \frac{3}{2} > 0$. Appling the Intermediate Value Theorem on $\left[\frac{1}{4}, 1\right]$ and then on $[1, 2]$, we see there are numbers c and d in $\left(\frac{1}{4}, 1\right)$ and $(1, 2)$ such that $f(c) = f(d) = 0$. Thus, f has at least two x-intercepts in $(0, 2)$.

65. ($\Rightarrow$) If f is continuous at a, then by Theorem 8 with $g(h) = a + h$, we have

$$\lim_{h \to 0} f(a+h) = f\left(\lim_{h \to 0} (a+h)\right) = f(a).$$

($\Leftarrow$) Let $\varepsilon > 0$. Since $\lim\limits_{h \to 0} f(a+h) = f(a)$, there exists $\delta > 0$ such that $0 < |h| < \delta \quad \Rightarrow$ $|f(a+h) - f(a)| < \varepsilon$. So if $0 < |x - a| < \delta$, then $|f(x) - f(a)| = |f(a + (x-a)) - f(a)| < \varepsilon$. Thus, $\lim\limits_{x \to a} f(x) = f(a)$ and so f is continuous at a.

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66. $\lim_{h\to 0}\sin(a+h)=\lim_{h\to 0}(\sin a\cos h+\cos a\sin h)=\lim_{h\to 0}(\sin a\cos h)+\lim_{h\to 0}(\cos a\sin h)$

$$=\left(\lim_{h\to 0}\sin a\right)\left(\lim_{h\to 0}\cos h\right)+\left(\lim_{h\to 0}\cos a\right)\left(\lim_{h\to 0}\sin h\right)=(\sin a)(1)+(\cos a)(0)=\sin a$$

67. As in the previous exercise, we must show that $\lim_{h\to 0}\cos(a+h)=\cos a$ to prove that the cosine function is continuous.

$$\lim_{h\to 0}\cos(a+h)=\lim_{h\to 0}(\cos a\cos h-\sin a\sin h)=\lim_{h\to 0}(\cos a\cos h)-\lim_{h\to 0}(\sin a\sin h)$$

$$=\left(\lim_{h\to 0}\cos a\right)\left(\lim_{h\to 0}\cos h\right)-\left(\lim_{h\to 0}\sin a\right)\left(\lim_{h\to 0}\sin h\right)=(\cos a)(1)-(\sin a)(0)=\cos a$$

68. (a) Since f is continuous at a, $\lim_{x\to a}f(x)=f(a)$. Thus, using the Constant Multiple Law of Limits, we have

$\lim_{x\to a}(cf)(x)=\lim_{x\to a}cf(x)=c\lim_{x\to a}f(x)=cf(a)=(cf)(a)$. Therefore, cf is continuous at a.

(b) Since f and g are continuous at a, $\lim_{x\to a}f(x)=f(a)$ and $\lim_{x\to a}g(x)=g(a)$. Since $g(a)\neq 0$, we can use the Quotient Law of Limits: $\lim_{x\to a}\left(\dfrac{f}{g}\right)(x)=\lim_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}=\dfrac{f(a)}{g(a)}=\left(\dfrac{f}{g}\right)(a)$. Thus, $\dfrac{f}{g}$ is continuous at a.

69. *Proof of Law 6*: Let n be a positive integer. By Theorem 8 with $f(x)=x^n$, we have

$$\lim_{x\to a}[g(x)]^n=\lim_{x\to a}f(g(x))=f\left(\lim_{x\to a}g(x)\right)=\left[\lim_{x\to a}g(x)\right]^n$$

Proof of Law 7: Let n be a positive integer. By Theorem 8 with $f(x)=\sqrt[n]{x}$, we have

$$\lim_{x\to a}\sqrt[n]{g(x)}=\lim_{x\to a}f(g(x))=f\left(\lim_{x\to a}g(x)\right)=\sqrt[n]{\lim_{x\to a}g(x)}$$

70. If there is such a number, it satisfies the equation $x^3+1=x \quad\Leftrightarrow\quad x^3-x+1=0$. Let the left-hand side of this equation be called $f(x)$. Now $f(-2)=-5<0$, and $f(-1)=1>0$. Note also that $f(x)$ is a polynomial, and thus continuous. So by the Intermediate Value Theorem, there is a number c between -2 and -1 such that $f(c)=0$, so that $c=c^3+1$.

71. $f(x)=\begin{cases}0 & \text{if } x \text{ is rational}\\ 1 & \text{if } x \text{ is irrational}\end{cases}$ is continuous nowhere. For, given any number a and any $\delta>0$, the interval $(a-\delta,a+\delta)$ contains both infinitely many rational and infinitely many irrational numbers. Since $f(a)=0$ or 1, there are infinitely many numbers x with $0<|x-a|<\delta$ and $|f(x)-f(a)|=1$. Thus, $\lim_{x\to a}f(x)\neq f(a)$. [In fact, $\lim_{x\to a}f(x)$ does not even exist.]

72. $g(x)=\begin{cases}0 & \text{if } x \text{ is rational}\\ x & \text{if } x \text{ is irrational}\end{cases}$ is continuous at 0. To see why, note that $-|x|\le g(x)\le|x|$, so by the Squeeze Theorem $\lim_{x\to 0}g(x)=0=g(0)$. But g is continuous nowhere else. For if $a\neq 0$ and $\delta>0$, the interval $(a-\delta,a+\delta)$ contains both infinitely many rational and infinitely many irrational numbers. Since $g(a)=0$ or a, there are infinitely many numbers x with $0<|x-a|<\delta$ and $|g(x)-g(a)|>|a|/2$. Thus, $\lim_{x\to a}g(x)\neq g(a)$.

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73. $f(x) = x^4 \sin(1/x)$ is continuous on $(-\infty, 0) \cup (0, \infty)$ since it is the product of a polynomial and a composite of a trigonometric function and a rational function. Now since $-1 \le \sin(1/x) \le 1$, we have $-x^4 \le x^4 \sin(1/x) \le x^4$. Because $\lim_{x\to 0}(-x^4) = 0$ and $\lim_{x\to 0} x^4 = 0$, the Squeeze Theorem gives us $\lim_{x\to 0}(x^4 \sin(1/x)) = 0$, which equals $f(0)$. Thus, f is continuous at 0 and, hence, on $(-\infty, \infty)$.

74. $\dfrac{a}{x^3 + 2x^2 - 1} + \dfrac{b}{x^3 + x - 2} = 0 \quad \Rightarrow \quad a(x^3 + x - 2) + b(x^3 + 2x^2 - 1) = 0$. Let $p(x)$ denote the left side of the last equation. Since p is continuous on $[-1, 1]$, $p(-1) = -4a < 0$, and $p(1) = 2b > 0$, there exists a c in $(-1, 1)$ such that $p(c) = 0$ by the Intermediate Value Theorem. Note that the only solution of either denominator that is in $(-1, 1)$ is $(-1 + \sqrt{5}\,)/2 = r$, but $p(r) = (3\sqrt{5} - 9)a/2 \ne 0$. Thus, c is not a solution of either denominator, so $p(c) = 0 \quad \Rightarrow$ $x = c$ is a solution of the given equation.

75. Define $u(t)$ to be the monk's distance from the monastery, as a function of time t (in hours), on the first day, and define $d(t)$ to be his distance from the monastery, as a function of time, on the second day. Let D be the distance from the monastery to the top of the mountain. From the given information we know that $u(0) = 0$, $u(12) = D$, $d(0) = D$ and $d(12) = 0$. Now consider the function $u - d$, which is clearly continuous. We calculate that $(u - d)(0) = -D$ and $(u - d)(12) = D$. So by the Intermediate Value Theorem, there must be some time t_0 between 0 and 12 such that $(u - d)(t_0) = 0 \quad \Leftrightarrow$ $u(t_0) = d(t_0)$. So at time t_0 after 7:00 AM, the monk will be at the same place on both days.

76. (a) $\lim_{x\to 0^+} F(x) = 0$ and $\lim_{x\to 0^-} F(x) = 0$, so $\lim_{x\to 0} F(x) = 0$, which is $F(0)$, and hence F is continuous at $x = a$ if $a = 0$. For $a > 0$, $\lim_{x\to a} F(x) = \lim_{x\to a} x = a = F(a)$. For $a < 0$, $\lim_{x\to a} F(x) = \lim_{x\to a}(-x) = -a = F(a)$. Thus, F is continuous at $x = a$; that is, continuous everywhere.

(b) Assume that f is continuous on the interval I. Then for $a \in I$, $\lim_{x\to a} |f(x)| = \left|\lim_{x\to a} f(x)\right| = |f(a)|$ by Theorem 8. (If a is an endpoint of I, use the appropriate one-sided limit.) So $|f|$ is continuous on I.

(c) No, the converse is false. For example, the function $f(x) = \begin{cases} 1 & \text{if } x \ge 0 \\ -1 & \text{if } x < 0 \end{cases}$ is not continuous at $x = 0$, but $|f(x)| = 1$ is continuous on $\mathbb{R}$.

1 Review

TRUE-FALSE QUIZ

1. False. Let $f(x) = x^2$, $s = -1$, and $t = 1$. Then $f(s + t) = (-1 + 1)^2 = 0^2 = 0$, but $f(s) + f(t) = (-1)^2 + 1^2 = 2 \ne 0 = f(s + t)$.

2. False. Let $f(x) = x^2$. Then $f(-2) = 4 = f(2)$, but $-2 \ne 2$.

3. False. Let $f(x) = x^2$. Then $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So $f(3x) \ne 3f(x)$.

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4. True. If $x_1 < x_2$ and f is a decreasing function, then the y-values get smaller as we move from left to right. Thus, $f(x_1) > f(x_2)$.

5. True. See the Vertical Line Test.

6. False. For example, if $x = -3$, then $\sqrt{(-3)^2} = \sqrt{9} = 3$, not -3.

7. False. Limit Law 2 applies only if the individual limits exist (these don't).

8. False. Limit Law 5 cannot be applied if the limit of the denominator is 0 (it is).

9. True. Limit Law 5 applies.

10. False. $\dfrac{x^2-9}{x-3}$ is not defined when $x = 3$, but $x + 3$ is.

11. True. $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = \lim\limits_{x\to 3} \dfrac{(x+3)(x-3)}{(x-3)} = \lim\limits_{x\to 3}(x+3)$

12. True. The limit doesn't exist since $f(x)/g(x)$ doesn't approach any real number as x approaches 5. (The denominator approaches 0 and the numerator doesn't.)

13. False. Consider $\lim\limits_{x\to 5} \dfrac{x(x-5)}{x-5}$ or $\lim\limits_{x\to 5} \dfrac{\sin(x-5)}{x-5}$. The first limit exists and is equal to 5. By Example 1.5.3, we know that the latter limit exists (and it is equal to 1).

14. False. If $f(x) = 1/x$, $g(x) = -1/x$, and $a = 0$, then $\lim\limits_{x\to 0} f(x)$ does not exist, $\lim\limits_{x\to 0} g(x)$ does not exist, but $\lim\limits_{x\to 0} [f(x) + g(x)] = \lim\limits_{x\to 0} 0 = 0$ exists.

15. True. Suppose that $\lim\limits_{x\to a} [f(x) + g(x)]$ exists. Now $\lim\limits_{x\to a} f(x)$ exists and $\lim\limits_{x\to a} g(x)$ does not exist, but $\lim\limits_{x\to a} g(x) = \lim\limits_{x\to a} \{[f(x) + g(x)] - f(x)\} = \lim\limits_{x\to a} [f(x) + g(x)] - \lim\limits_{x\to a} f(x)$ [by Limit Law 2], which exists, and we have a contradiction. Thus, $\lim\limits_{x\to a} [f(x) + g(x)]$ does not exist.

16. False. Consider $\lim\limits_{x\to 6} [f(x)g(x)] = \lim\limits_{x\to 6} \left[(x-6)\dfrac{1}{x-6}\right]$. It exists (its value is 1) but $f(6) = 0$ and $g(6)$ does not exist, so $f(6)g(6) \neq 1$.

17. True. A polynomial is continuous everywhere, so $\lim\limits_{x\to b} p(x)$ exists and is equal to $p(b)$.

18. False. Consider $\lim\limits_{x\to 0} [f(x) - g(x)] = \lim\limits_{x\to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{x^4}\right)$. This limit is $-\infty$ (not 0), but each of the individual functions approaches ∞.

19. False. Consider $f(x) = \begin{cases} 1/(x-1) & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$

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20. False. The function f must be *continuous* in order to use the Intermediate Value Theorem. For example, let $f(x) = \begin{cases} 1 & \text{if } 0 \le x < 3 \\ -1 & \text{if } x = 3 \end{cases}$ There is no number $c \in [0, 3]$ with $f(c) = 0$.

21. True. Use Theorem 1.8.8 with $a = 2$, $b = 5$, and $g(x) = 4x^2 - 11$. Note that $f(4) = 3$ is not needed.

22. True. Use the Intermediate Value Theorem with $a = -1$, $b = 1$, and $N = \pi$, since $3 < \pi < 4$.

23. True, by the definition of a limit with $\varepsilon = 1$.

24. False. For example, let $f(x) = \begin{cases} x^2 + 1 & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

Then $f(x) > 1$ for all x, but $\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(x^2 + 1\right) = 1$.

25. True. $f(x) = x^{10} - 10x^2 + 5$ is continuous on the interval $[0, 2]$, $f(0) = 5$, $f(1) = -4$, and $f(2) = 989$. Since $-4 < 0 < 5$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $x^{10} - 10x^2 + 5 = 0$ in the interval $(0, 1)$. Similarly, there is a solution in $(1, 2)$.

26. True. See Exercise 1.8.76(b).

27. False. See Exercise 1.8.76(c).

EXERCISES

1. (a) When $x = 2$, $y \approx 2.7$. Thus, $f(2) \approx 2.7$.

(b) $f(x) = 3 \quad \Rightarrow \quad x \approx 2.3, 5.6$

(c) The domain of f is $-6 \le x \le 6$, or $[-6, 6]$.

(d) The range of f is $-4 \le y \le 4$, or $[-4, 4]$.

(e) f is increasing on $[-4, 4]$, that is, on $-4 \le x \le 4$.

(f) f is odd because its graph is symmetric about the origin.

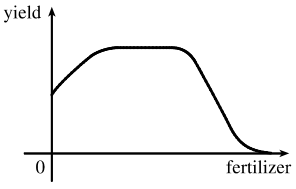
2. (a) This curve *is not* the graph of a function of x since it *fails* the Vertical Line Test.

(b) This curve *is* the graph of a function of x since it *passes* the Vertical Line Test. The domain is $[-3, 3]$ and the range is $[-2, 3]$.

3. $f(x) = x^2 - 2x + 3$, so $f(a + h) = (a + h)^2 - 2(a + h) + 3 = a^2 + 2ah + h^2 - 2a - 2h + 3$, and

$$\frac{f(a+h) - f(a)}{h} = \frac{(a^2 + 2ah + h^2 - 2a - 2h + 3) - (a^2 - 2a + 3)}{h} = \frac{h(2a + h - 2)}{h} = 2a + h - 2.$$

4. There will be some yield with no fertilizer, increasing yields with increasing fertilizer use, a leveling-off of yields at some point, and disaster with too much fertilizer use.



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5. $f(x) = 2/(3x-1)$.

Domain: $3x-1 \neq 0 \;\Rightarrow\; 3x \neq 1 \;\Rightarrow\; x \neq \frac{1}{3}$. $D = \left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$

Range: all reals except 0 ($y = 0$ is the horizontal asymptote for f.)

$R = (-\infty, 0) \cup (0, \infty)$

6. $g(x) = \sqrt{16 - x^4}$.

Domain: $16 - x^4 \geq 0 \;\Rightarrow\; x^4 \leq 16 \;\Rightarrow\; |x| \leq \sqrt[4]{16} \;\Rightarrow\; |x| \leq 2$. $D = [-2, 2]$

Range: $y \geq 0$ and $y \leq \sqrt{16} \;\Rightarrow\; 0 \leq y \leq 4$.

$R = [0, 4]$

7. $y = 1 + \sin x$.

Domain: $\mathbb{R}$.

Range: $-1 \leq \sin x \leq 1 \;\Rightarrow\; 0 \leq 1 + \sin x \leq 2 \;\Rightarrow\; 0 \leq y \leq 2$. $R = [0, 2]$

8. $y = F(t) = 3 + \cos 2t$.

Domain: $\mathbb{R}$. $D = (-\infty, \infty)$

Range: $-1 \leq \cos 2t \leq 1 \;\Rightarrow\; 2 \leq 3 + \cos 2t \leq 4 \;\Rightarrow\; 2 \leq y \leq 4$.

$R = [2, 4]$

9. (a) To obtain the graph of $y = f(x) + 5$, we shift the graph of $y = f(x)$ 5 units upward.

(b) To obtain the graph of $y = f(x + 5)$, we shift the graph of $y = f(x)$ 5 units to the left.

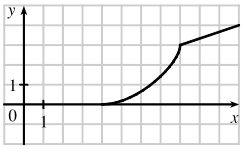
(c) To obtain the graph of $y = 1 + 2f(x)$, we stretch the graph of $y = f(x)$ vertically by a factor of 2, and then shift the resulting graph 1 unit upward.

(d) To obtain the graph of $y = f(x - 2) - 2$, we shift the graph of $y = f(x)$ 2 units to the right (for the "-2" inside the parentheses), and then shift the resulting graph 2 units downward.

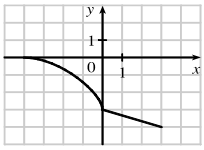
(e) To obtain the graph of $y = -f(x)$, we reflect the graph of $y = f(x)$ about the x-axis.

(f) To obtain the graph of $y = 3 - f(x)$, we reflect the graph of $y = f(x)$ about the x-axis, and then shift the resulting graph 3 units upward.

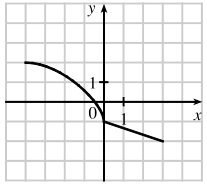
10. (a) To obtain the graph of $y = f(x - 8)$, we shift the graph of $y = f(x)$ right 8 units.



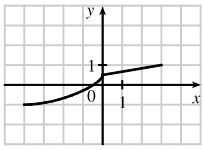
(b) To obtain the graph of $y = -f(x)$, we reflect the graph of $y = f(x)$ about the x-axis.



(c) To obtain the graph of $y = 2 - f(x)$, we reflect the graph of $y = f(x)$ about the x-axis, and then shift the resulting graph 2 units upward.



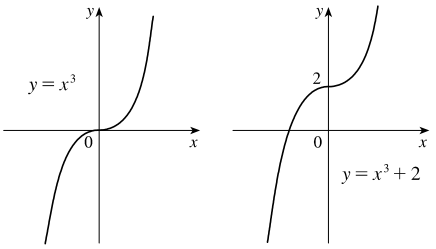
(d) To obtain the graph of $y = \frac{1}{2}f(x) - 1$, we shrink the graph of $y = f(x)$ by a factor of 2, and then shift the resulting graph 1 unit downward.



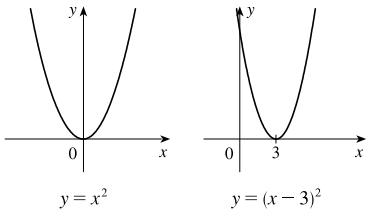
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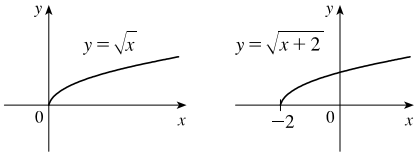
11. $f(x) = x^3 + 2$. Start with the graph of $y = x^3$ and shift 2 units upward.



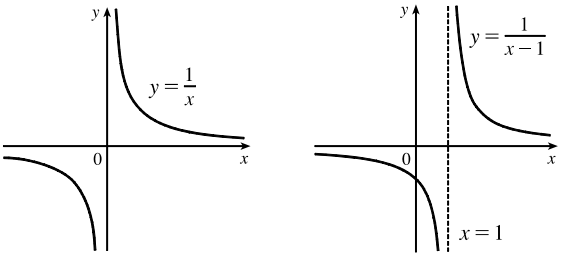
12. $f(x) = (x-3)^2$. Start with the graph of $y = x^2$ and shift 3 units to the right.



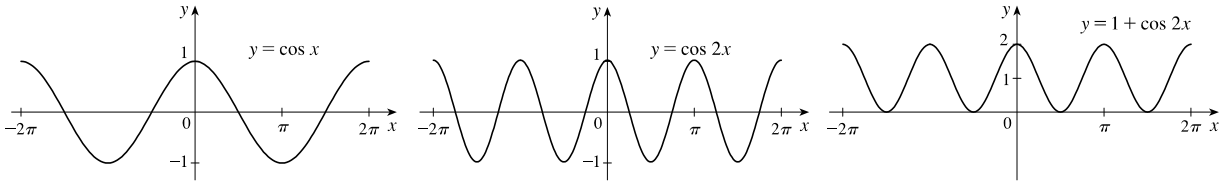
13. $y = \sqrt{x+2}$. Start with the graph of $y = \sqrt{x}$ and shift 2 units to the left.



14. $y = \dfrac{1}{x-1}$: Start with the graph of $y = \dfrac{1}{x}$ and shift 1 unit to the right.



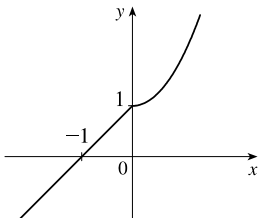
15. $g(x) = 1 + \cos 2x$. Start with the graph of $y = \cos x$, compress horizontally by a factor of 2, and then shift 1 unit upward.



16. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ 1+x^2 & \text{if } x \geq 0 \end{cases}$

On $(-\infty, 0)$, graph $y = 1 + x$ (the line with slope 1 and y-intercept 1) with open endpoint $(0, 1)$.

On $[0, \infty)$, graph $y = 1 + x^2$ (the rightmost half of the parabola $y = x^2$ shifted 1 unit upward) with closed endpoint $(0, 1)$.



17. (a) $f(x) = 2x^5 - 3x^2 + 2 \quad \Rightarrow \quad f(-x) = 2(-x)^5 - 3(-x)^2 + 2 = -2x^5 - 3x^2 + 2$. Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, f is neither even nor odd.

(b) $f(x) = x^3 - x^7 \quad \Rightarrow \quad f(-x) = (-x)^3 - (-x)^7 = -x^3 + x^7 = -(x^3 - x^7) = -f(x)$, so f is odd.

(c) $f(x) = 1 - \cos 2x \quad \Rightarrow \quad f(-x) = 1 - \cos[2(-x)] = 1 - \cos(-2x) = 1 - \cos 2x = f(x)$, so f is even.

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(d) $f(x) = 1 + \sin x \quad \Rightarrow \quad f(-x) = 1 + \sin(-x) = 1 - \sin x$. Now $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, so f is neither even nor odd.

(e) $f(x) = (x+1)^2 = x^2 + 2x + 1$. Now $f(-x) = (-x)^2 + 2(-x) + 1 = x^2 - 2x + 1$. Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, f is neither even nor odd.

18. For the line segment from $(-2, 2)$ to $(-1, 0)$, the slope is $\dfrac{0-2}{-1+2} = -2$, and an equation is $y - 0 = -2(x+1)$ or, equivalently, $y = -2x - 2$. The circle has equation $x^2 + y^2 = 1$; the top half has equation $y = \sqrt{1 - x^2}$ (we have solved for positive y). Thus, $f(x) = \begin{cases} -2x - 2 & \text{if } -2 \le x \le -1 \\ \sqrt{1 - x^2} & \text{if } -1 < x \le 1 \end{cases}$.

19. $f(x) = \sqrt{x}, \quad D = [0, \infty); \quad g(x) = \sin x, \quad D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f(\sin x) = \sqrt{\sin x}$. For $\sqrt{\sin x}$ to be defined, we must have $\sin x \ge 0 \quad \Leftrightarrow$ $x \in [0, \pi], [2\pi, 3\pi], [-2\pi, -\pi], [4\pi, 5\pi], [-4\pi, -3\pi], \ldots$, so $D = \{x \mid x \in [2n\pi, \pi + 2n\pi]$, where n is an integer$\}$.

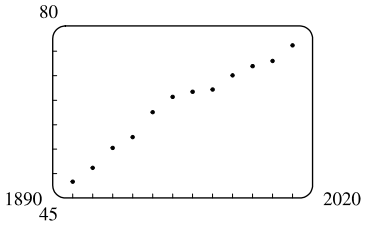
(b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sin\sqrt{x}$. x must be greater than or equal to 0 for $\sqrt{x}$ to be defined, so $D = [0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. $D = [0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g(\sin x) = \sin(\sin x)$. $D = \mathbb{R}$.

20. Let $h(x) = x + \sqrt{x}$, $g(x) = \sqrt{x}$, and $f(x) = 1/x$. Then $(f \circ g \circ h)(x) = \dfrac{1}{\sqrt{x + \sqrt{x}}} = F(x)$.

21.

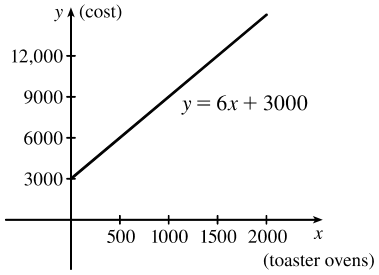


More than one model appears to be plausible. Your choice of model depends on whether you think medical advances will keep increasing life expectancy, or if there is bound to be a natural leveling-off of life expectancy. A linear model, $y = 0.2441x - 413.3960$, gives us an estimate of 82.1 years for the year 2030.

22. (a) Let x denote the number of toaster ovens produced in one week and y the associated cost. Using the points $(1000, 9000)$ and $(1500, 12{,}000)$, we get an equation of a line:

$y - 9000 = \dfrac{12{,}000 - 9000}{1500 - 1000}(x - 1000) \quad \Rightarrow$

$y = 6(x - 1000) + 9000 \quad \Rightarrow \quad y = 6x + 3000$.



(b) The slope of 6 means that each additional toaster oven produced adds \$6 to the weekly production cost.

(c) The y-intercept of 3000 represents the overhead cost—the cost incurred without producing anything.

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23. (a) (i) $\lim_{x\to 2^+} f(x) = 3$ (ii) $\lim_{x\to -3^+} f(x) = 0$

(iii) $\lim_{x\to -3} f(x)$ does not exist since the left and right limits are not equal. (The left limit is -2.)

(iv) $\lim_{x\to 4} f(x) = 2$

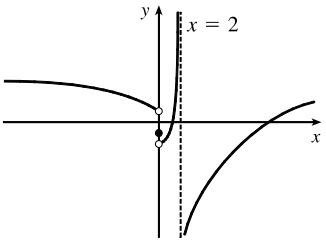
(v) $\lim_{x\to 0} f(x) = \infty$ (vi) $\lim_{x\to 2^-} f(x) = -\infty$

(b) The equations of the vertical asymptotes are $x = 0$ and $x = 2$.

(c) f is discontinuous at $x = -3$, 0, 2, and 4. The discontinuities are jump, infinite, infinite, and removable, respectively.

24. $\lim_{x\to 0^+} f(x) = -2$, $\lim_{x\to 0^-} f(x) = 1$, $f(0) = -1$,

$\lim_{x\to 2^-} f(x) = \infty$, $\lim_{x\to 2^+} f(x) = -\infty$



25. Since the cosine function is continuous on $(-\infty, \infty)$, $\lim_{x\to 0} \cos(x^3 + 3x) = \cos(0^3 + 3\cdot 0) = \cos 0 = 1$.

26. Since a rational function is continuous on its domain, $\lim_{x\to 3} \dfrac{x^2-9}{x^2+2x-3} = \dfrac{3^2-9}{3^2+2(3)-3} = \dfrac{0}{12} = 0$.

27. $\lim_{x\to -3} \dfrac{x^2-9}{x^2+2x-3} = \lim_{x\to -3} \dfrac{(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x\to -3} \dfrac{x-3}{x-1} = \dfrac{-3-3}{-3-1} = \dfrac{-6}{-4} = \dfrac{3}{2}$

28. $\lim_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3} = -\infty$ since $x^2+2x-3 \to 0^+$ as $x \to 1^+$ and $\dfrac{x^2-9}{x^2+2x-3} < 0$ for $1 < x < 3$.

29. $\lim_{h\to 0} \dfrac{(h-1)^3+1}{h} = \lim_{h\to 0} \dfrac{\left(h^3-3h^2+3h-1\right)+1}{h} = \lim_{h\to 0} \dfrac{h^3-3h^2+3h}{h} = \lim_{h\to 0} \left(h^2-3h+3\right) = 3$

Another solution: Factor the numerator as a sum of two cubes and then simplify.

$$\lim_{h\to 0} \frac{(h-1)^3+1}{h} = \lim_{h\to 0} \frac{(h-1)^3+1^3}{h} = \lim_{h\to 0} \frac{[(h-1)+1]\left[(h-1)^2-1(h-1)+1^2\right]}{h}$$
$$= \lim_{h\to 0} \left[(h-1)^2-h+2\right] = 1-0+2 = 3$$

30. $\lim_{t\to 2} \dfrac{t^2-4}{t^3-8} = \lim_{t\to 2} \dfrac{(t+2)(t-2)}{(t-2)(t^2+2t+4)} = \lim_{t\to 2} \dfrac{t+2}{t^2+2t+4} = \dfrac{2+2}{4+4+4} = \dfrac{4}{12} = \dfrac{1}{3}$

31. $\lim_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4} = \infty$ since $(r-9)^4 \to 0^+$ as $r \to 9$ and $\dfrac{\sqrt{r}}{(r-9)^4} > 0$ for $r \neq 9$.

32. $\lim_{v\to 4^+} \dfrac{4-v}{|4-v|} = \lim_{v\to 4^+} \dfrac{4-v}{-(4-v)} = \lim_{v\to 4^+} \dfrac{1}{-1} = -1$

33. $\lim_{r\to -1} \dfrac{r^2-3r-4}{4r^2+r-3} = \lim_{r\to -1} \dfrac{(r-4)(r+1)}{(4r-3)(r+1)} = \lim_{r\to -1} \dfrac{r-4}{4r-3} = \dfrac{-1-4}{4(-1)-3} = \dfrac{-5}{-7} = \dfrac{5}{7}$

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34. $\lim\limits_{t\to5}\dfrac{3-\sqrt{t+4}}{t-5}=\lim\limits_{t\to5}\dfrac{3-\sqrt{t+4}}{t-5}\left(\dfrac{3+\sqrt{t+4}}{3+\sqrt{t+4}}\right)=\lim\limits_{t\to5}\dfrac{9-(t+4)}{(t-5)\left(3+\sqrt{t+4}\right)}$

$$=\lim_{t\to5}\frac{5-t}{(t-5)\left(3+\sqrt{t+4}\right)}=\lim_{t\to5}\frac{-1}{3+\sqrt{t+4}}=\frac{-1}{3+\sqrt{5+4}}=-\frac{1}{6}$$

35. $\lim\limits_{s\to16}\dfrac{4-\sqrt{s}}{s-16}=\lim\limits_{s\to16}\dfrac{4-\sqrt{s}}{(\sqrt{s}+4)(\sqrt{s}-4)}=\lim\limits_{s\to16}\dfrac{-1}{\sqrt{s}+4}=\dfrac{-1}{\sqrt{16}+4}=-\dfrac{1}{8}$

36. $\lim\limits_{v\to2}\dfrac{v^2+2v-8}{v^4-16}=\lim\limits_{v\to2}\dfrac{(v+4)(v-2)}{(v+2)(v-2)(v^2+4)}=\lim\limits_{v\to2}\dfrac{v+4}{(v+2)(v^2+4)}=\dfrac{2+4}{(2+2)(2^2+4)}=\dfrac{3}{16}$

37. $\lim\limits_{x\to0}\dfrac{1-\sqrt{1-x^2}}{x}\cdot\dfrac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}=\lim\limits_{x\to0}\dfrac{1-(1-x^2)}{x\left(1+\sqrt{1-x^2}\right)}=\lim\limits_{x\to0}\dfrac{x^2}{x\left(1+\sqrt{1-x^2}\right)}=\lim\limits_{x\to0}\dfrac{x}{1+\sqrt{1-x^2}}=0$

38. $\lim\limits_{x\to1}\left(\dfrac{1}{x-1}+\dfrac{1}{x^2-3x+2}\right)=\lim\limits_{x\to1}\left[\dfrac{1}{x-1}+\dfrac{1}{(x-1)(x-2)}\right]=\lim\limits_{x\to1}\left[\dfrac{x-2}{(x-1)(x-2)}+\dfrac{1}{(x-1)(x-2)}\right]$

$$=\lim_{x\to1}\left[\frac{x-1}{(x-1)(x-2)}\right]=\lim_{x\to1}\frac{1}{x-2}=\frac{1}{1-2}=-1$$

39. Since $2x-1\le f(x)\le x^2$ for $0<x<3$ and $\lim\limits_{x\to1}(2x-1)=1=\lim\limits_{x\to1}x^2$, we have $\lim\limits_{x\to1}f(x)=1$ by the Squeeze Theorem.

40. Let $f(x)=-x^2$, $g(x)=x^2\cos\left(1/x^2\right)$ and $h(x)=x^2$. Then since $\left|\cos\left(1/x^2\right)\right|\le1$ for $x\neq0$, we have $f(x)\le g(x)\le h(x)$ for $x\neq0$, and so $\lim\limits_{x\to0}f(x)=\lim\limits_{x\to0}h(x)=0\quad\Rightarrow\quad\lim\limits_{x\to0}g(x)=0$ by the Squeeze Theorem.

41. Given $\varepsilon>0$, we need $\delta>0$ such that if $0<|x-2|<\delta$, then $|(14-5x)-4|<\varepsilon$. But $|(14-5x)-4|<\varepsilon\quad\Leftrightarrow\quad|-5x+10|<\varepsilon\quad\Leftrightarrow\quad|-5|\,|x-2|<\varepsilon\quad\Leftrightarrow\quad|x-2|<\varepsilon/5$. So if we choose $\delta=\varepsilon/5$, then $0<|x-2|<\delta\quad\Rightarrow\quad|(14-5x)-4|<\varepsilon$. Thus, $\lim\limits_{x\to2}(14-5x)=4$ by the definition of a limit.

42. Given $\varepsilon>0$ we must find $\delta>0$ so that if $0<|x-0|<\delta$, then $|\sqrt[3]{x}-0|<\varepsilon$. Now $|\sqrt[3]{x}-0|=|\sqrt[3]{x}|<\varepsilon\quad\Rightarrow\quad|x|=|\sqrt[3]{x}|^3<\varepsilon^3$. So take $\delta=\varepsilon^3$. Then $0<|x-0|=|x|<\varepsilon^3\quad\Rightarrow\quad|\sqrt[3]{x}-0|=|\sqrt[3]{x}|=\sqrt[3]{|x|}<\sqrt[3]{\varepsilon^3}=\varepsilon$. Therefore, by the definition of a limit, $\lim\limits_{x\to0}\sqrt[3]{x}=0$.

43. Given $\varepsilon>0$, we need $\delta>0$ so that if $0<|x-2|<\delta$, then $\left|x^2-3x-(-2)\right|<\varepsilon$. First, note that if $|x-2|<1$, then $-1<x-2<1$, so $0<x-1<2\quad\Rightarrow\quad|x-1|<2$. Now let $\delta=\min\{\varepsilon/2,1\}$. Then $0<|x-2|<\delta\quad\Rightarrow\quad\left|x^2-3x-(-2)\right|=|(x-2)(x-1)|=|x-2|\,|x-1|<(\varepsilon/2)(2)=\varepsilon$.

Thus, $\lim\limits_{x\to2}(x^2-3x)=-2$ by the definition of a limit.

44. Given $M>0$, we need $\delta>0$ such that if $0<x-4<\delta$, then $2/\sqrt{x-4}>M$. This is true $\quad\Leftrightarrow\quad\sqrt{x-4}<2/M\quad\Leftrightarrow\quad x-4<4/M^2$. So if we choose $\delta=4/M^2$, then $0<x-4<\delta\quad\Rightarrow\quad2/\sqrt{x-4}>M$. So by the definition of a limit, $\lim\limits_{x\to4^+}\left(2/\sqrt{x-4}\right)=\infty$.

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45. (a) $f(x) = \sqrt{-x}$ if $x < 0$, $f(x) = 3 - x$ if $0 \le x < 3$, $f(x) = (x-3)^2$ if $x > 3$.

(i) $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3 - x) = 3$

(ii) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \sqrt{-x} = 0$

(iii) Because of (i) and (ii), $\lim_{x \to 0} f(x)$ does not exist.

(iv) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (3 - x) = 0$

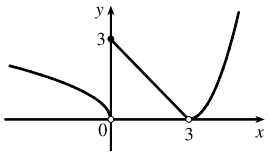
(v) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x-3)^2 = 0$

(vi) Because of (iv) and (v), $\lim_{x \to 3} f(x) = 0$.

(b) f is discontinuous at 0 since $\lim_{x \to 0} f(x)$ does not exist.

f is discontinuous at 3 since $f(3)$ does not exist.

(c)



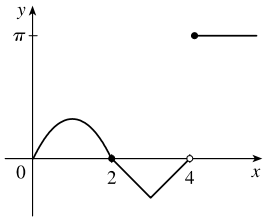
46. (a) $g(x) = 2x - x^2$ if $0 \le x \le 2$, $g(x) = 2 - x$ if $2 < x \le 3$, $g(x) = x - 4$ if $3 < x < 4$, $g(x) = \pi$ if $x \ge 4$.

Therefore, $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} \left(2x - x^2\right) = 0$ and $\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (2 - x) = 0$. Thus, $\lim_{x \to 2} g(x) = 0 = g(2)$, so g is continuous at 2. $\lim_{x \to 3^-} g(x) = \lim_{x \to 3^-} (2 - x) = -1$ and $\lim_{x \to 3^+} g(x) = \lim_{x \to 3^+} (x - 4) = -1$. Thus, $\lim_{x \to 3} g(x) = -1 = g(3)$, so g is continuous at 3.

$\lim_{x \to 4^-} g(x) = \lim_{x \to 4^-} (x - 4) = 0$ and $\lim_{x \to 4^+} g(x) = \lim_{x \to 4^+} \pi = \pi$.

Thus, $\lim_{x \to 4} g(x)$ does not exist, so g is discontinuous at 4. But $\lim_{x \to 4^+} g(x) = \pi = g(4)$, so g is continuous from the right at 4.

(b)



47. x^3 is continuous on $\mathbb{R}$ since it is a polynomial and $\cos x$ is also continuous on $\mathbb{R}$, so the product $x^3 \cos x$ is continuous on $\mathbb{R}$. The root function $\sqrt[4]{x}$ is continuous on its domain, $[0, \infty)$, and so the sum, $h(x) = \sqrt[4]{x} + x^3 \cos x$, is continuous on its domain, $[0, \infty)$.

48. $x^2 - 9$ is continuous on $\mathbb{R}$ since it is a polynomial and $\sqrt{x}$ is continuous on $[0, \infty)$ by Theorem 1.8.7, so the composition $\sqrt{x^2 - 9}$ is continuous on $\{x \mid x^2 - 9 \ge 0\} = (-\infty, -3] \cup [3, \infty)$ by Theorem 1.8.9. Note that $x^2 - 2 \ne 0$ on this set and so the quotient function $g(x) = \dfrac{\sqrt{x^2 - 9}}{x^2 - 2}$ is continuous on its domain, $(-\infty, -3] \cup [3, \infty)$ by Theorem 1.8.4.

49. $f(x) = x^5 - x^3 + 3x - 5$ is continuous on the interval $[1, 2]$, $f(1) = -2$, and $f(2) = 25$. Since $-2 < 0 < 25$, there is a number c in $(1, 2)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a solution of the equation $x^5 - x^3 + 3x - 5 = 0$ in the interval $(1, 2)$.

50. Let $f(x) = 2 \sin x - 3 + 2x$. Now f is continuous on $[0, 1]$ and $f(0) = -3 < 0$ and $f(1) = 2 \sin 1 - 1 \approx 0.68 > 0$. So by the Intermediate Value Theorem there is a number c in $(0, 1)$ such that $f(c) = 0$, that is, the equation $2 \sin x = 3 - 2x$ has a solution in $(0, 1)$.

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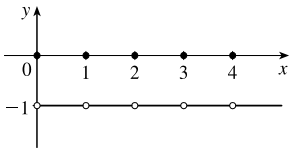
51. $|f(x)| \le g(x) \quad \Leftrightarrow \quad -g(x) \le f(x) \le g(x)$ and $\lim_{x \to a} g(x) = 0 = \lim_{x \to a} -g(x)$.

Thus, by the Squeeze Theorem, $\lim_{x \to a} f(x) = 0$.

52. (a) Note that f is an even function since $f(x) = f(-x)$. Now for any integer n,

$[\![n]\!] + [\![-n]\!] = n - n = 0$, and for any real number k which is not an integer,

$[\![k]\!] + [\![-k]\!] = [\![k]\!] + (-[\![k]\!] - 1) = -1$. So $\lim_{x \to a} f(x)$ exists (and is equal to -1) for all values of a.



(b) f is discontinuous at all integers.

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PRINCIPLES OF PROBLEM SOLVING

1. $\big|4x-|x+1|\,\big|=3 \quad\Rightarrow\quad 4x-|x+1|=-3$ (Equation 1) or $4x-|x+1|=3$ (Equation 2).

If $x+1<0$, or $x<-1$, then $|x+1|=-(x+1)=-x-1$. If $x+1\geq 0$, or $x\geq -1$, then $|x+1|=x+1$.

We thus consider two cases, $x<-1$ (Case 1) and $x\geq -1$ (Case 2), for each of Equations 1 and 2.

Equation 1, Case 1: $4x-|x+1|=-3 \quad\Rightarrow\quad 4x-(-x-1)=-3 \quad\Rightarrow\quad 5x+1=-3 \quad\Rightarrow$
$5x=-4 \quad\Rightarrow\quad x=-\frac{4}{5}$ which is invalid since $x<-1$.

Equation 1, Case 2: $4x-|x+1|=-3 \quad\Rightarrow\quad 4x-(x-1)=-3 \quad\Rightarrow\quad 3x-1=-3 \quad\Rightarrow$
$3x=-2 \quad\Rightarrow\quad x=-\frac{2}{3}$, which is valid since $x\geq -1$.

Equation 2, Case 1: $4x-|x+1|=3 \quad\Rightarrow\quad 4x-(-x-1)=3 \quad\Rightarrow\quad 5x+1=3 \quad\Rightarrow$
$5x=2 \quad\Rightarrow\quad x=\frac{2}{5}$, which is invalid since $x<-1$.

Equation 2, Case 2: $4x-|x+1|=3 \quad\Rightarrow\quad 4x-(x+1)=3 \quad\Rightarrow\quad 3x-1=3 \quad\Rightarrow$
$3x=4 \quad\Rightarrow\quad x=\frac{4}{3}$, which is valid since $x\geq -1$.

Thus, the solution set is $\left\{-\frac{2}{3},\frac{4}{3}\right\}$.

2. $|x-1|=\begin{cases} x-1 & \text{if } x\geq 1 \\ 1-x & \text{if } x<1 \end{cases}$ and $|x-3|=\begin{cases} x-3 & \text{if } x\geq 3 \\ 3-x & \text{if } x<3 \end{cases}$

Therefore, we consider the three cases $x<1$, $1\leq x<3$, and $x\geq 3$.

If $x<1$, we must have $1-x-(3-x)\geq 5 \quad\Leftrightarrow\quad 0\geq 7$, which is false.

If $1\leq x<3$, we must have $x-1-(3-x)\geq 5 \quad\Leftrightarrow\quad x\geq \frac{9}{2}$, which is false because $x<3$.

If $x\geq 3$, we must have $x-1-(x-3)\geq 5 \quad\Leftrightarrow\quad 2\geq 5$, which is false.

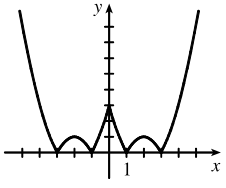
All three cases lead to falsehoods, so the inequality has no solution.

3. $f(x)=\big|x^2-4\,|x|+3\big|$. If $x\geq 0$, then $f(x)=\big|x^2-4x+3\big|=|(x-1)(x-3)|$.

Case (i): If $0<x\leq 1$, then $f(x)=x^2-4x+3$.

Case (ii): If $1<x\leq 3$, then $f(x)=-(x^2-4x+3)=-x^2+4x-3$.

Case (iii): If $x>3$, then $f(x)=x^2-4x+3$.



This enables us to sketch the graph for $x\geq 0$. Then we use the fact that f is an even function to reflect this part of the graph about the y-axis to obtain the entire graph. Or, we could consider also the cases $x<-3$, $-3\leq x<-1$, and $-1\leq x<0$.

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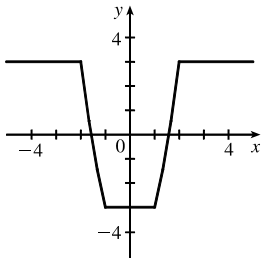
4. $g(x) = |x^2 - 1| - |x^2 - 4|$.

$$|x^2-1| = \begin{cases} x^2-1 & \text{if } |x| \geq 1 \\ 1-x^2 & \text{if } |x| < 1 \end{cases} \text{ and } |x^2-4| = \begin{cases} x^2-4 & \text{if } |x| \geq 2 \\ 4-x^2 & \text{if } |x| < 2 \end{cases}$$

So for $0 \leq |x| < 1, g(x) = 1 - x^2 - (4 - x^2) = -3$, for

$1 \leq |x| < 2, g(x) = x^2 - 1 - (4 - x^2) = 2x^2 - 5$, and for

$|x| \geq 2, g(x) = x^2 - 1 - (x^2 - 4) = 3$.



5. Remember that $|a| = a$ if $a \geq 0$ and that $|a| = -a$ if $a < 0$. Thus,

$$x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad \text{and} \quad y + |y| = \begin{cases} 2y & \text{if } y \geq 0 \\ 0 & \text{if } y < 0 \end{cases}$$

We will consider the equation $x + |x| = y + |y|$ in four cases.

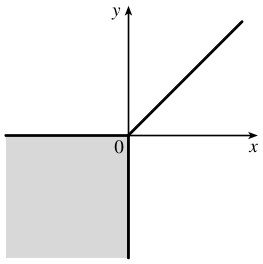
(1) $x \geq 0, y \geq 0$	(2) $x \geq 0, y < 0$	(3) $x < 0, y \geq 0$	(4) $x < 0, y < 0$
$2x = 2y$	$2x = 0$	$0 = 2y$	$0 = 0$
$x = y$	$x = 0$	$0 = y$	

Case 1 gives us the line $y = x$ with nonnegative x and y.

Case 2 gives us the portion of the y-axis with y negative.

Case 3 gives us the portion of the x-axis with x negative.

Case 4 gives us the entire third quadrant.



6. $|x - y| + |x| - |y| \leq 2$ [call this inequality $(\star)$]

Case (i): $x \geq y \geq 0$. Then $(\star) \Leftrightarrow x - y + x - y \leq 2 \Leftrightarrow x - y \leq 1 \Leftrightarrow y \geq x - 1$.

Case (ii): $y \geq x \geq 0$. Then $(\star) \Leftrightarrow y - x + x - y \leq 2 \Leftrightarrow 0 \leq 2$ (true).

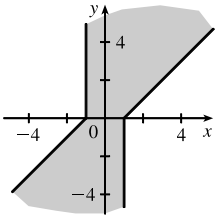
Case (iii): $x \geq 0$ and $y \leq 0$. Then $(\star) \Leftrightarrow x - y + x + y \leq 2 \Leftrightarrow 2x \leq 2 \Leftrightarrow x \leq 1$.

Case (iv): $x \leq 0$ and $y \geq 0$. Then $(\star) \Leftrightarrow y - x - x - y \leq 2 \Leftrightarrow -2x \leq 2 \Leftrightarrow x \geq -1$.

Case (v): $y \leq x \leq 0$. Then $(\star) \Leftrightarrow x - y - x + y \leq 2 \Leftrightarrow 0 \leq 2$ (true).

Case (vi): $x \leq y \leq 0$. Then $(\star) \Leftrightarrow y - x - x + y \leq 2 \Leftrightarrow y - x \leq 1 \Leftrightarrow y \leq x + 1$.

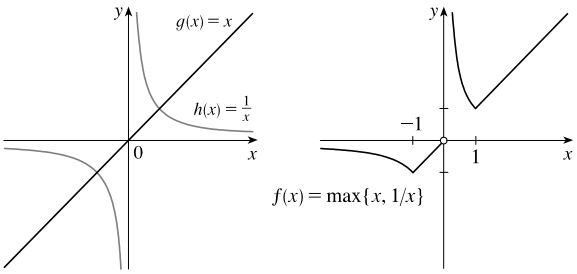
Note: Instead of considering cases (iv), (v), and (vi), we could have noted that the region is unchanged if x and y are replaced by $-x$ and $-y$, so the region is symmetric about the origin. Therefore, we need only draw cases (i), (ii), and (iii), and rotate through 180° about the origin.



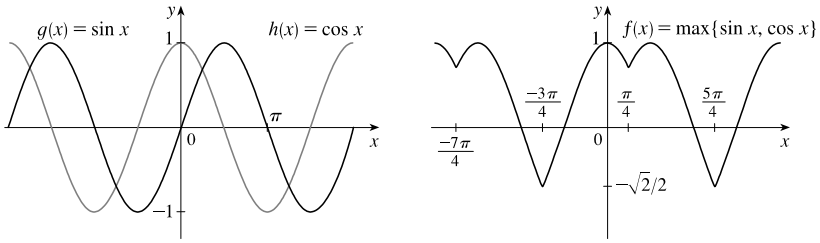
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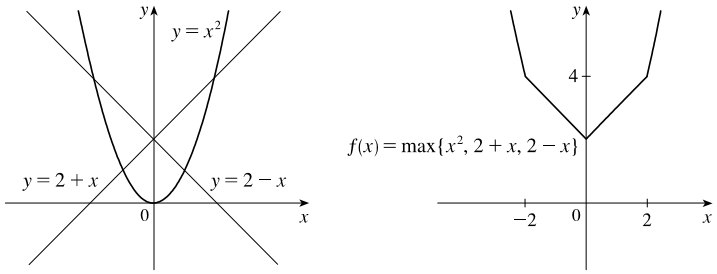
7. (a) To sketch the graph of $f(x) = \max\{x, 1/x\}$, we first graph $g(x) = x$ and $h(x) = 1/x$ on the same coordinate axes. Then create the graph of f by plotting the largest y-value of g and h for every value of x.



(b)

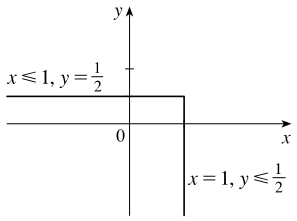


(c)

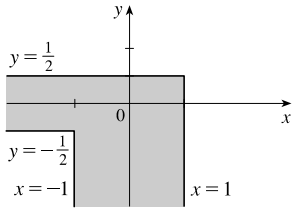


On the TI-84 Plus, max is found under `LIST`, then under `MATH`. To graph $f(x) = \max\{x^2, 2 + x, 2 - x\}$, use $\texttt{Y} = \max(x^2, \max(2 + x, 2 - x))$.

8. (a) If $\max\{x, 2y\} = 1$, then either $x = 1$ *and* $2y \le 1$ ***or*** $x \le 1$ *and* $2y = 1$. Thus, we obtain the set of points such that $x = 1$ *and* $y \le \frac{1}{2}$ [a vertical line with highest point $(1, \frac{1}{2})$] ***or*** $x \le 1$ *and* $y = \frac{1}{2}$ [a horizontal line with rightmost point $(1, \frac{1}{2})$].



(b) The graph of $\max\{x, 2y\} = 1$ is shown in part (a), and the graph of $\max\{x, 2y\} = -1$ can be found in a similar manner. The inequalities in $-1 \le \max\{x, 2y\} \le 1$ give us all the points on or inside the boundaries.

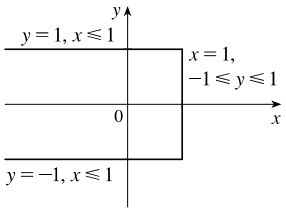


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(c) $\max\{x, y^2\} = 1 \quad \Leftrightarrow \quad x = 1$ *and* $y^2 \le 1$ $[-1 \le y \le 1]$

or $x \le 1$ *and* $y^2 = 1$ $[y = \pm 1]$.



9. By rearranging terms, we write the given expression as

$$\left(\sin\frac{\pi}{100} + \sin\frac{199\pi}{100}\right) + \left(\sin\frac{2\pi}{100} + \sin\frac{198\pi}{100}\right) + \cdots + \left(\sin\frac{99\pi}{100} + \sin\frac{101\pi}{100}\right) + \sin\frac{100\pi}{100} + \sin\frac{200\pi}{100}$$

Each grouped sum is of the form $\sin x + \sin y$ with $x + y = 2\pi$ so that $\dfrac{x+y}{2} = \dfrac{2\pi}{2} = \pi$. We now derive a useful identity from the product-to-sum identity $\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$. If in this identity we replace x with $\dfrac{x+y}{2}$ and y with $\dfrac{x-y}{2}$, we have

$$\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{2}\left[\sin\left(\frac{x+y}{2} + \frac{x-y}{2}\right) + \sin\left(\frac{x+y}{2} - \frac{x-y}{2}\right)\right] = \frac{1}{2}(\sin x + \sin y)$$

Multiplication of the left and right members of this equality by 2 gives the sum-to-product identity $\sin x + \sin y = 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x+y}{2}\right)$. Using this sum-to-product identity, we have each grouped sum equal to 0, since $\sin\left(\dfrac{x+y}{2}\right) = \sin\pi = 0$ is always a factor of the right side. Since $\sin\dfrac{100\pi}{100} = \sin\pi = 0$ and $\sin\dfrac{200\pi}{100} = \sin 2\pi = 0$, the sum of the given expression is 0.

Another approach: Since the sine function is odd, $\sin(-x) = -\sin x$. Because the period of the sine function is 2π, we have $\sin(-x + 2\pi) = -\sin x$. Multiplying each side by -1 and rearranging, we have $\sin x = -\sin(2\pi - x)$. This means that $\sin\dfrac{\pi}{100} = -\sin\left(2\pi - \dfrac{\pi}{100}\right) = -\sin\dfrac{199\pi}{100}$, $\sin\dfrac{2\pi}{100} = -\sin\left(2\pi - \dfrac{2\pi}{100}\right) = \sin\dfrac{198\pi}{100}$, and so on, until we have $\sin\dfrac{99\pi}{100} = -\sin\left(2\pi - \dfrac{99\pi}{100}\right) = -\sin\dfrac{101\pi}{100}$. As before we rearrange terms to write the given expression as

$$\left(\sin\frac{\pi}{100} + \sin\frac{199\pi}{100}\right) + \left(\sin\frac{2\pi}{100} + \sin\frac{198\pi}{100}\right) + \cdots + \left(\sin\frac{99\pi}{100} + \sin\frac{101\pi}{100}\right) + \sin\frac{100\pi}{100} + \sin\frac{200\pi}{100}$$

Each sum in parentheses is 0 since the two terms are opposites, and the last two terms again reduce to $\sin\pi$ and $\sin 2\pi$, respectively, each also 0. Thus, the value of the original expression is 0.

10. We note that $-1 \le \sin x \le 1$ for all x. Thus, any solution of $\sin x = x/100$ will have $-1 \le x/100 \le 1$, or $-100 \le x \le 100$. We next observe that the period of $\sin x$ is 2π, and $\sin x$ takes on each value in its range, except for -1 and 1, twice each cycle. We observe that $x = 0$ is a solution. Finally, we note that because $\sin x$ and $x/100$ are both odd functions, every solution on $0 \le x \le 100$ gives us a corresponding solution on $-100 \le x \le 0$.

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$100/2\pi \approx 15.9$, so there 15 full cycles of $\sin x$ on $[0, 100]$ .Each of the 15 intervals $[0, 2\pi]$, $[2\pi, 4\pi]$, . . ., $[28\pi, 30\pi]$ must contain two solutions of $\sin x = x/100$, as the graph of $\sin x$ will intersect the graph of $x/100$ twice each cycle. We must be careful with the next (16th) interval $[30\pi, 32\pi]$, because 100 is contained in the interval. A graph of $y_1 = \sin x$ and $y_2 = x/100$ over this interval reveals that two intersections occur within the interval with $x \le 100$.

Thus, there are $16 \cdot 2 = 32$ solutions of $\sin x = x/100$ on $[0, 100]$. There are also 32 solutions of the equation on $[-100, 0]$. Being careful to not count the solution $x = 0$ twice, we find that there are $32 + 32 - 1 = 63$ solutions of the equation $\sin x = x/100$.

11. Let d be the distance traveled on each half of the trip. Let t_1 and t_2 be the times taken for the first and second halves of the trip. For the first half of the trip we have $t_1 = d/30$ and for the second half we have $t_2 = d/60$. Thus, the average speed for the entire trip is $\dfrac{\text{total distance}}{\text{total time}} = \dfrac{2d}{t_1 + t_2} = \dfrac{2d}{\dfrac{d}{30} + \dfrac{d}{60}} \cdot \dfrac{60}{60} = \dfrac{120d}{2d + d} = \dfrac{120d}{3d} = 40$. The average speed for the entire trip is 40 mi/h.

12. Let $f(x) = \sin x$, $g(x) = x$, and $h(x) = x$. Then the left-hand side of the equation is $[f \circ (g + h)](x) = \sin(x + x) = \sin 2x = 2 \sin x \cos x$; and the right-hand side is $(f \circ g)(x) + (f \circ h)(x) = \sin x + \sin x = 2 \sin x$. The two sides are not equal, so the given statement is false.

13. Let S_n be the statement that $7^n - 1$ is divisible by 6.

- S_1 is true because $7^1 - 1 = 6$ is divisible by 6.
- Assume S_k is true, that is, $7^k - 1$ is divisible by 6. In other words, $7^k - 1 = 6m$ for some positive integer m. Then $7^{k+1} - 1 = 7^k \cdot 7 - 1 = (6m + 1) \cdot 7 - 1 = 42m + 6 = 6(7m + 1)$, which is divisible by 6, so S_{k+1} is true.
- Therefore, by mathematical induction, $7^n - 1$ is divisible by 6 for every positive integer n.

14. Let S_n be the statement that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

- S_1 is true because $[2(1) - 1] = 1 = 1^2$.
- Assume S_k is true, that is, $1 + 3 + 5 + \cdots + (2k - 1) = k^2$. Then

  $1 + 3 + 5 + \cdots + (2k - 1) + [2(k + 1) - 1] = 1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = k^2 + (2k + 1) = (k + 1)^2$

  which shows that S_{k+1} is true.
- Therefore, by mathematical induction, $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for every positive integer n.

15. $f_0(x) = x^2$ and $f_{n+1}(x) = f_0(f_n(x))$ for $n = 0, 1, 2, \ldots$.

$f_1(x) = f_0(f_0(x)) = f_0\left(x^2\right) = \left(x^2\right)^2 = x^4$, $f_2(x) = f_0(f_1(x)) = f_0(x^4) = (x^4)^2 = x^8$,

$f_3(x) = f_0(f_2(x)) = f_0(x^8) = (x^8)^2 = x^{16}, \ldots$. Thus, a general formula is $f_n(x) = x^{2^{n+1}}$.

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16. (a) $f_0(x) = 1/(2-x)$ and $f_{n+1} = f_0 \circ f_n$ for $n = 0, 1, 2, \ldots$.

$$f_1(x) = f_0\left(\frac{1}{2-x}\right) = \frac{1}{2-\dfrac{1}{2-x}} = \frac{2-x}{2(2-x)-1} = \frac{2-x}{3-2x},$$

$$f_2(x) = f_0\left(\frac{2-x}{3-2x}\right) = \frac{1}{2-\dfrac{2-x}{3-2x}} = \frac{3-2x}{2(3-2x)-(2-x)} = \frac{3-2x}{4-3x},$$

$$f_3(x) = f_0\left(\frac{3-2x}{4-3x}\right) = \frac{1}{2-\dfrac{3-2x}{4-3x}} = \frac{4-3x}{2(4-3x)-(3-2x)} = \frac{4-3x}{5-4x}, \ldots$$

Thus, we conjecture that the general formula is $f_n(x) = \dfrac{n+1-nx}{n+2-(n+1)x}$.

To prove this, we use the Principle of Mathematical Induction. We have already verified that f_n is true for $n = 1$.

Assume that the formula is true for $n = k$; that is, $f_k(x) = \dfrac{k+1-kx}{k+2-(k+1)x}$. Then

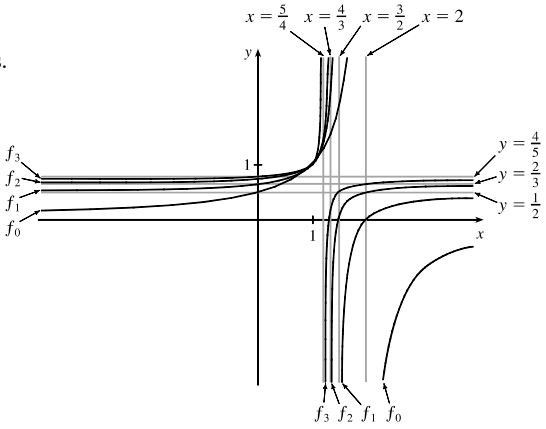
$$f_{k+1}(x) = (f_0 \circ f_k)(x) = f_0(f_k(x)) = f_0\left(\frac{k+1-kx}{k+2-(k+1)x}\right) = \frac{1}{2-\dfrac{k+1-kx}{k+2-(k+1)x}}$$

$$= \frac{k+2-(k+1)x}{2\,[k+2-(k+1)x]-(k+1-kx)} = \frac{k+2-(k+1)x}{k+3-(k+2)x}$$

This shows that the formula for f_n is true for $n = k+1$. Therefore, by mathematical induction, the formula is true for all positive integers n.

(b) From the graph, we can make several observations:

- The values at each fixed $x = a$ keep increasing as n increases.
- The vertical asymptote gets closer to $x = 1$ as n increases.
- The horizontal asymptote gets closer to $y = 1$ as n increases.
- The x-intercept for f_{n+1} is the value of the vertical asymptote for f_n.
- The y-intercept for f_n is the value of the horizontal asymptote for f_{n+1}.



17. Let $t = \sqrt[6]{x}$, so $x = t^6$. Then $t \to 1$ as $x \to 1$, so

$$\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \lim_{t\to 1} \frac{t^2-1}{t^3-1} = \lim_{t\to 1} \frac{(t-1)(t+1)}{(t-1)\,(t^2+t+1)} = \lim_{t\to 1} \frac{t+1}{t^2+t+1} = \frac{1+1}{1^2+1+1} = \frac{2}{3}.$$

Another method: Multiply both the numerator and the denominator by $(\sqrt{x}+1)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)$.

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18. First rationalize the numerator: $\lim_{x\to 0}\dfrac{\sqrt{ax+b}-2}{x}\cdot\dfrac{\sqrt{ax+b}+2}{\sqrt{ax+b}+2}=\lim_{x\to 0}\dfrac{ax+b-4}{x\left(\sqrt{ax+b}+2\right)}$. Now since the denominator approaches 0 as $x\to 0$, the limit will exist only if the numerator also approaches 0 as $x\to 0$. So we require that $a(0)+b-4=0 \;\Rightarrow\; b=4$. So the equation becomes $\lim_{x\to 0}\dfrac{a}{\sqrt{ax+4}+2}=1 \;\Rightarrow\; \dfrac{a}{\sqrt{4}+2}=1 \;\Rightarrow\; a=4$.

Therefore, $a=b=4$.

19. For $-\frac{1}{2}<x<\frac{1}{2}$, we have $2x-1<0$ and $2x+1>0$, so $|2x-1|=-(2x-1)$ and $|2x+1|=2x+1$.

Therefore, $\lim_{x\to 0}\dfrac{|2x-1|-|2x+1|}{x}=\lim_{x\to 0}\dfrac{-(2x-1)-(2x+1)}{x}=\lim_{x\to 0}\dfrac{-4x}{x}=\lim_{x\to 0}(-4)=-4$.

20. Let R be the midpoint of OP, so the coordinates of R are $\left(\frac{1}{2}x,\frac{1}{2}x^2\right)$ since the coordinates of P are $\left(x,x^2\right)$. Let $Q=(0,a)$.

Since the slope $m_{OP}=\dfrac{x^2}{x}=x$, $m_{QR}=-\dfrac{1}{x}$ (negative reciprocal). But $m_{QR}=\dfrac{\frac{1}{2}x^2-a}{\frac{1}{2}x-0}=\dfrac{x^2-2a}{x}$, so we conclude that $-1=x^2-2a \;\Rightarrow\; 2a=x^2+1 \;\Rightarrow\; a=\frac{1}{2}x^2+\frac{1}{2}$. As $x\to 0$, $a\to\frac{1}{2}$, and the limiting position of Q is $\left(0,\frac{1}{2}\right)$.

21. (a) For $0<x<1$, $\llbracket x\rrbracket=0$, so $\dfrac{\llbracket x\rrbracket}{x}=0$, and $\lim_{x\to 0^+}\dfrac{\llbracket x\rrbracket}{x}=0$. For $-1<x<0$, $\llbracket x\rrbracket=-1$, so $\dfrac{\llbracket x\rrbracket}{x}=\dfrac{-1}{x}$, and $\lim_{x\to 0^-}\dfrac{\llbracket x\rrbracket}{x}=\lim_{x\to 0^-}\left(\dfrac{-1}{x}\right)=\infty$. Since the one-sided limits are not equal, $\lim_{x\to 0}\dfrac{\llbracket x\rrbracket}{x}$ does not exist.

(b) For $x>0$, $1/x-1\le\llbracket 1/x\rrbracket\le 1/x \;\Rightarrow\; x(1/x-1)\le x\llbracket 1/x\rrbracket\le x(1/x) \;\Rightarrow\; 1-x\le x\llbracket 1/x\rrbracket\le 1$.

As $x\to 0^+$, $1-x\to 1$, so by the Squeeze Theorem, $\lim_{x\to 0^+}x\llbracket 1/x\rrbracket=1$.

For $x<0$, $1/x-1\le\llbracket 1/x\rrbracket\le 1/x \;\Rightarrow\; x(1/x-1)\ge x\llbracket 1/x\rrbracket\ge x(1/x) \;\Rightarrow\; 1-x\ge x\llbracket 1/x\rrbracket\ge 1$.

As $x\to 0^-$, $1-x\to 1$, so by the Squeeze Theorem, $\lim_{x\to 0^-}x\llbracket 1/x\rrbracket=1$.

Since the one-sided limits are equal, $\lim_{x\to 0}x\llbracket 1/x\rrbracket=1$.

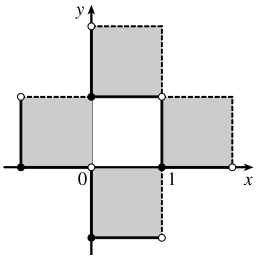
22. (a) $\llbracket x\rrbracket^2+\llbracket y\rrbracket^2=1$. Since $\llbracket x\rrbracket^2$ and $\llbracket y\rrbracket^2$ are positive integers or 0, there are only 4 cases:

Case (i): $\llbracket x\rrbracket=1$, $\llbracket y\rrbracket=0 \;\Rightarrow 1\le x<2$ and $0\le y<1$

Case (ii): $\llbracket x\rrbracket=-1$, $\llbracket y\rrbracket=0 \;\Rightarrow -1\le x<0$ and $0\le y<1$

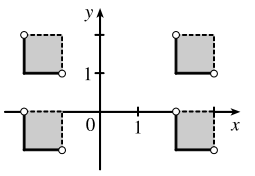
Case (iii): $\llbracket x\rrbracket=0$, $\llbracket y\rrbracket=1 \;\Rightarrow 0\le x<1$ and $1\le y<2$

Case (iv): $\llbracket x\rrbracket=0$, $\llbracket y\rrbracket=-1 \;\Rightarrow 0\le x<1$ and $-1\le y<0$



(b) $\llbracket x\rrbracket^2-\llbracket y\rrbracket^2=3$. The only integral solution of $n^2-m^2=3$ is $n=\pm 2$ and $m=\pm 1$. So the graph is

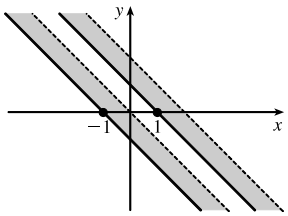
$$\{(x,y)\mid\llbracket x\rrbracket=\pm 2,\ \llbracket y\rrbracket=\pm 1\}=\left\{(x,y)\;\middle|\;\begin{array}{l}2\le x\le 3\ \text{ or }\ -2\le x<1,\\ 1\le y<2\ \text{ or }\ -1\le y<0\end{array}\right\}.$$



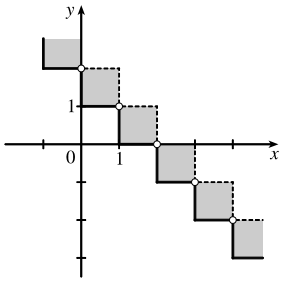
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(c) $[\![x+y]\!]^2 = 1 \quad \Rightarrow \quad [\![x+y]\!] = \pm 1 \quad \Rightarrow \quad 1 \le x+y < 2$ or $-1 \le x+y < 0$



(d) For $n \le x < n+1$, $[\![x]\!] = n$. Then $[\![x]\!] + [\![y]\!] = 1 \quad \Rightarrow \quad [\![y]\!] = 1-n \quad \Rightarrow$ $1-n \le y < 2-n$. Choosing integer values for n produces the graph.

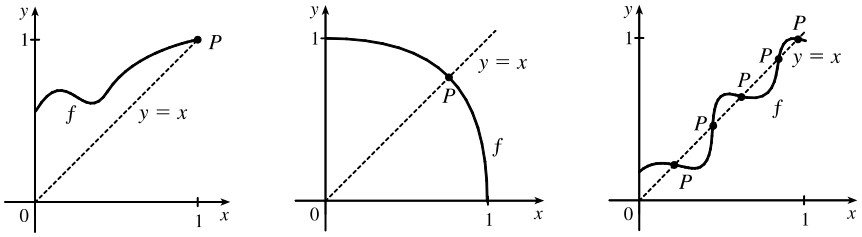


23. f is continuous on $(-\infty, a)$ and (a, ∞). To make f continuous on $\mathbb{R}$, we must have continuity at a. Thus,

$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \quad \Rightarrow \quad \lim_{x \to a^+} x^2 = \lim_{x \to a^-} (x+1) \quad \Rightarrow \quad a^2 = a+1 \quad \Rightarrow \quad a^2 - a - 1 = 0 \quad \Rightarrow$

[by the quadratic formula] $a = \left(1 \pm \sqrt{5}\right)/2 \approx 1.618$ or -0.618.

24. (a) Here are a few possibilities:



(b) The "obstacle" is the line $x = y$ (see diagram). Any intersection of the graph of f with the line $y = x$ constitutes a fixed point, and if the graph of the function does not cross the line somewhere in $(0, 1)$, then it must either start at $(0, 0)$ (in which case 0 is a fixed point) or finish at $(1, 1)$ (in which case 1 is a fixed point).

(c) Consider the function $F(x) = f(x) - x$, where f is any continuous function with domain $[0, 1]$ and range in $[0, 1]$. We shall prove that f has a fixed point. Now if $f(0) = 0$ then we are done: f has a fixed point (the number 0), which is what we are trying to prove. So assume $f(0) \neq 0$. For the same reason we can assume that $f(1) \neq 1$. Then $F(0) = f(0) > 0$ and $F(1) = f(1) - 1 < 0$. So by the Intermediate Value Theorem, there exists some number c in the interval $(0, 1)$ such that $F(c) = f(c) - c = 0$. So $f(c) = c$, and therefore f has a fixed point.

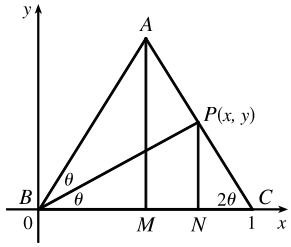
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25. $\begin{cases} \lim\limits_{x\to a}[f(x)+g(x)]=2 \\ \lim\limits_{x\to a}[f(x)-g(x)]=1 \end{cases} \Rightarrow \begin{cases} \lim\limits_{x\to a} f(x)+\lim\limits_{x\to a} g(x)=2 \quad \textbf{(1)} \\ \lim\limits_{x\to a} f(x)-\lim\limits_{x\to a} g(x)=1 \quad \textbf{(2)} \end{cases}$

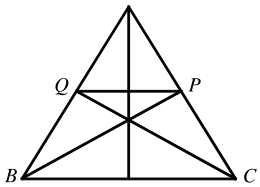
Adding equations **(1)** and **(2)** gives us $2\lim\limits_{x\to a} f(x)=3 \Rightarrow \lim\limits_{x\to a} f(x)=\frac{3}{2}$. From equation **(1)**, $\lim\limits_{x\to a} g(x)=\frac{1}{2}$. Thus, $\lim\limits_{x\to a}[f(x)\,g(x)]=\lim\limits_{x\to a} f(x)\cdot\lim\limits_{x\to a} g(x)=\frac{3}{2}\cdot\frac{1}{2}=\frac{3}{4}$.

26. (a) *Solution 1:* We introduce a coordinate system and drop a perpendicular from P, as shown. We see from $\angle NCP$ that $\tan 2\theta=\dfrac{y}{1-x}$, and from $\angle NBP$ that $\tan\theta=y/x$. Using the double-angle formula for tangents, we get $\dfrac{y}{1-x}=\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}=\dfrac{2(y/x)}{1-(y/x)^2}$. After a bit of simplification, this becomes $\dfrac{1}{1-x}=\dfrac{2x}{x^2-y^2} \Leftrightarrow y^2=x(3x-2)$.

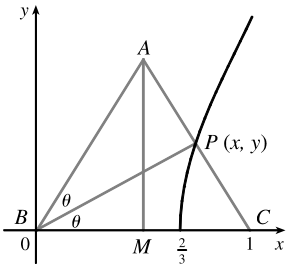


As the altitude AM decreases in length, the point P will approach the x-axis, that is, $y\to 0$, so the limiting location of P must be one of the roots of the equation $x(3x-2)=0$. Obviously it is not $x=0$ (the point P can never be to the left of the altitude AM, which it would have to be in order to approach 0) so it must be $3x-2=0$, that is, $x=\frac{2}{3}$.

Solution 2: We add a few lines to the original diagram, as shown. Now note that $\angle BPQ=\angle PBC$ (alternate angles; $QP\parallel BC$ by symmetry) and similarly $\angle CQP=\angle QCB$. So ΔBPQ and ΔCQP are isosceles, and the line segments BQ, QP and PC are all of equal length. As $|AM|\to 0$, P and Q approach points on the base, and the point P is seen to approach a position two-thirds of the way between B and C, as above.



(b) The equation $y^2=x(3x-2)$ calculated in part (a) is the equation of the curve traced out by P. Now as $|AM|\to\infty$, $2\theta\to\frac{\pi}{2}$, $\theta\to\frac{\pi}{4}$, $x\to 1$, and since $\tan\theta=y/x$, $y\to 1$. Thus, P only traces out the part of the curve with $0\le y<1$.



27. (a) Consider $G(x)=T(x+180°)-T(x)$. Fix any number a. If $G(a)=0$, we are done: Temperature at $a=$ Temperature at $a+180°$. If $G(a)>0$, then $G(a+180°)=T(a+360°)-T(a+180°)=T(a)-T(a+180°)=-G(a)<0$. Also, G is continuous since temperature varies continuously. So, by the Intermediate Value Theorem, G has a zero on the interval $[a,a+180°]$. If $G(a)<0$, then a similar argument applies.

(b) Yes. The same argument applies.

(c) The same argument applies for quantities that vary continuously, such as barometric pressure. But one could argue that altitude above sea level is sometimes discontinuous, so the result might not always hold for that quantity.

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Preface

I once facilitated a mandatory workshop on the teaching of calculus that was attended by a diverse mix of professors and teaching assistants. Before the seminar began, I asked for written answers to the following question: "Ideally, what would you like to get out of the next two days' activities?"

Their responses formed a collection of contradictory expectations. Some were cautionary: "You can tell me *what* to teach, but don't tell me *how*." Some wanted help with group work and cooperative learning, while others just wanted a general idea of what it meant to teach calculus "with modern pedagogy." And many wanted specifics: "How can you teach the Chain Rule 'reform-style'?" "How much homework should I assign?"

This Instructor's Guide tries to address the issues brought out by the above comments. The overall goal was not to write a reference book for a shelf, but to provide a user-friendly source of suggestions and activities for any teacher of calculus within a typical calculus curriculum. Instructors that have used previous editions of this book have said that it saved them a great deal of time, and helped them to teach a more student-oriented course. They have also reported that their classes have become more "fun," but agreed that this unfortunate by-product of an engaged student population can't always be avoided.

This guide should be used together with *Calculus, Ninth Edition* as a source of both supplementary and complementary material. Depending on individual preference, instructors can choose from occasionally glancing through the Guide for content ideas and alternate approaches, or using the material from the Instructor's Guide as a major component in planning their day-to-day classes as well as to set homework assignments and reading quizzes. There are student activities and worksheets, sample exam questions, and examples for every section.

Some of the continuing debates about changes in calculus content and pedagogy are rendered moot by adopting the principle that the instruction of any topic in calculus can be enhanced by using a wider range of approaches. This guide includes some conceptual and geometric problems in topics as mundane as rules for differentiation, and as traditional as ε-δ limits. Whether a class consists of a straight lecture or an hour of group work, the materials provided are meant to help.

I value reactions from all my colleagues who are teaching calculus from this guide, both to correct any errors and to suggest additional material for future editions. I am especially interested in which particular parts of the guide are the most and the least useful. Please email any feedback to `calculus@dougshaw.com`.

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This guide could not have been completed without the help of many people. I especially want to thank the late Jim Stewart for his belief in this project and trust in me. It's funny, you think I would get sick of reading tributes to him, and people writing about how great he was, and how much they miss him. But I never get sick of it. Jim Stewart was one of those rare, kind people who lived up to the hype.

My editor, Lynh Pham, has been wonderful. Previous versions of this guide have benefitted from the input of Virge Cornelius, Tom Hull, Joe Mercer, Melissa Pfohl, Michael Prophet, Suzanne Riehl, and John Samons. Over the years, I've had students read through the guide and offer suggestions from their perspective. Thanks go to Kate Degner, Ken Doss, Job Evers, Slade Hovick, Patricia Kloeckner, Jordan Meyer, Ben Nicholson, Paul Schou, Laura Waechter, and Cody Wilson. Further thanks go to James Stewart, John Hall, Robert Hesse, Harvey Keynes, Michael Lawler, and Dan O'Loughlin for their contributions to earlier incarnations of this guide. The book's typesetter and proofreader, Andy Bulman-Fleming, again went above and beyond the call of duty, both in his work on the book, and in keeping me humble by regularly trouncing me at online Scrabble as we produced it. The talents of these people and others at Cengage have truly helped to make writing this guide a learning experience.

This guide is dedicated to Dr. Francesca Gandini, who went from employee to colleague to friend in a very short time, and whose feedback on the eighth edition and suggestions for this one were valuable and insightful.

Doug Shaw

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How to Use the Instructor's Guide

For each section of *Calculus, Ninth Edition*, this *Instructor's Guide* provides information on the items listed below.

1. **Suggested Time and Emphasis** Here are suggestions for the amount of time to spend in a class of "average" students, and whether or not the material is essential to the rest of the course. If a section is labeled optional, the time range given is the amount of time for the material in the event that it is covered.
2. **Points to Stress** This is a short summary of the main topics to be covered. The stress is on the big ideas, rather than specific details.
3. **Quiz Questions** Some instructors have reported that they like to open or close class by handing out a single question, either as a quiz or to start a discussion. Two types are included:
   - **Text Question** This question is designed for students who have done the reading, but haven't yet seen the material in class. These questions can be used to help ensure that the students are reading the textbook carefully.
   - **Drill Question** These questions are designed to be straightforward "right down the middle" questions for students who have tried, but not necessarily mastered, the material.
4. **Materials for Lecture** These suggestions are meant to work along with the text to create a classroom atmosphere of experimentation and inquiry. They have a theoretical bent to help the students understand the material at a deep conceptual level. In a course with a "lecture and discussion" format, these ideas can be used during the lectures.
5. **Workshop/Discussion** These suggestions are interesting examples and applications aimed at motivating the material and helping the students master it. In a course with a "lecture and discussion" format, these ideas can be used during the discussions.
6. **Group Work** One of the main difficulties instructors have in presenting group work to their classes is that of choosing an appropriate group task. Suggestions for implementation and answers to the group activities are provided first, followed by photocopy-ready handouts on separate pages. The guide's main philosophy of group work is that there should be a solid introduction to each exercise ("What are we supposed to do?") and good closure before class is dismissed ("Why did we just do that?")
7. **Sample Exam Questions** I recommend that tests in a calculus course have a mix of routine and non-routine questions. The sample exam questions provided are meant to inspire the "non-routine" portion of a calculus test. We do not recommend that calculus tests be composed entirely of questions from this section. One strategy is to announce to the students that one-third of the text questions will be based on homework, one-third will be based on in-class group work, and one-third will not be immediately familiar.
8. **Web Resources** Useful resources can be found on the website for *Calculus, Ninth Edition* (`http://www.stewartcalculus.com`).

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How to Implement the Projects

One exciting yet intimidating aspect of teaching a calculus course is projects. An extended assignment gives students the chance to take a focused problem or project and explore it in-depth — making conjectures, discussing them, eventually drawing conclusions and writing them up in a clear, precise format. *Calculus, Ninth Edition* has many possible projects throughout its chapters. Here are some tips on ensuring that your students have a successful experience.

Time Students should have two to three weeks to work on any extended out-of-class assignment. This is not because they will need all this time to complete them! But a fifteen-to-twenty-day deadline allows the students to be flexible in structuring their time wisely, and allows the instructors to apply fairly strict standards in grading the work.

Groups Students usually work in teams and are expected to have team meetings. The main problem students have in setting up these meetings is scheduling. Four randomly selected undergraduates will probably find it very hard to get together for more than a few hours, which may not be sufficient. One way to help your students is to clearly specify a minimum number of meetings, and have one or all group members turn in summaries of what was accomplished at each meeting.On a commuter campus, a good first grouping might be by location.

Studies have shown that the optimal group size is three people, followed by four, then two. I advocate groups of four whenever possible. That way, if someone doesn't show up to a team meeting, there are still three people there to discuss the problems.

Before the first project, students should discuss the different roles that are assumed in a team. Who will be responsible for keeping people informed of where and when they meet? Who will be responsible for making sure that the final copy of the report is all together when it is supposed to be? These types of jobs can be assigned within the team, or by the teacher at the outset.

Tell the students that you will be grading on both content and presentation. They should gear their work toward an audience that is bright, but not necessarily up-to-speed on this problem. For example, they can think of themselves as professional mathematicians writing for a manager, or as research assistants writing for a professor who is not necessarily a mathematician.

If the students are expected to put some effort into the project, it is important to let them know that some effort was put into the grading. Both form and content should be commented on, and recognition of good aspects of their work should be included along with criticism.

One way to help ensure cooperation is to let the students know that there will be an exam question based on the project. If every member of the group does well on that particular question, then they can all get a bonus, either on the exam or on the project grade.

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Providing assistance Make sure that the students know when you are available to help them, and what kind of help you are willing to provide. Students may be required to hand in a rough draft ten days before the due date, to give them a little more structure and to make sure they have a solid week to write up the assignment.

Individual Accountability It is important that the students are individually accountable for the output of their group. Giving each student a different grade is a dangerous solution, because it does not necessarily encourage the students to discuss the material, and may actually discourage their working together. A better alternative might be to create a feedback form. If the students are given a copy of the feedback form ahead of time, and they know that their future group placement will be based on what they do in their present group, then they are given an incentive to work hard. One surprising result is that when a group consists of students who were previously slackers, that group often does quite well. The exam question idea discussed earlier also gives individuals an incentive to keep up with their colleagues.

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How to Use the Review Sections

Review sections for chapters of a calculus book are often assigned to students the weekend before a test, but never graded. Students realize that they won't be evaluated on this work and often skip the exercises, instead reworking previously done homework problems or quizzes, if they study at all. A more useful activity for students is to use the review sections in *Calculus, Ninth Edition* to discover their precise areas of difficulty. Implemented carefully, these are a useful resource for the students, particularly for helping them to retain the skills and concepts they've learned. To encourage more student usage, try the following alternatives:

1. Instead of giving a review session where you reiterate previous lectures, make notes of the types of problems students had difficulty with during the quarter and assign students to work on these exercises in the review sections and go over them at the end of class.
2. Use the review section problems to create a game. For instance, break students into groups and have a contest where the group that correctly answers the most randomly picked review questions "wins". One fun technique is to create a math "bingo" game. Give each group a 5×5 grid with answers to review problems. If you laminate the cards and give the students dry-erase markers, then you can use them year after year. Randomly pick review problems, and write the questions on the board. Make sure that for a group to win, they must not only have the correct answers to the problems, but be able to give sound explanations as to how they got the answers.
3. A simple way to give students an incentive to look at a review section is to use one of the problems, verbatim, for an exam question, and make no secret of your intention to do so. It is important that students have an opportunity to get answers to any questions they have on the review problems before the exam is given; otherwise, this technique loses a great deal of its value.

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How to Use the Problem-Solving Sections

Principles of Problem Solving (after Chapter 1) and Problems Plus (after subsequent chapters) are designed to help students build their critical thinking skills on calculus problems with significant depth. Some of these problems are more open-ended than the typical problems calculus students usually solve, while others are more challenging computationally. Many can be used as small-scale projects, suitable for an individual or a group to chew over at home or in an extended classroom session. If you are looking for problems to create your own projects, or to assign to students who would benefit from a challenge, then these sections provide excellent choices.

Because of the depth of these questions, students who come up with interesting partial solutions, or approaches leading to a possible solution, should be rewarded.

Before the students start one of these problems, or if they reach an impasse, it may be helpful for them to look at the Principles of Problem Solving section for ideas.

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Tips on In-Class Group Work

This *Instructor's Guide* gives classroom-tested group work activities for every section of *Calculus, Ninth Edition*. One reason for the popularity of in-class group work is that *it is effective*. When students are engaged in doing mathematics, and talking about mathematics with others, they tend to learn better and retain the material longer. Think back to your own career: didn't you learn a lot of mathematics when you began teaching it to other people? Many skeptics experiment by trying group work for one semester, and then they get hooked. Pick a group activity from the guide that you like, make some photocopies, and dive in!

1. Mechanics Books and seminars on in-class group work abound. I have conducted many such seminars myself. What follows are some tips to give you a good start:

(a) **Do it on the first day.**
The sources all agree on this one. If you want your students to believe that group work is an important part of the course, you have to start them on the first day. My rule of thumb is "at least three times the first week, and then at least once a week thereafter." I mention this first because it is the most important.

(b) **Make them move.**
Ideally, students should be eye-to-eye and knee-to-knee. If this isn't possible, do the best you can. But it is important to have them move. If your groups are randomly selected, then they will have to get up and sit in a different chair. If your groups are organized by where they are seated in the classroom, make them move their chairs so they face each other. There needs to be a "break" between sitting-and-writing mode and talking-to-colleagues mode.

(c) **Use the ideal group size.**
Research has shown that the ideal group size is three students, with four-student groups next. I like to use groups of four: if one of them is absent (physically or otherwise), the group still has three participating members.

(d) **Fixed versus random groups.**
There is a lot of disagreement here. Fixed groups allow each student to find her or his niche, and allow you to be thoughtful when you assign groups or reassign them after exams. Random groups allow students to have the experience of working with a variety of people. I believe the best thing to do is to try both methods, and see which works best for you and your students.

(e) **Should students hand in their work?**
The advantage of handing in group works is accountability. My philosophy is that I want the group work to have obvious, intrinsic benefit. I try to make the experience such that it is obvious to the student that they get a lot out of participating, so I don't need the threat of "I'm grading this" to get them to focus. I sometimes have the students hand in the group work, but only as a last resort.

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2. **Closure** As stated above, I want my students to understand the value of working together actively in their groups. Once you win this battle, you will find that a lot of motivation and discipline problems simply go away. I've found the best way to ensure that the students understand why they've done an activity is to tell them. The students should leave the room having seen the solutions and knowing why they did that particular activity. You can have the students present answers or present them yourself, whatever suits your teaching style. I've had success with having groups write their results on transparencies and present them to the class (after I've checked their accuracy).

   Here is another way to think about closure: Once in a while, give a future homework problem out as a group work. When the students realize that participating fully in the group work helps them in the homework, they get a solid feeling about the whole process.
3. **Introduction** The most important part of a group activity, in my opinion, is closure. The second most important is the introduction. A big killer of group work is that awful time between you telling your students they can start, and the first move of pencil on paper—the "what on earth do we do now?" moment. A good introduction should be focused on getting them past that moment. You don't want to give too much away, but you also don't want to throw them into the deep end of the swimming pool. In some classes, you will have to say very little, and in some you may have to do the first problem with them. Experiment with your introductions, but never neglect them.
4. **Help when you are needed** Some group work methods involve giving absolutely no help when the students are working. Again, you will have to find what is best for you. If you give help too freely, the students have no incentive to talk to each other. If you are too stingy, the students can wind up frustrated. When a student asks me for help, I first ask the group what they think, and if it is clear they are all stuck at the same point, I give a hint.
5. **Make understanding a goal in itself** Convey to the students (again, directness is a virtue here) that their goal is not just to get the answer written down, but to ensure that every student in their group understands the answer. Their work is not done until they are sure that every one of their colleagues can leave the room knowing how to do the problem. You don't have to sell every single student on this idea for it to work.
6. **Bring it back when you can** Many of the group works in this guide foreshadow future material. When you are lecturing, try to make reference to past group works when it is appropriate. You will find that your students more easily recall a particular problem they discussed with their friends than a particular statement that you made during a lecture.

The above is just the tip of the iceberg. There are plenty of resources available, both online and in print. Don't be intimidated by the literature—start it on the first day of the next semester, and once you are into it, you may naturally want to read what other people have to say!

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1 Functions and Limits

1.1 Four Ways to Represent a Function

SUGGESTED TIME AND EMPHASIS

1 class Essential material

POINTS TO STRESS

1. Definition of function, including piecewise functions.
2. Understanding the interplay between the four ways of representing a function (verbally, numerically, visually, algebraically) perhaps using the concepts of increasing and decreasing functions as an example.
3. Finding the domain and range of a function, regardless of representation.
4. Investigating even and odd functions.

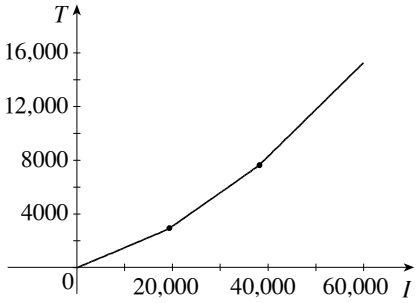
QUIZ QUESTIONS

- **TEXT QUESTIONS**
  - The authors describe the area function of a circle as $A(r) = \pi r^2$. Why do they say that the domain of this function is $(0, \infty)$?
    ANSWER The radius of a circle is always positive.
  - Fill in the blanks: $|x| = \begin{cases} ___ & \text{if } x \geq 0 \\ ___ & \text{if } x < 0 \end{cases}$
    ANSWER x, $-x$
- **DRILL QUESTION** What is the domain of the function $f(x) = \sqrt{1 - \sqrt{x}}$?
  ANSWER $0 \leq x \leq 1$

MATERIALS FOR LECTURE

- Draw a graph of electrical power consumption in the classroom versus time on a typical weekday, pointing out important features throughout, and using the vocabulary of this section as much as possible.
- In 1984, United States President Ronald Reagan proposed a plan to change the United States personal income tax system. According to his plan, the income tax would be 15% on the first $19,300 earned, 25% on the next $18,800, and 35% on all income above and beyond that. Describe this situation to the class, and have them graph (marginal) tax rate and tax owed versus income for incomes ranging from $0 to $80,000. Then have them try to come up with equations describing this situation.

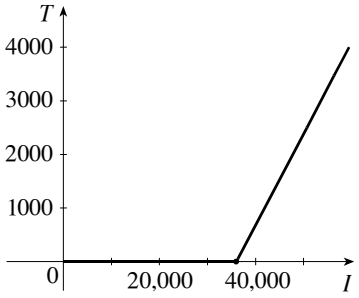
ANSWER



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- In the year 2000, Presidential candidate Steve Forbes proposed a "flat tax" model: 0% on the first $36,000 and 17% on the rest. Have the students do the same analysis, and compare the two models. As an extension, perhaps have the students look at a current tax table and draw similar graphs.

  ANSWER



- Let $f(x)$ be the leftmost nonzero digit of x. So $f(386.6) = 3$ and $f(0.000451) = 4$. Have the students try to find the domain and range of f.

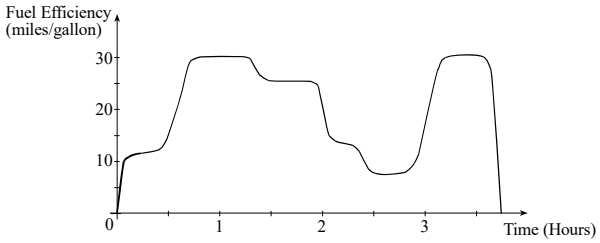
  ANSWER The domain seems to be all real numbers except zero, and the range seems to be the set of integers from 1 through 9. Although the graph of this "function" cannot be drawn, ask the students to verify that it passes the Vertical Line Test. It turns out that it does not (and in fact is not even a function), for a subtle reason. For example, let $x = \frac{1}{5}$. If we write x as 0.2, then $f(x) = 2$, but if we write x as $0.1\overline{9}$, then $f(x) = 1$. Therefore, f is not a function.

WORKSHOP/DISCUSSION

- Present graphs of even and odd functions, such as $\sin x$, $\cos x + x^2$, and $\cos(\sin x)$, and check with the standard algebraic tests.
- Start with a table of values for the function $f(x) = \frac{1}{4}x^2 + x$:

x	0	1	2	3	4
$f(x)$	0	1.25	3	5.25	8

  First, have the class describe the behavior of the function in words, trying to elicit the information that the function is increasing, and that its rate of increase is also increasing. Then, have them try to extrapolate the function in both directions, debating whether or not the function is always positive and increasing. Plot the points and connect the dots, then have them try to concoct a formula (not necessarily expecting them to succeed).
- Draw a graph of fuel efficiency versus time on a trip, such as the one below. Lead a discussion of what could have happened on the trip.



- Discuss the domain and range of a function such as $f(x) = \begin{cases} \sqrt{x} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

  Also talk about why f is neither increasing nor decreasing for $x > 0$. Stress that when dealing with new

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sorts of functions, it becomes important to know the precise mathematical definitions of such terms.

- Define "difference quotient" as done in the text. Define $f(x) = x^3$, and show that $\dfrac{f(a+h) - f(a)}{h} = 3a^2 + 3ah + h^2$. This example both reviews algebra skills and foreshadows future calculations.

GROUP WORK 1: EVERY PICTURE TELLS A STORY

Put the students in groups of four, and have them work on the exercise. If there are questions, encourage them to ask each other before asking you. After going through the correct matching with them, have each group tell their story to the class and see if it fits the remaining graph.

ANSWERS

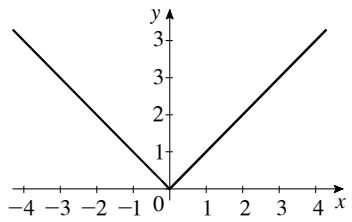
1. (b) **2.** (a) **3.** (c) **4.** The roast was cooked in the morning and put in the refrigerator in the afternoon.

GROUP WORK 2: A CHAIN OF FUNCTIONS

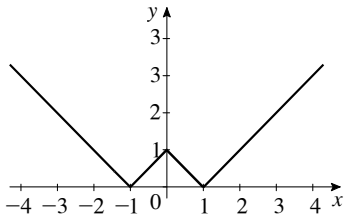
It is recommended that students not be allowed to use graphing technology to do this activity. The intention is to give them an opportunity to practice working with absolute values and order of operations, and to reinforce the idea of looking for mathematical patterns.

ANSWERS

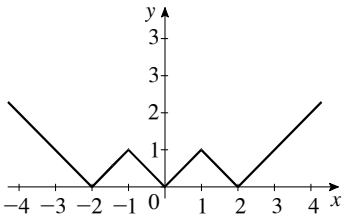
1.



$|x|$

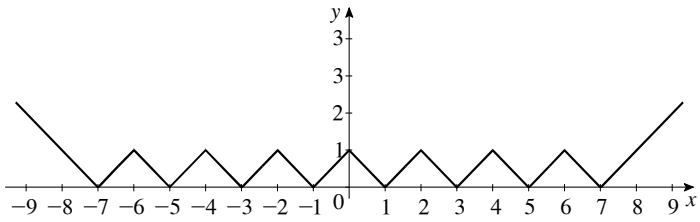


$|1 - |x||$



$|1 - |1 - |x|||$

2. (a) 1 (b)



$|1 - |1 - |1 - |1 - |1 - |1 - |1 - |x|||||||$

GROUP WORK 3: FINDING A FORMULA

Make sure that the students know the equation of a circle with radius r, and that they remember the notation for piecewise-defined functions. Split the students into groups of four. In each group, have half of the students work on each problem first, and then have them check each other's work. If the students find these problems difficult, have them work together on each problem.

ANSWERS

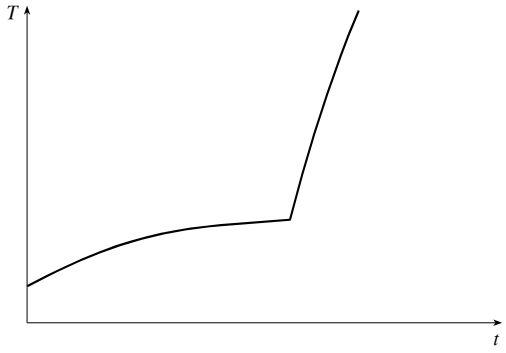
1. $f(x) = \begin{cases} -x - 2 & \text{if } x \le -2 \\ x + 2 & \text{if } -2 < x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$ **2.** $g(x) = \begin{cases} x + 4 & \text{if } x \le -2 \\ 2 & \text{if } -2 < x \le 0 \\ \sqrt{4 - x^2} & \text{if } 0 < x \le 2 \\ x - 2 & \text{if } x > 2 \end{cases}$

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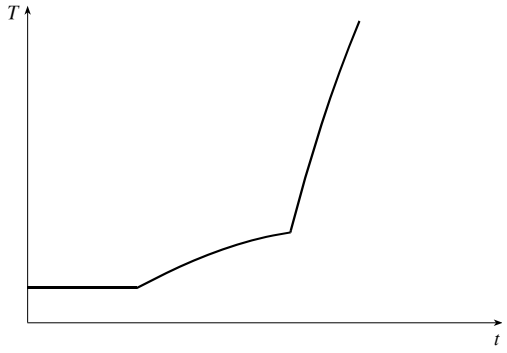
GROUP WORK 1, SECTION 1.1

Every Picture Tells a Story

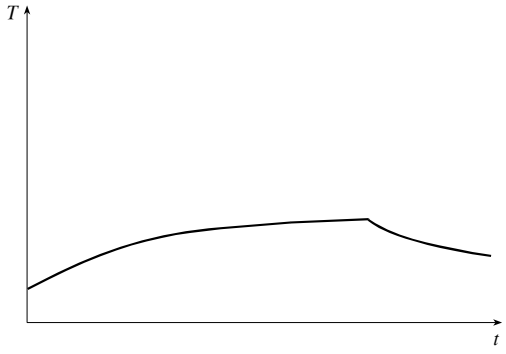
One of the skills you will be learning in this course is the ability to take a description of a real-world occurrence, and translate it into mathematics. Conversely, given a mathematical description of a phenomenon, you will learn how to describe what is happening in plain language. Here follow four graphs of temperature versus time and three stories. Match the stories with the graphs. When finished, write a similar story that would correspond to the final graph.



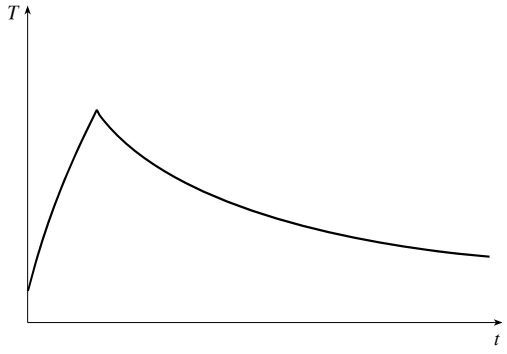
Graph 1



Graph 2



Graph 3



Graph 4

(a) I took my roast out of the freezer at noon, and left it on the counter to thaw. Then I cooked it in the oven when I got home.

(b) I took my roast out of the freezer this morning, and left it on the counter to thaw. Then I cooked it in the oven when I got home.

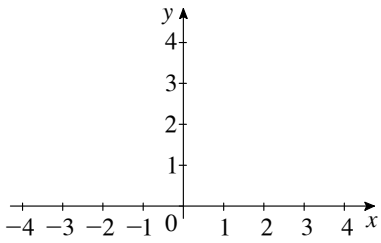
(c) I took my roast out of the freezer this morning, and left it on the counter to thaw. I forgot about it, and went out for Chinese food on my way home from work. I put it in the refrigerator when I finally got home.

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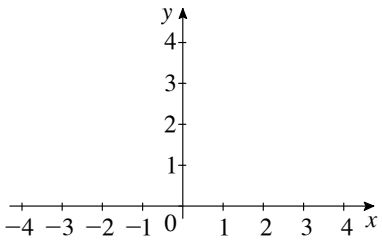
GROUP WORK 2, SECTION 1.1

A Chain of Functions

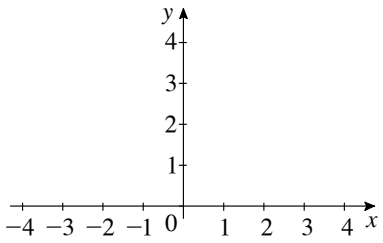
1. Sketch graphs of the following three functions.



$f_1(x) = |x|$



$f_2(x) = |1 - |x||$

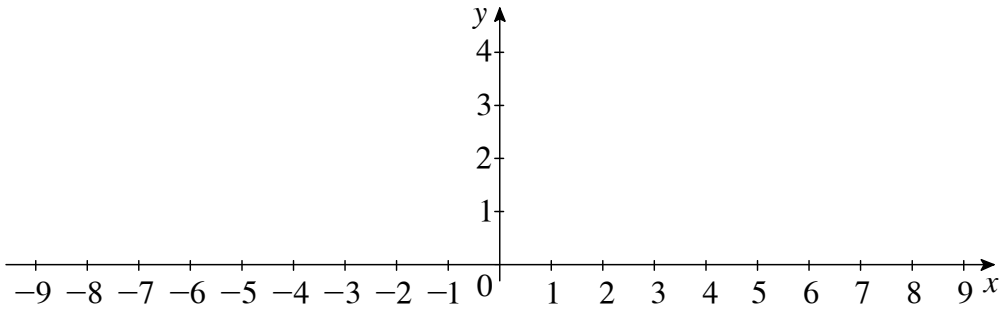


$f_3(x) = |1 - |1 - |x|||$

2. Continuing with the pattern, we get $f_8(x) = |1 - |1 - |1 - |1 - |1 - |1 - |1 - |x|||||||$.

(a) Compute $f_8(0)$.

(b) Using your graphs from Part 1 as a guide, sketch the graph of $f_8(x)$.



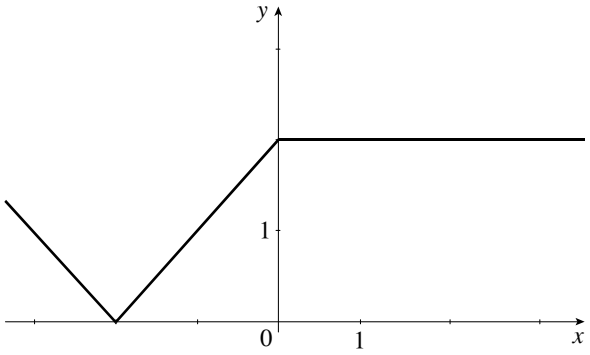
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GROUP WORK 3, SECTION 1.1

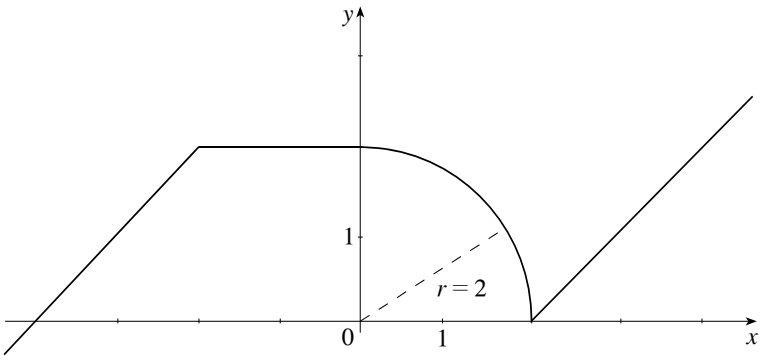
Finding a Formula

Find formulas for the following functions:

1.



2.



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1.2 Mathematical Models: A Catalog of Essential Functions

SUGGESTED TIME AND EMPHASIS

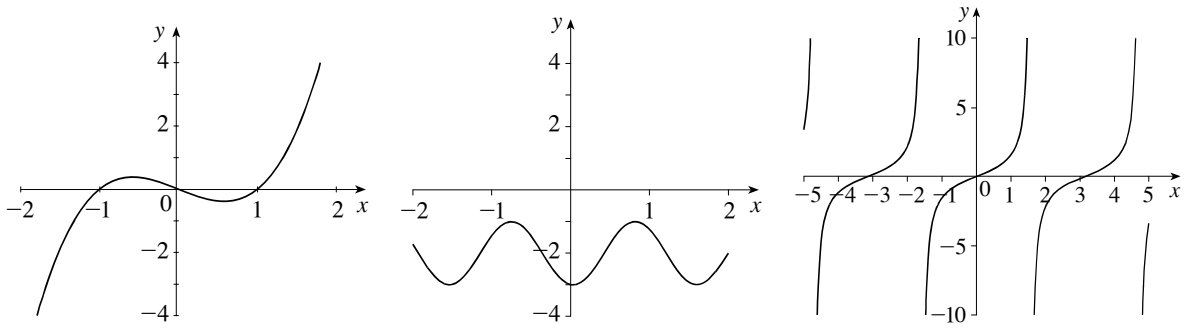
1 class Recommended material

POINTS TO STRESS

1. The modeling process: developing, analyzing, and interpreting a mathematical model.
2. Classes of functions: linear, power, rational, algebraic, and trigonometric functions. Include the special characteristics of each class of functions.

QUIZ QUESTIONS

- **TEXT QUESTION** What is the difference between a power function x^n with $n = 3$ and a cubic function?
  ANSWER A cubic function can have lower order terms, whereas a power function has just one term.
- **DRILL QUESTION** Classify each function graphed below as a power function, root function, polynomial, rational function, algebraic function, or trigonometric function. Explain your reasoning.



ANSWER Polynomial, trigonometric, trigonometric

MATERIALS FOR LECTURE

- Show that linear functions have constant differences in y-values for equally spaced x-values. This example illustrates the point:

Linear function (difference = 1.2)

x	$f(x)$
−2	−2.0
0	−0.8
2	0.4
4	1.6

- Discuss the shape, symmetries, and general "flatness" near 0 of the power functions x^n for various values of n. Similarly discuss $\sqrt[n]{x}$ for n even and n odd. A blackline master is provided at the end of this section, before the group work handouts.
- If Exercises 29–32 (concerning regression) are to be assigned, Exercise 29 can be done in class, discussing part (c) in the context of the technology available to the students.

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WORKSHOP/DISCUSSION

- Have the students use technology to graph 2^x, $\sin x$, $\sin 2^x$, and $2^{\sin x}$. Discuss why the latter two look the way that they do. Notice that for this discussion, students don't need to know anything about exponential functions; the emphasis is on how functions combine.
- Some of the figures in this section show noncontinuous functions expressible as simple formulas. Discuss these curves with your students, trying to get them to describe the idea of a break in a graph.
- Linear regression is a nice, clearly useful application of mathematics. Have your students fill out anonymous cards stating their shoe size and height. Depending on the technology available to you and your students, you can have them create a scatter plot and regression, or do it yourself. There are many websites that will do this nicely.

GROUP WORK 1: THE SMALL SHALL GROW LARGE

If a group finishes early, ask them to similarly compare x^3 and x^4.

ANSWERS **1.** $x^6 \geq x^8$ for $-1 \leq x \leq 1$ **2.** $x^3 \geq x^5$ for $-\infty < x \leq -1, 0 \leq x \leq 1$ **3.** $x^3 \geq x^{105}$ for $-\infty < x \leq -1, 0 \leq x \leq 1$. If the exponents are both even, the answer is the same as for Problem 1, if the exponents are both odd, the answer is the same as for Problem 2.

GROUP WORK 2: FUN WITH FOURIER

This activity will get students looking at combinations of sine curves, while at the same time foreshadowing the concepts of infinite series and Fourier series.

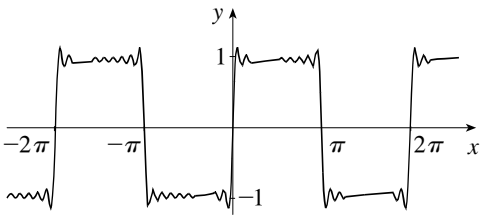
ANSWERS

1. No

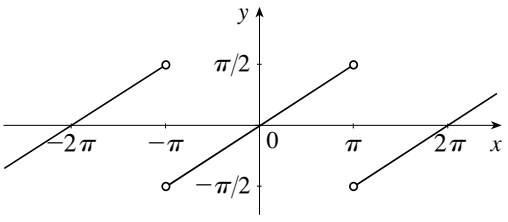
2. $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x\right)$

3. $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \frac{1}{9}\sin 9x\right)$

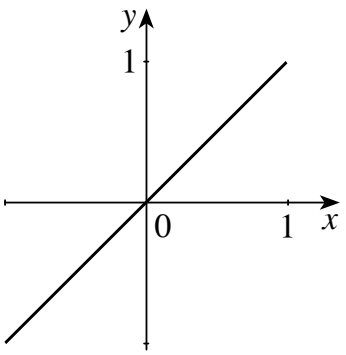
4. $\dfrac{4}{\pi}\left(\sin x + \dfrac{\sin 3x}{3} + \dfrac{\sin 5x}{5} + \dfrac{\sin 7x}{7} + \dfrac{\sin 9x}{9} + \dfrac{\sin 11x}{11} + \dfrac{\sin 13x}{13} + \dfrac{\sin 15x}{15} + \dfrac{\sin 17x}{17} + \dfrac{\sin 19x}{19}\right)$



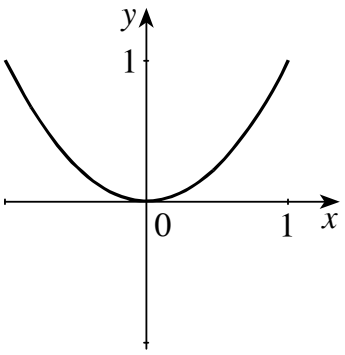
5.



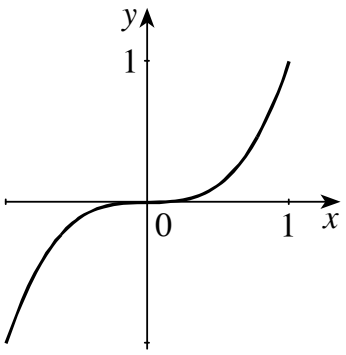
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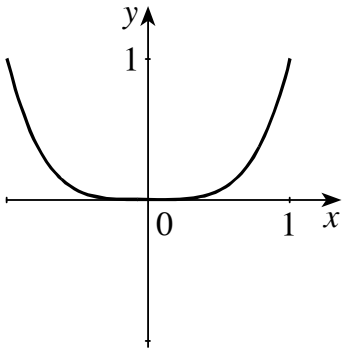
x



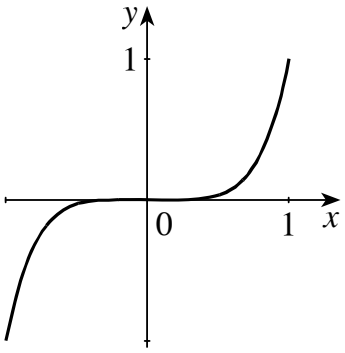
x^2



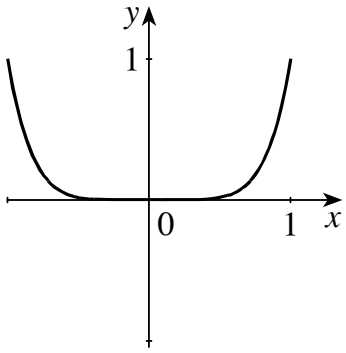
x^3



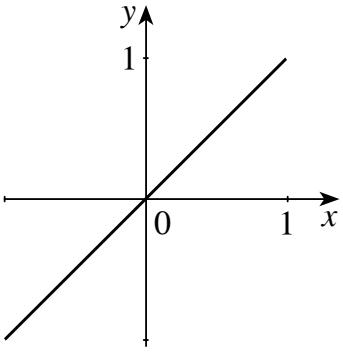
x^4



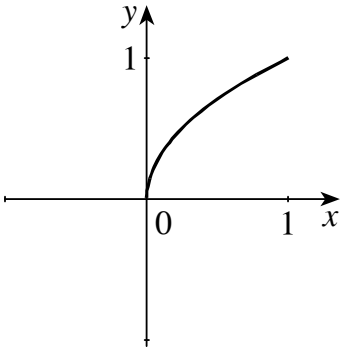
x^5



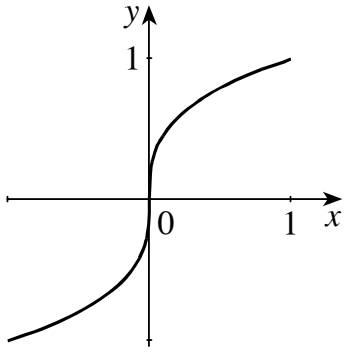
x^6



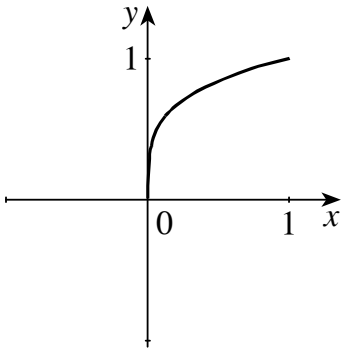
x



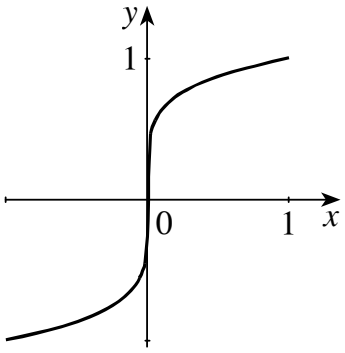
$\sqrt{x}$



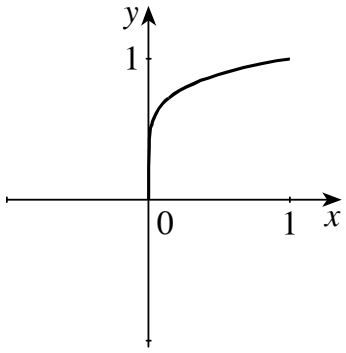
$\sqrt[3]{x}$



$\sqrt[4]{x}$



$\sqrt[5]{x}$



$\sqrt[6]{x}$

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GROUP WORK 1, SECTION 1.2
The Small Shall Grow Large

1. For what values of x is $x^6 \geq x^8$? For what values is $x^6 \leq x^8$?

2. For what values of x is $x^3 \geq x^5$?

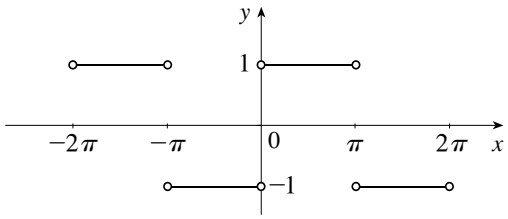
3. For what values of x is $x^3 \geq x^{105}$? Can you generalize your results?

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GROUP WORK 2, SECTION 1.2

Fun with Fourier

The following function $S(x)$ is called a "square wave".



1. Can you find a function on your calculator which has the given graph?

2. Select the function from among the following which gives the best approximation to the square wave: $\frac{4}{\pi}\sin x$, $\frac{4}{\pi}(\sin x + \sin 3x)$, and $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x\right)$.

3. Select the function from among the following which gives the best approximation to the square wave: $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x\right)$, $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x\right)$, $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x\right)$, and $\frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \frac{1}{9}\sin 9x\right)$.

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4. A *Fourier approximation* of a function is an approximation of the form

$$F(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots + a_n \cos nx + b_n \sin nx$$

You have just discovered the Fourier approximation to $S(x)$ with five terms. Find the Fourier approximation to $S(x)$ with ten terms, and sketch its graph.

5. The following expressions are Fourier approximations to a different function, $T(x)$:

$$\begin{aligned}
T(x) &\approx \sin x \\
T(x) &\approx \sin x - \tfrac{1}{2} \sin 2x \\
T(x) &\approx \sin x - \tfrac{1}{2} \sin 2x + \tfrac{1}{3} \sin 3x \\
T(x) &\approx \sin x - \tfrac{1}{2} \sin 2x + \tfrac{1}{3} \sin 3x - \tfrac{1}{4} \sin 4x \\
T(x) &\approx \sin x - \tfrac{1}{2} \sin 2x + \tfrac{1}{3} \sin 3x - \tfrac{1}{4} \sin 4x + \tfrac{1}{5} \sin 5x
\end{aligned}$$

Sketch $T(x)$.

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1.3 New Functions from Old Functions

SUGGESTED TIME AND EMPHASIS

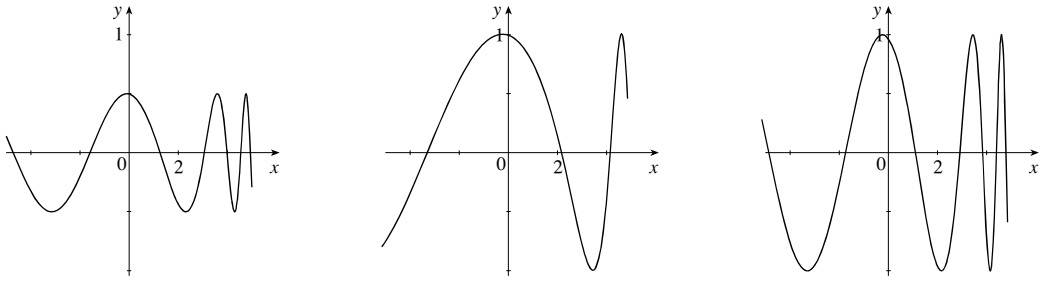
1 class Essential material

POINTS TO STRESS

1. The mechanics and geometry of transforming functions.
2. The mechanics and geometry of adding, subtracting, multiplying, and dividing functions.
3. The mechanics and geometry of composing functions.

QUIZ QUESTIONS

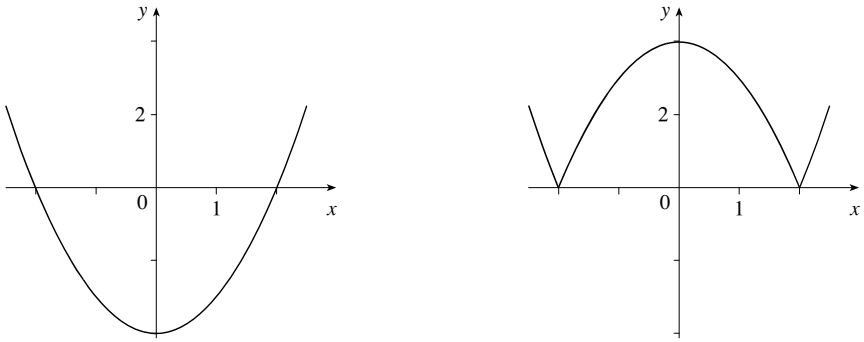
- **TEXT QUESTION** Label the following graphs: $f(x)$, $\frac{1}{2}f(x)$, $f\left(\frac{1}{2}x\right)$.



ANSWER $\frac{1}{2}f(x)$, $f\left(\frac{1}{2}x\right)$, $f(x)$

- **DRILL QUESTION** How can we construct the graph of $y = |f(x)|$ from the graph of $y = f(x)$? Explain in words, and demonstrate with the graph of $y = x^2 - 4$.

ANSWER We leave the positive values of $f(x)$ alone, and reflect the negative values about the x-axis.



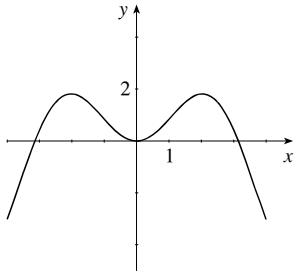
$y = x^2 - 4$ $y = |x^2 - 4|$

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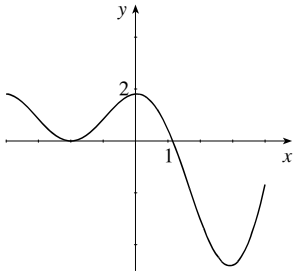
MATERIALS FOR LECTURE

- Using $f(x) = x \sin x$, explore graphs of $f(x+2)$, $f(x)+2$, $-f(x)$, $f(-x)$, $|f(x)|$. Note why $f(-x) = f(x)$.

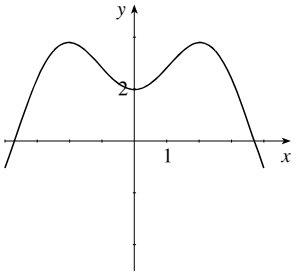
  ANSWER



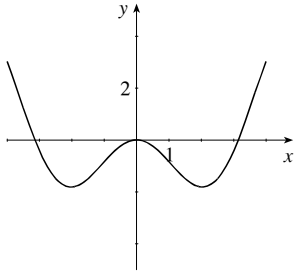
$y = f(x)$



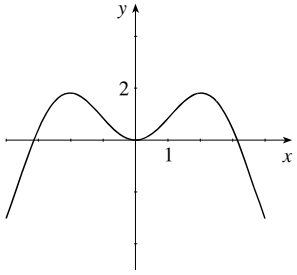
$y = f(x+2)$



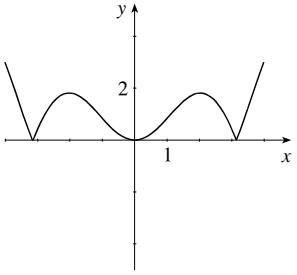
$y = f(x) + 2$



$y = -f(x)$



$y = f(-x)$



$y = |f(x)|$

  $f(-x) = f(x)$ because f is even.

- Graph $f(x) = \sin\left(\sqrt{x}\right)$ and $g(x) = \sqrt{\sin x}$. Draw diagrams like Figure 11 in the text and then write them in the forms $l \circ k$ and $k \circ l$. Then discuss reasons for the differences in their graphs.

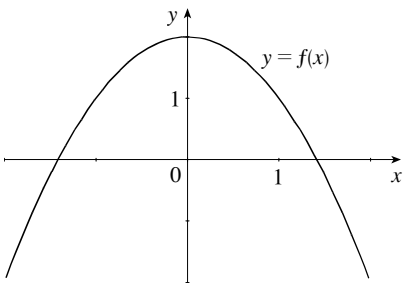
  ANSWER f is a sine function whose argument grows larger more and more slowly as we move away from the origin. g is a root function whose argument oscillates, causing g to oscillate as well.

WORKSHOP/DISCUSSION

- Using $f(x) = 1/x^2$ and $g(x) = \cos x$, compute the domains of $f+g$, f/g, g/f, $f \circ g$, and $g \circ f$, and the range of $g \circ f$. Pay particular attention to the domain of g/f, as many students will think it is $\mathbb{R}$.

  ANSWER $f+g$ has domain $\{x \mid x \neq 0\}$, f/g has domain $\left\{x \mid x \neq 0, x \neq \frac{\pi}{2} + k\pi\right\}$, g/f has domain $\{x \mid x \neq 0\}$, $f \circ g$ has domain $\left\{x \mid x \neq 0, x \neq \frac{\pi}{2} + k\pi\right\}$, $g \circ f$ has domain $\{x \mid x \neq 0\}$, and $g \circ f$ has range $[-1, 1]$.

- Do the following problem with the students:



  From the graph of $y = f(x) = -x^2 + 2$ shown above, compute $f \circ f$ at $x = -1$, 0, and 1. First do it

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graphically (as in Exercise 57), then algebraically.

- After doing a few basic examples of composition, it is possible to foreshadow the idea of inverses, which is covered in Chapter 6. Let $f(x) = 2x^3 + 3$ and $g(x) = x^2 - x$. Compute $f \circ g$ and $g \circ f$ for your students. Then ask them to come up with a function $h(x)$ with the property that $(f \circ h)(x) = x$. They may not be used to the idea of finding examples by themselves; important hints they might need are "Don't give up," "When in doubt, just try something and see what happens," and "I'm not expecting you to get it in fifteen seconds." If the class is really stuck, have them try $f(x) = 2x^3$ to get a feel for how the game is played. Once they have determined that $h(x) = \sqrt[3]{\dfrac{x-3}{2}}$, have them first compute $(h \circ f)(x)$, then conjecture whether $(f \circ g)(x) = x$ implies $(g \circ f)(x) = x$ in general.

GROUP WORK 1: WHICH IS THE ORIGINAL?

ANSWERS **1.** $2f(x+2)$, $2f(x)$, $f(2x)$, $f(x+2)$, $f(x)$ **2.** $2f(x)$, $f(x)$, $f(x+2)$, $f(2x)$, $2f(x+2)$

GROUP WORK 2: LABEL LABEL LABEL, I MADE IT OUT OF CLAY

Some of these transformations were not covered directly in the book. If the students are urged not to give up, and to use the process of elimination and testing individual points, they should be able to complete this activity.

ANSWERS **1.** (d) **2.** (a) **3.** (f) **4.** (e) **5.** (i) **6.** (j) **7.** (b) **8.** (c) **9.** (g) **10.** (h)

GROUP WORK 3: IT'S MORE FUN TO COMPUTE

Each group gets one copy of the graph. During each round, one representative from each group stands, and one of the questions below is asked. The representatives write their answer down, and all display their answers at the same time. Each representative has the choice of consulting with their group or not. A correct solo answer is worth two points, and a correct answer after a consult is worth one point.

ANSWERS **1.** 0 **2.** 0 **3.** 1 **4.** 5 **5.** 1 **6.** 1 **7.** 1 **8.** 0 **9.** 2 **10.** 1 **11.** 1 **12.** 1

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