Calculus Concepts and Contexts Enhanced Edition 4th edition Stewart Solutions Manual

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Calculus Concepts and Contexts Enhanced Edition 4th edition Stewart Solutions Manual

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COMPLETE SOLUTIONS MANUAL

for Stewart's

SINGLE VARIABLE CALCULUS CONCEPTS AND CONTEXTS

FOURTH EDITION

JEFFERY A. COLE
Anoka Ramsey Community College

TIMOTHY J. FLAHERTY
Carnegie Mellon University

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PREFACE

This Complete Solutions Manual contains solutions to all exercises in the texts *Single Variable Calculus: Concepts and Contexts,* Fourth Edition, and Chapters 1–8 of *Calculus: Concepts and Contexts,* Fourth Edition, by James Stewart. A student version of this manual is also available; it contains solutions to the odd-numbered exercises in each chapter section, the review sections, the True-False Quizzes, and the Focus on Problem Solving sections, as well as solutions to all the exercises in the Concept Checks. No solutions to the Projects appear in the student version. It is our hope that by browsing through the solutions, professors will save time in determining appropriate assignments for their particular classes.

Some nonstandard notation is used in order to save space. If you see a symbol that you don't recognize, refer to the Table of Abbreviations and Symbols on page v.

We appreciate feedback concerning errors, solution correctness or style, and manual style. Any comments may be sent directly to us at jeff.cole@anokaramsey.edu or tim@andrew.cmu.edu, or in care of the publisher: Cengage Learning Brooks/Cole, 20 Davis Drive, Belmont, CA 94002.

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Jeffery A. Cole
Anoka Ramsey Community College

Timothy J. Flaherty
Carnegie Mellon University

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ABBREVIATIONS AND SYMBOLS

CD	concave downward
CU	concave upward
D	the domain of f
FDT	First Derivative Test
HA	horizontal asymptote(s)
I	interval of convergence
I/D	Increasing/Decreasing Test
IP	inflection point(s)
R	radius of convergence
VA	vertical asymptote(s)
$\overset{\text{CAS}}{=}$	indicates the use of a computer algebra system.
$\overset{\text{H}}{=}$	indicates the use of l'Hospital's Rule.
$\overset{j}{=}$	indicates the use of Formula j in the Table of Integrals in the back endpapers.
$\overset{\text{s}}{=}$	indicates the use of the substitution $\{u = \sin x, du = \cos x\, dx\}$.
$\overset{\text{c}}{=}$	indicates the use of the substitution $\{u = \cos x, du = -\sin x\, dx\}$.

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DIAGNOSTIC TESTS

Test A Algebra

1. (a) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

(b) $-3^4 = -(3)(3)(3)(3) = -81$

(c) $3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

(d) $\dfrac{5^{23}}{5^{21}} = 5^{23-21} = 5^2 = 25$

(e) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

(f) $16^{-3/4} = \dfrac{1}{16^{3/4}} = \dfrac{1}{\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

2. (a) Note that $\sqrt{200} = \sqrt{100 \cdot 2} = 10\sqrt{2}$ and $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$. Thus $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$.

(b) $(3a^3b^3)(4ab^2)^2 = 3a^3b^3 16a^2b^4 = 48a^5b^7$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(\dfrac{x^2y^{-1/2}}{3x^{3/2}y^3}\right)^2 = \dfrac{(x^2y^{-1/2})^2}{(3x^{3/2}y^3)^2} = \dfrac{x^4y^{-1}}{9x^3y^6} = \dfrac{x^4}{9x^3y^6y} = \dfrac{x}{9y^7}$

3. (a) $3(x+6) + 4(2x-5) = 3x + 18 + 8x - 20 = 11x - 2$

(b) $(x+3)(4x-5) = 4x^2 - 5x + 12x - 15 = 4x^2 + 7x - 15$

(c) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \sqrt{a}\sqrt{b} + \sqrt{a}\sqrt{b} - \left(\sqrt{b}\right)^2 = a - b$

Or: Use the formula for the difference of two squares to see that $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$.

(d) $(2x+3)^2 = (2x+3)(2x+3) = 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$.

Note: A quicker way to expand this binomial is to use the formula $(a+b)^2 = a^2 + 2ab + b^2$ with $a = 2x$ and $b = 3$:
$(2x+3)^2 = (2x)^2 + 2(2x)(3) + 3^2 = 4x^2 + 12x + 9$

(e) See Reference Page 1 for the binomial formula $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. Using it, we get
$(x+2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3 = x^3 + 6x^2 + 12x + 8$.

4. (a) Using the difference of two squares formula, $a^2 - b^2 = (a+b)(a-b)$, we have
$4x^2 - 25 = (2x)^2 - 5^2 = (2x+5)(2x-5)$.

(b) Factoring by trial and error, we get $2x^2 + 5x - 12 = (2x-3)(x+4)$.

(c) Using factoring by grouping and the difference of two squares formula, we have
$x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x^2-4)(x-3) = (x-2)(x+2)(x-3)$.

(d) $x^4 + 27x = x(x^3+27) = x(x+3)(x^2-3x+9)$

This last expression was obtained using the sum of two cubes formula, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ with $a = x$ and $b = 3$. [See Reference Page 1 in the textbook.]

(e) The smallest exponent on x is $-\frac{1}{2}$, so we will factor out $x^{-1/2}$.
$3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = 3x^{-1/2}(x^2 - 3x + 2) = 3x^{-1/2}(x-1)(x-2)$

(f) $x^3y - 4xy = xy(x^2-4) = xy(x-2)(x+2)$

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5. (a) $\dfrac{x^2+3x+2}{x^2-x-2} = \dfrac{(x+1)(x+2)}{(x+1)(x-2)} = \dfrac{x+2}{x-2}$

(b) $\dfrac{2x^2-x-1}{x^2-9} \cdot \dfrac{x+3}{2x+1} = \dfrac{(2x+1)(x-1)}{(x-3)(x+3)} \cdot \dfrac{x+3}{2x+1} = \dfrac{x-1}{x-3}$

(c) $\dfrac{x^2}{x^2-4} - \dfrac{x+1}{x+2} = \dfrac{x^2}{(x-2)(x+2)} - \dfrac{x+1}{x+2} = \dfrac{x^2}{(x-2)(x+2)} - \dfrac{x+1}{x+2} \cdot \dfrac{x-2}{x-2} = \dfrac{x^2-(x+1)(x-2)}{(x-2)(x+2)}$

$= \dfrac{x^2-(x^2-x-2)}{(x+2)(x-2)} = \dfrac{x+2}{(x+2)(x-2)} = \dfrac{1}{x-2}$

(d) $\dfrac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} = \dfrac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} \cdot \dfrac{xy}{xy} = \dfrac{y^2-x^2}{x-y} = \dfrac{(y-x)(y+x)}{-(y-x)} = \dfrac{y+x}{-1} = -(x+y)$

6. (a) $\dfrac{\sqrt{10}}{\sqrt{5}-2} = \dfrac{\sqrt{10}}{\sqrt{5}-2} \cdot \dfrac{\sqrt{5}+2}{\sqrt{5}+2} = \dfrac{\sqrt{50}+2\sqrt{10}}{(\sqrt{5})^2-2^2} = \dfrac{5\sqrt{2}+2\sqrt{10}}{5-4} = 5\sqrt{2}+2\sqrt{10}$

(b) $\dfrac{\sqrt{4+h}-2}{h} = \dfrac{\sqrt{4+h}-2}{h} \cdot \dfrac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \dfrac{4+h-4}{h(\sqrt{4+h}+2)} = \dfrac{h}{h(\sqrt{4+h}+2)} = \dfrac{1}{\sqrt{4+h}+2}$

7. (a) $x^2+x+1 = \left(x^2+x+\frac{1}{4}\right)+1-\frac{1}{4} = \left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

(b) $2x^2-12x+11 = 2(x^2-6x)+11 = 2(x^2-6x+9-9)+11 = 2(x^2-6x+9)-18+11 = 2(x-3)^2-7$

8. (a) $x+5 = 14-\frac{1}{2}x \quad\Leftrightarrow\quad x+\frac{1}{2}x = 14-5 \quad\Leftrightarrow\quad \frac{3}{2}x = 9 \quad\Leftrightarrow\quad x = \frac{2}{3}\cdot 9 \quad\Leftrightarrow\quad x = 6$

(b) $\dfrac{2x}{x+1} = \dfrac{2x-1}{x} \quad\Rightarrow\quad 2x^2 = (2x-1)(x+1) \quad\Leftrightarrow\quad 2x^2 = 2x^2+x-1 \quad\Leftrightarrow\quad x = 1$

(c) $x^2-x-12 = 0 \quad\Leftrightarrow\quad (x+3)(x-4) = 0 \quad\Leftrightarrow\quad x+3 = 0$ or $x-4 = 0 \quad\Leftrightarrow\quad x = -3$ or $x = 4$

(d) By the quadratic formula, $2x^2+4x+1 = 0 \quad\Leftrightarrow$

$x = \dfrac{-4\pm\sqrt{4^2-4(2)(1)}}{2(2)} = \dfrac{-4\pm\sqrt{8}}{4} = \dfrac{-4\pm 2\sqrt{2}}{4} = \dfrac{2\left(-2\pm\sqrt{2}\right)}{4} = \dfrac{-2\pm\sqrt{2}}{2} = -1\pm\frac{1}{2}\sqrt{2}.$

(e) $x^4-3x^2+2 = 0 \quad\Leftrightarrow\quad (x^2-1)(x^2-2) = 0 \quad\Leftrightarrow\quad x^2-1 = 0$ or $x^2-2 = 0 \quad\Leftrightarrow\quad x^2 = 1$ or $x^2 = 2 \quad\Leftrightarrow$

$x = \pm 1$ or $x = \pm\sqrt{2}$

(f) $3\,|x-4| = 10 \quad\Leftrightarrow\quad |x-4| = \frac{10}{3} \quad\Leftrightarrow\quad x-4 = -\frac{10}{3}$ or $x-4 = \frac{10}{3} \quad\Leftrightarrow\quad x = \frac{2}{3}$ or $x = \frac{22}{3}$

(g) Multiplying through $2x(4-x)^{-1/2}-3\sqrt{4-x} = 0$ by $(4-x)^{1/2}$ gives $2x-3(4-x) = 0 \quad\Leftrightarrow$

$2x-12+3x = 0 \quad\Leftrightarrow\quad 5x-12 = 0 \quad\Leftrightarrow\quad 5x = 12 \quad\Leftrightarrow\quad x = \frac{12}{5}$.

9. (a) $-4 < 5-3x \le 17 \quad\Leftrightarrow\quad -9 < -3x \le 12 \quad\Leftrightarrow\quad 3 > x \ge -4$ or $-4 \le x < 3$.

In interval notation, the answer is $[-4, 3)$.

(b) $x^2 < 2x+8 \quad\Leftrightarrow\quad x^2-2x-8 < 0 \quad\Leftrightarrow\quad (x+2)(x-4) < 0$. Now, $(x+2)(x-4)$ will change sign at the critical values $x = -2$ and $x = 4$. Thus the possible intervals of solution are $(-\infty, -2)$, $(-2, 4)$, and $(4, \infty)$. By choosing a single test value from each interval, we see that $(-2, 4)$ is the only interval that satisfies the inequality.

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(c) The inequality $x(x-1)(x+2) > 0$ has critical values of $-2, 0$, and 1. The corresponding possible intervals of solution are $(-\infty, -2)$, $(-2, 0)$, $(0, 1)$ and $(1, \infty)$. By choosing a single test value from each interval, we see that both intervals $(-2, 0)$ and $(1, \infty)$ satisfy the inequality. Thus, the solution is the union of these two intervals: $(-2, 0) \cup (1, \infty)$.

(d) $|x-4| < 3 \quad \Leftrightarrow \quad -3 < x-4 < 3 \quad \Leftrightarrow \quad 1 < x < 7$. In interval notation, the answer is $(1, 7)$.

(e) $\dfrac{2x-3}{x+1} \le 1 \quad \Leftrightarrow \quad \dfrac{2x-3}{x+1} - 1 \le 0 \quad \Leftrightarrow \quad \dfrac{2x-3}{x+1} - \dfrac{x+1}{x+1} \le 0 \quad \Leftrightarrow \quad \dfrac{2x-3-x-1}{x+1} \le 0 \quad \Leftrightarrow \quad \dfrac{x-4}{x+1} \le 0.$

Now, the expression $\dfrac{x-4}{x+1}$ may change signs at the critical values $x = -1$ and $x = 4$, so the possible intervals of solution are $(-\infty, -1)$, $(-1, 4]$, and $[4, \infty)$. By choosing a single test value from each interval, we see that $(-1, 4]$ is the only interval that satisfies the inequality.

10. (a) False. In order for the statement to be true, it must hold for all real numbers, so, to show that the statement is false, pick $p = 1$ and $q = 2$ and observe that $(1+2)^2 \neq 1^2 + 2^2$. In general, $(p+q)^2 = p^2 + 2pq + q^2$.

(b) True as long as a and b are nonnegative real numbers. To see this, think in terms of the laws of exponents: $\sqrt{ab} = (ab)^{1/2} = a^{1/2}b^{1/2} = \sqrt{a}\,\sqrt{b}$.

(c) False. To see this, let $p = 1$ and $q = 2$, then $\sqrt{1^2 + 2^2} \neq 1 + 2$.

(d) False. To see this, let $T = 1$ and $C = 2$, then $\dfrac{1 + 1(2)}{2} \neq 1 + 1$.

(e) False. To see this, let $x = 2$ and $y = 3$, then $\dfrac{1}{2-3} \neq \dfrac{1}{2} - \dfrac{1}{3}$.

(f) True since $\dfrac{1/x}{a/x - b/x} \cdot \dfrac{x}{x} = \dfrac{1}{a-b}$, as long as $x \neq 0$ and $a - b \neq 0$.

Test B Analytic Geometry

1. (a) Using the point $(2, -5)$ and $m = -3$ in the point-slope equation of a line, $y - y_1 = m(x - x_1)$, we get $y - (-5) = -3(x-2) \quad \Rightarrow \quad y + 5 = -3x + 6 \quad \Rightarrow \quad y = -3x + 1$.

(b) A line parallel to the x-axis must be horizontal and thus have a slope of 0. Since the line passes through the point $(2, -5)$, the y-coordinate of every point on the line is -5, so the equation is $y = -5$.

(c) A line parallel to the y-axis is vertical with undefined slope. So the x-coordinate of every point on the line is 2 and so the equation is $x = 2$.

(d) Note that $2x - 4y = 3 \quad \Rightarrow \quad -4y = -2x + 3 \quad \Rightarrow \quad y = \frac{1}{2}x - \frac{3}{4}$. Thus the slope of the given line is $m = \frac{1}{2}$. Hence, the slope of the line we're looking for is also $\frac{1}{2}$ (since the line we're looking for is required to be parallel to the given line). So the equation of the line is $y - (-5) = \frac{1}{2}(x-2) \quad \Rightarrow \quad y + 5 = \frac{1}{2}x - 1 \quad \Rightarrow \quad y = \frac{1}{2}x - 6$.

2. First we'll find the distance between the two given points in order to obtain the radius, r, of the circle: $r = \sqrt{[3-(-1)]^2 + (-2-4)^2} = \sqrt{4^2 + (-6)^2} = \sqrt{52}$. Next use the standard equation of a circle, $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the center, to get $(x+1)^2 + (y-4)^2 = 52$.

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3. We must rewrite the equation in standard form in order to identify the center and radius. Note that $x^2 + y^2 - 6x + 10y + 9 = 0 \quad \Rightarrow \quad x^2 - 6x + 9 + y^2 + 10y = 0$. For the left-hand side of the latter equation, we factor the first three terms and complete the square on the last two terms as follows: $x^2 - 6x + 9 + y^2 + 10y = 0 \quad \Rightarrow$ $(x-3)^2 + y^2 + 10y + 25 = 25 \quad \Rightarrow \quad (x-3)^2 + (y+5)^2 = 25$. Thus, the center of the circle is $(3, -5)$ and the radius is 5.

4. (a) $A(-7, 4)$ and $B(5, -12) \quad \Rightarrow \quad m_{AB} = \dfrac{-12-4}{5-(-7)} = \dfrac{-16}{12} = -\dfrac{4}{3}$

(b) $y - 4 = -\frac{4}{3}[x - (-7)] \quad \Rightarrow \quad y - 4 = -\frac{4}{3}x - \frac{28}{3} \quad \Rightarrow \quad 3y - 12 = -4x - 28 \quad \Rightarrow \quad 4x + 3y + 16 = 0$. Putting $y = 0$, we get $4x + 16 = 0$, so the x-intercept is -4, and substituting 0 for x results in a y-intercept of $-\frac{16}{3}$.

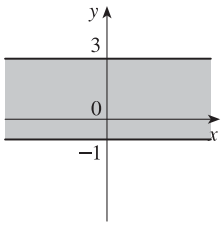
(c) The midpoint is obtained by averaging the corresponding coordinates of both points: $\left(\frac{-7+5}{2}, \frac{4+(-12)}{2}\right) = (-1, -4)$.

(d) $d = \sqrt{[5-(-7)]^2 + (-12-4)^2} = \sqrt{12^2 + (-16)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$

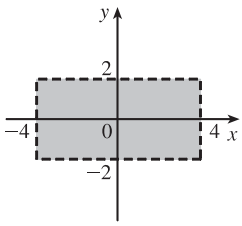
(e) The perpendicular bisector is the line that intersects the line segment $\overline{AB}$ at a right angle through its midpoint. Thus the perpendicular bisector passes through $(-1, -4)$ and has slope $\frac{3}{4}$ [the slope is obtained by taking the negative reciprocal of the answer from part (a)]. So the perpendicular bisector is given by $y + 4 = \frac{3}{4}[x - (-1)]$ or $3x - 4y = 13$.

(f) The center of the required circle is the midpoint of $\overline{AB}$, and the radius is half the length of $\overline{AB}$, which is 10. Thus, the equation is $(x+1)^2 + (y+4)^2 = 100$.

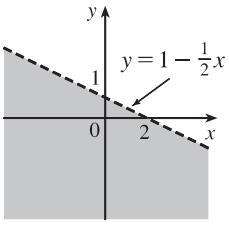
5. (a) Graph the corresponding horizontal lines (given by the equations $y = -1$ and $y = 3$) as solid lines. The inequality $y \geq -1$ describes the points (x, y) that lie on or *above* the line $y = -1$. The inequality $y \leq 3$ describes the points (x, y) that lie on or *below* the line $y = 3$. So the pair of inequalities $-1 \leq y \leq 3$ describes the points that lie on or *between* the lines $y = -1$ and $y = 3$.



(b) Note that the given inequalities can be written as $-4 < x < 4$ and $-2 < y < 2$, respectively. So the region lies between the vertical lines $x = -4$ and $x = 4$ and between the horizontal lines $y = -2$ and $y = 2$. As shown in the graph, the region common to both graphs is a rectangle (minus its edges) centered at the origin.



(c) We first graph $y = 1 - \frac{1}{2}x$ as a dotted line. Since $y < 1 - \frac{1}{2}x$, the points in the region lie *below* this line.



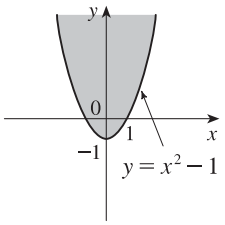
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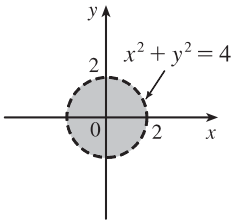
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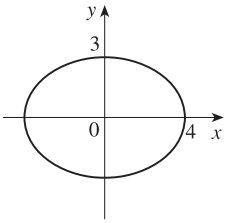
(d) We first graph the parabola $y = x^2 - 1$ using a solid curve. Since $y \geq x^2 - 1$, the points in the region lie on or *above* the parabola.



(e) We graph the circle $x^2 + y^2 = 4$ using a dotted curve. Since $\sqrt{x^2 + y^2} < 2$, the region consists of points whose distance from the origin is less than 2, that is, the points that lie *inside* the circle.



(f) The equation $9x^2 + 16y^2 = 144$ is an ellipse centered at $(0, 0)$. We put it in standard form by dividing by 144 and get $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$. The x-intercepts are located at a distance of $\sqrt{16} = 4$ from the center while the y-intercepts are a distance of $\sqrt{9} = 3$ from the center (see the graph).



Test C Functions

1. (a) Locate -1 on the x-axis and then go down to the point on the graph with an x-coordinate of -1. The corresponding y-coordinate is the value of the function at $x = -1$, which is -2. So, $f(-1) = -2$.

(b) Using the same technique as in part (a), we get $f(2) \approx 2.8$.

(c) Locate 2 on the y-axis and then go left and right to find all points on the graph with a y-coordinate of 2. The corresponding x-coordinates are the x-values we are searching for. So $x = -3$ and $x = 1$.

(d) Using the same technique as in part (c), we get $x \approx -2.5$ and $x \approx 0.3$.

(e) The domain is all the x-values for which the graph exists, and the range is all the y-values for which the graph exists. Thus, the domain is $[-3, 3]$, and the range is $[-2, 3]$.

2. Note that $f(2 + h) = (2 + h)^3$ and $f(2) = 2^3 = 8$. So the difference quotient becomes

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^3 - 8}{h} = \frac{8 + 12h + 6h^2 + h^3 - 8}{h} = \frac{12h + 6h^2 + h^3}{h} = \frac{h(12 + 6h + h^2)}{h} = 12 + 6h + h^2.$$

3. (a) Set the denominator equal to 0 and solve to find restrictions on the domain: $x^2 + x - 2 = 0 \quad \Rightarrow$ $(x - 1)(x + 2) = 0 \quad \Rightarrow \quad x = 1$ or $x = -2$. Thus, the domain is all real numbers except 1 or -2 or, in interval notation, $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

(b) Note that the denominator is always greater than or equal to 1, and the numerator is defined for all real numbers. Thus, the domain is $(-\infty, \infty)$.

(c) Note that the function h is the sum of two root functions. So h is defined on the intersection of the domains of these two root functions. The domain of a square root function is found by setting its radicand greater than or equal to 0. Now,

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$4 - x \geq 0 \quad \Rightarrow \quad x \leq 4$ and $x^2 - 1 \geq 0 \quad \Rightarrow \quad (x-1)(x+1) \geq 0 \quad \Rightarrow \quad x \leq -1$ or $x \geq 1$. Thus, the domain of h is $(-\infty, -1] \cup [1, 4]$.

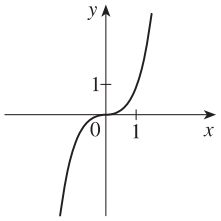
4. (a) Reflect the graph of f about the x-axis.

(b) Stretch the graph of f vertically by a factor of 2, then shift 1 unit downward.

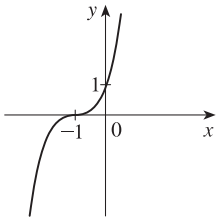
(c) Shift the graph of f right 3 units, then up 2 units.

5. (a) Make a table and then connect the points with a smooth curve:

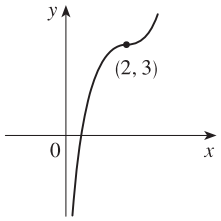
x	-2	-1	0	1	2
y	-8	-1	0	1	8



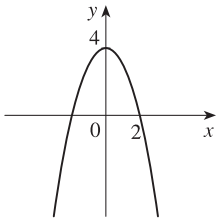
(b) Shift the graph from part (a) left 1 unit.



(c) Shift the graph from part (a) right 2 units and up 3 units.

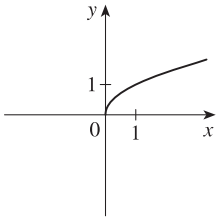


(d) First plot $y = x^2$. Next, to get the graph of $f(x) = 4 - x^2$, reflect f about the x-axis and then shift it upward 4 units.

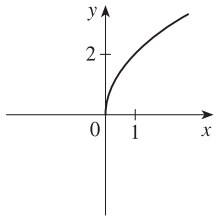


(e) Make a table and then connect the points with a smooth curve:

x	0	1	4	9
y	0	1	2	3



(f) Stretch the graph from part (e) vertically by a factor of two.



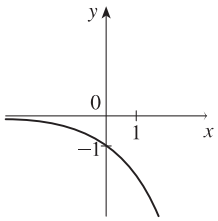
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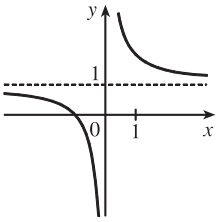
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(g) First plot $y = 2^x$. Next, get the graph of $y = -2^x$ by reflecting the graph of $y = 2^x$ about the x-axis.

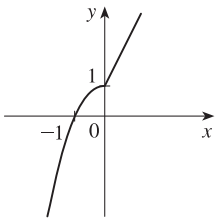


(h) Note that $y = 1 + x^{-1} = 1 + 1/x$. So first plot $y = 1/x$ and then shift it upward 1 unit.



6. (a) $f(-2) = 1 - (-2)^2 = -3$ and $f(1) = 2(1) + 1 = 3$

(b) For $x \le 0$ plot $f(x) = 1 - x^2$ and, on the same plane, for $x > 0$ plot the graph of $f(x) = 2x + 1$.



7. (a) $(f \circ g)(x) = f(g(x)) = f(2x-3) = (2x-3)^2 + 2(2x-3) - 1 = 4x^2 - 12x + 9 + 4x - 6 - 1 = 4x^2 - 8x + 2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 + 2x - 1) = 2(x^2 + 2x - 1) - 3 = 2x^2 + 4x - 2 - 3 = 2x^2 + 4x - 5$

(c) $(g \circ g \circ g)(x) = g(g(g(x))) = g(g(2x-3)) = g(2(2x-3) - 3) = g(4x - 9) = 2(4x - 9) - 3$
$= 8x - 18 - 3 = 8x - 21$

Test D Trigonometry

1. (a) $300^\circ = 300^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{300\pi}{180} = \dfrac{5\pi}{3}$ (b) $-18^\circ = -18^\circ\left(\dfrac{\pi}{180^\circ}\right) = -\dfrac{18\pi}{180} = -\dfrac{\pi}{10}$

2. (a) $\dfrac{5\pi}{6} = \dfrac{5\pi}{6}\left(\dfrac{180^\circ}{\pi}\right) = 150^\circ$ (b) $2 = 2\left(\dfrac{180^\circ}{\pi}\right) = \dfrac{360^\circ}{\pi} \approx 114.6^\circ$

3. We will use the arc length formula, $s = r\theta$, where s is arc length, r is the radius of the circle, and θ is the measure of the central angle in radians. First, note that $30^\circ = 30^\circ\left(\dfrac{\pi}{180^\circ}\right) = \dfrac{\pi}{6}$. So $s = (12)\left(\dfrac{\pi}{6}\right) = 2\pi$ cm.

4. (a) $\tan(\pi/3) = \sqrt{3}$ [You can read the value from a right triangle with sides 1, 2, and $\sqrt{3}$.]

(b) Note that $7\pi/6$ can be thought of as an angle in the third quadrant with reference angle $\pi/6$. Thus, $\sin(7\pi/6) = -\frac{1}{2}$, since the sine function is negative in the third quadrant.

(c) Note that $5\pi/3$ can be thought of as an angle in the fourth quadrant with reference angle $\pi/3$. Thus, $\sec(5\pi/3) = \dfrac{1}{\cos(5\pi/3)} = \dfrac{1}{1/2} = 2$, since the cosine function is positive in the fourth quadrant.

5. $\sin\theta = a/24 \;\Rightarrow\; a = 24\sin\theta$ and $\cos\theta = b/24 \;\Rightarrow\; b = 24\cos\theta$

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6. $\sin x = \frac{1}{3}$ and $\sin^2 x + \cos^2 x = 1 \quad \Rightarrow \quad \cos x = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$. Also, $\cos y = \frac{4}{5} \quad \Rightarrow \quad \sin y = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$.

So, using the sum identity for the sine, we have

$$\sin(x + y) = \sin x \cos y + \cos x \sin y = \frac{1}{3} \cdot \frac{4}{5} + \frac{2\sqrt{2}}{3} \cdot \frac{3}{5} = \frac{4 + 6\sqrt{2}}{15} = \frac{1}{15}\left(4 + 6\sqrt{2}\right)$$

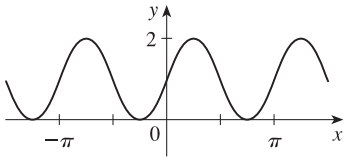
7. (a) $\tan\theta \sin\theta + \cos\theta = \dfrac{\sin\theta}{\cos\theta}\sin\theta + \cos\theta = \dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

(b) $\dfrac{2\tan x}{1 + \tan^2 x} = \dfrac{2\sin x/(\cos x)}{\sec^2 x} = 2\dfrac{\sin x}{\cos x}\cos^2 x = 2\sin x \cos x = \sin 2x$

8. $\sin 2x = \sin x \quad \Leftrightarrow \quad 2\sin x \cos x = \sin x \quad \Leftrightarrow \quad 2\sin x \cos x - \sin x = 0 \quad \Leftrightarrow \quad \sin x\,(2\cos x - 1) = 0 \quad \Leftrightarrow$

$\sin x = 0$ or $\cos x = \frac{1}{2} \quad \Rightarrow \quad x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$.

9. We first graph $y = \sin 2x$ (by compressing the graph of $\sin x$ by a factor of 2) and then shift it upward 1 unit.



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1 ☐ FUNCTIONS AND MODELS

1.1 Four Ways To Represent a Function

In exercises requiring estimations or approximations, your answers may vary slightly from the answers given here.

1. (a) The point $(1, 3)$ is on the graph of f, so $f(1) = 3$.

(b) When $x = -1$, y is about -0.2, so $f(-1) \approx -0.2$.

(c) $f(x) = 1$ is equivalent to $y = 1$. When $y = 1$, we have $x = 0$ and $x = 3$.

(d) A reasonable estimate for x when $y = 0$ is $x = -0.8$.

(e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-2 \le x \le 4$, or $[-2, 4]$. The range of f consists of all y-values on the graph of f. For this function, the range is $-1 \le y \le 3$, or $[-1, 3]$.

(f) As x increases from -2 to 1, y increases from -1 to 3. Thus, f is increasing on the interval $[-2, 1]$.

2. (a) The point $(-4, -2)$ is on the graph of f, so $f(-4) = -2$. The point $(3, 4)$ is on the graph of g, so $g(3) = 4$.

(b) We are looking for the values of x for which the y-values are equal. The y-values for f and g are equal at the points $(-2, 1)$ and $(2, 2)$, so the desired values of x are -2 and 2.

(c) $f(x) = -1$ is equivalent to $y = -1$. When $y = -1$, we have $x = -3$ and $x = 4$.

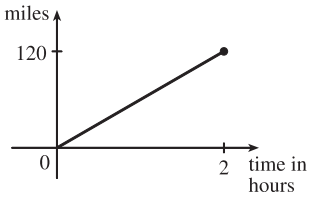
(d) As x increases from 0 to 4, y decreases from 3 to -1. Thus, f is decreasing on the interval $[0, 4]$.

(e) The domain of f consists of all x-values on the graph of f. For this function, the domain is $-4 \le x \le 4$, or $[-4, 4]$. The range of f consists of all y-values on the graph of f. For this function, the range is $-2 \le y \le 3$, or $[-2, 3]$.

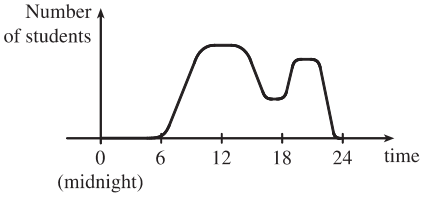
(f) The domain of g is $[-4, 3]$ and the range is $[0.5, 4]$.

3. From Figure 1 in the text, the lowest point occurs at about $(t, a) = (12, -85)$. The highest point occurs at about $(17, 115)$. Thus, the range of the vertical ground acceleration is $-85 \le a \le 115$. Written in interval notation, we get $[-85, 115]$.

4. *Example 1:* A car is driven at 60 mi/h for 2 hours. The distance d traveled by the car is a function of the time t. The domain of the function is $\{t \mid 0 \le t \le 2\}$, where t is measured in hours. The range of the function is $\{d \mid 0 \le d \le 120\}$, where d is measured in miles.



Example 2: At a certain university, the number of students N on campus at any time on a particular day is a function of the time t after midnight. The domain of the function is $\{t \mid 0 \le t \le 24\}$, where t is measured in hours. The range of the function is $\{N \mid 0 \le N \le k\}$, where N is an integer and k is the largest number of students on campus at once.

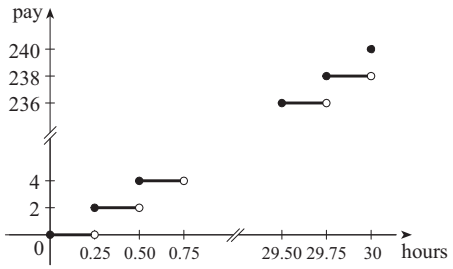


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Example 3: A certain employee is paid \$8.00 per hour and works a maximum of 30 hours per week. The number of hours worked is rounded down to the nearest quarter of an hour. This employee's gross weekly pay P is a function of the number of hours worked h. The domain of the function is $[0, 30]$ and the range of the function is $\{0, 2.00, 4.00, \ldots, 238.00, 240.00\}$.



5. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.

6. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-2, 2]$ and the range is $[-1, 2]$.

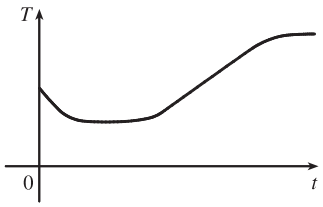
7. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-3, -2) \cup [-1, 3]$.

8. No, the curve is not the graph of a function since for $x = 0$, ± 1, and ± 2, there are infinitely many points on the curve.

9. The person's weight increased to about 160 pounds at age 20 and stayed fairly steady for 10 years. The person's weight dropped to about 120 pounds for the next 5 years, then increased rapidly to about 170 pounds. The next 30 years saw a gradual increase to 190 pounds. Possible reasons for the drop in weight at 30 years of age: diet, exercise, health problems.

10. First, the tub was filled with water to a height of 15 in. Then a person got into the tub, raising the water level to 20 in. At around 12 minutes, the person stood up in the tub but then immediately sat down. Finally, at around 17 minutes, the person got out of the tub, and then drained the water.

11. The water will cool down almost to freezing as the ice melts. Then, when the ice has melted, the water will slowly warm up to room temperature.



12. Runner A won the race, reaching the finish line at 100 meters in about 15 seconds, followed by runner B with a time of about 19 seconds, and then by runner C who finished in around 23 seconds. B initially led the race, followed by C, and then A. C then passed B to lead for a while. Then A passed first B, and then passed C to take the lead and finish first. Finally, B passed C to finish in second place. All three runners completed the race.

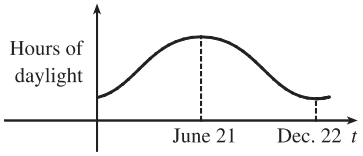
13. (a) The power consumption at 6 AM is 500 MW, which is obtained by reading the value of power P when $t = 6$ from the graph. At 6 PM we read the value of P when $t = 18$, obtaining approximately 730 MW.

(b) The minimum power consumption is determined by finding the time for the lowest point on the graph, $t = 4$, or 4 AM. The maximum power consumption corresponds to the highest point on the graph, which occurs just before $t = 12$, or right before noon. These times are reasonable, considering the power consumption schedules of most individuals and businesses.

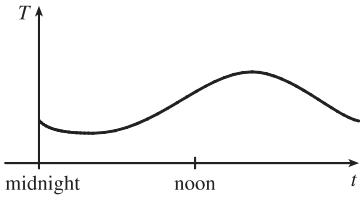
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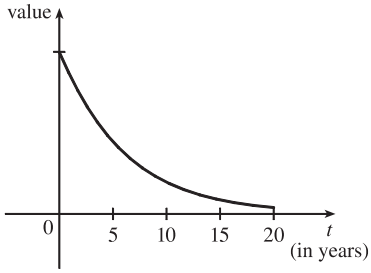
14. The summer solstice (the longest day of the year) is around June 21, and the winter solstice (the shortest day) is around December 22. (Exchange the dates for the southern hemisphere.)



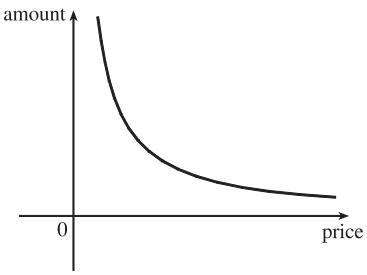
15. Of course, this graph depends strongly on the geographical location!



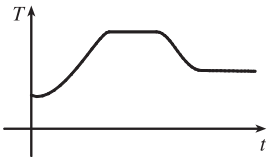
16. The value of the car decreases fairly rapidly initially, then somewhat less rapidly.



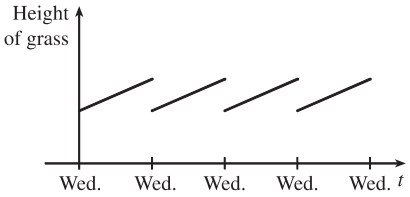
17. As the price increases, the amount sold decreases.



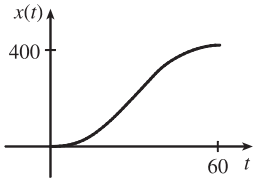
18. The temperature of the pie would increase rapidly, level off to oven temperature, decrease rapidly, and then level off to room temperature.



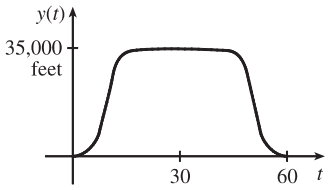
19.



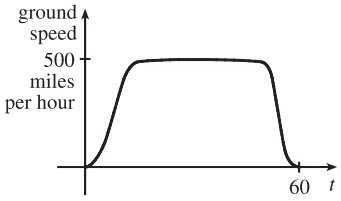
20. (a)



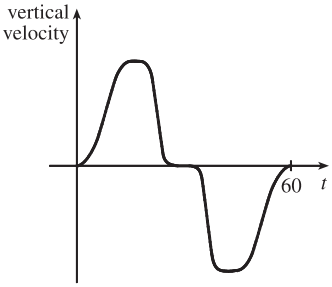
(b)



(c)



(d)



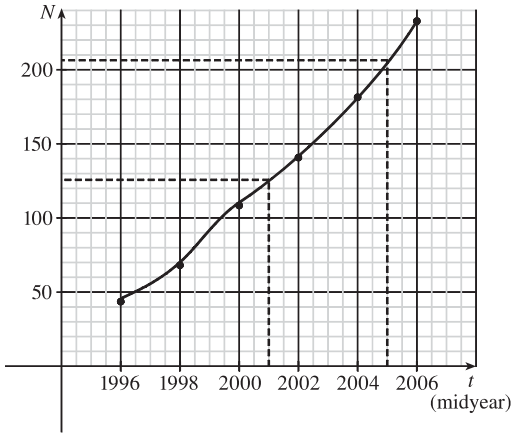
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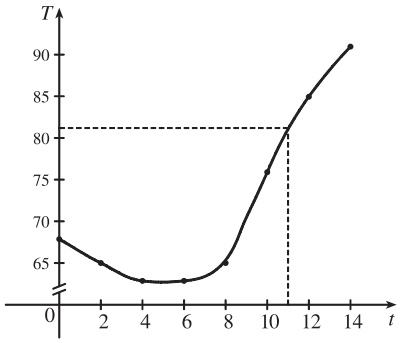
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21. (a)



(b) From the graph, we estimate the number of US cell-phone subscribers to be about 126 million in 2001 and 207 million in 2005.

22. (a)



(b) From the graph in part (a), we estimate the temperature at 11:00 AM to be about 81°F.

23. $f(x) = 3x^2 - x + 2$.

$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12$.

$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16$.

$f(a) = 3a^2 - a + 2$.

$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2$.

$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 = 3a^2 + 5a + 4$.

$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4$.

$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2$.

$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2$.

$$[f(a)]^2 = \left[3a^2 - a + 2\right]^2 = \left(3a^2 - a + 2\right)\left(3a^2 - a + 2\right)$$
$$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4.$$

$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2$.

24. A spherical balloon with radius $r+1$ has volume $V(r+1) = \frac{4}{3}\pi(r+1)^3 = \frac{4}{3}\pi(r^3 + 3r^2 + 3r + 1)$. We wish to find the amount of air needed to inflate the balloon from a radius of r to $r+1$. Hence, we need to find the difference $V(r+1) - V(r) = \frac{4}{3}\pi(r^3 + 3r^2 + 3r + 1) - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3r^2 + 3r + 1)$.

25. $f(x) = 4 + 3x - x^2$, so $f(3+h) = 4 + 3(3+h) - (3+h)^2 = 4 + 9 + 3h - (9 + 6h + h^2) = 4 - 3h - h^2$,

and $\dfrac{f(3+h) - f(3)}{h} = \dfrac{(4 - 3h - h^2) - 4}{h} = \dfrac{h(-3-h)}{h} = -3 - h$.

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26. $f(x) = x^3$, so $f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$,

and $\dfrac{f(a+h)-f(a)}{h} = \dfrac{(a^3+3a^2h+3ah^2+h^3)-a^3}{h} = \dfrac{h(3a^2+3ah+h^2)}{h} = 3a^2+3ah+h^2.$

27. $\dfrac{f(x)-f(a)}{x-a} = \dfrac{\frac{1}{x}-\frac{1}{a}}{x-a} = \dfrac{\frac{a-x}{xa}}{x-a} = \dfrac{a-x}{xa(x-a)} = \dfrac{-1(x-a)}{xa(x-a)} = -\dfrac{1}{ax}$

28. $\dfrac{f(x)-f(1)}{x-1} = \dfrac{\frac{x+3}{x+1}-2}{x-1} = \dfrac{\frac{x+3-2(x+1)}{x+1}}{x-1} = \dfrac{x+3-2x-2}{(x+1)(x-1)}$

$= \dfrac{-x+1}{(x+1)(x-1)} = \dfrac{-(x-1)}{(x+1)(x-1)} = -\dfrac{1}{x+1}$

29. $f(x) = (x+4)/(x^2-9)$ is defined for all x except when $0 = x^2-9 \;\Leftrightarrow\; 0 = (x+3)(x-3) \;\Leftrightarrow\; x = -3$ or 3, so the domain is $\{x \in \mathbb{R} \mid x \neq -3, 3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$.

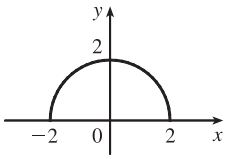
30. $f(x) = (2x^3-5)/(x^2+x-6)$ is defined for all x except when $0 = x^2+x-6 \;\Leftrightarrow\; 0 = (x+3)(x-2) \;\Leftrightarrow$ $x = -3$ or 2, so the domain is $\{x \in \mathbb{R} \mid x \neq -3, 2\} = (-\infty,-3) \cup (-3,2) \cup (2,\infty)$.

31. $f(t) = \sqrt[3]{2t-1}$ is defined for all real numbres. In fact $\sqrt[3]{p(t)}$, where $p(t)$ is a polynomial, is defined for all real numbers. Thus, the domain is $\mathbb{R}$, or $(-\infty,\infty)$.

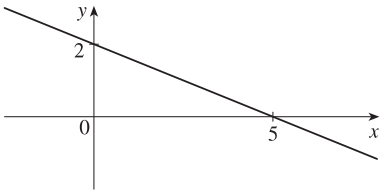
32. $g(t) = \sqrt{3-t} - \sqrt{2+t}$ is defined when $3-t \geq 0 \;\Leftrightarrow\; t \leq 3$ and $2+t \geq 0 \;\Leftrightarrow\; t \geq -2$. Thus, the domain is $-2 \leq t \leq 3$, or $[-2,3]$.

33. $h(x) = 1/\sqrt[4]{x^2-5x}$ is defined when $x^2-5x > 0 \;\Leftrightarrow\; x(x-5) > 0$. Note that $x^2-5x \neq 0$ since that would result in division by zero. The expression $x(x-5)$ is positive if $x < 0$ or $x > 5$. (See Appendix A for methods for solving inequalities.) Thus, the domain is $(-\infty,0) \cup (5,\infty)$.

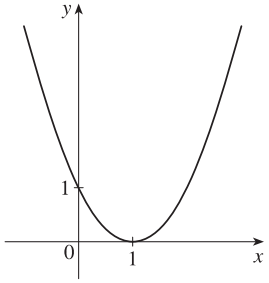
34. $h(x) = \sqrt{4-x^2}$. Now $y = \sqrt{4-x^2} \;\Rightarrow\; y^2 = 4-x^2 \;\Leftrightarrow\; x^2+y^2 = 4$, so the graph is the top half of a circle of radius 2 with center at the origin. The domain is $\{x \mid 4-x^2 \geq 0\} = \{x \mid 4 \geq x^2\} = \{x \mid 2 \geq |x|\} = [-2,2]$. From the graph, the range is $0 \leq y \leq 2$, or $[0,2]$.



35. $f(x) = 2 - 0.4x$ is defined for all real numbers, so the domain is $\mathbb{R}$, or $(-\infty,\infty)$. The graph of f is a line with slope -0.4 and y-intercept 2.



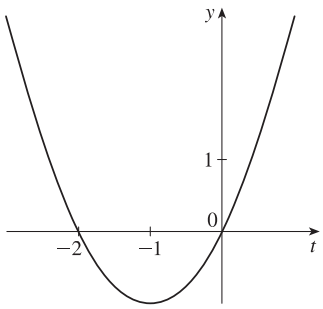
36. $F(x) = x^2 - 2x + 1 = (x-1)^2$ is defined for all real numbers, so the domain is $\mathbb{R}$, or $(-\infty,\infty)$. The graph of F is a parabola with vertex $(1,0)$.



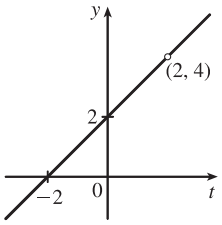
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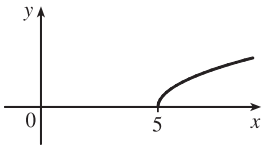
37. $f(t) = 2t + t^2$ is defined for all real numbers, so the domain is $\mathbb{R}$, or $(-\infty, \infty)$. The graph of f is a parabola opening upward since the coefficient of t^2 is positive. To find the t-intercepts, let $y = 0$ and solve for t. $0 = 2t + t^2 = t(2 + t) \quad \Rightarrow \quad t = 0$ or $t = -2$. The t-coordinate of the vertex is halfway between the t-intercepts, that is, at $t = -1$. Since $f(-1) = 2(-1) + (-1)^2 = -2 + 1 = -1$, the vertex is $(-1, -1)$.



38. $H(t) = \dfrac{4 - t^2}{2 - t} = \dfrac{(2 + t)(2 - t)}{2 - t}$, so for $t \neq 2$, $H(t) = 2 + t$. The domain is $\{t \mid t \neq 2\}$. So the graph of H is the same as the graph of the function $f(t) = t + 2$ (a line) except for the hole at $(2, 4)$.



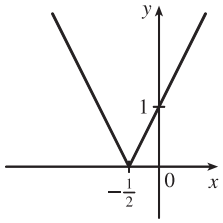
39. $g(x) = \sqrt{x - 5}$ is defined when $x - 5 \geq 0$ or $x \geq 5$, so the domain is $[5, \infty)$. Since $y = \sqrt{x - 5} \quad \Rightarrow \quad y^2 = x - 5 \quad \Rightarrow \quad x = y^2 + 5$, we see that g is the top half of a parabola.



40. $F(x) = |2x + 1| = \begin{cases} 2x + 1 & \text{if } 2x + 1 \geq 0 \\ -(2x + 1) & \text{if } 2x + 1 < 1 \end{cases}$

$= \begin{cases} 2x + 1 & \text{if } x \geq -\frac{1}{2} \\ -2x - 1 & \text{if } x < -\frac{1}{2} \end{cases}$

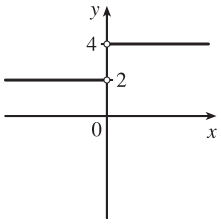
The domain is $\mathbb{R}$, or $(-\infty, \infty)$.



41. $G(x) = \dfrac{3x + |x|}{x}$. Since $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$, we have

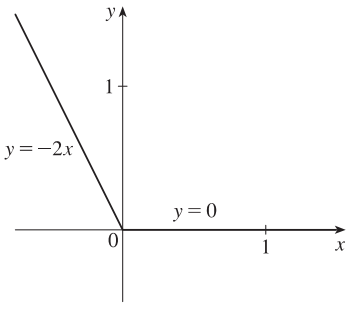
$$G(x) = \begin{cases} \dfrac{3x + x}{x} & \text{if } x > 0 \\ \dfrac{3x - x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} \dfrac{4x}{x} & \text{if } x > 0 \\ \dfrac{2x}{x} & \text{if } x < 0 \end{cases} = \begin{cases} 4 & \text{if } x > 0 \\ 2 & \text{if } x < 0 \end{cases}$$

Note that G is not defined for $x = 0$. The domain is $(-\infty, 0) \cup (0, \infty)$.



42. $g(x) = |x| - x = \begin{cases} x - x & \text{if } x \geq 0 \\ -x - x & \text{if } x < 0 \end{cases} = \begin{cases} 0 & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$.

The domain is $\mathbb{R}$, or $(-\infty, \infty)$.

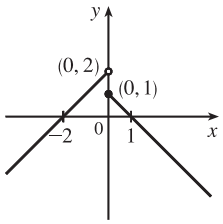


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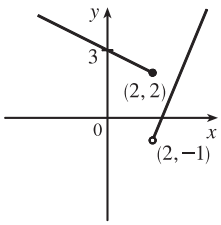
43. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$

The domain is $\mathbb{R}$.



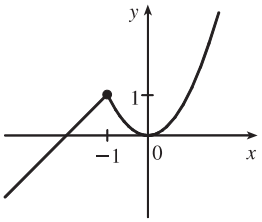
44. $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x \leq 2 \\ 2x - 5 & \text{if } x > 2 \end{cases}$

The domain is $\mathbb{R}$.



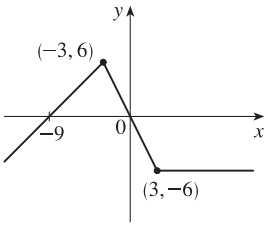
45. $f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

Note that for $x = -1$, both $x + 2$ and x^2 are equal to 1. The domain is $\mathbb{R}$.



46. $f(x) = \begin{cases} x+9 & \text{if } \quad x < -3 \\ -2x & \text{if } |x| \leq 3 \\ -6 & \text{if } \quad x > 3 \end{cases}$

Note that for $x = -3$, both $x + 9$ and $-2x$ are equal to 6; and for $x = 3$, both $-2x$ and -6 are equal to -6. The domain is $\mathbb{R}$.



47. Recall that the slope m of a line between the two points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and an equation of the line connecting those two points is $y - y_1 = m(x - x_1)$. The slope of the line segment joining the points $(1, -3)$ and $(5, 7)$ is $\dfrac{7 - (-3)}{5 - 1} = \dfrac{5}{2}$, so an equation is $y - (-3) = \frac{5}{2}(x - 1)$. The function is $f(x) = \frac{5}{2}x - \frac{11}{2}$, $1 \leq x \leq 5$.

48. The slope of the line segment joining the points $(-5, 10)$ and $(7, -10)$ is $\dfrac{-10 - 10}{7 - (-5)} = -\dfrac{5}{3}$, so an equation is $y - 10 = -\frac{5}{3}[x - (-5)]$. The function is $f(x) = -\frac{5}{3}x + \frac{5}{3}$, $-5 \leq x \leq 7$.

49. We need to solve the given equation for y. $x + (y-1)^2 = 0 \quad \Leftrightarrow \quad (y-1)^2 = -x \quad \Leftrightarrow \quad y - 1 = \pm\sqrt{-x} \quad \Leftrightarrow$ $y = 1 \pm \sqrt{-x}$. The expression with the positive radical represents the top half of the parabola, and the one with the negative radical represents the bottom half. Hence, we want $f(x) = 1 - \sqrt{-x}$. Note that the domain is $x \leq 0$.

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50. $x^2+(y-2)^2=4 \;\Leftrightarrow\; (y-2)^2=4-x^2 \;\Leftrightarrow\; y-2=\pm\sqrt{4-x^2} \;\Leftrightarrow\; y=2\pm\sqrt{4-x^2}$. The top half is given by the function $f(x)=2+\sqrt{4-x^2}$, $-2\le x\le 2$.

51. For $0\le x\le 3$, the graph is the line with slope -1 and y-intercept 3, that is, $y=-x+3$. For $3<x\le 5$, the graph is the line with slope 2 passing through $(3,0)$; that is, $y-0=2(x-3)$, or $y=2x-6$. So the function is

$$f(x)=\begin{cases}-x+3 & \text{if } 0\le x\le 3\\ 2x-6 & \text{if } 3<x\le 5\end{cases}$$

52. For $-4\le x\le -2$, the graph is the line with slope $-\frac{3}{2}$ passing through $(-2,0)$; that is, $y-0=-\frac{3}{2}[x-(-2)]$, or $y=-\frac{3}{2}x-3$. For $-2<x<2$, the graph is the top half of the circle with center $(0,0)$ and radius 2. An equation of the circle is $x^2+y^2=4$, so an equation of the top half is $y=\sqrt{4-x^2}$. For $2\le x\le 4$, the graph is the line with slope $\frac{3}{2}$ passing through $(2,0)$; that is, $y-0=\frac{3}{2}(x-2)$, or $y=\frac{3}{2}x-3$. So the function is

$$f(x)=\begin{cases}-\frac{3}{2}x-3 & \text{if } -4\le x\le -2\\ \sqrt{4-x^2} & \text{if } -2<x<2\\ \frac{3}{2}x-3 & \text{if } \quad 2\le x\le 4\end{cases}$$

53. Let the length and width of the rectangle be L and W. Then the perimeter is $2L+2W=20$ and the area is $A=LW$. Solving the first equation for W in terms of L gives $W=\dfrac{20-2L}{2}=10-L$. Thus, $A(L)=L(10-L)=10L-L^2$. Since lengths are positive, the domain of A is $0<L<10$. If we further restrict L to be larger than W, then $5<L<10$ would be the domain.

54. Let the length and width of the rectangle be L and W. Then the area is $LW=16$, so that $W=16/L$. The perimeter is $P=2L+2W$, so $P(L)=2L+2(16/L)=2L+32/L$, and the domain of P is $L>0$, since lengths must be positive quantities. If we further restrict L to be larger than W, then $L>4$ would be the domain.

55. Let the length of a side of the equilateral triangle be x. Then by the Pythagorean Theorem, the height y of the triangle satisfies $y^2+\left(\frac{1}{2}x\right)^2=x^2$, so that $y^2=x^2-\frac{1}{4}x^2=\frac{3}{4}x^2$ and $y=\frac{\sqrt{3}}{2}x$. Using the formula for the area A of a triangle, $A=\frac{1}{2}(\text{base})(\text{height})$, we obtain $A(x)=\frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right)=\frac{\sqrt{3}}{4}x^2$, with domain $x>0$.

56. Let the volume of the cube be V and the length of an edge be L. Then $V=L^3$ so $L=\sqrt[3]{V}$, and the surface area is $S(V)=6\left(\sqrt[3]{V}\right)^2=6V^{2/3}$, with domain $V>0$.

57. Let each side of the base of the box have length x, and let the height of the box be h. Since the volume is 2, we know that $2=hx^2$, so that $h=2/x^2$, and the surface area is $S=x^2+4xh$. Thus, $S(x)=x^2+4x(2/x^2)=x^2+(8/x)$, with domain $x>0$.

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58. The area of the window is $A = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = xh + \dfrac{\pi x^2}{8}$, where h is the height of the rectangular portion of the window.

The perimeter is $P = 2h + x + \frac{1}{2}\pi x = 30 \quad \Leftrightarrow \quad 2h = 30 - x - \frac{1}{2}\pi x \quad \Leftrightarrow \quad h = \frac{1}{4}(60 - 2x - \pi x)$. Thus,

$$A(x) = x\,\frac{60 - 2x - \pi x}{4} + \frac{\pi x^2}{8} = 15x - \tfrac{1}{2}x^2 - \tfrac{\pi}{4}x^2 + \tfrac{\pi}{8}x^2 = 15x - \tfrac{4}{8}x^2 - \tfrac{\pi}{8}x^2 = 15x - x^2\left(\frac{\pi + 4}{8}\right).$$

Since the lengths x and h must be positive quantities, we have $x > 0$ and $h > 0$. For $h > 0$, we have $2h > 0 \quad \Leftrightarrow$

$30 - x - \frac{1}{2}\pi x > 0 \quad \Leftrightarrow \quad 60 > 2x + \pi x \quad \Leftrightarrow \quad x < \dfrac{60}{2 + \pi}$. Hence, the domain of A is $0 < x < \dfrac{60}{2 + \pi}$.

59. The height of the box is x and the length and width are $L = 20 - 2x$, $W = 12 - 2x$. Then $V = LWx$ and so

$$V(x) = (20 - 2x)(12 - 2x)(x) = 4(10 - x)(6 - x)(x) = 4x(60 - 16x + x^2) = 4x^3 - 64x^2 + 240x.$$

The sides L, W, and x must be positive. Thus, $L > 0 \quad \Leftrightarrow \quad 20 - 2x > 0 \quad \Leftrightarrow \quad x < 10$;

$W > 0 \quad \Leftrightarrow \quad 12 - 2x > 0 \quad \Leftrightarrow \quad x < 6$; and $x > 0$. Combining these restrictions gives us the domain $0 < x < 6$.

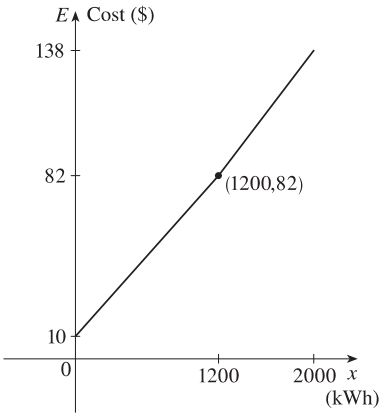
60. For the first 1200 kWh, $E(x) = 10 + 0.06x$.

For usage over 1200 kWh, the cost is

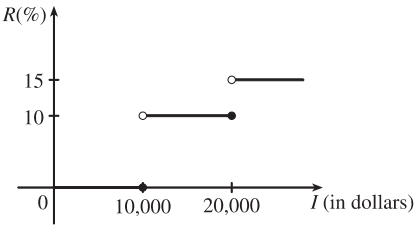
$E(x) = 10 + 0.06(1200) + 0.07(x - 1200) = 82 + 0.07(x - 1200)$.

Thus,

$$E(x) = \begin{cases} 10 + 0.06x & \text{if } 0 \le x \le 1200 \\ 82 + 0.07(x - 1200) & \text{if } x > 1200 \end{cases}$$



61. (a)

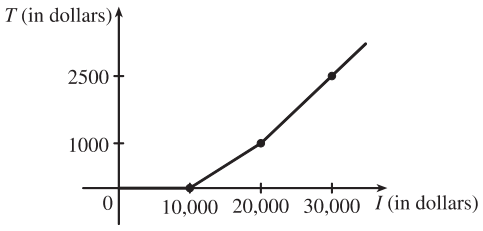


(b) On \$14,000, tax is assessed on \$4000, and $10\%(\$4000) = \$400$.

On \$26,000, tax is assessed on \$16,000, and

$10\%(\$10{,}000) + 15\%(\$6000) = \$1000 + \$900 = \$1900$.

(c) As in part (b), there is \$1000 tax assessed on \$20,000 of income, so the graph of T is a line segment from $(10{,}000, 0)$ to $(20{,}000, 1000)$. The tax on \$30,000 is \$2500, so the graph of T for $x > 20{,}000$ is the ray with initial point $(20{,}000, 1000)$ that passes through $(30{,}000, 2500)$.



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62. One example is the amount paid for cable or telephone system repair in the home, usually measured to the nearest quarter hour. Another example is the amount paid by a student in tuition fees, if the fees vary according to the number of credits for which the student has registered.

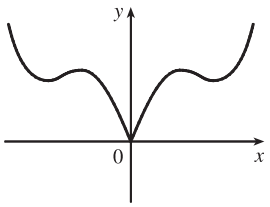
63. f is an odd function because its graph is symmetric about the origin. g is an even function because its graph is symmetric with respect to the y-axis.

64. f is not an even function since it is not symmetric with respect to the y-axis. f is not an odd function since it is not symmetric about the origin. Hence, f is *neither* even nor odd. g is an even function because its graph is symmetric with respect to the y-axis.

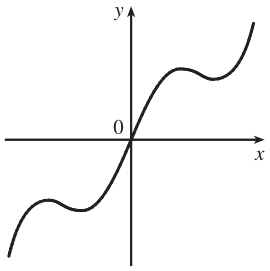
65. (a) Because an even function is symmetric with respect to the y-axis, and the point $(5, 3)$ is on the graph of this even function, the point $(-5, 3)$ must also be on its graph.

(b) Because an odd function is symmetric with respect to the origin, and the point $(5, 3)$ is on the graph of this odd function, the point $(-5, -3)$ must also be on its graph.

66. (a) If f is even, we get the rest of the graph by reflecting about the y-axis.



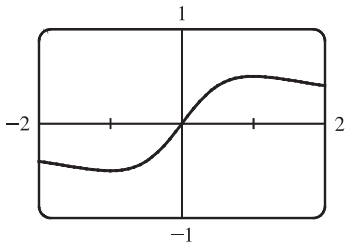
(b) If f is odd, we get the rest of the graph by rotating 180° about the origin.



67. $f(x) = \dfrac{x}{x^2+1}$.

$$f(-x) = \frac{-x}{(-x)^2+1} = \frac{-x}{x^2+1} = -\frac{x}{x^2+1} = -f(x).$$

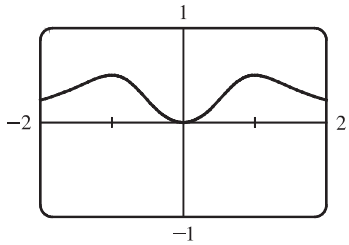
So f is an odd function.



68. $f(x) = \dfrac{x^2}{x^4+1}$.

$$f(-x) = \frac{(-x)^2}{(-x)^4+1} = \frac{x^2}{x^4+1} = f(x).$$

So f is an even function.

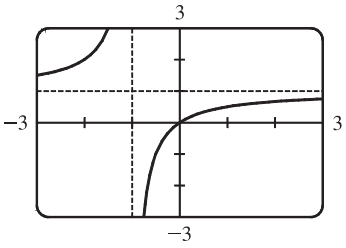


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69. $f(x) = \dfrac{x}{x+1}$, so $f(-x) = \dfrac{-x}{-x+1} = \dfrac{x}{x-1}$.

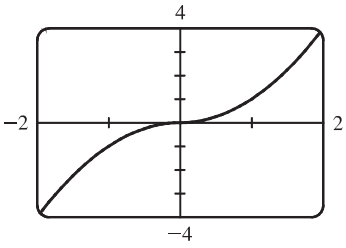
Since this is neither $f(x)$ nor $-f(x)$, the function f is neither even nor odd.



70. $f(x) = x\,|x|$.

$$f(-x) = (-x)\,|-x| = (-x)\,|x| = -(x\,|x|)$$
$$= -f(x)$$

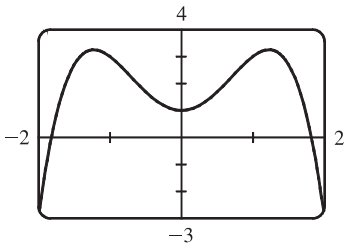
So f is an odd function.



71. $f(x) = 1 + 3x^2 - x^4$.

$f(-x) = 1+3(-x)^2-(-x)^4 = 1+3x^2-x^4 = f(x)$.

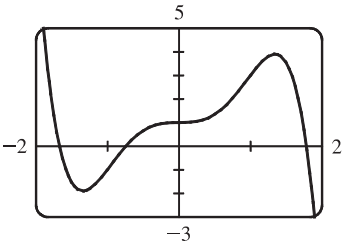
So f is an even function.



72. $f(x) = 1 + 3x^3 - x^5$, so

$$f(-x) = 1 + 3(-x)^3 - (-x)^5 = 1 + 3(-x^3) - (-x^5)$$
$$= 1 - 3x^3 + x^5$$

Since this is neither $f(x)$ nor $-f(x)$, the function f is neither even nor odd.



73. (i) If f and g are both even functions, then $f(-x) = f(x)$ and $g(-x) = g(x)$. Now $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$, so $f+g$ is an *even* function.

(ii) If f and g are both odd functions, then $f(-x) = -f(x)$ and $g(-x) = -g(x)$. Now $(f+g)(-x) = f(-x) + g(-x) = -f(x) + [-g(x)] = -[f(x) + g(x)] = -(f+g)(x)$, so $f+g$ is an *odd* function.

(iii) If f is an even function and g is an odd function, then $(f+g)(-x) = f(-x)+g(-x) = f(x)+[-g(x)] = f(x)-g(x)$, which is not $(f+g)(x)$ nor $-(f+g)(x)$, so $f+g$ is *neither* even nor odd. (Exception: if f is the zero function, then $f+g$ will be *odd*. If g is the zero function, then $f+g$ will be *even*.)

74. (i) If f and g are both even functions, then $f(-x) = f(x)$ and $g(-x) = g(x)$. Now $(fg)(-x) = f(-x)g(-x) = f(x)g(x) = (fg)(x)$, so fg is an *even* function.

(ii) If f and g are both odd functions, then $f(-x) = -f(x)$ and $g(-x) = -g(x)$. Now $(fg)(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = (fg)(x)$, so fg is an *even* function.

(iii) If f is an even function and g is an odd function, then $(fg)(-x) = f(-x)g(-x) = f(x)[-g(x)] = -[f(x)g(x)] = -(fg)(x)$, so fg is an *odd* function.

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1.2 Mathematical Models: A Catalog of Essential Functions

1. (a) $f(x) = \log_2 x$ is a logarithmic function.

(b) $g(x) = \sqrt[4]{x}$ is a root function with $n = 4$.

(c) $h(x) = \dfrac{2x^3}{1 - x^2}$ is a rational function because it is a ratio of polynomials.

(d) $u(t) = 1 - 1.1t + 2.54t^2$ is a polynomial of degree 2 (also called a *quadratic function*).

(e) $v(t) = 5^t$ is an exponential function.

(f) $w(\theta) = \sin\theta \cos^2\theta$ is a trigonometric function.

2. (a) $y = \pi^x$ is an exponential function (notice that x is the *exponent*).

(b) $y = x^\pi$ is a power function (notice that x is the *base*).

(c) $y = x^2(2 - x^3) = 2x^2 - x^5$ is a polynomial of degree 5.

(d) $y = \tan t - \cos t$ is a trigonometric function.

(e) $y = s/(1 + s)$ is a rational function because it is a ratio of polynomials.

(f) $y = \sqrt{x^3 - 1}/(1 + \sqrt[3]{x})$ is an algebraic function because it involves polynomials and roots of polynomials.

3. We notice from the figure that g and h are even functions (symmetric with respect to the y-axis) and that f is an odd function (symmetric with respect to the origin). So (b) $\left[y = x^5\right]$ must be f. Since g is flatter than h near the origin, we must have (c) $\left[y = x^8\right]$ matched with g and (a) $\left[y = x^2\right]$ matched with h.

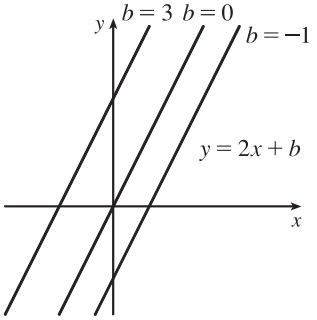
4. (a) The graph of $y = 3x$ is a line (choice G).

(b) $y = 3^x$ is an exponential function (choice f).

(c) $y = x^3$ is an odd polynomial function or power function (choice F).

(d) $y = \sqrt[3]{x} = x^{1/3}$ is a root function (choice g).

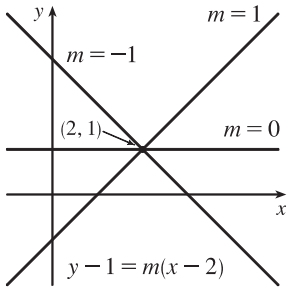
5. (a) An equation for the family of linear functions with slope 2 is $y = f(x) = 2x + b$, where b is the y-intercept.



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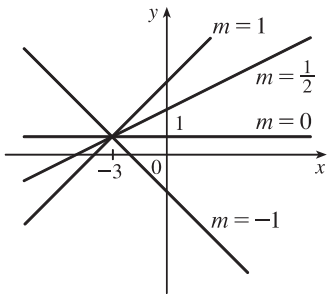
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(b) $f(2) = 1$ means that the point $(2, 1)$ is on the graph of f. We can use the point-slope form of a line to obtain an equation for the family of linear functions through the point $(2, 1)$. $y - 1 = m(x - 2)$, which is equivalent to $y = mx + (1 - 2m)$ in slope-intercept form.

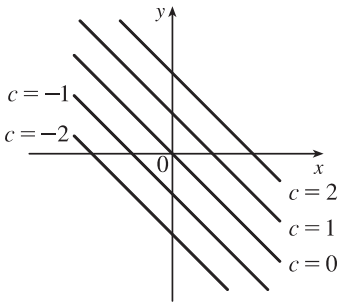


(c) To belong to both families, an equation must have slope $m = 2$, so the equation in part (b), $y = mx + (1 - 2m)$, becomes $y = 2x - 3$. It is the *only* function that belongs to both families.

6. All members of the family of linear functions $f(x) = 1 + m(x + 3)$ have graphs that are lines passing through the point $(-3, 1)$.



7. All members of the family of linear functions $f(x) = c - x$ have graphs that are lines with slope -1. The y-intercept is c.



8. The vertex of the parabola on the left is $(3, 0)$, so an equation is $y = a(x - 3)^2 + 0$. Since the point $(4, 2)$ is on the parabola, we'll substitute 4 for x and 2 for y to find a. $2 = a(4 - 3)^2 \Rightarrow a = 2$, so an equation is $f(x) = 2(x - 3)^2$.

The y-intercept of the parabola on the right is $(0, 1)$, so an equation is $y = ax^2 + bx + 1$. Since the points $(-2, 2)$ and $(1, -2.5)$ are on the parabola, we'll substitute -2 for x and 2 for y as well as 1 for x and -2.5 for y to obtain two equations with the unknowns a and b.

$$(-2, 2): \quad 2 = 4a - 2b + 1 \quad \Rightarrow \quad 4a - 2b = 1 \qquad \textbf{(1)}$$

$$(1, -2.5): \quad -2.5 = a + b + 1 \quad \Rightarrow \quad a + b = -3.5 \qquad \textbf{(2)}$$

$2 \cdot$ **(2)** + **(1)** gives us $6a = -6 \Rightarrow a = -1$. From **(2)**, $-1 + b = -3.5 \Rightarrow b = -2.5$, so an equation is $g(x) = -x^2 - 2.5x + 1$.

9. Since $f(-1) = f(0) = f(2) = 0$, f has zeros of -1, 0, and 2, so an equation for f is $f(x) = a[x - (-1)](x - 0)(x - 2)$, or $f(x) = ax(x + 1)(x - 2)$. Because $f(1) = 6$, we'll substitute 1 for x and 6 for $f(x)$.

$6 = a(1)(2)(-1) \Rightarrow -2a = 6 \Rightarrow a = -3$, so an equation for f is $f(x) = -3x(x + 1)(x - 2)$.

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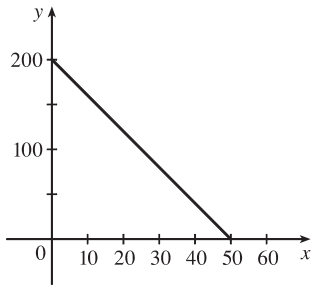
10. (a) For $T = 0.02t + 8.50$, the slope is 0.02, which means that the average surface temperature of the world is increasing at a rate of $0.02\,^\circ\text{C}$ per year. The T-intercept is 8.50, which represents the average surface temperature in $^\circ\text{C}$ in the year 1900.

(b) $t = 2100 - 1900 = 200 \quad \Rightarrow \quad T = 0.02(200) + 8.50 = 12.50\,^\circ\text{C}$

11. (a) $D = 200$, so $c = 0.0417D(a + 1) = 0.0417(200)(a + 1) = 8.34a + 8.34$. The slope is 8.34, which represents the change in mg of the dosage for a child for each change of 1 year in age.

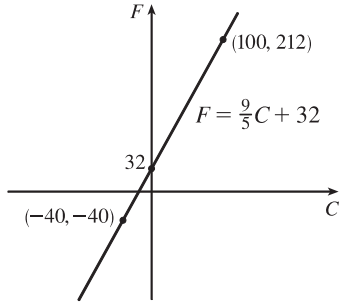
(b) For a newborn, $a = 0$, so $c = 8.34$ mg.

12. (a)



(b) The slope of -4 means that for each increase of 1 dollar for a rental space, the number of spaces rented *decreases* by 4. The y-intercept of 200 is the number of spaces that would be occupied if there were no charge for each space. The x-intercept of 50 is the smallest rental fee that results in no spaces rented.

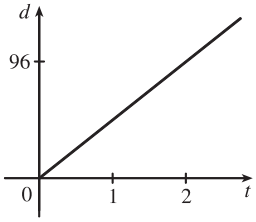
13. (a)



(b) The slope of $\frac{9}{5}$ means that F increases $\frac{9}{5}$ degrees for each increase of 1°C. (Equivalently, F increases by 9 when C increases by 5 and F decreases by 9 when C decreases by 5.) The F-intercept of 32 is the Fahrenheit temperature corresponding to a Celsius temperature of 0.

14. (a) Let $d =$ distance traveled (in miles) and $t =$ time elapsed (in hours). At $t = 0$, $d = 0$ and at $t = 50$ minutes $= 50 \cdot \frac{1}{60} = \frac{5}{6}$ h, $d = 40$. Thus we have two points: $(0, 0)$ and $\left(\frac{5}{6}, 40\right)$, so $m = \dfrac{40 - 0}{\frac{5}{6} - 0} = 48$ and so $d = 48t$.

(b)



(c) The slope is 48 and represents the car's speed in mi/h.

15. (a) Using N in place of x and T in place of y, we find the slope to be $\dfrac{T_2 - T_1}{N_2 - N_1} = \dfrac{80 - 70}{173 - 113} = \dfrac{10}{60} = \dfrac{1}{6}$. So a linear equation is $T - 80 = \frac{1}{6}(N - 173) \quad \Leftrightarrow \quad T - 80 = \frac{1}{6}N - \frac{173}{6} \quad \Leftrightarrow \quad T = \frac{1}{6}N + \frac{307}{6} \quad \left[\frac{307}{6} = 51.1\overline{6}\right]$.

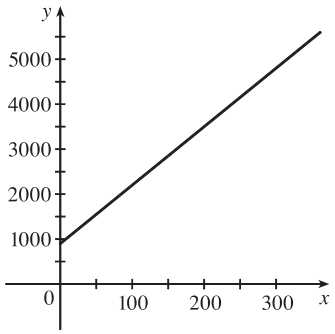
(b) The slope of $\frac{1}{6}$ means that the temperature in Fahrenheit degrees increases one-sixth as rapidly as the number of cricket chirps per minute. Said differently, each increase of 6 cricket chirps per minute corresponds to an increase of 1°F.

(c) When $N = 150$, the temperature is given approximately by $T = \frac{1}{6}(150) + \frac{307}{6} = 76.1\overline{6}\,^\circ\text{F} \approx 76\,^\circ\text{F}$.

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16. (a) Let x denote the number of chairs produced in one day and y the associated cost. Using the points $(100, 2200)$ and $(300, 4800)$, we get the slope $\frac{4800-2200}{300-100} = \frac{2600}{200} = 13$. So $y - 2200 = 13(x - 100) \quad \Leftrightarrow$ $y = 13x + 900$.



(b) The slope of the line in part (a) is 13 and it represents the cost (in dollars) of producing each additional chair.

(c) The y-intercept is 900 and it represents the fixed daily costs of operating the factory.

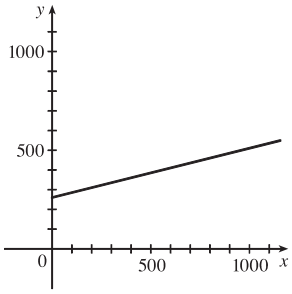
17. (a) We are given $\dfrac{\text{change in pressure}}{\text{10 feet change in depth}} = \dfrac{4.34}{10} = 0.434$. Using P for pressure and d for depth with the point $(d, P) = (0, 15)$, we have the slope-intercept form of the line, $P = 0.434d + 15$.

(b) When $P = 100$, then $100 = 0.434d + 15 \quad \Leftrightarrow \quad 0.434d = 85 \quad \Leftrightarrow \quad d = \frac{85}{0.434} \approx 195.85$ feet. Thus, the pressure is 100 lb/in^2 at a depth of approximately 196 feet.

18. (a) Using d in place of x and C in place of y, we find the slope to be $\dfrac{C_2 - C_1}{d_2 - d_1} = \dfrac{460 - 380}{800 - 480} = \dfrac{80}{320} = \dfrac{1}{4}$. So a linear equation is $C - 460 = \frac{1}{4}(d - 800) \quad \Leftrightarrow \quad C - 460 = \frac{1}{4}d - 200 \quad \Leftrightarrow \quad C = \frac{1}{4}d + 260$.

(b) Letting $d = 1500$ we get $C = \frac{1}{4}(1500) + 260 = 635$.

The cost of driving 1500 miles is \$635.

(c) 

The slope of the line represents the cost per mile, \$0.25.

(d) The y-intercept represents the fixed cost, \$260.

(e) A linear function gives a suitable model in this situation because you have fixed monthly costs such as insurance and car payments, as well as costs that increase as you drive, such as gasoline, oil, and tires, and the cost of these for each additional mile driven is a constant.

19. (a) The data appear to be periodic and a sine or cosine function would make the best model. A model of the form $f(x) = a\cos(bx) + c$ seems appropriate.

(b) The data appear to be decreasing in a linear fashion. A model of the form $f(x) = mx + b$ seems appropriate.

20. (a) The data appear to be increasing exponentially. A model of the form $f(x) = a \cdot b^x$ or $f(x) = a \cdot b^x + c$ seems appropriate.

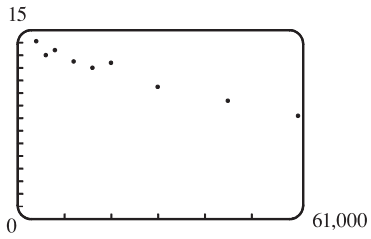
(b) The data appear to be decreasing similarly to the values of the reciprocal function. A model of the form $f(x) = a/x$ seems appropriate.

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Exercises 21 – 24: Some values are given to many decimal places. These are the results given by several computer algebra systems — rounding is left to the reader.

21. (a)

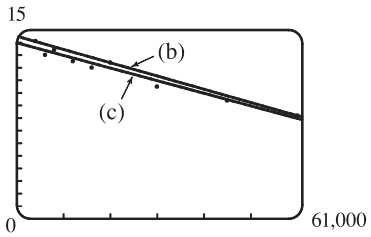


A linear model does seem appropriate.

(b) Using the points $(4000, 14.1)$ and $(60{,}000, 8.2)$, we obtain

$y - 14.1 = \dfrac{8.2 - 14.1}{60{,}000 - 4000}\,(x - 4000)$ or, equivalently,

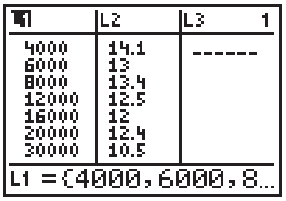
$y \approx -0.000105357x + 14.521429$.

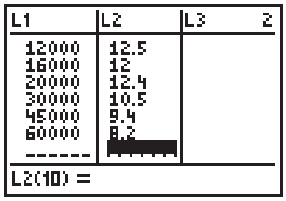


(c) Using a computing device, we obtain the least squares regression line $y = -0.0000997855x + 13.950764$.

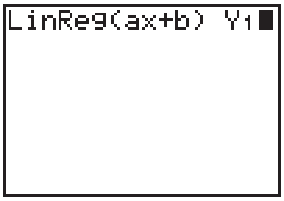
The following commands and screens illustrate how to find the least squares regression line on a TI-84 Plus.

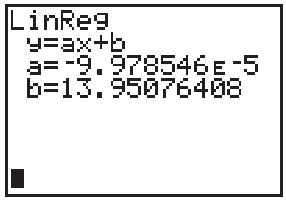
Enter the data into list one (L1) and list two (L2). Press STAT 1 to enter the editor.

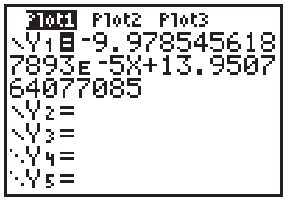




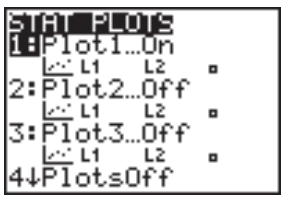
Find the regession line and store it in Y_1. Press 2nd QUIT STAT ▸ 4 VARS ▸ 1 1 ENTER.

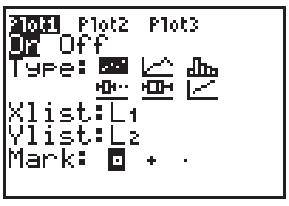




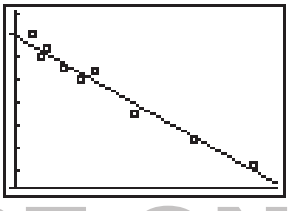


Note from the last figure that the regression line has been stored in Y_1 and that Plot1 has been turned on (Plot1 is highlighted). You can turn on Plot1 from the Y= menu by placing the cursor on Plot1 and pressing ENTER or by pressing 2nd STAT PLOT 1 ENTER.





Now press ZOOM 9 to produce a graph of the data and the regression line. Note that choice 9 of the ZOOM menu automatically selects a window that displays all of the data.



(d) When $x = 25{,}000$, $y \approx 11.456$; or about 11.5 per 100 population.

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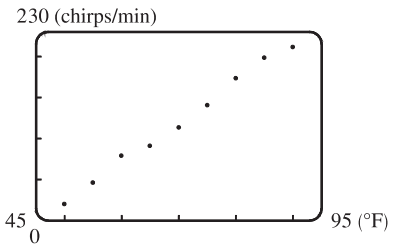
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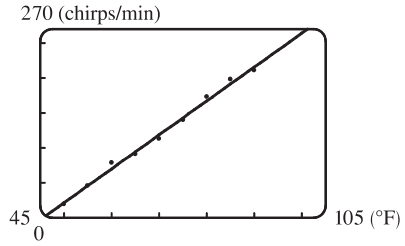
(e) When $x = 80{,}000$, $y \approx 5.968$; or about a 6% chance.

(f) When $x = 200{,}000$, y is negative, so the model does not apply.

22. (a)



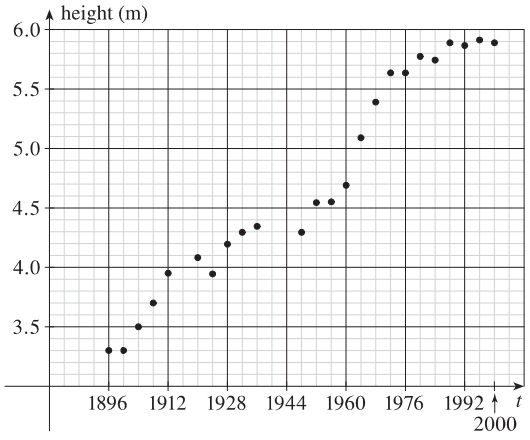
(b)



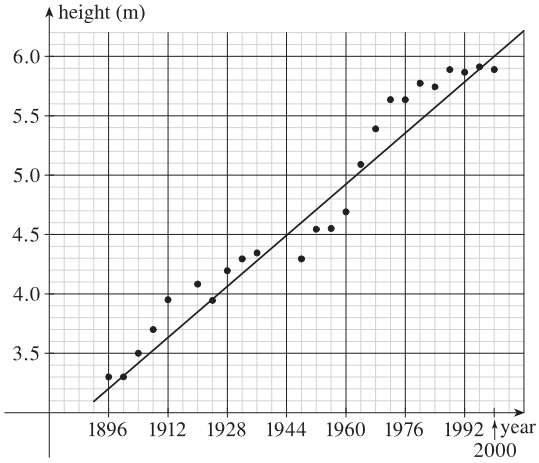
Using a computing device, we obtain the least squares regression line $y = 4.85\overline{6}x - 220.9\overline{6}$.

(c) When $x = 100°\text{F}$, $y = 264.7 \approx 265$ chirps/min.

23. (a) A linear model seems appropriate over the time interval considered.



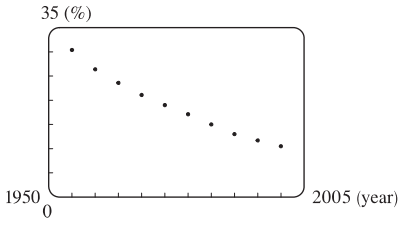
(b) Using a computing device, we obtain the least squares regression line $y \approx 0.027t - 47.758$.



(c) When $t = 2004$, $y = 6.35$, which is higher than the actual winning height of $5.95\ \text{m}$.

(d) No, since the times appear to be leveling off and getting further away from the model.

24. By looking at the scatter plot of the data, we rule out the power and logarithmic models.



Scatter plot

We try various models:

Linear $y = -0.430\,545\,454\,5x + 870.183\,636\,4$

Quadratic: $y = 0.004\,893\,939\,4x^2 - 19.786\,075\,76x + 20\,006.954\,85$

Cubic: $y = -0.000\,073\,193\,47x^3 + 0.439\,114\,219\,1x^2 - 878.429\,871\,8x + 585\,960.983$

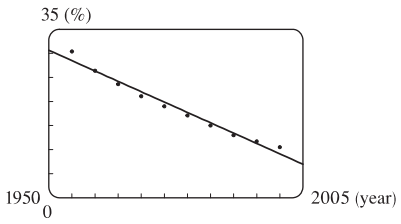
Quartic: $y = 0.000\,007\,902\,097\,9x^4 - 0.062\,578\,787\,9x^3 + 185.842\,283\,8x^2 - 245\,290.9304x + 121\,409\,472.7$

Exponential: $y = 2.618\,230\,2 \times 10^{21}(0.976\,789\,309\,4)^x$

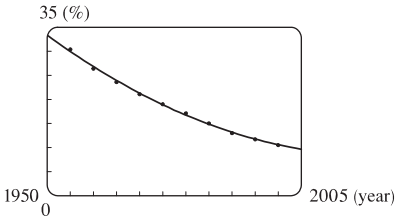
[continued]

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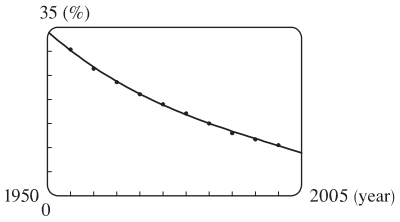
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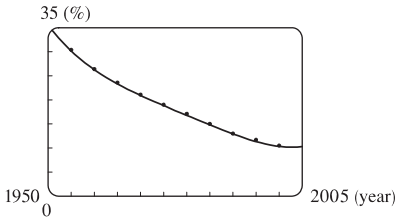
Linear model



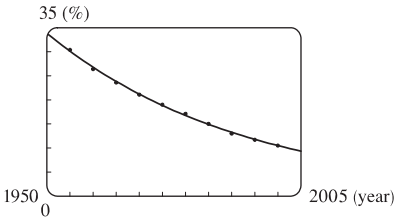
Quadratic model



Cubic model



Quartic model

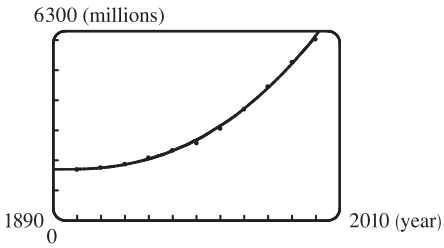


Exponential model

After examining the graphs of these models, we see that all the models are good and the quartic model is the best.

Using this model, we obtain estimates 13.6% and 10.2% for the rural percentages in 1988 and 2002 respectively.

25.



Using a computing device, we obtain the cubic function $y = ax^3 + bx^2 + cx + d$ with $a = 0.0012937$, $b = -7.06142$, $c = 12{,}823$, and $d = -7{,}743{,}770$. When $x = 1925$, $y \approx 1914$ (million).

26. (a) $T = 1.000\,431\,227 d^{\,1.499\,528\,750}$

(b) The power model in part (a) is approximately $T = d^{1.5}$. Squaring both sides gives us $T^2 = d^3$, so the model matches Kepler's Third Law, $T^2 = kd^3$.

1.3 New Functions from Old Functions

1. (a) If the graph of f is shifted 3 units upward, its equation becomes $y = f(x) + 3$.

(b) If the graph of f is shifted 3 units downward, its equation becomes $y = f(x) - 3$.

(c) If the graph of f is shifted 3 units to the right, its equation becomes $y = f(x - 3)$.

(d) If the graph of f is shifted 3 units to the left, its equation becomes $y = f(x + 3)$.

(e) If the graph of f is reflected about the x-axis, its equation becomes $y = -f(x)$.

(f) If the graph of f is reflected about the y-axis, its equation becomes $y = f(-x)$.

(g) If the graph of f is stretched vertically by a factor of 3, its equation becomes $y = 3f(x)$.

(h) If the graph of f is shrunk vertically by a factor of 3, its equation becomes $y = \frac{1}{3} f(x)$.

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2. (a) To obtain the graph of $y = f(x) + 8$ from the graph of $y = f(x)$, shift the graph 8 units upward.

(b) To obtain the graph of $y = f(x + 8)$ from the graph of $y = f(x)$, shift the graph 8 units to the left.

(c) To obtain the graph of $y = 8f(x)$ from the graph of $y = f(x)$, stretch the graph vertically by a factor of 8.

(d) To obtain the graph of $y = f(8x)$ from the graph of $y = f(x)$, shrink the graph horizontally by a factor of 8.

(e) To obtain the graph of $y = -f(x) - 1$ from the graph of $y = f(x)$, first reflect the graph about the x-axis, and then shift it 1 unit downward.

(f) To obtain the graph of $y = 8f(\frac{1}{8}x)$ from the graph of $y = f(x)$, stretch the graph horizontally and vertically by a factor of 8.

3. (a) (graph 3) The graph of f is shifted 4 units to the right and has equation $y = f(x - 4)$.

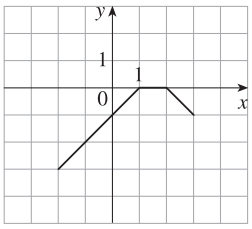
(b) (graph 1) The graph of f is shifted 3 units upward and has equation $y = f(x) + 3$.

(c) (graph 4) The graph of f is shrunk vertically by a factor of 3 and has equation $y = \frac{1}{3}f(x)$.

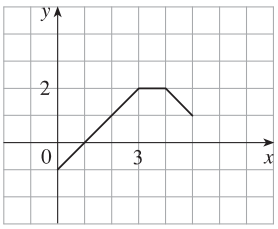
(d) (graph 5) The graph of f is shifted 4 units to the left and reflected about the x-axis. Its equation is $y = -f(x + 4)$.

(e) (graph 2) The graph of f is shifted 6 units to the left and stretched vertically by a factor of 2. Its equation is $y = 2f(x + 6)$.

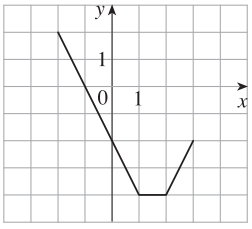
4. (a) To graph $y = f(x) - 2$, we shift the graph of f, 2 units downward.The point $(1, 2)$ on the graph of f corresponds to the point $(1, 2 - 2) = (1, 0)$.



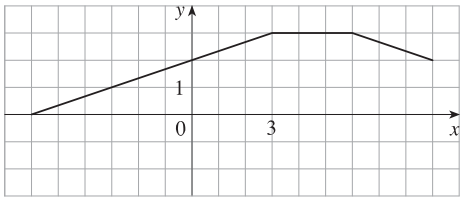
(b) To graph $y = f(x - 2)$, we shift the graph of f, 2 units to the right.The point $(1, 2)$ on the graph of f corresponds to the point $(1 + 2, 2) = (3, 2)$.



(c) To graph $y = -2f(x)$, we reflect the graph about the x-axis and stretch the graph vertically by a factor of 2. The point $(1, 2)$ on the graph of f corresponds to the point $(1, -2 \cdot 2) = (1, -4)$.



(d) To graph $y = f(\frac{1}{3}x) + 1$, we stretch the graph horizontally by a factor of 3 and shift it 1 unit upward. The point $(1, 2)$ on the graph of f corresponds to the point $(1 \cdot 3, 2 + 1) = (3, 3)$.



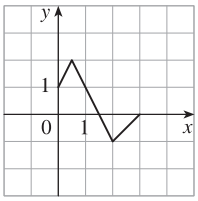
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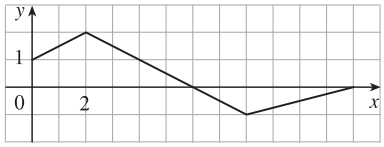
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5. (a) To graph $y = f(2x)$ we shrink the graph of f horizontally by a factor of 2.



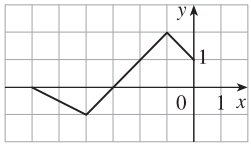
The point $(4, -1)$ on the graph of f corresponds to the point $\left(\frac{1}{2} \cdot 4, -1\right) = (2, -1)$.

(b) To graph $y = f\left(\frac{1}{2}x\right)$ we stretch the graph of f horizontally by a factor of 2.



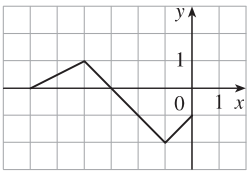
The point $(4, -1)$ on the graph of f corresponds to the point $(2 \cdot 4, -1) = (8, -1)$.

(c) To graph $y = f(-x)$ we reflect the graph of f about the y-axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1) = (-4, -1)$.

(d) To graph $y = -f(-x)$ we reflect the graph of f about the y-axis, then about the x-axis.



The point $(4, -1)$ on the graph of f corresponds to the point $(-1 \cdot 4, -1 \cdot -1) = (-4, 1)$.

6. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 2 units to the right and stretched vertically by a factor of 2. Thus, a function describing the graph is

$$y = 2f(x-2) = 2\sqrt{3(x-2) - (x-2)^2} = 2\sqrt{3x - 6 - (x^2 - 4x + 4)} = 2\sqrt{-x^2 + 7x - 10}$$

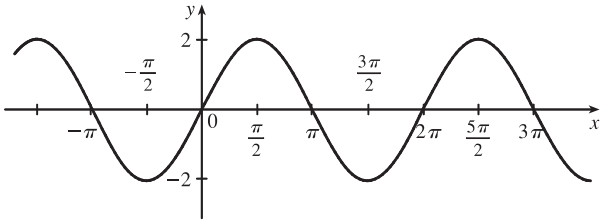
7. The graph of $y = f(x) = \sqrt{3x - x^2}$ has been shifted 4 units to the left, reflected about the x-axis, and shifted downward 1 unit. Thus, a function describing the graph is

$$y = \underbrace{-1 \cdot}_{\substack{\text{reflect} \\ \text{about } x\text{-axis}}} f \underbrace{(x+4)}_{\substack{\text{shift} \\ \text{4 units left}}} \underbrace{-\ 1}_{\substack{\text{shift} \\ \text{1 unit left}}}$$

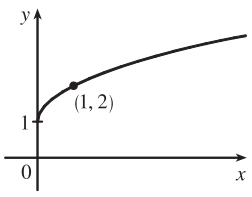
This function can be written as

$$y = -f(x+4) - 1 = -\sqrt{3(x+4) - (x+4)^2} - 1 = -\sqrt{3x + 12 - (x^2 + 8x + 16)} - 1 = -\sqrt{-x^2 - 5x - 4} - 1$$

8. (a) The graph of $y = 2\sin x$ can be obtained from the graph of $y = \sin x$ by stretching it vertically by a factor of 2.



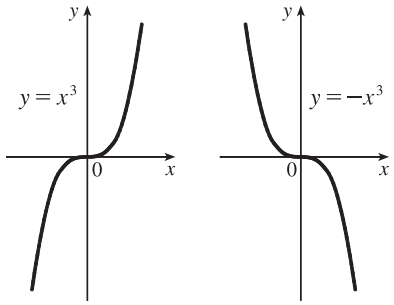
(b) The graph of $y = 1 + \sqrt{x}$ can be obtained from the graph of $y = \sqrt{x}$ by shifting it upward 1 unit.



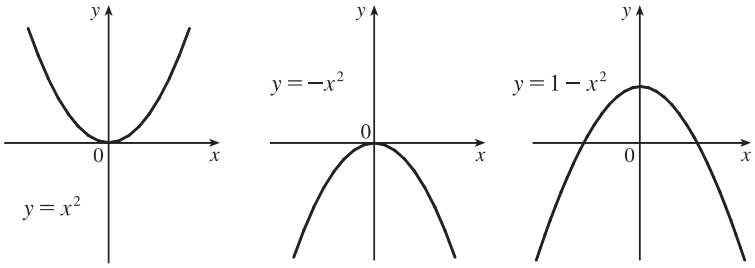
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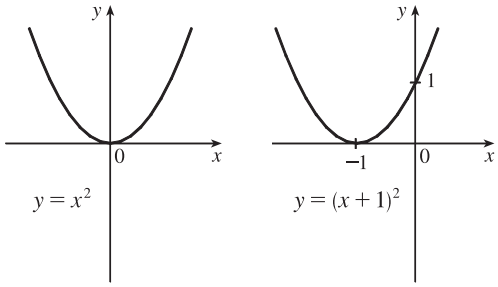
9. $y = -x^3$: Start with the graph of $y = x^3$ and reflect about the x-axis. Note: Reflecting about the y-axis gives the same result since substituting $-x$ for x gives us $y = (-x)^3 = -x^3$.



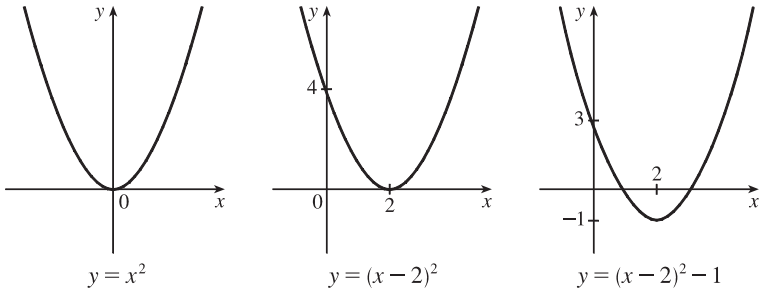
10. $y = 1 - x^2 = -x^2 + 1$: Start with the graph of $y = x^2$, reflect about the x-axis, and then shift 1 unit upward.



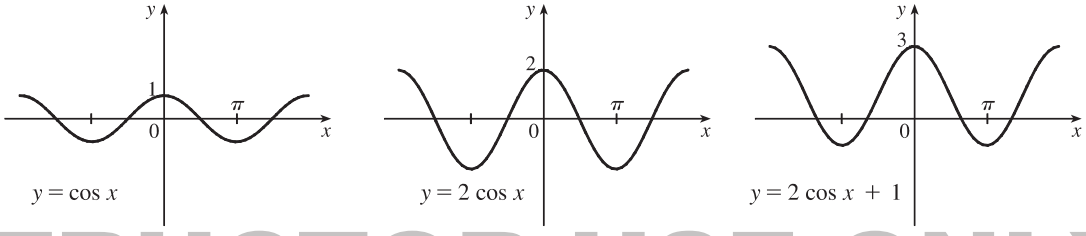
11. $y = (x + 1)^2$: Start with the graph of $y = x^2$ and shift 1 unit to the left.



12. $y = x^2 - 4x + 3 = (x^2 - 4x + 4) - 1 = (x - 2)^2 - 1$: Start with the graph of $y = x^2$, shift 2 units to the right, and then shift 1 unit downward.



13. $y = 1 + 2\cos x$: Start with the graph of $y = \cos x$, stretch vertically by a factor of 2, and then shift 1 unit upward.



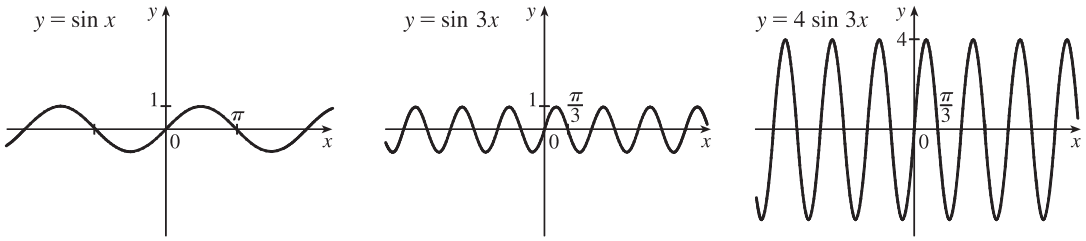
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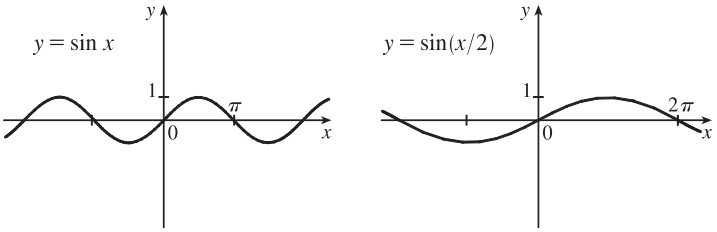
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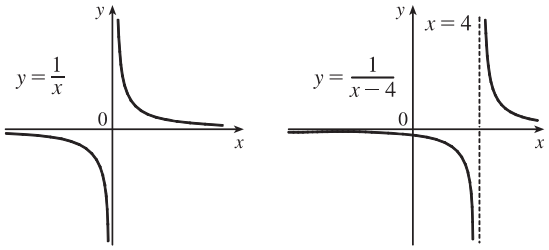
14. $y = 4\sin 3x$: Start with the graph of $y = \sin x$, compress horizontally by a factor of 3, and then stretch vertically by a factor of 4.



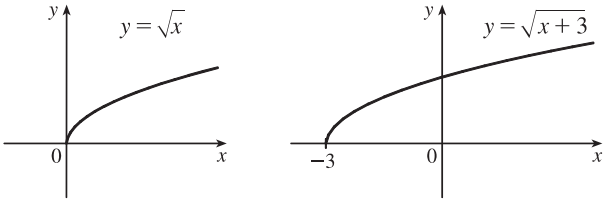
15. $y = \sin(x/2)$: Start with the graph of $y = \sin x$ and stretch horizontally by a factor of 2.



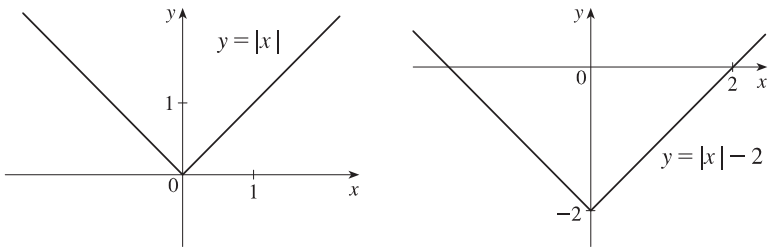
16. $y = 1/(x - 4)$: Start with the graph of $y = 1/x$ and shift 4 units to the right.



17. $y = \sqrt{x+3}$: Start with the graph of $y = \sqrt{x}$ and shift 3 units to the left.



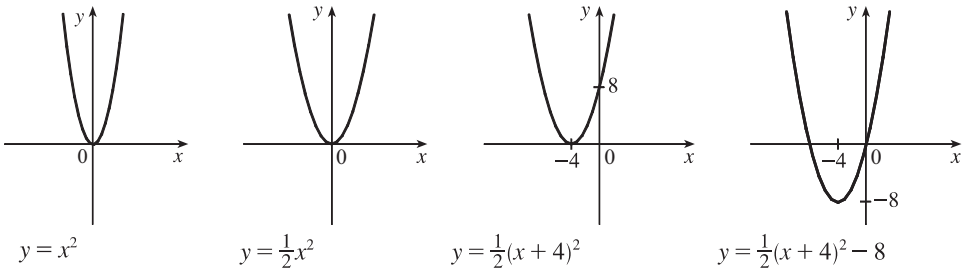
18. $y = |x| - 2$: Start with the graph of $y = |x|$ and shift 2 units downward.



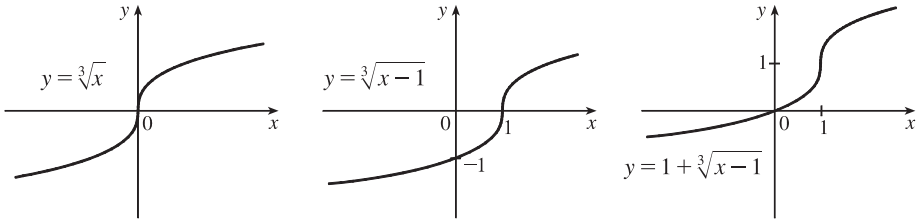
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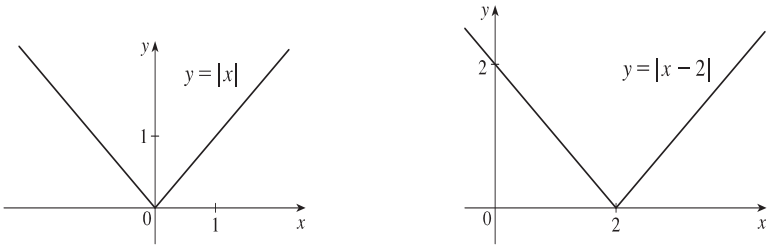
19. $y = \frac{1}{2}(x^2 + 8x) = \frac{1}{2}(x^2 + 8x + 16) - 8 = \frac{1}{2}(x + 4)^2 - 8$: Start with the graph of $y = x^2$, compress vertically by a factor of 2, shift 4 units to the left, and then shift 8 units downward.



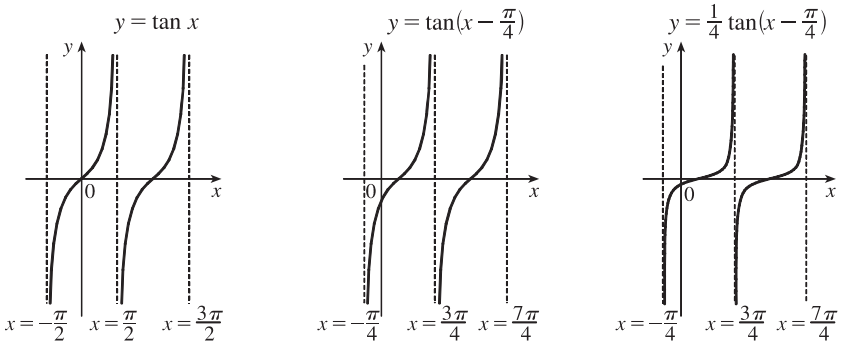
20. $y = 1 + \sqrt[3]{x - 1}$: Start with the graph of $y = \sqrt[3]{x}$, shift 1 unit to the right, and then shift 1 unit upward.



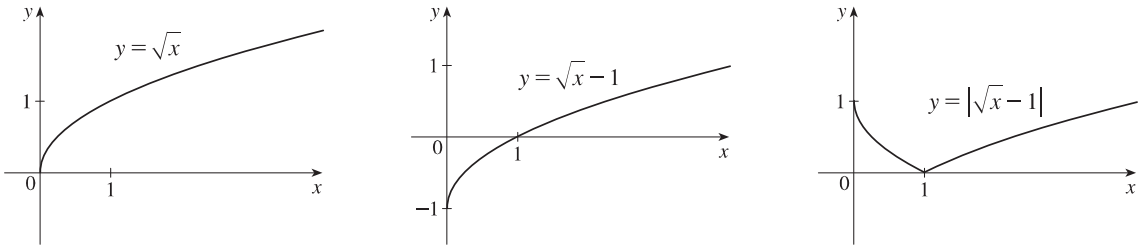
21. $y = |x - 2|$: Start with the graph of $y = |x|$ and shift 2 units to the right.



22. $y = \frac{1}{4}\tan(x - \frac{\pi}{4})$: Start with the graph of $y = \tan x$, shift $\frac{\pi}{4}$ units to the right, and then compress vertically by a factor of 4.



23. $y = |\sqrt{x} - 1|$: Start with the graph of $y = \sqrt{x}$, shift it 1 unit downward, and then reflect the portion of the graph below the x-axis about the x-axis.



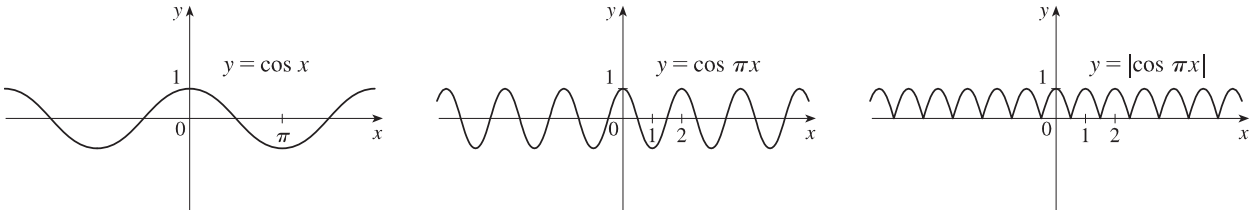
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24. $y = |\cos \pi x|$: Start with the graph of $y = \cos x$, shrink it horizontally by a factor of π, and reflect all the parts of the graph below the x-axis about the x-axis.

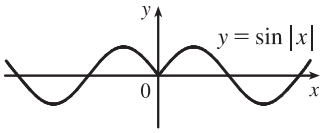


25. This is just like the solution to Example 4 except the amplitude of the curve (the 30°N curve in Figure 9 on June 21) is $14 - 12 = 2$. So the function is $L(t) = 12 + 2\sin\left[\frac{2\pi}{365}(t - 80)\right]$. March 31 is the 90th day of the year, so the model gives $L(90) \approx 12.34$ h. The daylight time (5:51 AM to 6:18 PM) is 12 hours and 27 minutes, or 12.45 h. The model value differs from the actual value by $\frac{12.45 - 12.34}{12.45} \approx 0.009$, less than 1%.

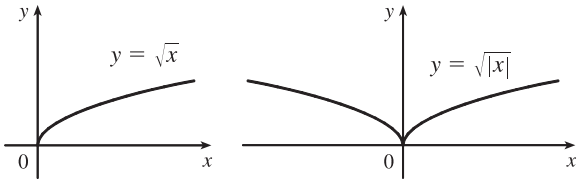
26. Using a sine function to model the brightness of Delta Cephei as a function of time, we take its period to be 5.4 days, its amplitude to be 0.35 (on the scale of magnitude), and its average magnitude to be 4.0. If we take $t = 0$ at a time of average brightness, then the magnitude (brightness) as a function of time t in days can be modeled by the formula $M(t) = 4.0 + 0.35\sin\left(\frac{2\pi}{5.4}t\right)$.

27. (a) To obtain $y = f(|x|)$, the portion of the graph of $y = f(x)$ to the right of the y-axis is reflected about the y-axis.

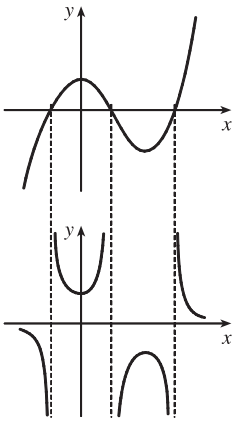
(b) $y = \sin|x|$



(c) $y = \sqrt{|x|}$



28. The most important features of the given graph are the x-intercepts and the maximum and minimum points. The graph of $y = 1/f(x)$ has vertical asymptotes at the x-values where there are x-intercepts on the graph of $y = f(x)$. The maximum of 1 on the graph of $y = f(x)$ corresponds to a minimum of $1/1 = 1$ on $y = 1/f(x)$. Similarly, the minimum on the graph of $y = f(x)$ corresponds to a maximum on the graph of $y = 1/f(x)$. As the values of y get large (positively or negatively) on the graph of $y = f(x)$, the values of y get close to zero on the graph of $y = 1/f(x)$.



29. $f(x) = x^3 + 2x^2$; $g(x) = 3x^2 - 1$. $D = \mathbb{R}$ for both f and g.

(a) $(f + g)(x) = (x^3 + 2x^2) + (3x^2 - 1) = x^3 + 5x^2 - 1$, $D = \mathbb{R}$.

(b) $(f - g)(x) = (x^3 + 2x^2) - (3x^2 - 1) = x^3 - x^2 + 1$, $D = \mathbb{R}$.

(c) $(fg)(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2$, $D = \mathbb{R}$.

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^3 + 2x^2}{3x^2 - 1}$, $D = \left\{x \mid x \neq \pm\frac{1}{\sqrt{3}}\right\}$ since $3x^2 - 1 \neq 0$.

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30. $f(x) = \sqrt{3-x},\ D = (-\infty, 3]; \quad g(x) = \sqrt{x^2-1},\ D = (-\infty,-1] \cup [1,\infty)$.

(a) $(f+g)(x) = \sqrt{3-x} + \sqrt{x^2-1},\ D = (-\infty,-1] \cup [1,3]$, which is the intersection of the domains of f and g.

(b) $(f-g)(x) = \sqrt{3-x} - \sqrt{x^2-1},\ D = (-\infty,-1] \cup [1,3]$.

(c) $(fg)(x) = \sqrt{3-x} \cdot \sqrt{x^2-1},\ D = (-\infty,-1] \cup [1,3]$.

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{3-x}}{\sqrt{x^2-1}},\ D = (-\infty,-1) \cup (1,3]$. We must exclude $x = \pm 1$ since these values would make $\dfrac{f}{g}$ undefined.

31. $f(x) = x^2 - 1,\ D = \mathbb{R}; \quad g(x) = 2x+1,\ D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x+1) = (2x+1)^2 - 1 = (4x^2+4x+1) - 1 = 4x^2 + 4x,\ D = \mathbb{R}$.

(b) $(g \circ f)(x) = g(f(x)) = g(x^2-1) = 2(x^2-1) + 1 = (2x^2-2) + 1 = 2x^2 - 1,\ D = \mathbb{R}$.

(c) $(f \circ f)(x) = f(f(x)) = f(x^2-1) = (x^2-1)^2 - 1 = (x^4 - 2x^2 + 1) - 1 = x^4 - 2x^2,\ D = \mathbb{R}$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x+1) = 2(2x+1) + 1 = (4x+2) + 1 = 4x + 3,\ D = \mathbb{R}$.

32. $f(x) = x - 2;\ g(x) = x^2 + 3x + 4$. $D = \mathbb{R}$ for both f and g, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(x^2+3x+4) = (x^2+3x+4) - 2 = x^2 + 3x + 2$.

(b) $(g \circ f)(x) = g(f(x)) = g(x-2) = (x-2)^2 + 3(x-2) + 4 = x^2 - 4x + 4 + 3x - 6 + 4 = x^2 - x + 2$.

(c) $(f \circ f)(x) = f(f(x)) = f(x-2) = (x-2) - 2 = x - 4$.

(d) $(g \circ g)(x) = g(g(x)) = g(x^2+3x+4) = (x^2+3x+4)^2 + 3(x^2+3x+4) + 4$

$$= (x^4 + 9x^2 + 16 + 6x^3 + 8x^2 + 24x) + 3x^2 + 9x + 12 + 4$$

$$= x^4 + 6x^3 + 20x^2 + 33x + 32$$

33. $f(x) = 1 - 3x;\ g(x) = \cos x$. $D = \mathbb{R}$ for both f and g, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(\cos x) = 1 - 3\cos x$.

(b) $(g \circ f)(x) = g(f(x)) = g(1-3x) = \cos(1-3x)$.

(c) $(f \circ f)(x) = f(f(x)) = f(1-3x) = 1 - 3(1-3x) = 1 - 3 + 9x = 9x - 2$.

(d) $(g \circ g)(x) = g(g(x)) = g(\cos x) = \cos(\cos x)$ [Note that this is *not* $\cos x \cdot \cos x$.]

34. $f(x) = \sqrt{x},\ D = [0,\infty); \quad g(x) = \sqrt[3]{1-x},\ D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{1-x}\right) = \sqrt{\sqrt[3]{1-x}} = \sqrt[6]{1-x}$.

The domain of $f \circ g$ is $\{x \mid \sqrt[3]{1-x} \geq 0\} = \{x \mid 1 - x \geq 0\} = \{x \mid x \leq 1\} = (-\infty, 1]$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \sqrt[3]{1-\sqrt{x}}$.

The domain of $g \circ f$ is $\{x \mid x$ is in the domain of f and $f(x)$ is in the domain of $g\}$. This is the domain of f, that is, $[0,\infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x}\right) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. The domain of $f \circ f$ is $\{x \mid x \geq 0$ and $\sqrt{x} \geq 0\} = [0,\infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g\left(\sqrt[3]{1-x}\right) = \sqrt[3]{1 - \sqrt[3]{1-x}}$, and the domain is $(-\infty, \infty)$.

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35. $f(x) = x + \dfrac{1}{x},\ D = \{x \mid x \neq 0\}; \quad g(x) = \dfrac{x+1}{x+2},\ D = \{x \mid x \neq -2\}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x+1}{x+2}\right) = \dfrac{x+1}{x+2} + \dfrac{1}{\dfrac{x+1}{x+2}} = \dfrac{x+1}{x+2} + \dfrac{x+2}{x+1}$

$$= \frac{(x+1)(x+1)+(x+2)(x+2)}{(x+2)(x+1)} = \frac{(x^2+2x+1)+(x^2+4x+4)}{(x+2)(x+1)} = \frac{2x^2+6x+5}{(x+2)(x+1)}$$

Since $g(x)$ is not defined for $x = -2$ and $f(g(x))$ is not defined for $x = -2$ and $x = -1$,

the domain of $(f \circ g)(x)$ is $D = \{x \mid x \neq -2, -1\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(x + \dfrac{1}{x}\right) = \dfrac{\left(x + \dfrac{1}{x}\right)+1}{\left(x + \dfrac{1}{x}\right)+2} = \dfrac{\dfrac{x^2+1+x}{x}}{\dfrac{x^2+1+2x}{x}} = \dfrac{x^2+x+1}{x^2+2x+1} = \dfrac{x^2+x+1}{(x+1)^2}$

Since $f(x)$ is not defined for $x = 0$ and $g(f(x))$ is not defined for $x = -1$,
the domain of $(g \circ f)(x)$ is $D = \{x \mid x \neq -1, 0\}$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(x + \dfrac{1}{x}\right) = \left(x + \dfrac{1}{x}\right) + \dfrac{1}{x + \frac{1}{x}} = x + \dfrac{1}{x} + \dfrac{1}{\frac{x^2+1}{x}} = x + \dfrac{1}{x} + \dfrac{x}{x^2+1}$

$$= \frac{x(x)(x^2+1)+1\,(x^2+1)+x(x)}{x(x^2+1)} = \frac{x^4+x^2+x^2+1+x^2}{x(x^2+1)}$$

$$= \frac{x^4+3x^2+1}{x(x^2+1)}, \quad D = \{x \mid x \neq 0\}$$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{x+1}{x+2}\right) = \dfrac{\dfrac{x+1}{x+2}+1}{\dfrac{x+1}{x+2}+2} = \dfrac{\dfrac{x+1+1(x+2)}{x+2}}{\dfrac{x+1+2(x+2)}{x+2}} = \dfrac{x+1+x+2}{x+1+2x+4} = \dfrac{2x+3}{3x+5}$

Since $g(x)$ is not defined for $x = -2$ and $g(g(x))$ is not defined for $x = -\frac{5}{3}$,

the domain of $(g \circ g)(x)$ is $D = \left\{x \mid x \neq -2, -\frac{5}{3}\right\}$.

36. $f(x) = \dfrac{x}{1+x},\ D = \{x \mid x \neq -1\}; \quad g(x) = \sin 2x,\ D = \mathbb{R}.$

(a) $(f \circ g)(x) = f(g(x)) = f(\sin 2x) = \dfrac{\sin 2x}{1 + \sin 2x}$

Domain: $1 + \sin 2x \neq 0 \quad \Rightarrow \quad \sin 2x \neq -1 \quad \Rightarrow \quad 2x \neq \dfrac{3\pi}{2} + 2\pi n \quad \Rightarrow \quad x \neq \dfrac{3\pi}{4} + \pi n$ [n an integer].

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x}{1+x}\right) = \sin\left(\dfrac{2x}{1+x}\right)$.

Domain: $\{x \mid x \neq -1\}$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{x}{1+x}\right) = \dfrac{\dfrac{x}{1+x}}{1 + \dfrac{x}{1+x}} = \dfrac{\left(\dfrac{x}{1+x}\right) \cdot (1+x)}{\left(1 + \dfrac{x}{1+x}\right) \cdot (1+x)} = \dfrac{x}{1+x+x} = \dfrac{x}{2x+1}$

Since $f(x)$ is not defined for $x = -1$, and $f(f(x))$ is not defined for $x = -\frac{1}{2}$,

the domain of $(f \circ f)(x)$ is $D = \left\{x \mid x \neq -1, -\frac{1}{2}\right\}$.

(d) $(g \circ g)(g) = g(g(x)) = g(\sin 2x) = \sin(2 \sin 2x)$.
Domain: $\mathbb{R}$

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37. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x-1)) = f(2(x-1)) = 2(x-1)+1 = 2x-1$

38. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(1-x)) = f((1-x)^2) = 2(1-x)^2 - 1 = 2x^2 - 4x + 1$

39. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^3+2)) = f[(x^3+2)^2]$
$= f(x^6+4x^3+4) = \sqrt{(x^6+4x^3+4)-3} = \sqrt{x^6+4x^3+1}$

40. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt[3]{x})) = f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) = \tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

41. Let $g(x) = 2x + x^2$ and $f(x) = x^4$. Then $(f \circ g)(x) = f(g(x)) = f(2x+x^2) = (2x+x^2)^4 = F(x)$.

42. Let $g(x) = \cos x$ and $f(x) = x^2$. Then $(f \circ g)(x) = f(g(x)) = f(\cos x) = (\cos x)^2 = \cos^2 x = F(x)$.

43. Let $g(x) = \sqrt[3]{x}$ and $f(x) = \dfrac{x}{1+x}$. Then $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x}) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}} = F(x)$.

44. Let $g(x) = \dfrac{x}{1+x}$ and $f(x) = \sqrt[3]{x}$. Then $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x}{1+x}\right) = \sqrt[3]{\dfrac{x}{1+x}} = G(x)$.

45. Let $g(t) = \cos t$ and $f(t) = \sqrt{t}$. Then $(f \circ g)(t) = f(g(t)) = f(\cos t) = \sqrt{\cos t} = u(t)$.

46. Let $g(t) = \tan t$ and $f(t) = \dfrac{t}{1+t}$. Then $(f \circ g)(t) = f(g(t)) = f(\tan t) = \dfrac{\tan t}{1+\tan t} = u(t)$.

47. Let $h(x) = x^2$, $g(x) = 3^x$, and $f(x) = 1 - x$. Then
$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f\left(3^{x^2}\right) = 1 - 3^{x^2} = H(x)$.

48. Let $h(x) = |x|$, $g(x) = 2 + x$, and $f(x) = \sqrt[8]{x}$. Then
$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(|x|)) = f(2+|x|) = \sqrt[8]{2+|x|} = H(x)$.

49. Let $h(x) = \sqrt{x}$, $g(x) = \sec x$, and $f(x) = x^4$. Then
$(f \circ g \circ h)(x) = f(g(h(x))) = f(g(\sqrt{x})) = f(\sec\sqrt{x}) = (\sec\sqrt{x})^4 = \sec^4(\sqrt{x}) = H(x)$.

50. (a) $f(g(1)) = f(6) = 5$

(b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$

(d) $g(g(1)) = g(6) = 3$

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

51. (a) $g(2) = 5$, because the point $(2, 5)$ is on the graph of g. Thus, $f(g(2)) = f(5) = 4$, because the point $(5, 4)$ is on the graph of f.

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$. This value is not defined, because there is no point on the graph of g that has x-coordinate 6.

(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

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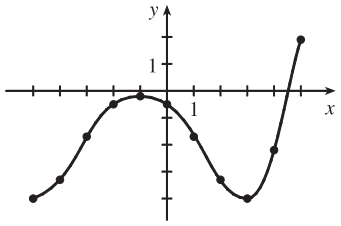
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52. To find a particular value of $f(g(x))$, say for $x = 0$, we note from the graph that $g(0) \approx 2.8$ and $f(2.8) \approx -0.5$. Thus, $f(g(0)) \approx f(2.8) \approx -0.5$. The other values listed in the table were obtained in a similar fashion.

x	$g(x)$	$f(g(x))$
-5	-0.2	-4
-4	1.2	-3.3
-3	2.2	-1.7
-2	2.8	-0.5
-1	3	-0.2

x	$g(x)$	$f(g(x))$
0	2.8	-0.5
1	2.2	-1.7
2	1.2	-3.3
3	-0.2	-4
4	-1.9	-2.2
5	-4.1	1.9



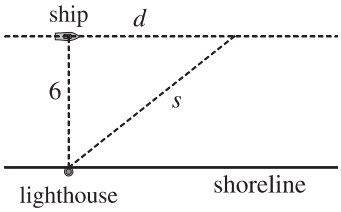
53. (a) Using the relationship *distance* = *rate* · *time* with the radius r as the distance, we have $r(t) = 60t$.

(b) $A = \pi r^2 \quad \Rightarrow \quad (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$. This formula gives us the extent of the rippled area (in cm^2) at any time t.

54. (a) The radius r of the balloon is increasing at a rate of 2 cm/s, so $r(t) = (2 \text{ cm/s})(t \text{ s}) = 2t$ (in cm).

(b) Using $V = \frac{4}{3}\pi r^3$, we get $(V \circ r)(t) = V(r(t)) = V(2t) = \frac{4}{3}\pi(2t)^3 = \frac{32}{3}\pi t^3$.
The result, $V = \frac{32}{3}\pi t^3$, gives the volume of the balloon (in cm^3) as a function of time (in s).

55. (a) From the figure, we have a right triangle with legs 6 and d, and hypotenuse s.
By the Pythagorean Theorem, $d^2 + 6^2 = s^2 \quad \Rightarrow \quad s = f(d) = \sqrt{d^2 + 36}$.



(b) Using $d = rt$, we get $d = (30 \text{ km/h})(t \text{ hours}) = 30t$ (in km). Thus, $d = g(t) = 30t$.

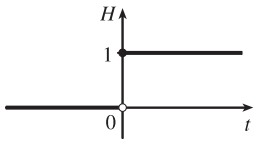
(c) $(f \circ g)(t) = f(g(t)) = f(30t) = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$. This function represents the distance between the lighthouse and the ship as a function of the time elapsed since noon.

56. (a) $d = rt \quad \Rightarrow \quad d(t) = 350t$

(b) There is a Pythagorean relationship involving the legs with lengths d and 1 and the hypotenuse with length s: $d^2 + 1^2 = s^2$. Thus, $s(d) = \sqrt{d^2 + 1}$.

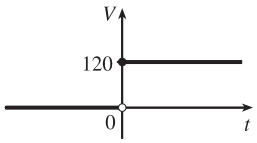
(c) $(s \circ d)(t) = s(d(t)) = s(350t) = \sqrt{(350t)^2 + 1}$

57. (a)



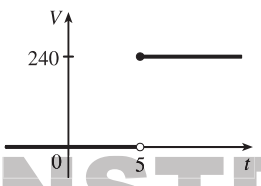
$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

(b)



$$V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 120 & \text{if } t \geq 0 \end{cases} \quad \text{so } V(t) = 120H(t).$$

(c)

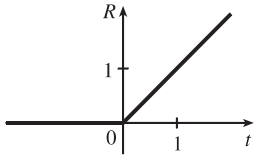


Starting with the formula in part (b), we replace 120 with 240 to reflect the different voltage. Also, because we are starting 5 units to the right of $t = 0$, we replace t with $t - 5$. Thus, the formula is $V(t) = 240H(t - 5)$.

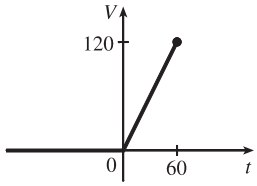
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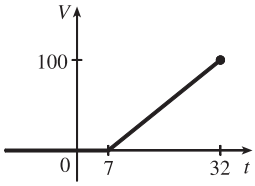
58. (a) $R(t) = tH(t)$
$= \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}$



(b) $V(t) = \begin{cases} 0 & \text{if } t < 0 \\ 2t & \text{if } 0 \leq t \leq 60 \end{cases}$
so $V(t) = 2tH(t)$, $t \leq 60$.



(c) $V(t) = \begin{cases} 0 & \text{if } t < 7 \\ 4\,(t-7) & \text{if } 7 \leq t \leq 32 \end{cases}$
so $V(t) = 4(t-7)H(t-7)$, $t \leq 32$.



59. If $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$, then

$(f \circ g)(x) = f(g(x)) = f(m_2x + b_2) = m_1(m_2x + b_2) + b_1 = m_1m_2x + m_1b_2 + b_1$.

So $f \circ g$ is a linear function with slope m_1m_2.

60. If $A(x) = 1.04x$, then

$(A \circ A)(x) = A(A(x)) = A(1.04x) = 1.04(1.04x) = (1.04)^2x$,

$(A \circ A \circ A)(x) = A((A \circ A)(x)) = A((1.04)^2x) = 1.04(1.04)^2x = (1.04)^3x$, and

$(A \circ A \circ A \circ A)(x) = A((A \circ A \circ A)(x)) = A((1.04)^3x) = 1.04(1.04)^3x, = (1.04)^4x$.

These compositions represent the amount of the investment after 2, 3, and 4 years.

Based on this pattern, when we compose n copies of A, we get the formula $\underbrace{(A \circ A \circ \cdots \circ A)}_{n\ A\text{'s}}(x) = (1.04)^n x$.

61. (a) By examining the variable terms in g and h, we deduce that we must square g to get the terms $4x^2$ and $4x$ in h. If we let $f(x) = x^2 + c$, then $(f \circ g)(x) = f(g(x)) = f(2x+1) = (2x+1)^2 + c = 4x^2 + 4x + (1+c)$. Since $h(x) = 4x^2 + 4x + 7$, we must have $1 + c = 7$. So $c = 6$ and $f(x) = x^2 + 6$.

(b) We need a function g so that $f(g(x)) = 3(g(x)) + 5 = h(x)$. But $h(x) = 3x^2 + 3x + 2 = 3(x^2 + x) + 2 = 3(x^2 + x - 1) + 5$, so we see that $g(x) = x^2 + x - 1$.

62. We need a function g so that $g(f(x)) = g(x+4) = h(x) = 4x - 1 = 4(x+4) - 17$. So we see that the function g must be $g(x) = 4x - 17$.

63. We need to examine $h(-x)$.

$$h(-x) = (f \circ g)(-x) = f(g(-x)) = f(g(x)) \quad \text{[because } g \text{ is even]} \quad = h(x)$$

Because $h(-x) = h(x)$, h is an even function.

64. $h(-x) = f(g(-x)) = f(-g(x))$. At this point, we can't simplify the expression, so we might try to find a counterexample to show that h is not an odd function. Let $g(x) = x$, an odd function, and $f(x) = x^2 + x$. Then $h(x) = x^2 + x$, which is neither even nor odd.

Now suppose f is an odd function. Then $f(-g(x)) = -f(g(x)) = -h(x)$. Hence, $h(-x) = -h(x)$, and so h is odd if both f and g are odd.

Now suppose f is an even function. Then $f(-g(x)) = f(g(x)) = h(x)$. Hence, $h(-x) = h(x)$, and so h is even if g is odd and f is even.

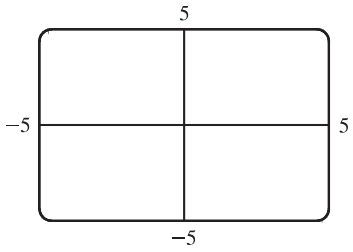
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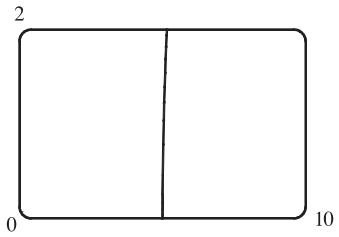
1.4 Graphing Calculators and Computers

1. $f(x) = \sqrt{x^3 - 5x^2}$

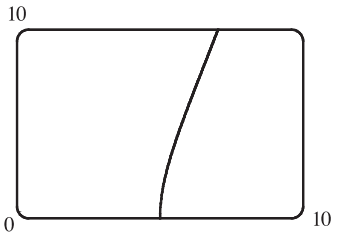
(a) $[-5, 5]$ by $[-5, 5]$ (There is no graph shown.)



(b) $[0, 10]$ by $[0, 2]$



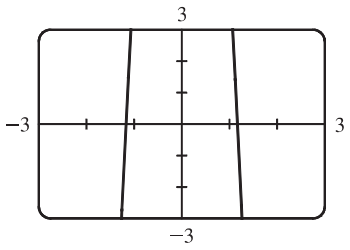
(c) $[0, 10]$ by $[0, 10]$



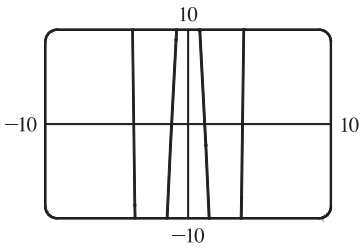
The most appropriate graph is produced in viewing rectangle (c).

2. $f(x) = x^4 - 16x^2 + 20$

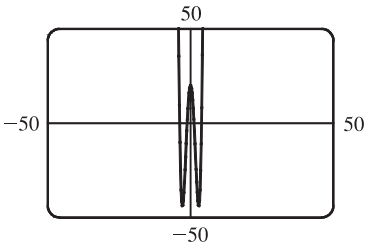
(a) $[-3, 3]$ by $[-3, 3]$



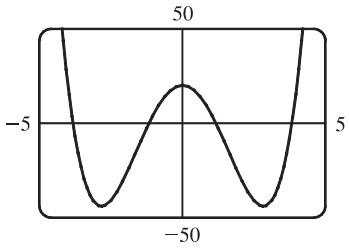
(b) $[-10, 10]$ by $[-10, 10]$



(c) $[-50, 50]$ by $[-50, 50]$

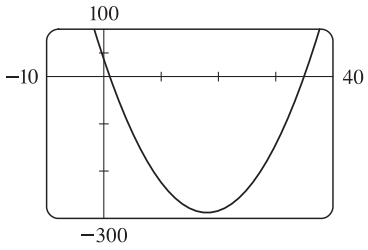


(d) $[-5, 5]$ by $[-50, 50]$

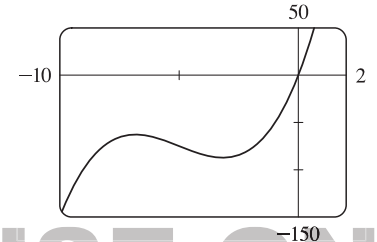


The most appropriate graph is produced in viewing rectangle (d).

3. Since the graph of $f(x) = x^2 - 36x + 32$ is a parabola opening upward, an appropriate viewing rectangle should include the minimum point. Completing the square, we get $f(x) = (x - 18)^2 - 292$, and so the minimum point is $(18, -292)$.



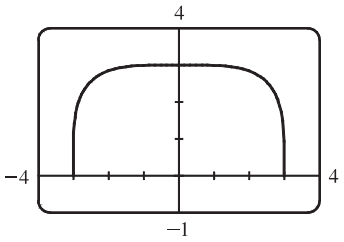
4. An appropriate viewing rectangle for $f(x) = x^3 + 15x^2 + 65x$ should include the high and low points.



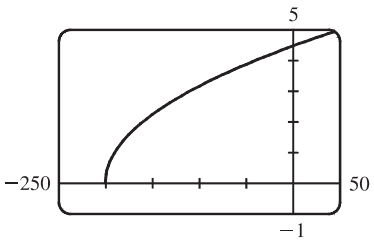
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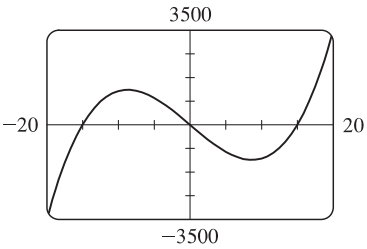
5. $f(x) = \sqrt[4]{81 - x^4}$ is defined when $81 - x^4 \geq 0 \quad \Leftrightarrow \quad x^4 \leq 81 \quad \Leftrightarrow$ $|x| \leq 3$, so the domain of f is $[-3, 3]$. Also $0 \leq \sqrt[4]{81 - x^4} \leq \sqrt[4]{81} = 3$, so the range is $[0, 3]$.



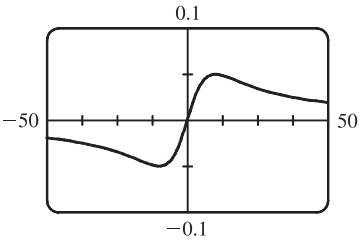
6. $f(x) = \sqrt{0.1x + 20}$ is defined when $0.1x + 20 \geq 0 \quad \Leftrightarrow \quad x \geq -200$, so the domain of f is $[-200, \infty)$.



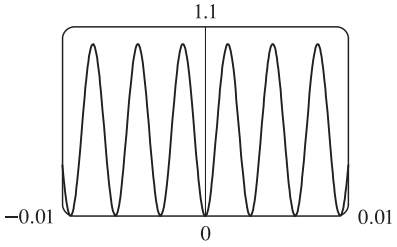
7. The graph of $f(x) = x^3 - 225x$ is symmetric with respect to the origin. Since $f(x) = x^3 - 225x = x(x^2 - 225) = x(x + 15)(x - 15)$, there are x-intercepts at 0, -15, and 15. $f(20) = 3500$.



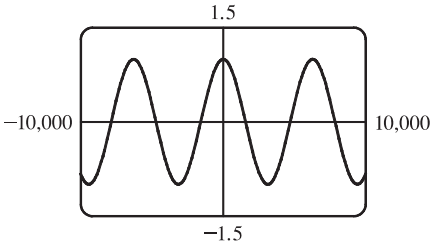
8. The graph of $f(x) = x/(x^2 + 100)$ is symmetric with respect to the origin.



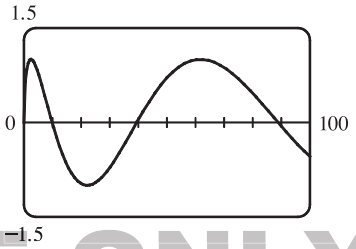
9. The period of $g(x) = \sin(1000x)$ is $\frac{2\pi}{1000} \approx 0.0063$ and its range is $[-1, 1]$. Since $f(x) = \sin^2(1000x)$ is the square of g, its range is $[0, 1]$ and a viewing rectangle of $[-0.01, 0.01]$ by $[0, 1.1]$ seems appropriate.



10. The period of $f(x) = \cos(0.001x)$ is $\frac{2\pi}{0.001} \approx 6300$ and its range is $[-1, 1]$, so a viewing rectangle of $[-10{,}000, 10{,}000]$ by $[-1.5, 1.5]$ seems appropriate.



11. The domain of $y = \sqrt{x}$ is $x \geq 0$, so the domain of $f(x) = \sin\sqrt{x}$ is $[0, \infty)$ and the range is $[-1, 1]$. With a little trial-and-error experimentation, we find that an Xmax of 100 illustrates the general shape of f, so an appropriate viewing rectangle is $[0, 100]$ by $[-1.5, 1.5]$.



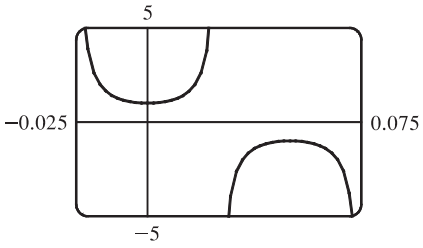
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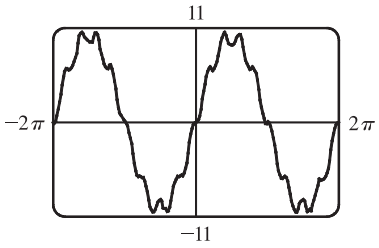
12. One period of $y = \sec x$ occurs on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

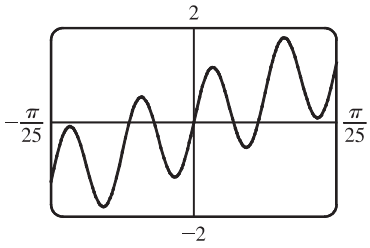
$-\frac{\pi}{2} < 20\pi x < \frac{3\pi}{2} \quad \Rightarrow \quad -\frac{1}{40} < x < \frac{3}{40}$, or equivalently,

$-0.025 < x < 0.075$.

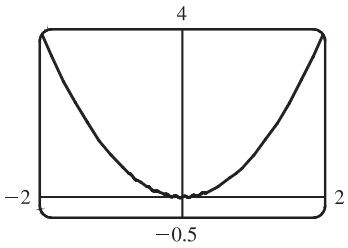


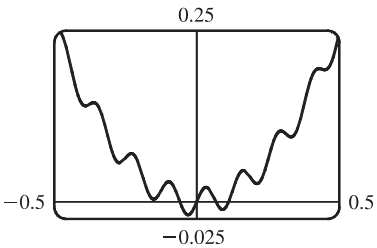
13. The first term, $10 \sin x$, has period 2π and range $[-10, 10]$. It will be the dominant term in any "large" graph of $y = 10 \sin x + \sin 100x$, as shown in the first figure. The second term, $\sin 100x$, has period $\frac{2\pi}{100} = \frac{\pi}{50}$ and range $[-1, 1]$. It causes the bumps in the first figure and will be the dominant term in any "small" graph, as shown in the view near the origin in the second figure.



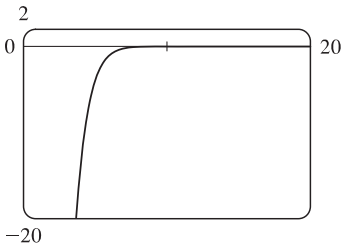


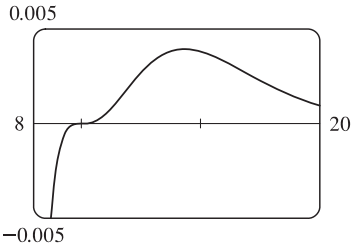
14. $y = x^2 + 0.02 \sin(50x)$





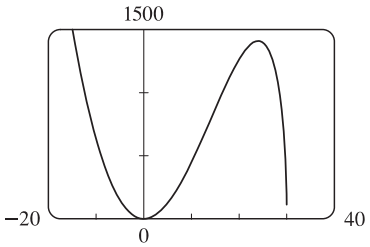
15. (a) The first figure shows the "big picture" for $f(x) = (x - 10)^3 2^{-x}$. The second figure shows a maximum near $x = 10$.





(b) You need more than one window because no single window can show what the function looks like globally and the detail of the function near $x = 10$.

16. The function $f(x) = x^2\sqrt{30 - x}$ has domain $(-\infty, 30]$. Its graph is very steep near $x = 30$, so part of the graph may appear to be missing.



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Calculus Concepts and Contexts Enhanced Edition 4th edition Stewart Solutions Manual

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COMPLETE SOLUTIONS MANUAL

for Stewart's

MULTIVARIABLE CALCULUS: CONCEPTS AND CONTEXTS

FOURTH EDITION

DAN CLEGG
Palomar College

BROOKS/COLE
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PREFACE

This *Complete Solutions Manual* contains detailed solutions to all exercises in the text *Multivariable Calculus: Concepts and Contexts,* Fourth Edition (Chapters 8–13 of *Calculus: Concepts and Contexts,* Fourth Edition) by James Stewart. A *Student Solutions Manual* is also available, which contains solutions to the odd-numbered exercises in each chapter section, review section, True-False Quiz, and Focus on Problem Solving section as well as all solutions to the Concept Check questions. (It does not, however, include solutions to any of the projects.)

While I have extended every effort to ensure the accuracy of the solutions presented, I would appreciate correspondence regarding any errors that may exist. Other suggestions or comments are also welcome, and can be sent to me at the email address or mailing address below.

I would like to thank James Stewart for entrusting me with the writing of this manual and offering suggestions, Kathi Townes, Stephanie Kuhns, and Rebekah Steele of TECH-arts for typesetting and producing this manual, and Brian Betsill of TECH-arts for creating the illustrations. Brian Karasek prepared solutions for comparison of accuracy and style in addition to proofreading manuscript; his assistance and suggestions were very helpful and much appreciated. Finally, I would like to thank Richard Stratton and Elizabeth Neustaetter of Brooks/Cole, Cengage Learning for their trust, assistance, and patience.

DAN CLEGG

dclegg@palomar.edu
Palomar College
Department of Mathematics
1140 West Mission Road
San Marcos, CA 92069

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8 □ INFINITE SEQUENCES AND SERIES

8.1 Sequences

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.

(b) The terms a_n approach 8 as n becomes large. In fact, we can make a_n as close to 8 as we like by taking n sufficiently large.

(c) The terms a_n become large as n becomes large. In fact, we can make a_n as large as we like by taking n sufficiently large.

2. (a) From Definition 1, a convergent sequence is a sequence for which $\lim_{n\to\infty} a_n$ exists. Examples: $\{1/n\}$, $\{1/2^n\}$

(b) A divergent sequence is a sequence for which $\lim_{n\to\infty} a_n$ *does not* exist. Examples: $\{n\}$, $\{\sin n\}$

3. The first six terms of $a_n = \dfrac{n}{2n+1}$ are $\dfrac{1}{3}, \dfrac{2}{5}, \dfrac{3}{7}, \dfrac{4}{9}, \dfrac{5}{11}, \dfrac{6}{13}$. It appears that the sequence is approaching $\dfrac{1}{2}$.

$$\lim_{n\to\infty} \frac{n}{2n+1} = \lim_{n\to\infty} \frac{1}{2+1/n} = \frac{1}{2}$$

4. $\{\cos(n\pi/3)\}_{n=1}^{9} = \left\{\frac{1}{2}, -\frac{1}{2}, -1, -\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, -\frac{1}{2}, -1\right\}$. The sequence does not appear to have a limit. The values will cycle through the first six numbers in the sequence—never approaching a particular number.

5. $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\right\}$. The denominator of the *n*th term is the *n*th positive odd integer, so $a_n = \dfrac{1}{2n-1}$.

6. $\left\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots\right\}$. The denominator of the *n*th term is the $(n-1)$st power of 3, so $a_n = \dfrac{1}{3^{n-1}}$.

7. $\{2, 7, 12, 17, \ldots\}$. Each term is larger than the preceding one by 5, so $a_n = a_1 + d(n-1) = 2 + 5(n-1) = 5n - 3$.

8. $\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \ldots\right\}$. The numerator of the nth term is n and its denominator is $(n+1)^2$. Including the alternating signs, we get $a_n = (-1)^n \dfrac{n}{(n+1)^2}$.

9. $\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots\right\}$. Each term is $-\frac{2}{3}$ times the preceding one, so $a_n = \left(-\frac{2}{3}\right)^{n-1}$.

10. $\{5, 1, 5, 1, 5, 1, \ldots\}$. The average of 5 and 1 is 3, so we can think of the sequence as alternately adding 2 and -2 to 3. Thus, $a_n = 3 + (-1)^{n+1} \cdot 2$.

11. $a_n = \dfrac{3+5n^2}{n+n^2} = \dfrac{(3+5n^2)/n^2}{(n+n^2)/n^2} = \dfrac{5+3/n^2}{1+1/n}$, so $a_n \to \dfrac{5+0}{1+0} = 5$ as $n \to \infty$. Converges

12. $a_n = \dfrac{n^3}{n^3+1} = \dfrac{n^3/n^3}{(n^3+1)/n^3} = \dfrac{1}{1+1/n^3}$, so $a_n \to \dfrac{1}{1+0} = 1$ as $n \to \infty$. Converges

13. $a_n = 1 - (0.2)^n$, so $\lim_{n\to\infty} a_n = 1 - 0 = 1$ by (7). Converges

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14. $a_n = \dfrac{n^3}{n+1} = \dfrac{n^3/n}{(n+1)/n} = \dfrac{n^2}{1+1/n^2}$, so $a_n \to \infty$ as $n \to \infty$ since $\lim\limits_{n\to\infty} n^2 = \infty$ and $\lim\limits_{n\to\infty} (1+1/n^2) = 1$. Diverges

15. Because the natural exponential function is continuous at 0, Theorem 5 enables us to write

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} e^{1/n} = e^{\lim_{n\to\infty}(1/n)} = e^0 = 1$. Converges

16. $a_n = \dfrac{3^{n+2}}{5^n} = \dfrac{3^2 3^n}{5^n} = 9\left(\frac{3}{5}\right)^n$, so $\lim\limits_{n\to\infty} a_n = 9 \lim\limits_{n\to\infty} \left(\frac{3}{5}\right)^n = 9 \cdot 0 = 0$ by (7) with $r = \frac{3}{5}$. Converges

17. If $b_n = \dfrac{2n\pi}{1+8n}$, then $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{(2n\pi)/n}{(1+8n)/n} = \lim\limits_{n\to\infty} \dfrac{2\pi}{1/n+8} = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$. Since tan is continuous at $\frac{\pi}{4}$, by Theorem 5, $\lim\limits_{n\to\infty} \tan\left(\dfrac{2n\pi}{1+8n}\right) = \tan\left(\lim\limits_{n\to\infty} \dfrac{2n\pi}{1+8n}\right) = \tan\dfrac{\pi}{4} = 1$. Converges

18. Using the last limit law for sequences and the continuity of the square root function,

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \sqrt{\dfrac{n+1}{9n+1}} = \sqrt{\lim\limits_{n\to\infty} \dfrac{n+1}{9n+1}} = \sqrt{\lim\limits_{n\to\infty} \dfrac{1+1/n}{9+1/n}} = \sqrt{\dfrac{1}{9}} = \dfrac{1}{3}$. Converges

19. $a_n = \dfrac{(-1)^{n-1}\, n}{n^2+1} = \dfrac{(-1)^{n-1}}{n+1/n}$, so $0 \le |a_n| = \dfrac{1}{n+1/n} \le \dfrac{1}{n} \to 0$ as $n \to \infty$, so $a_n \to 0$ by the Squeeze Theorem and Theorem 4. Converges

20. $a_n = \dfrac{(-1)^n n^3}{n^3+2n^2+1}$. Now $|a_n| = \dfrac{n^3}{n^3+2n^2+1} = \dfrac{1}{1+\frac{2}{n}+\frac{1}{n^3}} \to 1$ as $n \to \infty$, but the terms of the sequence $\{a_n\}$ alternate in sign, so the sequence $a_1, a_3, a_5, \ldots$ converges to -1 and the sequence $a_2, a_4, a_6, \ldots$ converges to $+1$. This shows that the given sequence diverges since its terms don't approach a single real number.

21. $a_n = \dfrac{e^n + e^{-n}}{e^{2n}-1} \cdot \dfrac{e^{-n}}{e^{-n}} = \dfrac{1+e^{-2n}}{e^n - e^{-n}} \to 0$ as $n \to \infty$ because $1 + e^{-2n} \to 1$ and $e^n - e^{-n} \to \infty$. Converges

22. $a_n = \cos(2/n)$. As $n \to \infty$, $2/n \to 0$, so $\cos(2/n) \to \cos 0 = 1$ because cos is continuous. Converges

23. $a_n = n^2 e^{-n} = \dfrac{n^2}{e^n}$. Since $\lim\limits_{x\to\infty} \dfrac{x^2}{e^x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{2x}{e^x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{2}{e^x} = 0$, it follows from Theorem 2 that $\lim\limits_{n\to\infty} a_n = 0$. Converges

24. $2n \to \infty$ as $n \to \infty$, so since $\lim\limits_{x\to\infty} \arctan x = \frac{\pi}{2}$, we have $\lim\limits_{n\to\infty} \arctan 2n = \frac{\pi}{2}$. Converges

25. $0 \le \dfrac{\cos^2 n}{2^n} \le \dfrac{1}{2^n}$ [since $0 \le \cos^2 n \le 1$], so since $\lim\limits_{n\to\infty} \dfrac{1}{2^n} = 0$, $\left\{\dfrac{\cos^2 n}{2^n}\right\}$ converges to 0 by the Squeeze Theorem.

26. $a_n = n \cos n\pi = n(-1)^n$. Since $|a_n| = n \to \infty$ as $n \to \infty$, the given sequence diverges.

27. $y = \left(1+\dfrac{2}{x}\right)^x \quad \Rightarrow \quad \ln y = x \ln\left(1+\dfrac{2}{x}\right)$, so

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(1+2/x)}{1/x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{\left(\frac{1}{1+2/x}\right)\left(-\frac{2}{x^2}\right)}{-1/x^2} = \lim_{x\to\infty} \frac{2}{1+2/x} = 2 \quad \Rightarrow$$

$\lim\limits_{x\to\infty} \left(1+\dfrac{2}{x}\right)^x = \lim\limits_{x\to\infty} e^{\ln y} = e^2$, so by Theorem 2, $\lim\limits_{n\to\infty} \left(1+\dfrac{2}{n}\right)^n = e^2$. Convergent

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28. $a_n = \sqrt[n]{2^{1+3n}} = (2^{1+3n})^{1/n} = (2^1 2^{3n})^{1/n} = 2^{1/n}2^3 = 8 \cdot 2^{1/n}$, so

$\lim_{n\to\infty} a_n = 8 \lim_{n\to\infty} 2^{1/n} = 8 \cdot 2^{\lim_{n\to\infty}(1/n)} = 8 \cdot 2^0 = 8$ by Theorem 5, since the function $f(x) = 2^x$ is continuous at 0.

Convergent

29. $a_n = \dfrac{(2n-1)!}{(2n+1)!} = \dfrac{(2n-1)!}{(2n+1)(2n)(2n-1)!} = \dfrac{1}{(2n+1)(2n)} \to 0$ as $n \to \infty$. Converges

30. $a_n = \dfrac{\sin 2n}{1+\sqrt{n}}$. $|a_n| \le \dfrac{1}{1+\sqrt{n}}$ and $\lim_{n\to\infty} \dfrac{1}{1+\sqrt{n}} = 0$, so $\dfrac{-1}{1+\sqrt{n}} \le a_n \le \dfrac{1}{1+\sqrt{n}} \quad \Rightarrow \quad \lim_{n\to\infty} a_n = 0$ by the Squeeze Theorem. Converges

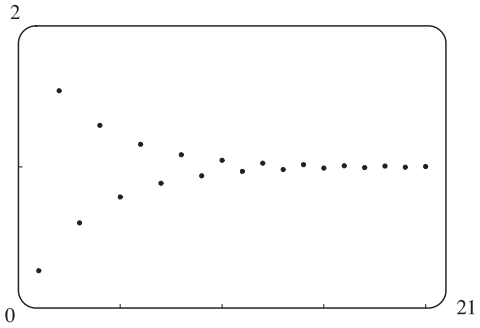
31. $\{0, 1, 0, 0, 1, 0, 0, 0, 1, \ldots\}$ diverges since the sequence takes on only two values, 0 and 1, and never stays arbitrarily close to either one (or any other value) for n sufficiently large.

32. $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x} \overset{\text{H}}{=} \lim_{x\to\infty} \dfrac{2(\ln x)(1/x)}{1} = 2 \lim_{x\to\infty} \dfrac{\ln x}{x} \overset{\text{H}}{=} 2 \lim_{x\to\infty} \dfrac{1/x}{1} = 0$, so by Theorem 3, $\lim_{n\to\infty} \dfrac{(\ln n)^2}{n} = 0$. Convergent

33. $a_n = \ln(2n^2+1) - \ln(n^2+1) = \ln\left(\dfrac{2n^2+1}{n^2+1}\right) = \ln\left(\dfrac{2+1/n^2}{1+1/n^2}\right) \to \ln 2$ as $n \to \infty$. Convergent

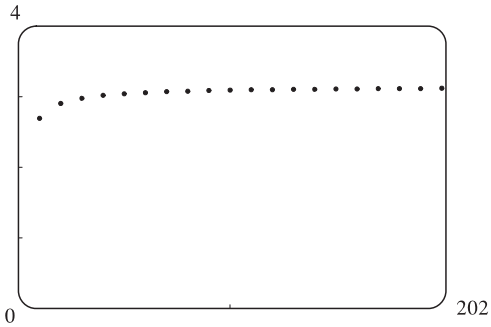
34. $0 < |a_n| = \dfrac{3^n}{n!} = \dfrac{3}{1} \cdot \dfrac{3}{2} \cdot \dfrac{3}{3} \cdot \cdots \cdot \dfrac{3}{(n-1)} \cdot \dfrac{3}{n} \le \dfrac{3}{1} \cdot \dfrac{3}{2} \cdot \dfrac{3}{n}$ [for $n > 2$] $= \dfrac{27}{2n} \to 0$ as $n \to \infty$, so by the Squeeze Theorem and Theorem 4, $\{(-3)^n/n!\}$ converges to 0.

35.



From the graph, it appears that the sequence converges to 1. $\{(-2/e)^n\}$ converges to 0 by (7), and hence $\{1 + (-2/e)^n\}$ converges to $1 + 0 = 1$.

36.



From the graph, it appears that the sequence converges to a number greater than 3.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{n}\sin\left(\frac{\pi}{\sqrt{n}}\right) = \lim_{n\to\infty} \frac{\sin\left(\pi/\sqrt{n}\right)}{\pi/\sqrt{n}} \cdot \pi$$

$$= \lim_{x\to 0^+} \frac{\sin x}{x} \cdot \pi \quad \left[x = \pi/\sqrt{n}\right] \quad = 1 \cdot \pi = \pi.$$

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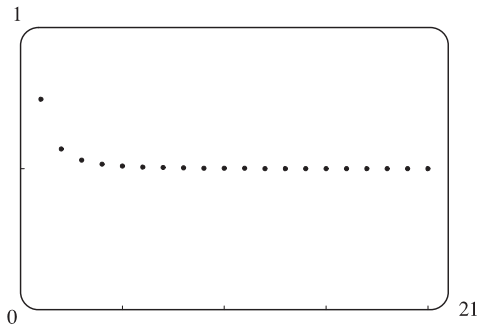
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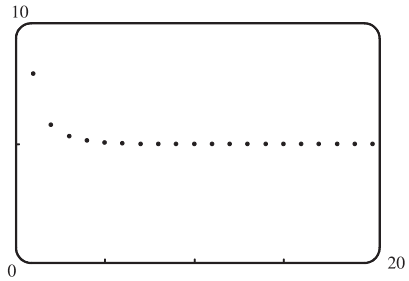
From the graph, it appears that the sequence converges to $\frac{1}{2}$.

As $n \to \infty$,

$$a_n = \sqrt{\frac{3+2n^2}{8n^2+n}} = \sqrt{\frac{3/n^2+2}{8+1/n}} \quad \Rightarrow \quad \sqrt{\frac{0+2}{8+0}} = \sqrt{\frac{1}{4}} = \frac{1}{2},$$

so $\lim_{n\to\infty} a_n = \frac{1}{2}$.

38.



From the graph, it appears that the sequence converges to 5.

$$5 = \sqrt[n]{5^n} \le \sqrt[n]{3^n+5^n} \le \sqrt[n]{5^n+5^n} = \sqrt[n]{2}\,\sqrt[n]{5^n}$$

$$= \sqrt[n]{2}\cdot 5 \to 5 \text{ as } n \to \infty \quad \left[\lim_{n\to\infty} 2^{1/n} = 2^0 = 1\right]$$

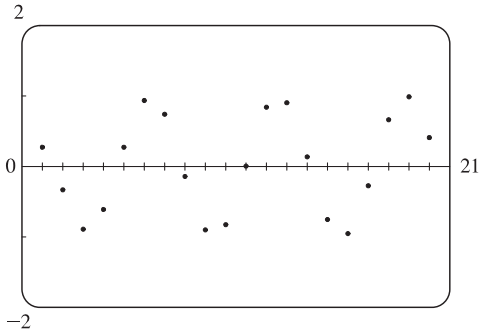
Hence, $a_n \to 5$ by the Squeeze Theorem.

Alternate solution: Let $y = (3^x+5^x)^{1/x}$. Then

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(3^x+5^x)}{x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{3^x\ln 3 + 5^x \ln 5}{3^x+5^x} = \lim_{x\to\infty} \frac{\left(\frac{3}{5}\right)^x \ln 3 + \ln 5}{\left(\frac{3}{5}\right)^x + 1} = \ln 5,$$

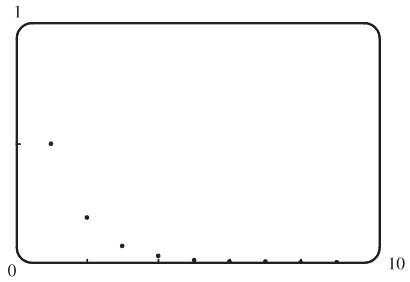
so $\lim_{x\to\infty} y = e^{\ln 5} = 5$, and so $\left\{\sqrt[n]{3^n+5^n}\right\}$ converges to 5.

39.



From the graph, it appears that the sequence $\{a_n\} = \left\{\dfrac{n^2\cos n}{1+n^2}\right\}$ is divergent, since it oscillates between 1 and -1 (approximately). To prove this, suppose that $\{a_n\}$ converges to L. If $b_n = \dfrac{n^2}{1+n^2}$, then $\{b_n\}$ converges to 1, and $\lim_{n\to\infty} \dfrac{a_n}{b_n} = \dfrac{L}{1} = L$. But $\dfrac{a_n}{b_n} = \cos n$, so $\lim_{n\to\infty} \dfrac{a_n}{b_n}$ does not exist. This contradiction shows that $\{a_n\}$ diverges.

40.



From the graph, it appears that the sequence approaches 0.

$$0 < a_n = \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{(2n)^n} = \frac{1}{2n}\cdot\frac{3}{2n}\cdot\frac{5}{2n}\cdot\cdots\cdot\frac{2n-1}{2n}$$

$$\le \frac{1}{2n}\cdot(1)\cdot(1)\cdot\cdots\cdot(1) = \frac{1}{2n} \to 0 \text{ as } n \to \infty$$

So by the Squeeze Theorem, $\left\{\dfrac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{(2n)^n}\right\}$ converges to 0.

41. (a) $a_n = 1000(1.06)^n \quad \Rightarrow \quad a_1 = 1060$, $a_2 = 1123.60$, $a_3 = 1191.02$, $a_4 = 1262.48$, and $a_5 = 1338.23$.

(b) $\lim_{n\to\infty} a_n = 1000 \lim_{n\to\infty} (1.06)^n$, so the sequence diverges by (7) with $r = 1.06 > 1$.

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42. (a) Substitute 1 to 6 for n in $I_n = 100\left(\dfrac{1.0025^n - 1}{0.0025} - n\right)$ to get $I_1 = \$0$, $I_2 = \$0.25$, $I_3 = \$0.75$, $I_4 = \$1.50$, $I_5 = \$2.51$, and $I_6 = \$3.76$.

(b) For two years, use $2 \cdot 12 = 24$ for n to get $\$70.28$.

43. (a) We are given that the initial population is 5000, so $P_0 = 5000$. The number of catfish increases by 8% per month and is decreased by 300 per month, so $P_1 = P_0 + 8\%P_0 - 300 = 1.08P_0 - 300$, $P_2 = 1.08P_1 - 300$, and so on. Thus, $P_n = 1.08P_{n-1} - 300$.

(b) Using the recursive formula with $P_0 = 5000$, we get $P_1 = 5100$, $P_2 = 5208$, $P_3 = 5325$ (rounding any portion of a catfish), $P_4 = 5451$, $P_5 = 5587$, and $P_6 = 5734$, which is the number of catfish in the pond after six months.

44. $a_{n+1} = \begin{cases} \frac{1}{2}a_n & \text{if } a_n \text{ is an even number} \\ 3a_n + 1 & \text{if } a_n \text{ is an odd number} \end{cases}$ When $a_1 = 11$, the first 40 terms are 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4. When $a_1 = 25$, the first 40 terms are 25, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4.
The famous Collatz conjecture is that this sequence always reaches 1, regardless of the starting point a_1.

45. (a) $a_1 = 1$, $a_{n+1} = 4 - a_n$ for $n \geq 1$. $a_1 = 1$, $a_2 = 4 - a_1 = 4 - 1 = 3$, $a_3 = 4 - a_2 = 4 - 3 = 1$, $a_4 = 4 - a_3 = 4 - 1 = 3$, $a_5 = 4 - a_4 = 4 - 3 = 1$. Since the terms of the sequence alternate between 1 and 3, the sequence is divergent.

(b) $a_1 = 2$, $a_2 = 4 - a_1 = 4 - 2 = 2$, $a_3 = 4 - a_2 = 4 - 2 = 2$. Since all of the terms are 2, $\lim\limits_{n\to\infty} a_n = 2$ and hence, the sequence is convergent.

46. (a) Since $\lim\limits_{n\to\infty} a_n = L$, the terms a_n approach L as n becomes large. Because we can make a_n as close to L as we wish, a_{n+1} will also be close, and so $\lim\limits_{n\to\infty} a_{n+1} = L$.

(b) $a_1 = 1$, $a_2 = \dfrac{1}{1+a_1} = \dfrac{1}{1+1} = \dfrac{1}{2} = 0.5$, $a_3 = \dfrac{1}{1+a_2} = \dfrac{1}{1+\frac{1}{2}} = \dfrac{2}{3} \approx 0.66667$,

$a_4 = \dfrac{1}{1+a_3} = \dfrac{1}{1+\frac{2}{3}} = \dfrac{3}{5} = 0.6$, $a_5 = \dfrac{1}{1+a_4} = \dfrac{1}{1+\frac{3}{5}} = \dfrac{5}{8} = 0.625$,

$a_6 = \dfrac{1}{1+a_5} = \dfrac{1}{1+\frac{5}{8}} = \dfrac{8}{13} \approx 0.61538$, $a_7 = \dfrac{1}{1+a_6} = \dfrac{1}{1+\frac{8}{13}} = \dfrac{13}{21} \approx 0.61905$,

$a_8 = \dfrac{1}{1+a_7} = \dfrac{1}{1+\frac{13}{21}} = \dfrac{21}{34} \approx 0.61765$, $a_9 = \dfrac{1}{1+a_8} = \dfrac{1}{1+\frac{21}{34}} = \dfrac{34}{55} \approx 0.61818$,

$a_{10} = \dfrac{1}{1+a_9} = \dfrac{1}{1+\frac{34}{55}} = \dfrac{55}{89} \approx 0.61800$. It appears that $\lim\limits_{n\to\infty} a_n \approx 0.618$; hence, the sequence is convergent.

(c) If $L = \lim\limits_{n\to\infty} a_n$ then $\lim\limits_{n\to\infty} a_{n+1} = L$ also, so L must satisfy

$L = 1/(1+L) \quad \Rightarrow \quad L^2 + L - 1 = 0 \quad \Rightarrow \quad L = \frac{-1+\sqrt{5}}{2} \approx 0.618$ (since L has to be non-negative if it exists).

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47. (a) Let a_n be the number of rabbit pairs in the nth month. Clearly $a_1 = 1 = a_2$. In the nth month, each pair that is 2 or more months old (that is, a_{n-2} pairs) will produce a new pair to add to the a_{n-1} pairs already present. Thus, $a_n = a_{n-1} + a_{n-2}$, so that $\{a_n\} = \{f_n\}$, the Fibonacci sequence.

(b) $a_n = \dfrac{f_{n+1}}{f_n} \;\Rightarrow\; a_{n-1} = \dfrac{f_n}{f_{n-1}} = \dfrac{f_{n-1}+f_{n-2}}{f_{n-1}} = 1 + \dfrac{f_{n-2}}{f_{n-1}} = 1 + \dfrac{1}{f_{n-1}/f_{n-2}} = 1 + \dfrac{1}{a_{n-2}}$. If $L = \lim\limits_{n\to\infty} a_n$, then $L = \lim\limits_{n\to\infty} a_{n-1}$ and $L = \lim\limits_{n\to\infty} a_{n-2}$, so L must satisfy $L = 1 + \dfrac{1}{L} \;\Rightarrow\; L^2 - L - 1 = 0 \;\Rightarrow\; L = \frac{1+\sqrt{5}}{2}$ [since L must be positive].

48. For $\left\{\sqrt{2},\ \sqrt{2\sqrt{2}},\ \sqrt{2\sqrt{2\sqrt{2}}},\ \ldots\right\}$, $a_1 = 2^{1/2}$, $a_2 = 2^{3/4}$, $a_3 = 2^{7/8}$, …, so $a_n = 2^{(2^n-1)/2^n} = 2^{1-(1/2^n)}$.

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} 2^{1-(1/2^n)} = 2^1 = 2$.

Alternate solution: Let $L = \lim\limits_{n\to\infty} a_n$. (We could show the limit exists by showing that $\{a_n\}$ is bounded and increasing.)

Then L must satisfy $L = \sqrt{2 \cdot L} \;\Rightarrow\; L^2 = 2L \;\Rightarrow\; L(L-2) = 0$. $L \neq 0$ since the sequence increases, so $L = 2$.

49. $a_n = \dfrac{1}{2n+3}$ is decreasing since $a_{n+1} = \dfrac{1}{2(n+1)+3} = \dfrac{1}{2n+5} < \dfrac{1}{2n+3} = a_n$ for each $n \geq 1$. The sequence is bounded since $0 < a_n \leq \frac{1}{5}$ for all $n \geq 1$. Note that $a_1 = \frac{1}{5}$.

50. $a_n = \dfrac{2n-3}{3n+4}$ defines an increasing sequence since for $f(x) = \dfrac{2x-3}{3x+4}$, $f'(x) = \dfrac{(3x+4)(2) - (2x-3)(3)}{(3x+4)^2} = \dfrac{17}{(3x+4)^2} > 0$. The sequence is bounded since $a_n \geq a_1 = -\frac{1}{7}$ for $n \geq 1$, and $a_n < \dfrac{2n-3}{3n} < \dfrac{2n}{3n} = \dfrac{2}{3}$ for $n \geq 1$.

51. The terms of $a_n = n(-1)^n$ alternate in sign, so the sequence is not monotonic. The first five terms are -1, 2, -3, 4, and -5. Since $\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} n = \infty$, the sequence is not bounded.

52. $a_n = n + \dfrac{1}{n}$ defines an increasing sequence since the function $g(x) = x + \dfrac{1}{x}$ is increasing for $x > 1$. [$g'(x) = 1 - 1/x^2 > 0$ for $x > 1$.] The sequence is unbounded since $a_n \to \infty$ as $n \to \infty$. (It is, however, bounded below by $a_1 = 2$.)

53. Since $\{a_n\}$ is a decreasing sequence, $a_n > a_{n+1}$ for all $n \geq 1$. Because all of its terms lie between 5 and 8, $\{a_n\}$ is a bounded sequence. By the Monotonic Sequence Theorem, $\{a_n\}$ is convergent; that is, $\{a_n\}$ has a limit L. L must be less than 8 since $\{a_n\}$ is decreasing, so $5 \leq L < 8$.

54. (a) Let P_n be the statement that $a_{n+1} \geq a_n$ and $a_n \leq 3$. P_1 is obviously true. We will assume that P_n is true and then show that as a consequence P_{n+1} must also be true. $a_{n+2} \geq a_{n+1} \;\Leftrightarrow\; \sqrt{2+a_{n+1}} \geq \sqrt{2+a_n} \;\Leftrightarrow$

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$2 + a_{n+1} \geq 2 + a_n \quad \Leftrightarrow \quad a_{n+1} \geq a_n$, which is the induction hypothesis. $a_{n+1} \leq 3 \quad \Leftrightarrow \quad \sqrt{2 + a_n} \leq 3 \quad \Leftrightarrow$ $2 + a_n \leq 9 \quad \Leftrightarrow \quad a_n \leq 7$, which is certainly true because we are assuming that $a_n \leq 3$. So P_n is true for all n, and so $a_1 \leq a_n \leq 3$ (showing that the sequence is bounded), and hence by the Monotonic Sequence Theorem, $\lim_{n\to\infty} a_n$ exists.

(b) If $L = \lim_{n\to\infty} a_n$, then $\lim_{n\to\infty} a_{n+1} = L$ also, so $L = \sqrt{2 + L} \quad \Rightarrow \quad L^2 = 2 + L \quad \Leftrightarrow \quad L^2 - L - 2 = 0 \quad \Leftrightarrow$ $(L + 1)(L - 2) = 0 \quad \Leftrightarrow \quad L = 2$ [since L can't be negative].

55. $a_1 = 1, a_{n+1} = 3 - \dfrac{1}{a_n}$. We show by induction that $\{a_n\}$ is increasing and bounded above by 3. Let P_n be the proposition that $a_{n+1} > a_n$ and $0 < a_n < 3$. Clearly P_1 is true. Assume that P_n is true. Then $a_{n+1} > a_n \quad \Rightarrow \quad \dfrac{1}{a_{n+1}} < \dfrac{1}{a_n} \quad \Rightarrow$ $-\dfrac{1}{a_{n+1}} > -\dfrac{1}{a_n}$. Now $a_{n+2} = 3 - \dfrac{1}{a_{n+1}} > 3 - \dfrac{1}{a_n} = a_{n+1} \quad \Leftrightarrow \quad P_{n+1}$. This proves that $\{a_n\}$ is increasing and bounded above by 3, so $1 = a_1 < a_n < 3$, that is, $\{a_n\}$ is bounded, and hence convergent by the Monotonic Sequence Theorem.

If $L = \lim_{n\to\infty} a_n$, then $\lim_{n\to\infty} a_{n+1} = L$ also, so L must satisfy $L = 3 - 1/L \quad \Rightarrow \quad L^2 - 3L + 1 = 0 \quad \Rightarrow \quad L = \frac{3 \pm \sqrt{5}}{2}$. But $L > 1$, so $L = \frac{3 + \sqrt{5}}{2}$.

56. $a_1 = 2, a_{n+1} = \dfrac{1}{3 - a_n}$. We use induction. Let P_n be the statement that $0 < a_{n+1} \leq a_n \leq 2$. Clearly P_1 is true, since $a_2 = 1/(3 - 2) = 1$. Now assume that P_n is true. Then $a_{n+1} \leq a_n \quad \Rightarrow \quad -a_{n+1} \geq -a_n \quad \Rightarrow \quad 3 - a_{n+1} \geq 3 - a_n \quad \Rightarrow$ $a_{n+2} = \dfrac{1}{3 - a_{n+1}} \leq \dfrac{1}{3 - a_n} = a_{n+1}$. Also $a_{n+2} > 0$ [since $3 - a_{n+1}$ is positive] and $a_{n+1} \leq 2$ by the induction hypothesis, so P_{n+1} is true. To find the limit, we use the fact that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} \quad \Rightarrow \quad L = \frac{1}{3 - L} \quad \Rightarrow$ $L^2 - 3L + 1 = 0 \quad \Rightarrow \quad L = \frac{3 \pm \sqrt{5}}{2}$. But $L \leq 2$, so we must have $L = \frac{3 - \sqrt{5}}{2}$.

57. $(0.8)^n < 0.000001 \quad \Rightarrow \quad \ln(0.8)^n < \ln(0.000001) \quad \Rightarrow \quad n \ln(0.8) < \ln(0.000001) \quad \Rightarrow \quad n > \dfrac{\ln(0.000001)}{\ln(0.8)} \quad \Rightarrow$ $n > 61.9$, so n must be at least 62 to satisfy the given inequality.

58. (a) If f is continuous, then $f(L) = f\left(\lim_{n\to\infty} a_n\right) = \lim_{n\to\infty} f(a_n) = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n = L$ by Exercise 46(a).

(b) By repeatedly pressing the cosine key on the calculator (that is, taking cosine of the previous answer) until the displayed value stabilizes, we see that $L \approx 0.73909$.

59. (a) Suppose $\{p_n\}$ converges to p. Then $p_{n+1} = \dfrac{bp_n}{a + p_n} \quad \Rightarrow \quad \lim_{n\to\infty} p_{n+1} = \dfrac{b \lim_{n\to\infty} p_n}{a + \lim_{n\to\infty} p_n} \quad \Rightarrow \quad p = \dfrac{bp}{a + p} \quad \Rightarrow$ $p^2 + ap = bp \quad \Rightarrow \quad p(p + a - b) = 0 \quad \Rightarrow \quad p = 0$ or $p = b - a$.

(b) $p_{n+1} = \dfrac{bp_n}{a + p_n} = \dfrac{\left(\frac{b}{a}\right)p_n}{1 + \frac{p_n}{a}} < \left(\dfrac{b}{a}\right)p_n$ since $1 + \dfrac{p_n}{a} > 1$.

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(c) By part (b), $p_1 < \left(\frac{b}{a}\right)p_0$, $p_2 < \left(\frac{b}{a}\right)p_1 < \left(\frac{b}{a}\right)^2 p_0$, $p_3 < \left(\frac{b}{a}\right)p_2 < \left(\frac{b}{a}\right)^3 p_0$, etc. In general, $p_n < \left(\frac{b}{a}\right)^n p_0$, so $\lim_{n\to\infty} p_n \le \lim_{n\to\infty} \left(\frac{b}{a}\right)^n \cdot p_0 = 0$ since $b < a$. $\left[\text{By (7), } \lim_{n\to\infty} r^n = 0 \text{ if } -1 < r < 1. \text{ Here } r = \frac{b}{a} \in (0,1).\right]$

(d) Let $a < b$. We first show, by induction, that if $p_0 < b - a$, then $p_n < b - a$ and $p_{n+1} > p_n$.

For $n = 0$, we have $p_1 - p_0 = \frac{bp_0}{a + p_0} - p_0 = \frac{p_0(b - a - p_0)}{a + p_0} > 0$ since $p_0 < b - a$. So $p_1 > p_0$.

Now we suppose the assertion is true for $n = k$, that is, $p_k < b - a$ and $p_{k+1} > p_k$. Then

$b - a - p_{k+1} = b - a - \frac{bp_k}{a + p_k} = \frac{a(b-a) + bp_k - ap_k - bp_k}{a + p_k} = \frac{a(b - a - p_k)}{a + p_k} > 0$ because $p_k < b - a$. So

$p_{k+1} < b - a$. And $p_{k+2} - p_{k+1} = \frac{bp_{k+1}}{a + p_{k+1}} - p_{k+1} = \frac{p_{k+1}(b - a - p_{k+1})}{a + p_{k+1}} > 0$ since $p_{k+1} < b - a$. Therefore,

$p_{k+2} > p_{k+1}$. Thus, the assertion is true for $n = k + 1$. It is therefore true for all n by mathematical induction.

A similar proof by induction shows that if $p_0 > b - a$, then $p_n > b - a$ and $\{p_n\}$ is decreasing.

In either case the sequence $\{p_n\}$ is bounded and monotonic, so it is convergent by the Monotonic Sequence Theorem. It then follows from part (a) that $\lim_{n\to\infty} p_n = b - a$.

60. $a_1 = 1$, $a_2 = 1 + \frac{1}{1+1} = \frac{3}{2} = 1.5$, $a_3 = 1 + \frac{1}{5/2} = \frac{7}{5} = 1.4$, $a_4 = 1 + \frac{1}{12/5} = \frac{17}{12} = 1.41\overline{6}$,

$a_5 = 1 + \frac{1}{29/12} = \frac{41}{29} \approx 1.413793$, $a_6 = 1 + \frac{1}{70/29} = \frac{99}{70} \approx 1.414286$, $a_7 = 1 + \frac{1}{169/70} = \frac{239}{169} \approx 1.414201$,

$a_8 = 1 + \frac{1}{408/169} = \frac{577}{408} \approx 1.414216$. Notice that $a_1 < a_3 < a_5 < a_7$ and $a_2 > a_4 > a_6 > a_8$. It appears that the odd terms are increasing and the even terms are decreasing. Let's prove that $a_{2n-2} > a_{2n}$ and $a_{2n-1} < a_{2n+1}$ by mathematical induction. Suppose that $a_{2k-2} > a_{2k}$. Then $1 + a_{2k-2} > 1 + a_{2k} \quad \Rightarrow$

$$\frac{1}{1 + a_{2k-2}} < \frac{1}{1 + a_{2k}} \quad \Rightarrow \quad 1 + \frac{1}{1 + a_{2k-2}} < 1 + \frac{1}{1 + a_{2k}} \quad \Rightarrow \quad a_{2k-1} < a_{2k+1} \quad \Rightarrow$$

$$1 + a_{2k-1} < 1 + a_{2k+1} \quad \Rightarrow \quad \frac{1}{1 + a_{2k-1}} > \frac{1}{1 + a_{2k+1}} \quad \Rightarrow \quad 1 + \frac{1}{1 + a_{2k-1}} > 1 + \frac{1}{1 + a_{2k+1}} \quad \Rightarrow \quad a_{2k} > a_{2k+2}.$$

We have thus shown, by induction, that the odd terms are increasing and the even terms are decreasing. Also all terms lie between 1 and 2, so both $\{a_n\}$ and $\{b_n\}$ are bounded monotonic sequences and therefore convergent by the Monotonic Sequence Theorem. Let $\lim_{n\to\infty} a_{2n} = L$. Then $\lim_{n\to\infty} a_{2n+2} = L$ also. We have

$a_{n+2} = 1 + \frac{1}{1 + 1 + 1/(1 + a_n)} = 1 + \frac{1}{(3 + 2a_n)/(1 + a_n)} = \frac{4 + 3a_n}{3 + 2a_n}$, so $a_{2n+2} = \frac{4 + 3a_{2n}}{3 + 2a_{2n}}$. Taking limits of both sides, we get $L = \frac{4 + 3L}{3 + 2L} \quad \Rightarrow \quad 3L + 2L^2 = 4 + 3L \quad \Rightarrow \quad L^2 = 2 \quad \Rightarrow \quad L = \sqrt{2}$ [since $L > 0$]. Thus, $\lim_{n\to\infty} a_{2n} = \sqrt{2}$.

Similarly, we find that $\lim_{n\to\infty} a_{2n+1} = \sqrt{2}$. Since the even terms approach $\sqrt{2}$ and the odd terms also approach $\sqrt{2}$, it follows that the sequence as a whole approaches $\sqrt{2}$, that is, $\lim_{n\to\infty} a_n = \sqrt{2}$.

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LABORATORY PROJECT Logistic Sequences

1. To write such a program in Maple it is best to calculate all the points first and then graph them. One possible sequence of commands [taking $p_0 = \frac{1}{2}$ and $k = 1.5$ for the difference equation] is

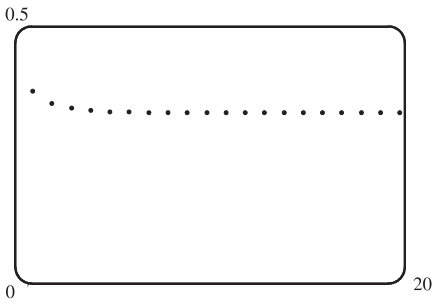
```
t:='t';p(0):=1/2;k:=1.5;
for j from 1 to 20  do p(j):=k*p(j-1)*(1-p(j-1)) od;
plot([seq([t,p(t)] t=0..20)],t=0..20,p=0..0.5,style=point);
```

In Mathematica, we can use the following program:

```
p[0]=1/2
k=1.5
p[j_]:=k*p[j-1]*(1-p[j-1])
P=Table[p[t],{t,20}]
ListPlot[P]
```

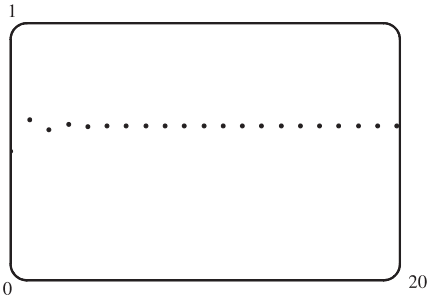
With $p_0 = \frac{1}{2}$ and $k = 1.5$:

n	p_n	n	p_n	n	p_n
0	0.5	7	0.3338465076	14	0.3333373303
1	0.375	8	0.3335895255	15	0.3333353318
2	0.3515625	9	0.3334613309	16	0.3333343326
3	0.3419494629	10	0.3333973076	17	0.3333338329
4	0.3375300416	11	0.3333653143	18	0.3333335831
5	0.3354052689	12	0.3333493223	19	0.3333334582
6	0.3343628617	13	0.3333413274	20	0.3333333958



With $p_0 = \frac{1}{2}$ and $k = 2.5$:

n	p_n	n	p_n	n	p_n
0	0.5	7	0.6004164790	14	0.5999967417
1	0.625	8	0.5997913269	15	0.6000016291
2	0.5859375	9	0.6001042277	16	0.5999991854
3	0.6065368651	10	0.5999478590	17	0.6000004073
4	0.5966247409	11	0.6000260637	18	0.5999997964
5	0.6016591486	12	0.5999869664	19	0.6000001018
6	0.5991635437	13	0.6000065164	20	0.5999999491



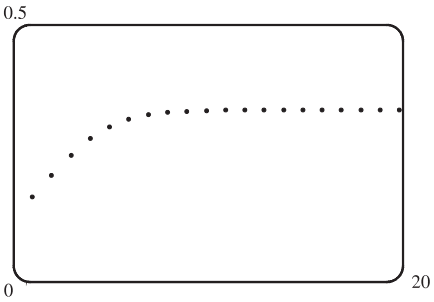
Both of these sequences seem to converge (the first to about $\frac{1}{3}$, the second to about 0.60).

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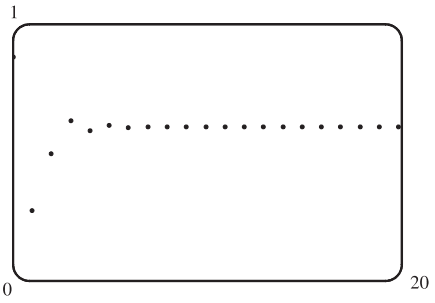
With $p_0 = \frac{7}{8}$ and $k = 1.5$:

n	p_n	n	p_n	n	p_n
0	0.875	7	0.3239166554	14	0.3332554829
1	0.1640625	8	0.3284919837	15	0.3332943990
2	0.2057189941	9	0.3308775005	16	0.3333138639
3	0.2450980344	10	0.3320963702	17	0.3333235980
4	0.2775374819	11	0.3327125567	18	0.3333284655
5	0.3007656421	12	0.3330223670	19	0.3333308994
6	0.3154585059	13	0.3331777051	20	0.3333321164



With $p_0 = \frac{7}{8}$ and $k = 2.5$:

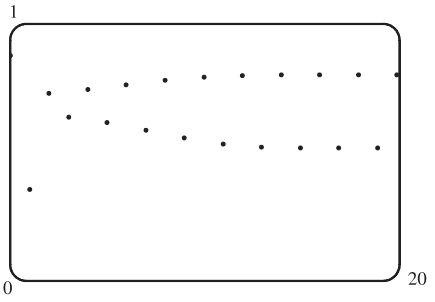
n	p_n	n	p_n	n	p_n
0	0.875	7	0.6016572368	14	0.5999869815
1	0.2734375	8	0.5991645155	15	0.6000065088
2	0.4966735840	9	0.6004159972	16	0.5999967455
3	0.6249723374	10	0.5997915688	17	0.6000016272
4	0.5859547872	11	0.6001041070	18	0.5999991864
5	0.6065294364	12	0.5999479194	19	0.6000004068
6	0.5966286980	13	0.6000260335	20	0.5999997966



The limit of the sequence seems to depend on k, but not on p_0.

2. With $p_0 = \frac{7}{8}$ and $k = 3.2$:

n	p_n	n	p_n	n	p_n
0	0.875	7	0.5830728495	14	0.7990633827
1	0.35	8	0.7779164854	15	0.5137954979
2	0.728	9	0.5528397669	16	0.7993909896
3	0.6336512	10	0.7910654689	17	0.5131681132
4	0.7428395416	11	0.5288988570	18	0.7994451225
5	0.6112926626	12	0.7973275394	19	0.5130643795
6	0.7603646184	13	0.5171082698	20	0.7994538304



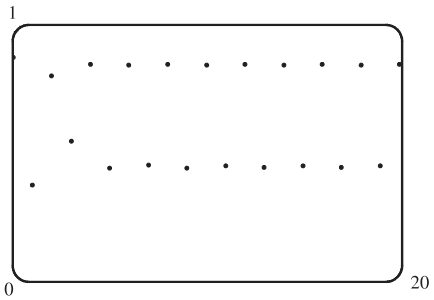
It seems that eventually the terms fluctuate between two values (about 0.5 and 0.8 in this case).

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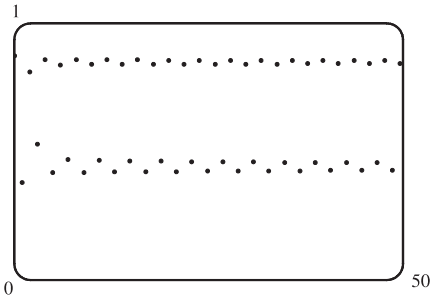
3. With $p_0 = \frac{7}{8}$ and $k = 3.42$:

n	p_n	n	p_n	n	p_n
0	0.875	7	0.4523028596	14	0.8442074951
1	0.3740625	8	0.8472194412	15	0.4498025048
2	0.8007579316	9	0.4426802161	16	0.8463823232
3	0.5456427596	10	0.8437633929	17	0.4446659586
4	0.8478752457	11	0.4508474156	18	0.8445284520
5	0.4411212220	12	0.8467373602	19	0.4490464985
6	0.8431438501	13	0.4438243545	20	0.8461207931

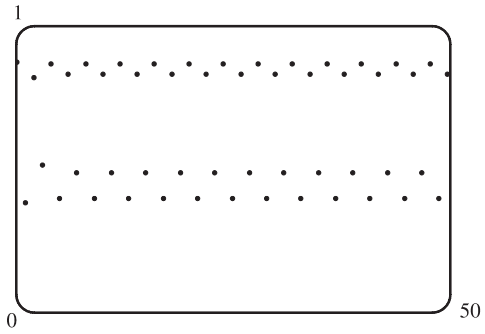


With $p_0 = \frac{7}{8}$ and $k = 3.45$:

n	p_n	n	p_n	n	p_n
0	0.875	7	0.4670259170	14	0.8403376122
1	0.37734375	8	0.8587488490	15	0.4628875685
2	0.8105962830	9	0.4184824586	16	0.8577482026
3	0.5296783241	10	0.8395743720	17	0.4209559716
4	0.8594612299	11	0.4646778983	18	0.8409445432
5	0.4167173034	12	0.8581956045	19	0.4614610237
6	0.8385707740	13	0.4198508858	20	0.8573758782



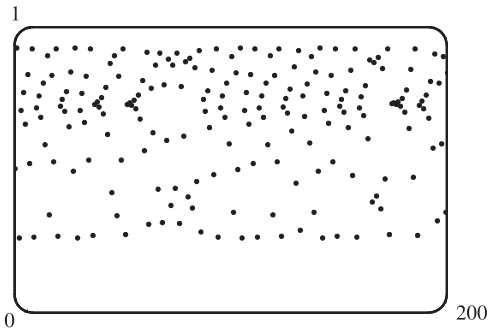
From the graphs above, it seems that for k between 3.4 and 3.5, the terms eventually fluctuate between four values. In the graph below, the pattern followed by the terms is $0.395, 0.832, 0.487, 0.869, 0.395, \ldots$. Note that even for $k = 3.42$ (as in the first graph), there are four distinct "branches"; even after 1000 terms, the first and third terms in the pattern differ by about 2×10^{-9}, while the first and fifth terms differ by only 2×10^{-10}. With $p_0 = \frac{7}{8}$ and $k = 3.48$:



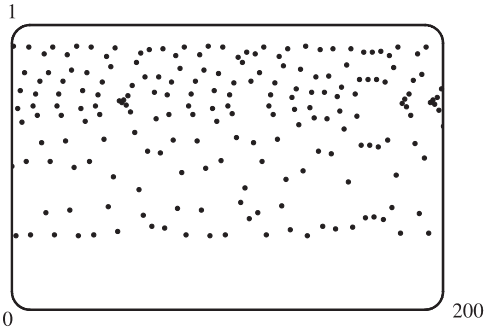
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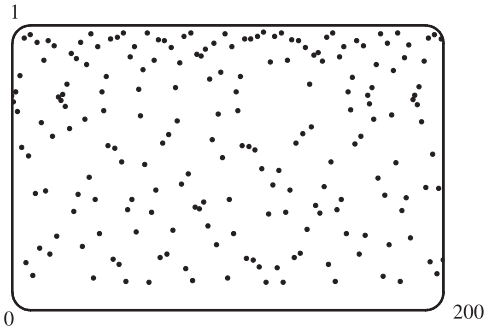
4.



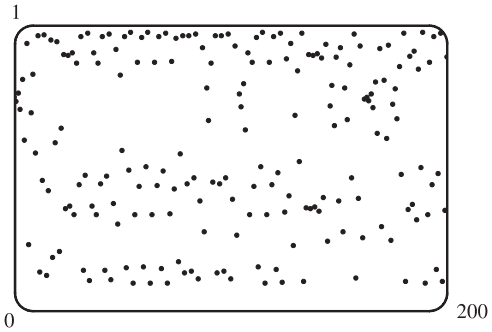
$p_0 = 0.5,\ k = 3.7$



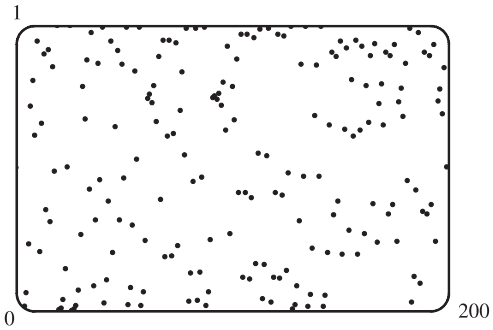
$p_0 = 0.501,\ k = 3.7$



$p_0 = 0.75,\ k = 3.9$



$p_0 = 0.749,\ k = 3.9$



$p_0 = 0.5,\ k = 3.999$

From the graphs, it seems that if p_0 is changed by 0.001, the whole graph changes completely. (Note, however, that this might be partially due to accumulated round-off error in the CAS. These graphs were generated by Maple with 100-digit accuracy, and different degrees of accuracy give different graphs.) There seem to be some some fleeting patterns in these graphs, but on the whole they are certainly very chaotic. As k increases, the graph spreads out vertically, with more extreme values close to 0 or 1.

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8.2 Series

1. (a) A sequence is an ordered list of numbers whereas a series is the *sum* of a list of numbers.

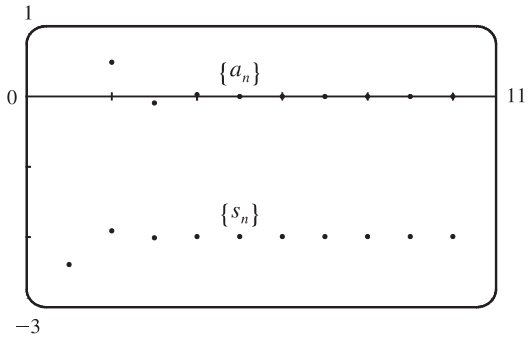
(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.

2. $\sum_{n=1}^{\infty} a_n = 5$ means that by adding sufficiently many terms of the series we can get as close as we like to the number 5.

In other words, it means that $\lim_{n\to\infty} s_n = 5$, where s_n is the nth partial sum, that is, $\sum_{i=1}^{n} a_i$.

3.

n	s_n
1	−2.40000
2	−1.92000
3	−2.01600
4	−1.99680
5	−2.00064
6	−1.99987
7	−2.00003
8	−1.99999
9	−2.00000
10	−2.00000



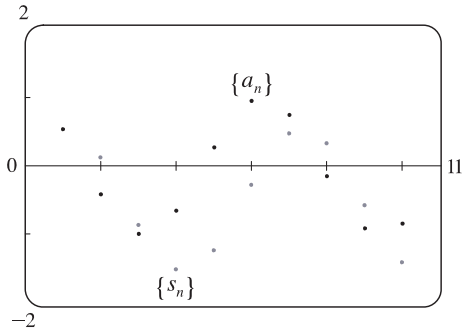
From the graph and the table, it seems that the series converges to −2. In fact, it is a geometric series with $a = -2.4$ and $r = -\frac{1}{5}$, so its sum is $\sum_{n=1}^{\infty} \frac{12}{(-5)^n} = \frac{-2.4}{1-\left(-\frac{1}{5}\right)} = \frac{-2.4}{1.2} = -2$.

Note that the dot corresponding to $n = 1$ is part of both $\{a_n\}$ and $\{s_n\}$.

TI-86 Note: To graph $\{a_n\}$ and $\{s_n\}$, set your calculator to Param mode and DrawDot mode. (DrawDot is under GRAPH, MORE, FORMT (F3).) Now under `E(t)=` make the assignments: `xt1=t, yt1=12/(-5)^t, xt2=t, yt2=sum seq(yt1,t,1,t,1)`. (sum and seq are under LIST, OPS (F5), MORE.) Under WIND use `1,10,1,0,10,1,-3,1,1` to obtain a graph similar to the one above. Then use TRACE (F4) to see the values.

4.

n	s_n
1	0.54030
2	0.12416
3	−0.86584
4	−1.51948
5	−1.23582
6	−0.27565
7	0.47825
8	0.33275
9	−0.57838
10	−1.41745



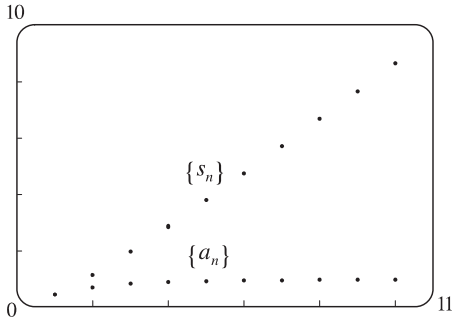
The series $\sum_{n=1}^{\infty} \cos n$ diverges, since its terms do not approach 0.

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5.

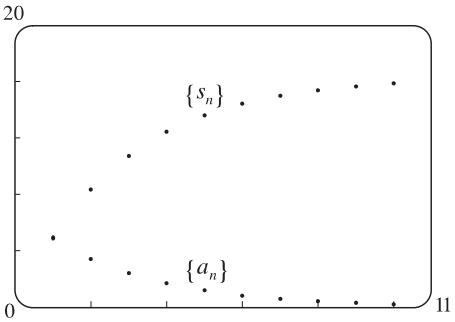
n	s_n
1	0.44721
2	1.15432
3	1.98637
4	2.88080
5	3.80927
6	4.75796
7	5.71948
8	6.68962
9	7.66581
10	8.64639



The series $\sum_{n=1}^{\infty} \dfrac{n}{\sqrt{n^2+4}}$ diverges, since its terms do not approach 0.

6.

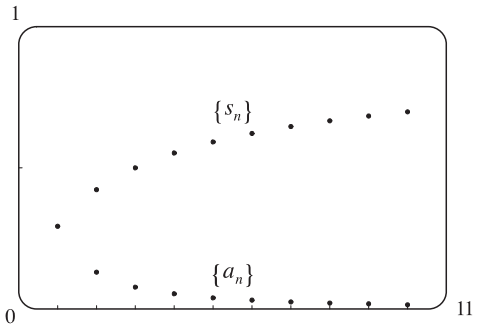
n	s_n
1	4.90000
2	8.33000
3	10.73100
4	12.41170
5	13.58819
6	14.41173
7	14.98821
8	15.39175
9	15.67422
10	15.87196



From the graph and the table, we see that the terms are getting smaller and may approach 0, and that the series approaches a value near 16. The series is geometric with $a_1 = 4.9$ and $r = 0.7$, so its sum is $\sum_{n=1}^{\infty} \dfrac{7^{n+1}}{10^n} = \dfrac{4.9}{1-0.7} = \dfrac{4.9}{0.3} = 16.\overline{3}$.

7.

n	s_n
1	0.29289
2	0.42265
3	0.50000
4	0.55279
5	0.59175
6	0.62204
7	0.64645
8	0.66667
9	0.68377
10	0.69849



From the graph and the table, it seems that the series converges.

$$\sum_{n=1}^{k}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)=\left(\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)+\cdots+\left(\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}\right)$$
$$=1-\frac{1}{\sqrt{k+1}},$$

so $\sum_{n=1}^{\infty}\left(\dfrac{1}{\sqrt{n}}-\dfrac{1}{\sqrt{n+1}}\right)=\lim_{k\to\infty}\left(1-\dfrac{1}{\sqrt{k+1}}\right)=1.$

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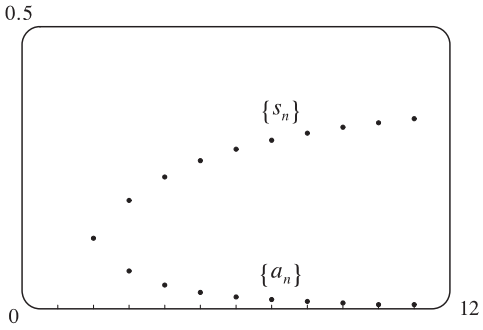
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8.

n	s_n
2	0.12500
3	0.19167
4	0.23333
5	0.26190
6	0.28274
7	0.29861
8	0.31111
9	0.32121
10	0.32955
11	0.33654



From the graph and the table, it seems that the series converges.

$$\frac{1}{n(n+2)} = \frac{1/2}{n} - \frac{1/2}{n+2} = \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n+2}\right), \text{ so}$$

$$\sum_{n=2}^{k} \frac{1}{n(n+2)} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{6}\right) + \cdots + \frac{1}{2}\left(\frac{1}{k} - \frac{1}{k+2}\right)$$
$$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{k+1} - \frac{1}{k+2}\right).$$

As $k \to \infty$, this sum approaches $\frac{1}{2}\left(\frac{5}{6} - 0\right) = \frac{5}{12}$.

9. (a) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{2n}{3n+1} = \dfrac{2}{3}$, so the *sequence* $\{a_n\}$ is convergent by (8.1.1).

(b) Since $\lim_{n\to\infty} a_n = \frac{2}{3} \neq 0$, the *series* $\sum_{n=1}^{\infty} a_n$ is divergent by the Test for Divergence.

10. (a) Both $\sum_{i=1}^{n} a_i$ and $\sum_{j=1}^{n} a_j$ represent the sum of the first n terms of the sequence $\{a_n\}$, that is, the nth partial sum.

(b) $\sum_{i=1}^{n} a_j = \underbrace{a_j + a_j + \cdots + a_j}_{n \text{ terms}} = na_j$, which, in general, is not the same as $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$.

11. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$ is a geometric series with ratio $r = -\frac{4}{3}$. Since $|r| = \frac{4}{3} > 1$, the series diverges.

12. $4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$ is a geometric series with ratio $\frac{3}{4}$. Since $|r| = \frac{3}{4} < 1$, the series converges to $\dfrac{a}{1-r} = \dfrac{4}{1-3/4} = 16$.

13. $10 - 2 + 0.4 - 0.08 + \cdots$ is a geometric series with ratio $-\frac{2}{10} = -\frac{1}{5}$. Since $|r| = \frac{1}{5} < 1$, the series converges to

$$\frac{a}{1-r} = \frac{10}{1-(-1/5)} = \frac{10}{6/5} = \frac{50}{6} = \frac{25}{3}.$$

14. $1 + 0.4 + 0.16 + 0.064 + \cdots$ is a geometric series with ratio $r = 0.4 = \frac{2}{5}$. Since $|r| = \frac{2}{5} < 1$, the series converges to

$$\frac{a}{1-r} = \frac{1}{1-2/5} = \frac{5}{3}.$$

15. $\sum_{n=1}^{\infty} 6(0.9)^{n-1}$ is a geometric series with first term $a = 6$ and ratio $r = 0.9$. Since $|r| = 0.9 < 1$, the series converges to

$$\frac{a}{1-r} = \frac{6}{1-0.9} = \frac{6}{0.1} = 60.$$

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16. $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}} = \sum_{n=1}^{\infty} \frac{10(10)^{n-1}}{(-9)^{n-1}} = 10 \sum_{n=1}^{\infty} \left(-\frac{10}{9}\right)^{n-1}$. The latter series is geometric with $a = 10$ and ratio $r = -\frac{10}{9}$.

Since $|r| = \frac{10}{9} > 1$, the series diverges.

17. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{\pi}{3}\right)^n$ is a geometric series with ratio $r = \frac{\pi}{3}$. Since $|r| > 1$, the series diverges.

18. $\sum_{n=0}^{\infty} \frac{1}{\left(\sqrt{2}\right)^n}$ is a geometric series with ratio $r = \frac{1}{\sqrt{2}}$. Since $|r| = \frac{1}{\sqrt{2}} < 1$, the series converges. Its sum is

$$\frac{1}{1 - 1/\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} - 1} = \frac{\sqrt{2}}{\sqrt{2} - 1} \cdot \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \sqrt{2}\left(\sqrt{2} + 1\right) = 2 + \sqrt{2}.$$

19. $\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$ diverges by the Test for Divergence since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n-1}{3n-1} = \frac{1}{3} \neq 0$.

20. $\sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2}$ diverges by the Test for Divergence since $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k(k+2)}{(k+3)^2} = \lim_{k\to\infty} \frac{1 \cdot (1 + 2/k)}{(1 + 3/k)^2} = 1 \neq 0$.

21. $\sum_{k=2}^{\infty} \frac{k^2}{k^2 - 1}$ diverges by the Test for Divergence since $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k^2}{k^2 - 1} = 1 \neq 0$.

22. $\sum_{n=1}^{\infty} \cos\frac{1}{n}$ diverges by the Test for Divergence since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \cos\frac{1}{n} = \cos 0 = 1 \neq 0$.

23. Converges.

$$\sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{1}{3^n} + \frac{2^n}{3^n}\right) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{3}\right)^n + \left(\frac{2}{3}\right)^n\right] \quad \text{[sum of two convergent geometric series]}$$

$$= \frac{1/3}{1 - 1/3} + \frac{2/3}{1 - 2/3} = \frac{1}{2} + 2 = \frac{5}{2}$$

24. $\sum_{n=1}^{\infty} \frac{1 + 3^n}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{3^n}{2^n}\right) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^n + \left(\frac{3}{2}\right)^n\right] = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$. The first series is a convergent geometric series ($|r| = \frac{1}{2} < 1$), but the second series is a divergent geometric series ($|r| = \frac{3}{2} \geq 1$), so the original series is divergent.

25. $\sum_{n=1}^{\infty} \sqrt[n]{2} = 2 + \sqrt{2} + \sqrt[3]{2} + \sqrt[4]{2} + \cdots$ diverges by the Test for Divergence since

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[n]{2} = \lim_{n\to\infty} 2^{1/n} = 2^0 = 1 \neq 0$.

26. $\sum_{k=1}^{\infty} (\cos 1)^k$ is a geometric series with ratio $r = \cos 1 \approx 0.540302$. It converges because $|r| < 1$. Its sum is

$$\frac{\cos 1}{1 - \cos 1} \approx 1.175343.$$

27. $\sum_{n=1}^{\infty} \arctan n$ diverges by the Test for Divergence since $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \arctan n = \frac{\pi}{2} \neq 0$.

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28. $\sum_{n=1}^{\infty}\left[(0.8)^{n-1}-(0.3)^n\right]=\sum_{n=1}^{\infty}(0.8)^{n-1}-\sum_{n=1}^{\infty}(0.3)^n$ [difference of two convergent geometric series]

$$=\frac{1}{1-0.8}-\frac{0.3}{1-0.3}=5-\frac{3}{7}=\frac{32}{7}$$

29. $\sum_{n=1}^{\infty}\frac{1}{e^n}=\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$ is a geometric series with first term $a=\frac{1}{e}$ and ratio $r=\frac{1}{e}$. Since $|r|=\frac{1}{e}<1$, the series converges to $\frac{1/e}{1-1/e}=\frac{1/e}{1-1/e}\cdot\frac{e}{e}=\frac{1}{e-1}$. By Example 6, $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=1$. Thus, by Theorem 8(ii),

$$\sum_{n=1}^{\infty}\left(\frac{1}{e^n}+\frac{1}{n(n+1)}\right)=\sum_{n=1}^{\infty}\frac{1}{e^n}+\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=\frac{1}{e-1}+1=\frac{1}{e-1}+\frac{e-1}{e-1}=\frac{e}{e-1}.$$

30. $\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)$ diverges because $\sum_{n=1}^{\infty}\frac{2}{n}=2\sum_{n=1}^{\infty}\frac{1}{n}$ diverges. (If it converged, then $\frac{1}{2}\cdot 2\sum_{n=1}^{\infty}\frac{1}{n}$ would also converge by Theorem 8(i), but we know from Example 7 that the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.) If the given series converges, then the difference $\sum_{n=1}^{\infty}\left(\frac{3}{5^n}+\frac{2}{n}\right)-\sum_{n=1}^{\infty}\frac{3}{5^n}$ must converge (since $\sum_{n=1}^{\infty}\frac{3}{5^n}$ is a convergent geometric series) and equal $\sum_{n=1}^{\infty}\frac{2}{n}$, but we have just seen that $\sum_{n=1}^{\infty}\frac{2}{n}$ diverges, so the given series must also diverge.

31. Using partial fractions, the partial sums of the series $\sum_{n=2}^{\infty}\frac{2}{n^2-1}$ are

$$s_n=\sum_{i=2}^{n}\frac{2}{(i-1)(i+1)}=\sum_{i=2}^{n}\left(\frac{1}{i-1}-\frac{1}{i+1}\right)$$

$$=\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{n-3}-\frac{1}{n-1}\right)+\left(\frac{1}{n-2}-\frac{1}{n}\right)$$

This sum is a telescoping series and $s_n=1+\frac{1}{2}-\frac{1}{n-1}-\frac{1}{n}$.

Thus, $\sum_{n=2}^{\infty}\frac{2}{n^2-1}=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(1+\frac{1}{2}-\frac{1}{n-1}-\frac{1}{n}\right)=\frac{3}{2}$.

32. For the series $\sum_{n=1}^{\infty}\frac{2}{n^2+4n+3}$, $s_n=\sum_{i=1}^{n}\frac{2}{i^2+4i+3}=\sum_{i=1}^{n}\left(\frac{1}{i+1}-\frac{1}{i+3}\right)$ [using partial fractions]. The latter sum is

$$\left(\tfrac{1}{2}-\tfrac{1}{4}\right)+\left(\tfrac{1}{3}-\tfrac{1}{5}\right)+\left(\tfrac{1}{4}-\tfrac{1}{6}\right)+\left(\tfrac{1}{5}-\tfrac{1}{7}\right)+\cdots+\left(\tfrac{1}{n}-\tfrac{1}{n+2}\right)+\left(\tfrac{1}{n+1}-\tfrac{1}{n+3}\right)=\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{n+2}-\tfrac{1}{n+3}$$

[telescoping series]

Thus, $\sum_{n=1}^{\infty}\frac{2}{n^2+4n+3}=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}\right)=\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$. Converges

33. For the series $\sum_{n=1}^{\infty}\frac{3}{n(n+3)}$, $s_n=\sum_{i=1}^{n}\frac{3}{i(i+3)}=\sum_{i=1}^{n}\left(\frac{1}{i}-\frac{1}{i+3}\right)$ [using partial fractions]. The latter sum is

$$\left(1-\tfrac{1}{4}\right)+\left(\tfrac{1}{2}-\tfrac{1}{5}\right)+\left(\tfrac{1}{3}-\tfrac{1}{6}\right)+\left(\tfrac{1}{4}-\tfrac{1}{7}\right)+\cdots+\left(\tfrac{1}{n-3}-\tfrac{1}{n}\right)+\left(\tfrac{1}{n-2}-\tfrac{1}{n+1}\right)+\left(\tfrac{1}{n-1}-\tfrac{1}{n+2}\right)+\left(\tfrac{1}{n}-\tfrac{1}{n+3}\right)$$

$$=1+\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{n+1}-\tfrac{1}{n+2}-\tfrac{1}{n+3}$$ [telescoping series]

Thus, $\sum_{n=1}^{\infty}\frac{3}{n(n+3)}=\lim_{n\to\infty}s_n=\lim_{n\to\infty}\left(1+\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{n+1}-\tfrac{1}{n+2}-\tfrac{1}{n+3}\right)=1+\tfrac{1}{2}+\tfrac{1}{3}=\tfrac{11}{6}$. Converges

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34. For the series $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$,

$s_n = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + \cdots + [\ln n - \ln(n+1)] = \ln 1 - \ln(n+1) = -\ln(n+1)$

[telescoping series]

Thus, $\lim_{n\to\infty} s_n = -\infty$, so the series is divergent.

35. (a) Many people would guess that $x < 1$, but note that x consists of an infinite number of 9s.

(b) $x = 0.99999\ldots = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10{,}000} + \cdots = \sum_{n=1}^{\infty} \frac{9}{10^n}$, which is a geometric series with $a_1 = 0.9$ and $r = 0.1$. Its sum is $\frac{0.9}{1-0.1} = \frac{0.9}{0.9} = 1$, that is, $x = 1$.

(c) The number 1 has two decimal representations, $1.00000\ldots$ and $0.99999\ldots$.

(d) Except for 0, all rational numbers that have a terminating decimal representation can be written in more than one way. For example, 0.5 can be written as $0.49999\ldots$ as well as $0.50000\ldots$.

36. $0.\overline{73} = \frac{73}{10^2} + \frac{73}{10^4} + \cdots = \frac{73/10^2}{1 - 1/10^2} = \frac{73/100}{99/100} = \frac{73}{99}$

37. $0.\overline{2} = \frac{2}{10} + \frac{2}{10^2} + \cdots$ is a geometric series with $a = \frac{2}{10}$ and $r = \frac{1}{10}$. It converges to $\frac{a}{1-r} = \frac{2/10}{1 - 1/10} = \frac{2}{9}$.

38. $6.2\overline{54} = 6.2 + \frac{54}{10^3} + \frac{54}{10^5} + \cdots = 6.2 + \frac{54/10^3}{1 - 1/10^2} = \frac{62}{10} + \frac{54}{990} = \frac{6192}{990} = \frac{344}{55}$

39. $1.53\overline{42} = 1.53 + \frac{42}{10^4} + \frac{42}{10^6} + \cdots$. Now $\frac{42}{10^4} + \frac{42}{10^6} + \cdots$ is a geometric series with $a = \frac{42}{10^4}$ and $r = \frac{1}{10^2}$.

It converges to $\frac{a}{1-r} = \frac{42/10^4}{1 - 1/10^2} = \frac{42/10^4}{99/10^2} = \frac{42}{9900}$.

Thus, $1.53\overline{42} = 1.53 + \frac{42}{9900} = \frac{153}{100} + \frac{42}{9900} = \frac{15{,}147}{9900} + \frac{42}{9900} = \frac{15{,}189}{9900}$ or $\frac{5063}{3300}$.

40. $7.\overline{12345} = 7 + \frac{12{,}345}{10^5} + \frac{12{,}345}{10^{10}} + \cdots$. Now $\frac{12{,}345}{10^5} + \frac{12{,}345}{10^{10}} + \cdots$ is a geometric series with $a = \frac{12{,}345}{10^5}$ and $r = \frac{1}{10^5}$.

It converges to $\frac{a}{1-r} = \frac{12{,}345/10^5}{1 - 1/10^5} = \frac{12{,}345/10^5}{99{,}999/10^5} = \frac{12{,}345}{99{,}999}$.

Thus, $7.\overline{12345} = 7 + \frac{12{,}345}{99{,}999} = \frac{699{,}993}{99{,}999} + \frac{12{,}345}{99{,}999} = \frac{712{,}338}{99{,}999}$ or $\frac{237{,}446}{33{,}333}$.

41. $\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n$ is a geometric series with $r = \frac{x}{3}$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $\frac{|x|}{3} < 1$ $\Leftrightarrow$ $|x| < 3$;

that is, $-3 < x < 3$. In that case, the sum of the series is $\frac{a}{1-r} = \frac{x/3}{1 - x/3} = \frac{x/3}{1 - x/3} \cdot \frac{3}{3} = \frac{x}{3-x}$.

42. $\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$ is a geometric series with $r = \frac{x+3}{2}$, so the series converges $\Leftrightarrow$ $|r| < 1$ $\Leftrightarrow$ $\frac{|x+3|}{2} < 1$ $\Leftrightarrow$

$|x+3| < 2$ $\Leftrightarrow$ $-5 < x < -1$. For these values of x, the sum of the series is $\frac{1}{1 - (x+3)/2} = \frac{2}{2 - (x+3)} = -\frac{2}{x+1}$.

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43. $\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$ is a geometric series with first term 1 and ratio $r = \frac{\cos x}{2}$, so it converges $\Leftrightarrow$ $|r| < 1$. But $|r| = \frac{|\cos x|}{2} \le \frac{1}{2}$ for all x. Thus, the series converges for all real values of x and the sum of the series is $\frac{1}{1-(\cos x)/2} = \frac{2}{2-\cos x}$.

44. Because $\frac{1}{n} \to 0$ and ln is continuous, we have $\lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right) = \ln 1 = 0$.

We now show that the series $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right) = \sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right) = \sum_{n=1}^{\infty} [\ln(n+1) - \ln n]$ diverges.

$s_n = (\ln 2 - \ln 1) + (\ln 3 - \ln 2) + \cdots + (\ln(n+1) - \ln n) = \ln(n+1) - \ln 1 = \ln(n+1)$.

As $n \to \infty$, $s_n = \ln(n+1) \to \infty$, so the series diverges.

45. After defining f, We use `convert(f,parfrac);` in Maple, `Apart` in Mathematica, or `Expand Rational` and `Simplify` in Derive to find that the general term is $\frac{3n^2+3n+1}{(n^2+n)^3} = \frac{1}{n^3} - \frac{1}{(n+1)^3}$. So the nth partial sum is

$$s_n = \sum_{k=1}^{n} \left(\frac{1}{k^3} - \frac{1}{(k+1)^3}\right) = \left(1 - \frac{1}{2^3}\right) + \left(\frac{1}{2^3} - \frac{1}{3^3}\right) + \cdots + \left(\frac{1}{n^3} - \frac{1}{(n+1)^3}\right) = 1 - \frac{1}{(n+1)^3}$$

The series converges to $\lim_{n\to\infty} s_n = 1$. This can be confirmed by directly computing the sum using `sum(f,1..infinity);` (in Maple), `Sum[f,{n,1,Infinity}]` (in Mathematica), or `Calculus Sum` (from 1 to ∞) and `Simplify` (in Derive).

46. See Exercise 45 for specific CAS commands. $\frac{1}{n^3-n} = \frac{1/2}{n-1} - \frac{1}{n} + \frac{1/2}{n+1}$. So the nth partial sum is

$$\begin{aligned} s_n &= \sum_{k=2}^{n} \left(\frac{1/2}{k-1} - \frac{1}{k} + \frac{1/2}{k+1}\right) \\ &= \left(\frac{1/2}{1} - \frac{1}{2} + \frac{1/2}{3}\right) + \left(\frac{1/2}{2} - \frac{1}{3} + \frac{1/2}{4}\right) + \left(\frac{1/2}{3} - \frac{1}{4} + \frac{1/2}{5}\right) \\ &\qquad + \left(\frac{1/2}{4} - \frac{1}{5} + \frac{1/2}{6}\right) + \cdots + \left(\frac{1/2}{n-2} - \frac{1}{n-1} + \frac{1/2}{n}\right) + \left(\frac{1/2}{n-1} - \frac{1}{n} + \frac{1/2}{n+1}\right) \\ &= \frac{1/2}{1} + \left(-\frac{1}{2} + \frac{1/2}{2}\right) + \left(\frac{1/2}{3} - \frac{1}{3} + \frac{1/2}{3}\right) + \left(\frac{1/2}{4} - \frac{1}{4} + \frac{1/2}{4}\right) + \cdots + \left(\frac{1/2}{n} - \frac{1}{n} + \frac{1/2}{n+1}\right) \\ &= \frac{1}{2} + \left(-\frac{1}{4}\right) + 0 + 0 + \cdots + \frac{1/2}{n} - \frac{1}{n} + \frac{1/2}{n+1} \end{aligned}$$

The series converges to $\lim_{n\to\infty} s_n = \frac{1}{4}$.

47. For $n = 1$, $a_1 = 0$ since $s_1 = 0$. For $n > 1$,

$$a_n = s_n - s_{n-1} = \frac{n-1}{n+1} - \frac{(n-1)-1}{(n-1)+1} = \frac{(n-1)n - (n+1)(n-2)}{(n+1)n} = \frac{2}{n(n+1)}$$

Also, $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1-1/n}{1+1/n} = 1$.

48. $a_1 = s_1 = 3 - \frac{1}{2} = \frac{5}{2}$. For $n \ne 1$,

$$a_n = s_n - s_{n-1} = \left(3 - n2^{-n}\right) - \left[3 - (n-1)2^{-(n-1)}\right] = -\frac{n}{2^n} + \frac{n-1}{2^{n-1}} \cdot \frac{2}{2} = \frac{2(n-1)}{2^n} - \frac{n}{2^n} = \frac{n-2}{2^n}$$

Also, $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \left(3 - \frac{n}{2^n}\right) = 3$ because $\lim_{x\to\infty} \frac{x}{2^x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{1}{2^x \ln 2} = 0$.

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49. (a) After the first pill is taken, 100 mg of the drug is in the body. After the second pill is taken, 100 mg plus $100(5\%)$ mg remains in the body. After the third pill is taken 100 mg plus $100(5\%)$ mg plus $[100(5\%)](5\%)$ mg remains in the body. This gives us $100 + 100(0.05) + 100(0.05)^2 = 105.25$ mg of the drug remaining in the body after the patient takes three pills.

(b) Continuing the pattern established in part (a), we get $100 + 100(0.05) + 100(0.05)^2 + \cdots + 100(0.05)^{n-1}$ mg after n pills are taken. By (3), this sum is $\dfrac{100(1 - 0.05^n)}{1 - 0.05}$.

(c) The amount of the drug remaining in the body in the long run can be approximated by summing the infinite series with $a_1 = 100$ and $r = 0.05$. This sum is $\dfrac{100}{1 - 0.05} = \dfrac{100}{0.95} \approx 105.26$ mg.

50. (a) We start with N flies being released. After 1 day, we have Ns^1 flies plus a new release of N flies. After 2 days, we have $Ns^2 + Ns^1 + N$ flies. After n days, we have $Ns^n + Ns^{n-1} + \cdots + Ns^1 + N = \sum_{i=0}^{n} Ns^i = \dfrac{N(1 - s^{n+1})}{1 - s}$ by (3). In the long run, we have a geometric series with $a_1 = N$ and $r = s$, so its sum is $\dfrac{N}{1 - s}$.

(b) We want the sum to be 10,000, so $10{,}000 = \dfrac{N}{1 - 0.9}$ [from part (a)] $\Rightarrow$ $N = 10{,}000(0.1) = 1000$.

51. (a) The first step in the chain occurs when the local government spends D dollars. The people who receive it spend a fraction c of those D dollars, that is, Dc dollars. Those who receive the Dc dollars spend a fraction c of it, that is, Dc^2 dollars. Continuing in this way, we see that the total spending after n transactions is

$S_n = D + Dc + Dc^2 + \cdots + Dc^{n-1} = \dfrac{D(1 - c^n)}{1 - c}$ by (3).

(b) $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \dfrac{D(1 - c^n)}{1 - c} = \dfrac{D}{1 - c} \lim\limits_{n\to\infty} (1 - c^n) = \dfrac{D}{1 - c}$ $\left[\text{since } 0 < c < 1 \quad \Rightarrow \quad \lim\limits_{n\to\infty} c^n = 0\right]$

$= \dfrac{D}{s}$ [since $c + s = 1$] $= kD$ [since $k = 1/s$]

If $c = 0.8$, then $s = 1 - c = 0.2$ and the multiplier is $k = 1/s = 5$.

52. (a) Initially, the ball falls a distance H, then rebounds a distance rH, falls rH, rebounds r^2H, falls r^2H, etc. The total distance it travels is

$$H + 2rH + 2r^2H + 2r^3H + \cdots = H\big(1 + 2r + 2r^2 + 2r^3 + \cdots\big) = H\big[1 + 2r\big(1 + r + r^2 + \cdots\big)\big]$$
$$= H\left[1 + 2r\left(\frac{1}{1 - r}\right)\right] = H\left(\frac{1 + r}{1 - r}\right) \text{ meters}$$

(b) From Example 3 in Section 2.1, we know that a ball falls $\frac{1}{2}gt^2$ meters in t seconds, where g is the gravitational acceleration. Thus, a ball falls h meters in $t = \sqrt{2h/g}$ seconds. The total travel time in seconds is

$$\sqrt{\frac{2H}{g}} + 2\sqrt{\frac{2H}{g}r} + 2\sqrt{\frac{2H}{g}r^2} + 2\sqrt{\frac{2H}{g}r^3} + \cdots = \sqrt{\frac{2H}{g}}\left[1 + 2\sqrt{r} + 2\sqrt{r}^{\,2} + 2\sqrt{r}^{\,3} + \cdots\right]$$
$$= \sqrt{\frac{2H}{g}}\left(1 + 2\sqrt{r}\left[1 + \sqrt{r} + \sqrt{r}^{\,2} + \cdots\right]\right)$$
$$= \sqrt{\frac{2H}{g}}\left[1 + 2\sqrt{r}\left(\frac{1}{1 - \sqrt{r}}\right)\right] = \sqrt{\frac{2H}{g}}\frac{1 + \sqrt{r}}{1 - \sqrt{r}}$$

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(c) It will help to make a chart of the time for each descent and each rebound of the ball, together with the velocity just before and just after each bounce. Recall that the time in seconds needed to fall h meters is $\sqrt{2h/g}$. The ball hits the ground with velocity $-g\sqrt{2h/g} = -\sqrt{2hg}$ (taking the upward direction to be positive) and rebounds with velocity $kg\sqrt{2h/g} = k\sqrt{2hg}$, taking time $k\sqrt{2h/g}$ to reach the top of its bounce, where its velocity is 0. At that point, its height is k^2h. All these results follow from the formulas for vertical motion with gravitational acceleration $-g$:

$$\frac{d^2y}{dt^2} = -g \quad \Rightarrow \quad v = \frac{dy}{dt} = v_0 - gt \quad \Rightarrow \quad y = y_0 + v_0t - \tfrac{1}{2}gt^2.$$

number of descent	time of descent	speed before bounce	speed after bounce	time of ascent	peak height
1	$\sqrt{2H/g}$	$\sqrt{2Hg}$	$k\sqrt{2Hg}$	$k\sqrt{2H/g}$	k^2H
2	$\sqrt{2k^2H/g}$	$\sqrt{2k^2Hg}$	$k\sqrt{2k^2Hg}$	$k\sqrt{2k^2H/g}$	k^4H
3	$\sqrt{2k^4H/g}$	$\sqrt{2k^4Hg}$	$k\sqrt{2k^4Hg}$	$k\sqrt{2k^4H/g}$	k^6H
...	...	...	...	...	...

The total travel time in seconds is

$$\begin{aligned}\sqrt{\frac{2H}{g}} + k\sqrt{\frac{2H}{g}} + k\sqrt{\frac{2H}{g}} + k^2\sqrt{\frac{2H}{g}} + k^2\sqrt{\frac{2H}{g}} + \cdots &= \sqrt{\frac{2H}{g}}\left(1 + 2k + 2k^2 + 2k^3 + \cdots\right) \\ &= \sqrt{\frac{2H}{g}}\left[1 + 2k(1 + k + k^2 + \cdots)\right] \\ &= \sqrt{\frac{2H}{g}}\left[1 + 2k\left(\frac{1}{1-k}\right)\right] = \sqrt{\frac{2H}{g}}\,\frac{1+k}{1-k}\end{aligned}$$

Another method: We could use part (b). At the top of the bounce, the height is $k^2h = rh$, so $\sqrt{r} = k$ and the result follows from part (b).

53. $\sum_{n=2}^{\infty}(1+c)^{-n}$ is a geometric series with $a = (1+c)^{-2}$ and $r = (1+c)^{-1}$, so the series converges when $\left|(1+c)^{-1}\right| < 1 \quad \Leftrightarrow \quad |1+c| > 1 \quad \Leftrightarrow \quad 1+c > 1$ or $1+c < -1 \quad \Leftrightarrow \quad c > 0$ or $c < -2$. We calculate the sum of the series and set it equal to 2: $\dfrac{(1+c)^{-2}}{1-(1+c)^{-1}} = 2 \quad \Leftrightarrow \quad \left(\dfrac{1}{1+c}\right)^2 = 2 - 2\left(\dfrac{1}{1+c}\right) \quad \Leftrightarrow \quad 1 = 2(1+c)^2 - 2(1+c) \quad \Leftrightarrow$ $2c^2 + 2c - 1 = 0 \quad \Leftrightarrow \quad c = \frac{-2 \pm \sqrt{12}}{4} = \frac{\pm\sqrt{3}-1}{2}$. However, the negative root is inadmissible because $-2 < \frac{-\sqrt{3}-1}{2} < 0$. So $c = \frac{\sqrt{3}-1}{2}$.

54. $\sum_{n=0}^{\infty} e^{nc} = \sum_{n=0}^{\infty}(e^c)^n$ is a geometric series with $a = (e^c)^0 = 1$ and $r = e^c$. If $e^c < 1$, it has sum $\dfrac{1}{1-e^c}$, so $\dfrac{1}{1-e^c} = 10 \quad \Rightarrow$ $\frac{1}{10} = 1 - e^c \quad \Rightarrow \quad e^c = \frac{9}{10} \quad \Rightarrow \quad c = \ln\frac{9}{10}$.

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55. $e^{s_n} = e^{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}} = e^1 e^{1/2} e^{1/3} \cdots e^{1/n} > (1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{n}\right) \qquad [e^x > 1+x]$

$$= \frac{2}{1}\frac{3}{2}\frac{4}{3}\cdots\frac{n+1}{n} = n+1$$

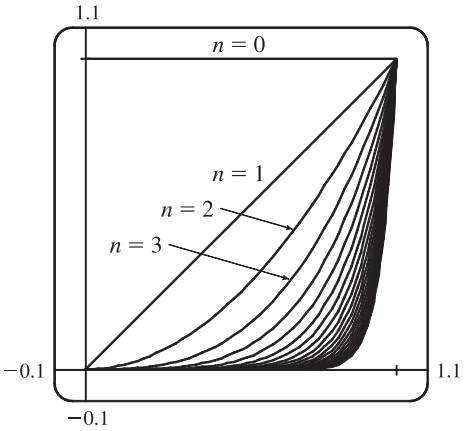
Thus, $e^{s_n} > n+1$ and $\lim_{n\to\infty} e^{s_n} = \infty$. Since $\{s_n\}$ is increasing, $\lim_{n\to\infty} s_n = \infty$, implying that the harmonic series is divergent.

56. The area between $y = x^{n-1}$ and $y = x^n$ for $0 \le x \le 1$ is

$$\int_0^1 (x^{n-1} - x^n)\,dx = \left[\frac{x^n}{n} - \frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n} - \frac{1}{n+1}$$

$$= \frac{(n+1)-n}{n(n+1)} = \frac{1}{n(n+1)}$$

We can see from the diagram that as $n \to \infty$, the sum of the areas between the successive curves approaches the area of the unit square, that is, 1. So $\sum_{n=1}^{\infty} \frac{1}{n\,(n+1)} = 1$.



57. Let d_n be the diameter of C_n. We draw lines from the centers of the C_i to the center of D (or C), and using the Pythagorean Theorem, we can write

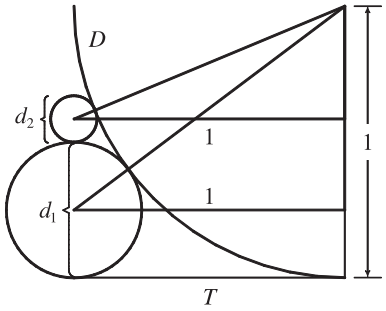
$1^2 + \left(1 - \frac{1}{2}d_1\right)^2 = \left(1 + \frac{1}{2}d_1\right)^2 \quad \Leftrightarrow$

$1 = \left(1 + \frac{1}{2}d_1\right)^2 - \left(1 - \frac{1}{2}d_1\right)^2 = 2d_1$ [difference of squares] $\quad \Rightarrow \quad d_1 = \frac{1}{2}$.

Similarly,

$$1 = \left(1 + \tfrac{1}{2}d_2\right)^2 - \left(1 - d_1 - \tfrac{1}{2}d_2\right)^2 = 2d_2 + 2d_1 - d_1^2 - d_1 d_2$$

$$= (2 - d_1)(d_1 + d_2) \quad \Leftrightarrow$$



$d_2 = \dfrac{1}{2-d_1} - d_1 = \dfrac{(1-d_1)^2}{2-d_1}$, $1 = \left(1 + \frac{1}{2}d_3\right)^2 - \left(1 - d_1 - d_2 - \frac{1}{2}d_3\right)^2 \quad \Leftrightarrow \quad d_3 = \dfrac{[1-(d_1+d_2)]^2}{2-(d_1+d_2)}$, and in general, $d_{n+1} = \dfrac{\left(1 - \sum_{i=1}^{n} d_i\right)^2}{2 - \sum_{i=1}^{n} d_i}$. If we actually calculate d_2 and d_3 from the formulas above, we find that they are $\dfrac{1}{6} = \dfrac{1}{2\cdot 3}$ and $\dfrac{1}{12} = \dfrac{1}{3\cdot 4}$ respectively, so we suspect that in general, $d_n = \dfrac{1}{n(n+1)}$. To prove this, we use induction: Assume that for all $k \le n$, $d_k = \dfrac{1}{k(k+1)} = \dfrac{1}{k} - \dfrac{1}{k+1}$. Then $\sum_{i=1}^{n} d_i = 1 - \dfrac{1}{n+1} = \dfrac{n}{n+1}$ [telescoping sum]. Substituting this into our formula for d_{n+1}, we get $d_{n+1} = \dfrac{\left[1 - \frac{n}{n+1}\right]^2}{2 - \left(\frac{n}{n+1}\right)} = \dfrac{\frac{1}{(n+1)^2}}{\frac{n+2}{n+1}} = \dfrac{1}{(n+1)(n+2)}$, and the induction is complete.

Now, we observe that the partial sums $\sum_{i=1}^{n} d_i$ of the diameters of the circles approach 1 as $n \to \infty$; that is,

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \dfrac{1}{n(n+1)} = 1$, which is what we wanted to prove.

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58. $|CD| = b\sin\theta$, $|DE| = |CD|\sin\theta = b\sin^2\theta$, $|EF| = |DE|\sin\theta = b\sin^3\theta, \ldots$. Therefore,

$|CD| + |DE| + |EF| + |FG| + \cdots = b\sum_{n=1}^{\infty}\sin^n\theta = b\left(\dfrac{\sin\theta}{1-\sin\theta}\right)$ since this is a geometric series with $r = \sin\theta$ and $|\sin\theta| < 1$ $\left[\text{because } 0 < \theta < \frac{\pi}{2}\right]$.

59. The series $1 - 1 + 1 - 1 + 1 - 1 + \cdots$ diverges (geometric series with $r = -1$) so we cannot say that $0 = 1 - 1 + 1 - 1 + 1 - 1 + \cdots$.

60. If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$ by Theorem 6, so $\lim_{n\to\infty}\dfrac{1}{a_n} \neq 0$, and so $\sum_{n=1}^{\infty}\dfrac{1}{a_n}$ is divergent by the Test for Divergence.

61. Suppose on the contrary that $\sum(a_n + b_n)$ converges. Then $\sum(a_n + b_n)$ and $\sum a_n$ are convergent series. So by Theorem 8(iii), $\sum[(a_n + b_n) - a_n]$ would also be convergent. But $\sum[(a_n + b_n) - a_n] = \sum b_n$, a contradiction, since $\sum b_n$ is given to be divergent.

62. No. For example, take $\sum a_n = \sum n$ and $\sum b_n = \sum(-n)$, which both diverge, yet $\sum(a_n + b_n) = \sum 0$, which converges with sum 0.

63. The partial sums $\{s_n\}$ form an increasing sequence, since $s_n - s_{n-1} = a_n > 0$ for all n. Also, the sequence $\{s_n\}$ is bounded since $s_n \le 1000$ for all n. So by the Monotonic Sequence Theorem, the sequence of partial sums converges, that is, the series $\sum a_n$ is convergent.

64. (a) $\text{RHS} = \dfrac{1}{f_{n-1}f_n} - \dfrac{1}{f_n f_{n+1}} = \dfrac{f_n f_{n+1} - f_n f_{n-1}}{f_n^2 f_{n-1} f_{n+1}} = \dfrac{f_{n+1} - f_{n-1}}{f_n f_{n-1} f_{n+1}} = \dfrac{(f_{n-1} + f_n) - f_{n-1}}{f_n f_{n-1} f_{n+1}} = \dfrac{1}{f_{n-1}f_{n+1}} = \text{LHS}$

(b) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{f_{n-1}f_{n+1}} = \sum_{n=2}^{\infty}\left(\frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}\right)$ [from part (a)]

$$= \lim_{n\to\infty}\left[\left(\frac{1}{f_1 f_2} - \frac{1}{f_2 f_3}\right) + \left(\frac{1}{f_2 f_3} - \frac{1}{f_3 f_4}\right) + \left(\frac{1}{f_3 f_4} - \frac{1}{f_4 f_5}\right) + \cdots + \left(\frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}\right)\right]$$

$$= \lim_{n\to\infty}\left(\frac{1}{f_1 f_2} - \frac{1}{f_n f_{n+1}}\right) = \frac{1}{f_1 f_2} - 0 = \frac{1}{1\cdot 1} = 1 \quad \text{because } f_n \to \infty \text{ as } n \to \infty.$$

(c) $\displaystyle\sum_{n=2}^{\infty}\frac{f_n}{f_{n-1}f_{n+1}} = \sum_{n=2}^{\infty}\left(\frac{f_n}{f_{n-1}f_n} - \frac{f_n}{f_n f_{n+1}}\right)$ [as above]

$$= \sum_{n=2}^{\infty}\left(\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}}\right)$$

$$= \lim_{n\to\infty}\left[\left(\frac{1}{f_1} - \frac{1}{f_3}\right) + \left(\frac{1}{f_2} - \frac{1}{f_4}\right) + \left(\frac{1}{f_3} - \frac{1}{f_5}\right) + \left(\frac{1}{f_4} - \frac{1}{f_6}\right) + \cdots + \left(\frac{1}{f_{n-1}} - \frac{1}{f_{n+1}}\right)\right]$$

$$= \lim_{n\to\infty}\left(\frac{1}{f_1} + \frac{1}{f_2} - \frac{1}{f_n} - \frac{1}{f_{n+1}}\right) = 1 + 1 - 0 - 0 = 2 \quad \text{because } f_n \to \infty \text{ as } n \to \infty.$$

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65. (a) At the first step, only the interval $\left(\frac{1}{3}, \frac{2}{3}\right)$ (length $\frac{1}{3}$) is removed. At the second step, we remove the intervals $\left(\frac{1}{9}, \frac{2}{9}\right)$ and $\left(\frac{7}{9}, \frac{8}{9}\right)$, which have a total length of $2 \cdot \left(\frac{1}{3}\right)^2$. At the third step, we remove 2^2 intervals, each of length $\left(\frac{1}{3}\right)^3$. In general, at the nth step we remove 2^{n-1} intervals, each of length $\left(\frac{1}{3}\right)^n$, for a length of $2^{n-1} \cdot \left(\frac{1}{3}\right)^n = \frac{1}{3}\left(\frac{2}{3}\right)^{n-1}$. Thus, the total length of all removed intervals is $\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^{n-1} = \frac{1/3}{1-2/3} = 1$ [geometric series with $a = \frac{1}{3}$ and $r = \frac{2}{3}$]. Notice that at the nth step, the leftmost interval that is removed is $\left(\left(\frac{1}{3}\right)^n, \left(\frac{2}{3}\right)^n\right)$, so we never remove 0, and 0 is in the Cantor set. Also, the rightmost interval removed is $\left(1 - \left(\frac{2}{3}\right)^n, 1 - \left(\frac{1}{3}\right)^n\right)$, so 1 is never removed. Some other numbers in the Cantor set are $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$.

(b) The area removed at the first step is $\frac{1}{9}$; at the second step, $8 \cdot \left(\frac{1}{9}\right)^2$; at the third step, $(8)^2 \cdot \left(\frac{1}{9}\right)^3$. In general, the area removed at the nth step is $(8)^{n-1}\left(\frac{1}{9}\right)^n = \frac{1}{9}\left(\frac{8}{9}\right)^{n-1}$, so the total area of all removed squares is

$$\sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{8}{9}\right)^{n-1} = \frac{1/9}{1 - 8/9} = 1.$$

66. (a)

a_1	1	2	4	1	1	1000
a_2	2	3	1	4	1000	1
a_3	1.5	2.5	2.5	2.5	500.5	500.5
a_4	1.75	2.75	1.75	3.25	750.25	250.75
a_5	1.625	2.625	2.125	2.875	625.375	375.625
a_6	1.6875	2.6875	1.9375	3.0625	687.813	313.188
a_7	1.65625	2.65625	2.03125	2.96875	656.594	344.406
a_8	1.67188	2.67188	1.98438	3.01563	672.203	328.797
a_9	1.66406	2.66406	2.00781	2.99219	664.398	336.602
a_{10}	1.66797	2.66797	1.99609	3.00391	668.301	332.699
a_{11}	1.66602	2.66602	2.00195	2.99805	666.350	334.650
a_{12}	1.66699	2.66699	1.99902	3.00098	667.325	333.675

The limits seem to be $\frac{5}{3}$, $\frac{8}{3}$, 2, 3, 667, and 334. Note that the limits appear to be "weighted" more toward a_2. In general, we guess that the limit is $\dfrac{a_1 + 2a_2}{3}$.

(b) $a_{n+1} - a_n = \frac{1}{2}(a_n + a_{n-1}) - a_n = -\frac{1}{2}(a_n - a_{n-1}) = -\frac{1}{2}\left[\frac{1}{2}(a_{n-1} + a_{n-2}) - a_{n-1}\right]$

$= -\frac{1}{2}\left[-\frac{1}{2}(a_{n-1} - a_{n-2})\right] = \cdots = \left(-\frac{1}{2}\right)^{n-1}(a_2 - a_1)$

Note that we have used the formula $a_k = \frac{1}{2}(a_{k-1} + a_{k-2})$ a total of $n - 1$ times in this calculation, once for each k between 3 and $n + 1$. Now we can write

$$a_n = a_1 + (a_2 - a_1) + (a_3 - a_2) + \cdots + (a_{n-1} - a_{n-2}) + (a_n - a_{n-1})$$

$$= a_1 + \sum_{k=1}^{n-1}(a_{k+1} - a_k) = a_1 + \sum_{k=1}^{n-1}\left(-\tfrac{1}{2}\right)^{k-1}(a_2 - a_1)$$

and so

$$\lim_{n\to\infty} a_n = a_1 + (a_2 - a_1)\sum_{k=1}^{\infty}\left(-\tfrac{1}{2}\right)^{k-1} = a_1 + (a_2 - a_1)\left[\frac{1}{1 - (-1/2)}\right] = a_1 + \tfrac{2}{3}(a_2 - a_1) = \frac{a_1 + 2a_2}{3}.$$

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67. (a) For $\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$, $s_1 = \frac{1}{1\cdot 2} = \frac{1}{2}$, $s_2 = \frac{1}{2} + \frac{2}{1\cdot 2\cdot 3} = \frac{5}{6}$, $s_3 = \frac{5}{6} + \frac{3}{1\cdot 2\cdot 3\cdot 4} = \frac{23}{24}$,

$s_4 = \frac{23}{24} + \frac{4}{1\cdot 2\cdot 3\cdot 4\cdot 5} = \frac{119}{120}$. The denominators are $(n+1)!$, so a guess would be $s_n = \frac{(n+1)!-1}{(n+1)!}$.

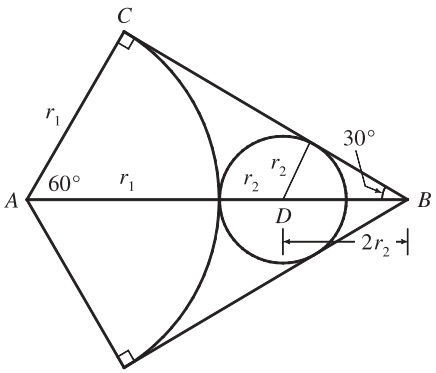
(b) For $n = 1$, $s_1 = \frac{1}{2} = \frac{2!-1}{2!}$, so the formula holds for $n = 1$. Assume $s_k = \frac{(k+1)!-1}{(k+1)!}$. Then

$$s_{k+1} = \frac{(k+1)!-1}{(k+1)!} + \frac{k+1}{(k+2)!} = \frac{(k+1)!-1}{(k+1)!} + \frac{k+1}{(k+1)!(k+2)} = \frac{(k+2)!-(k+2)+k+1}{(k+2)!}$$
$$= \frac{(k+2)!-1}{(k+2)!}$$

Thus, the formula is true for $n = k+1$. So by induction, the guess is correct.

(c) $\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{(n+1)!-1}{(n+1)!} = \lim_{n\to\infty} \left[1 - \frac{1}{(n+1)!}\right] = 1$ and so $\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$.

68.



Let r_1 = radius of the large circle, r_2 = radius of next circle, and so on. From the figure we have $\angle BAC = 60°$ and $\cos 60° = r_1/|AB|$, so $|AB| = 2r_1$ and $|DB| = 2r_2$. Therefore, $2r_1 = r_1 + r_2 + 2r_2 \Rightarrow r_1 = 3r_2$. In general, we have $r_{n+1} = \frac{1}{3}r_n$, so the total area is

$$A = \pi r_1^2 + 3\pi r_2^2 + 3\pi r_3^2 + \cdots = \pi r_1^2 + 3\pi r_2^2\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots\right)$$
$$= \pi r_1^2 + 3\pi r_2^2 \cdot \frac{1}{1 - 1/9} = \pi r_1^2 + \tfrac{27}{8}\pi r_2^2$$

Since the sides of the triangle have length 1, $|BC| = \frac{1}{2}$ and $\tan 30° = \frac{r_1}{1/2}$. Thus, $r_1 = \frac{\tan 30°}{2} = \frac{1}{2\sqrt{3}} \Rightarrow r_2 = \frac{1}{6\sqrt{3}}$,

so $A = \pi\left(\frac{1}{2\sqrt{3}}\right)^2 + \frac{27\pi}{8}\left(\frac{1}{6\sqrt{3}}\right)^2 = \frac{\pi}{12} + \frac{\pi}{32} = \frac{11\pi}{96}$. The area of the triangle is $\frac{\sqrt{3}}{4}$, so the circles occupy about 83.1% of the area of the triangle.

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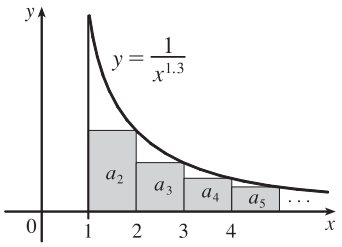
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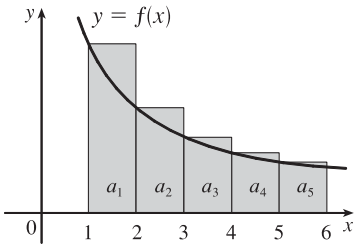
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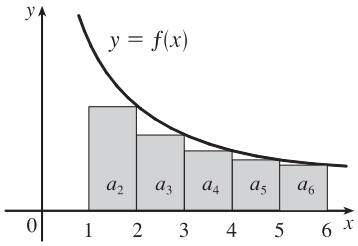
8.3 The Integral and Comparison Tests; Estimating Sums

1. The picture shows that $a_2 = \dfrac{1}{2^{1.3}} < \displaystyle\int_1^2 \frac{1}{x^{1.3}}\,dx$, $a_3 = \dfrac{1}{3^{1.3}} < \displaystyle\int_2^3 \frac{1}{x^{1.3}}\,dx$, and so on, so $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{1.3}} < \int_1^{\infty} \frac{1}{x^{1.3}}\,dx$. The integral converges by (5.10.2) with $p = 1.3 > 1$, so the series converges.



2. From the first figure, we see that $\int_1^6 f(x)\,dx < \displaystyle\sum_{i=1}^{5} a_i$. From the second figure, we see that $\displaystyle\sum_{i=2}^{6} a_i < \int_1^6 f(x)\,dx$. Thus, we have $\displaystyle\sum_{i=2}^{6} a_i < \int_1^6 f(x)\,dx < \sum_{i=1}^{5} a_i$.





3. (a) We cannot say anything about $\sum a_n$. If $a_n > b_n$ for all n and $\sum b_n$ is convergent, then $\sum a_n$ could be convergent or divergent. (See the note after Example 2.)

(b) If $a_n < b_n$ for all n, then $\sum a_n$ is convergent. [This is part (i) of the Comparison Test.]

4. (a) If $a_n > b_n$ for all n, then $\sum a_n$ is divergent. [This is part (ii) of the Comparison Test.]

(b) We cannot say anything about $\sum a_n$. If $a_n < b_n$ for all n and $\sum b_n$ is divergent, then $\sum a_n$ could be convergent or divergent.

5. $\displaystyle\sum_{n=1}^{\infty} n^b$ is a p-series with $p = -b$. $\displaystyle\sum_{n=1}^{\infty} b^n$ is a geometric series. By (1), the p-series is convergent if $p > 1$. In this case, $\displaystyle\sum_{n=1}^{\infty} n^b = \sum_{n=1}^{\infty} \left(1/n^{-b}\right)$, so $-b > 1 \quad \Leftrightarrow \quad b < -1$ are the values for which the series converge. A geometric series $\displaystyle\sum_{n=1}^{\infty} ar^{n-1}$ converges if $|r| < 1$, so $\displaystyle\sum_{n=1}^{\infty} b^n$ converges if $|b| < 1 \quad \Leftrightarrow \quad -1 < b < 1$.

6. The function $f(x) = 1/x^5$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$$\int_1^{\infty} \frac{1}{x^5}\,dx = \lim_{t\to\infty} \int_1^t x^{-5}\,dx = \lim_{t\to\infty} \left[\frac{x^{-4}}{-4}\right]_1^t = \lim_{t\to\infty} \left(-\frac{1}{4t^4} + \frac{1}{4}\right) = \frac{1}{4}.$$

Since this improper integral is convergent, the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$ is also convergent by the Integral Test.

7. The function $f(x) = 1/\sqrt[5]{x} = x^{-1/5}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$\int_1^{\infty} x^{-1/5}\,dx = \lim_{t\to\infty} \int_1^t x^{-1/5}\,dx = \lim_{t\to\infty} \left[\frac{5}{4}x^{4/5}\right]_1^t = \lim_{t\to\infty} \left(\frac{5}{4}t^{4/5} - \frac{5}{4}\right) = \infty$, so $\displaystyle\sum_{n=1}^{\infty} 1/\sqrt[5]{n}$ diverges.

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8. The function $f(x) = 1/\sqrt{x+4} = (x+4)^{-1/2}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$\int_1^\infty (x+4)^{-1/2}\,dx = \lim_{t\to\infty} \int_1^t (x+4)^{-1/2}\,dx = \lim_{t\to\infty} \left[2(x+4)^{1/2}\right]_1^t = \lim_{t\to\infty} \left(2\sqrt{t+4} - 2\sqrt{5}\right) = \infty$, so the series $\sum_{n=1}^{\infty} 1/\sqrt{n+4}$ diverges.

9. $\dfrac{n}{2n^3+1} < \dfrac{n}{2n^3} = \dfrac{1}{2n^2} < \dfrac{1}{n^2}$ for all $n \geq 1$, so $\sum_{n=1}^{\infty} \dfrac{n}{2n^3+1}$ converges by comparison with $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$, which converges because it is a p-series with $p = 2 > 1$.

10. $\dfrac{n^3}{n^4-1} > \dfrac{n^3}{n^4} = \dfrac{1}{n}$ for all $n \geq 2$, so $\sum_{n=2}^{\infty} \dfrac{n^3}{n^4-1}$ diverges by comparison with $\sum_{n=2}^{\infty} \dfrac{1}{n}$, which diverges because it is a p-series with $p = 1 \leq 1$ (the harmonic series).

11. The series $\sum_{n=1}^{\infty} \dfrac{1}{n^{0.85}}$ is a p-series with $p = 0.85 \leq 1$, so it diverges by (1). Therefore, the series $\sum_{n=1}^{\infty} \dfrac{2}{n^{0.85}}$ must also diverge, for if it converged, then $\sum_{n=1}^{\infty} \dfrac{1}{n^{0.85}}$ would have to converge [by Theorem 8(i) in Section 8.2].

12. $\sum_{n=1}^{\infty} n^{-1.4}$ and $\sum_{n=1}^{\infty} n^{-1.2}$ are p-series with $p > 1$, so they converge by (1). Thus, $\sum_{n=1}^{\infty} 3n^{-1.2}$ converges by Theorem 8(i) in Section 8.2. It follows from Theorem 8(ii) that the given series $\sum_{n=1}^{\infty} (n^{-1.4} + 3n^{-1.2})$ also converges.

13. $1 + \dfrac{1}{8} + \dfrac{1}{27} + \dfrac{1}{64} + \dfrac{1}{125} + \cdots = \sum_{n=1}^{\infty} \dfrac{1}{n^3}$. This is a p-series with $p = 3 > 1$, so it converges by (1).

14. $1 + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{3\sqrt{3}} + \dfrac{1}{4\sqrt{4}} + \dfrac{1}{5\sqrt{5}} + \cdots = \sum_{n=1}^{\infty} \dfrac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \dfrac{1}{n^{3/2}}$. This is a p-series with $p = \frac{3}{2} > 1$, so it converges by (1).

15. $f(x) = xe^{-x}$ is continuous and positive on $[1, \infty)$. $f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1-x) < 0$ for $x > 1$, so f is decreasing on $[1, \infty)$. Thus, the Integral Test applies.

$$\int_1^\infty xe^{-x}\,dx = \lim_{b\to\infty} \int_1^b xe^{-x}\,dx = \lim_{b\to\infty} \left[-xe^{-x} - e^{-x}\right]_1^b \quad \text{[by parts]} \quad = \lim_{b\to\infty} \left[-be^{-b} - e^{-b} + e^{-1} + e^{-1}\right] = 2/e$$

since $\lim_{b\to\infty} be^{-b} = \lim_{b\to\infty} (b/e^b) \overset{\text{H}}{=} \lim_{b\to\infty} (1/e^b) = 0$ and $\lim_{b\to\infty} e^{-b} = 0$. Thus, $\sum_{n=1}^{\infty} ne^{-n}$ converges.

16. $f(x) = \dfrac{x^2}{x^3+1}$ is continuous and positive on $[2, \infty)$, and also decreasing since $f'(x) = \dfrac{x(2-x^3)}{(x^3+1)^2} < 0$ for $x \geq 2$, so we can use the Integral Test [note that f is *not* decreasing on $[1, \infty)$].

$\displaystyle\int_2^\infty \frac{x^2}{x^3+1}\,dx = \lim_{t\to\infty} \left[\tfrac{1}{3}\ln(x^3+1)\right]_2^t = \tfrac{1}{3} \lim_{t\to\infty} \left[\ln(t^3+1) - \ln 9\right] = \infty$, so the series $\sum_{n=2}^{\infty} \dfrac{n^2}{n^3+1}$ diverges, and so does the given series, $\sum_{n=1}^{\infty} \dfrac{n^2}{n^3+1}$.

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17. $f(x) = \dfrac{1}{x \ln x}$ is continuous and positive on $[2, \infty)$, and also decreasing since $f'(x) = -\dfrac{1 + \ln x}{x^2 (\ln x)^2} < 0$ for $x > 2$, so we can use the Integral Test. $\displaystyle\int_2^\infty \frac{1}{x \ln x}\, dx = \lim_{t\to\infty} [\ln(\ln x)]_2^t = \lim_{t\to\infty} [\ln(\ln t) - \ln(\ln 2)] = \infty$, so the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ diverges.

18. $\dfrac{1}{n^2+9} < \dfrac{1}{n^2}$, so the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+9}$ converges by comparison with the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ $[p = 2 > 1]$.

Another solution: The function $f(x) = 1/(x^2+9)$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies. $\displaystyle\int_1^\infty \frac{1}{x^2+9}\, dx = \lim_{t\to\infty} \int_1^t \frac{1}{x^2+9}\, dx = \lim_{t\to\infty} \left[\frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right)\right]_1^t = \frac{1}{3} \lim_{t\to\infty} \left[\tan^{-1}\left(\frac{t}{3}\right) - \tan^{-1}\left(\frac{1}{3}\right)\right] =$ $\dfrac{1}{3}\left[\dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{1}{3}\right)\right]$. Since the integral converges, so does the series.

19. $\dfrac{\cos^2 n}{n^2+1} \le \dfrac{1}{n^2+1} < \dfrac{1}{n^2}$, so the series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ converges by comparison with the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ $[p = 2 > 1]$.

20. $\dfrac{n^2-1}{3n^4+1} < \dfrac{n^2}{3n^4+1} < \dfrac{n^2}{3n^4} = \dfrac{1}{3}\dfrac{1}{n^2}$. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{3n^4+1}$ converges by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^2}$, which converges because it is a constant multiple of a convergent p-series $[p = 2 > 1]$. The terms of the given series are positive for $n > 1$, which is good enough.

21. $\dfrac{n-1}{n\,4^n}$ is positive for $n > 1$ and $\dfrac{n-1}{n\,4^n} < \dfrac{n}{n\,4^n} = \dfrac{1}{4^n} = \left(\dfrac{1}{4}\right)^n$, so $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n\,4^n}$ converges by comparison with the convergent geometric series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$.

22. $\dfrac{4+3^n}{2^n} > \dfrac{3^n}{2^n} = \left(\dfrac{3}{2}\right)^n$ for all $n \ge 1$, so $\displaystyle\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$ diverges by comparison with the divergent geometric series $\displaystyle\sum_{n=1}^{\infty} \left(\tfrac{3}{2}\right)^n$.

23. $1 + \dfrac{1}{3} + \dfrac{1}{5} + \dfrac{1}{7} + \dfrac{1}{9} + \cdots = \displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$. The function $f(x) = \dfrac{1}{2x-1}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$\displaystyle\int_1^\infty \frac{1}{2x-1}\, dx = \lim_{t\to\infty} \int_1^t \frac{1}{2x-1}\, dx = \lim_{t\to\infty} \left[\tfrac{1}{2} \ln|2x-1|\right]_1^t = \tfrac{1}{2} \lim_{t\to\infty} (\ln(2t-1) - 0) = \infty$, so the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges.

24. $\dfrac{1}{5} + \dfrac{1}{8} + \dfrac{1}{11} + \dfrac{1}{14} + \dfrac{1}{17} + \cdots = \displaystyle\sum_{n=1}^{\infty} \frac{1}{3n+2}$. The function $f(x) = \dfrac{1}{3x+2}$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies.

$\displaystyle\int_1^\infty \frac{1}{3x+2}\, dx = \lim_{t\to\infty} \int_1^t \frac{1}{3x+2}\, dx = \lim_{t\to\infty} \left[\tfrac{1}{3} \ln|3x+2|\right]_1^t = \tfrac{1}{3} \lim_{t\to\infty} (\ln(3t+2) - \ln 5) = \infty$, so the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n+2}$ diverges.

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25. Use the Limit Comparison Test with $a_n = \dfrac{1+4^n}{1+3^n}$ and $b_n = \dfrac{4^n}{3^n}$:

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\dfrac{1+4^n}{1+3^n}}{\dfrac{4^n}{3^n}} = \lim_{n\to\infty}\frac{1+4^n}{1+3^n}\cdot\frac{3^n}{4^n} = \lim_{n\to\infty}\frac{1+4^n}{4^n}\cdot\frac{3^n}{1+3^n} = \lim_{n\to\infty}\left(\frac{1}{4^n}+1\right)\cdot\frac{1}{\dfrac{1}{3^n}+1} = 1 > 0$$

Since the geometric series $\sum b_n = \sum \left(\frac{4}{3}\right)^n$ diverges, so does $\sum_{n=1}^{\infty}\dfrac{1+4^n}{1+3^n}$. Alternatively, use the Comparison Test with

$\dfrac{1+4^n}{1+3^n} > \dfrac{1+4^n}{3^n+3^n} > \dfrac{4^n}{2(3^n)} = \dfrac{1}{2}\left(\dfrac{4}{3}\right)^n$ or use the Test for Divergence.

26. $\dfrac{1}{\sqrt{n^3+1}} < \dfrac{1}{\sqrt{n^3}} = \dfrac{1}{n^{3/2}}$, so $\sum_{n=1}^{\infty}\dfrac{1}{\sqrt{n^3+1}}$ converges by comparison with the convergent p-series

$\sum_{n=1}^{\infty}\dfrac{1}{n^{3/2}}$ $\quad\left[p = \frac{3}{2} > 1\right]$.

27. $\dfrac{2+(-1)^n}{n\sqrt{n}} \le \dfrac{3}{n\sqrt{n}}$, and $\sum_{n=1}^{\infty}\dfrac{3}{n\sqrt{n}}$ converges because it is a constant multiple of the convergent p-series $\sum_{n=1}^{\infty}\dfrac{1}{n\sqrt{n}}$

$\left[p = \frac{3}{2} > 1\right]$, so the given series converges by the Comparison Test.

28. $\dfrac{1+\sin n}{10^n} \le \dfrac{2}{10^n}$ and $\sum_{n=0}^{\infty}\dfrac{2}{10^n} = 2\sum_{n=0}^{\infty}\left(\dfrac{1}{10}\right)^n$, so the given series converges by comparison with a constant multiple of a convergent geometric series.

29. Use the Limit Comparison Test with $a_n = \sin\left(\dfrac{1}{n}\right)$ and $b_n = \dfrac{1}{n}$. Then $\sum a_n$ and $\sum b_n$ are series with positive terms and

$\lim_{n\to\infty}\dfrac{a_n}{b_n} = \lim_{n\to\infty}\dfrac{\sin(1/n)}{1/n} = \lim_{\theta\to 0}\dfrac{\sin\theta}{\theta} = 1 > 0$. Since $\sum_{n=1}^{\infty} b_n$ is the divergent harmonic series,

$\sum_{n=1}^{\infty}\sin(1/n)$ also diverges. [Note that we could also use l'Hospital's Rule to evaluate the limit:

$\lim_{x\to\infty}\dfrac{\sin(1/x)}{1/x} \overset{\text{H}}{=} \lim_{x\to\infty}\dfrac{\cos(1/x)\cdot(-1/x^2)}{-1/x^2} = \lim_{x\to\infty}\cos\dfrac{1}{x} = \cos 0 = 1$.]

30. If $a_n = \dfrac{n^2-5n}{n^3+n+1}$ and $b_n = \dfrac{1}{n}$, then $\lim_{n\to\infty}\dfrac{a_n}{b_n} = \lim_{n\to\infty}\dfrac{n^3-5n^2}{n^3+n+1} = \lim_{n\to\infty}\dfrac{1-5/n}{1+1/n^2+1/n^3} = 1 > 0$,

so $\sum_{n=1}^{\infty}\dfrac{n^2-5n}{n^3+n+1}$ diverges by the Limit Comparison Test with the divergent harmonic series $\sum_{n=1}^{\infty}\dfrac{1}{n}$.

(Note that $a_n > 0$ for $n \ge 6$.)

31. We have already shown (in Exercise 17) that when $p = 1$ the series $\sum_{n=2}^{\infty}\dfrac{1}{n(\ln n)^p}$ diverges, so assume that $p \ne 1$.

$f(x) = \dfrac{1}{x(\ln x)^p}$ is continuous and positive on $[2,\infty)$, and $f'(x) = -\dfrac{p+\ln x}{x^2(\ln x)^{p+1}} < 0$ if $x > e^{-p}$, so that f is eventually decreasing and we can use the Integral Test.

$$\int_2^{\infty}\frac{1}{x(\ln x)^p}\,dx = \lim_{t\to\infty}\left[\frac{(\ln x)^{1-p}}{1-p}\right]_2^t \quad [\text{for } p\ne 1] = \lim_{t\to\infty}\left[\frac{(\ln t)^{1-p}}{1-p} - \frac{(\ln 2)^{1-p}}{1-p}\right]$$

This limit exists whenever $1-p < 0 \iff p > 1$, so the series converges for $p > 1$.

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32. (a) $f(x) = 1/x^4$ is positive and continuous and $f'(x) = -4/x^5$ is negative for $x > 0$, and so the Integral Test applies.

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \approx s_{10} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots + \frac{1}{10^4} \approx 1.082037.$$

$$R_{10} \le \int_{10}^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \left[\frac{1}{-3x^3}\right]_{10}^{t} = \lim_{t\to\infty} \left(-\frac{1}{3t^3} + \frac{1}{3\,(10)^3}\right) = \frac{1}{3000}, \text{ so the error is at most } 0.000\overline{3}.$$

(b) $s_{10} + \displaystyle\int_{11}^{\infty} \frac{1}{x^4}\,dx \le s \le s_{10} + \int_{10}^{\infty} \frac{1}{x^4}\,dx \quad \Rightarrow \quad s_{10} + \frac{1}{3(11)^3} \le s \le s_{10} + \frac{1}{3(10)^3} \quad \Rightarrow$

$1.082037 + 0.000250 = 1.082287 \le s \le 1.082037 + 0.000333 = 1.082370$, so we get $s \approx 1.08233$ with error ≤ 0.00005.

(c) $R_n \le \displaystyle\int_n^{\infty} \frac{1}{x^4}\,dx = \frac{1}{3n^3}$. So $R_n < 0.00001 \quad \Rightarrow \quad \frac{1}{3n^3} < \frac{1}{10^5} \quad \Rightarrow \quad 3n^3 > 10^5 \quad \Rightarrow \quad n > \sqrt[3]{(10)^5/3} \approx 32.2,$

that is, for $n > 32$.

33. (a) $f(x) = \dfrac{1}{x^2}$ is positive and continuous and $f'(x) = -\dfrac{2}{x^3}$ is negative for $x > 0$, and so the Integral Test applies.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \approx s_{10} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{10^2} \approx 1.549768.$$

$$R_{10} \le \int_{10}^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \left[\frac{-1}{x}\right]_{10}^{t} = \lim_{t\to\infty} \left(-\frac{1}{t} + \frac{1}{10}\right) = \frac{1}{10}, \text{ so the error is at most } 0.1.$$

(b) $s_{10} + \displaystyle\int_{11}^{\infty} \frac{1}{x^2}\,dx \le s \le s_{10} + \int_{10}^{\infty} \frac{1}{x^2}\,dx \quad \Rightarrow \quad s_{10} + \frac{1}{11} \le s \le s_{10} + \frac{1}{10} \quad \Rightarrow$

$1.549768 + 0.090909 = 1.640677 \le s \le 1.549768 + 0.1 = 1.649768$, so we get $s \approx 1.64522$ (the average of 1.640677 and 1.649768) with error ≤ 0.005 (the maximum of $1.649768 - 1.64522$ and $1.64522 - 1.640677$, rounded up).

(c) $R_n \le \displaystyle\int_n^{\infty} \frac{1}{x^2}\,dx = \frac{1}{n}$. So $R_n < 0.001$ if $\dfrac{1}{n} < \dfrac{1}{1000} \quad \Leftrightarrow \quad n > 1000.$

34. $f(x) = 1/x^5$ is positive and continuous and $f'(x) = -5/x^6$ is negative for $x > 0$, and so the Integral Test applies. Using (3),

$R_n \le \displaystyle\int_n^{\infty} x^{-5}\,dx = \lim_{t\to\infty} \left[\frac{-1}{4x^4}\right]_n^{t} = \frac{1}{4n^4}$. If we take $n = 5$, then $s_5 \approx 1.036662$ and $R_5 \le 0.0004$. So $s \approx s_5 \approx 1.037$.

35. $f(x) = 1/(2x+1)^6$ is continuous, positive, and decreasing on $[1, \infty)$, so the Integral Test applies. Using (3),

$R_n \le \displaystyle\int_n^{\infty} (2x+1)^{-6}\,dx = \lim_{t\to\infty} \left[\frac{-1}{10(2x+1)^5}\right]_n^{t} = \frac{1}{10(2n+1)^5}$. To be correct to five decimal places, we want

$$\frac{1}{10(2n+1)^5} \le \frac{5}{10^6} \quad \Leftrightarrow \quad (2n+1)^5 \ge 20{,}000 \quad \Leftrightarrow \quad n \ge \tfrac{1}{2}\left(\sqrt[5]{20{,}000} - 1\right) \approx 3.12, \text{ so use } n = 4.$$

$$s_4 = \sum_{n=1}^{4} \frac{1}{(2n+1)^6} = \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \frac{1}{9^6} \approx 0.001\,446 \approx 0.00145.$$

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36. $f(x) = \dfrac{1}{x(\ln x)^2}$ is positive and continuous and $f'(x) = -\dfrac{\ln x + 2}{x^2(\ln x)^3}$ is negative for $x > 1$, so the Integral Test applies.

Using (3), we need $0.01 > \displaystyle\int_n^\infty \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty}\left[\frac{-1}{\ln x}\right]_n^t = \frac{1}{\ln n}$. This is true for $n > e^{100}$, so we would have to take this many terms, which would be problematic because $e^{100} \approx 2.7 \times 10^{43}$.

37. $\displaystyle\sum_{n=1}^{10} \frac{1}{\sqrt{n^4+1}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{17}} + \frac{1}{\sqrt{82}} + \cdots + \frac{1}{\sqrt{10{,}001}} \approx 1.24856$. Now $\dfrac{1}{\sqrt{n^4+1}} < \dfrac{1}{\sqrt{n^4}} = \dfrac{1}{n^2}$, so the error is

$$R_{10} \le T_{10} \le \int_{10}^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{x}\right]_{10}^t = \lim_{t\to\infty}\left(-\frac{1}{t} + \frac{1}{10}\right) = \frac{1}{10} = 0.1.$$

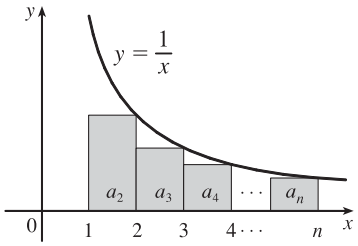
38. $\displaystyle\sum_{n=1}^{10} \frac{\sin^2 n}{n^3} = \frac{\sin^2 1}{1} + \frac{\sin^2 2}{8} + \frac{\sin^2 3}{27} + \cdots + \frac{\sin^2 10}{1000} \approx 0.83253$. Now $\dfrac{\sin^2 n}{n^3} \le \dfrac{1}{n^3}$, so the error is

$$R_{10} \le T_{10} \le \int_{10}^\infty \frac{1}{x^3}\,dx = \lim_{t\to\infty}\left[-\frac{1}{2x^2}\right]_{10}^t = \lim_{t\to\infty}\left(-\frac{1}{2t^2} + \frac{1}{200}\right) = \frac{1}{200} = 0.005.$$

39. (a) From the figure, $a_2 + a_3 + \cdots + a_n \le \int_1^n f(x)\,dx$, so with

$$f(x) = \frac{1}{x},\ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} \le \int_1^n \frac{1}{x}\,dx = \ln n.$$

Thus, $s_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + \cdots + \dfrac{1}{n} \le 1 + \ln n$.



(b) By part (a), $s_{10^6} \le 1 + \ln 10^6 \approx 14.82 < 15$ and

$s_{10^9} \le 1 + \ln 10^9 \approx 21.72 < 22$.

40. $\displaystyle\sum_{n=1}^{\infty} n^{-1.001} = \sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$ is a convergent p-series with $p = 1.001 > 1$. Using (3), we get

$$R_n \le \int_n^\infty x^{-1.001}\,dx = \lim_{t\to\infty}\left[\frac{x^{-0.001}}{-0.001}\right]_n^t = -1000\lim_{t\to\infty}\left[\frac{1}{x^{0.001}}\right]_n^t = -1000\left(-\frac{1}{n^{0.001}}\right) = \frac{1000}{n^{0.001}}.$$

We want $R_n < 0.000\,000\,005 \iff \dfrac{1000}{n^{0.001}} < 5 \times 10^{-9} \iff n^{0.001} > \dfrac{1000}{5 \times 10^{-9}} \iff$

$n > \left(2 \times 10^{11}\right)^{1000} = 2^{1000} \times 10^{11{,}000} \approx 1.07 \times 10^{301} \times 10^{11{,}000} = 1.07 \times 10^{11{,}301}$.

41. Since $\dfrac{d_n}{10^n} \le \dfrac{9}{10^n}$ for each n, and since $\displaystyle\sum_{n=1}^{\infty} \frac{9}{10^n}$ is a convergent geometric series $\left(|r| = \frac{1}{10} < 1\right)$, $0.d_1d_2d_3\ldots = \displaystyle\sum_{n=1}^{\infty} \frac{d_n}{10^n}$ will always converge by the Comparison Test.

42. First we observe that, by l'Hospital's Rule, $\displaystyle\lim_{x\to0} \frac{\ln(1+x)}{x} = \lim_{x\to0} \frac{1}{1+x} = 1$. Also, if $\sum a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$ by Theorem 8.2.6. Therefore, $\displaystyle\lim_{n\to\infty} \frac{\ln(1+a_n)}{a_n} = \lim_{x\to0} \frac{\ln(1+x)}{x} = 1 > 0$. We are given that $\sum a_n$ is convergent and $a_n > 0$. Thus, $\sum \ln(1+a_n)$ is convergent by the Limit Comparison Test.

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43. Yes. Since $\sum a_n$ is a convergent series with positive terms, $\lim_{n\to\infty} a_n = 0$ by Theorem 8.2.6, and $\sum b_n = \sum \sin(a_n)$ is a series with positive terms (for large enough n). We have $\lim_{n\to\infty} \dfrac{b_n}{a_n} = \lim_{n\to\infty} \dfrac{\sin(a_n)}{a_n} = 1 > 0$ by Theorem 3.3.2. Thus, $\sum b_n$ is also convergent by the Limit Comparison Test.

44. $b^{\ln n} = \left(e^{\ln b}\right)^{\ln n} = \left(e^{\ln n}\right)^{\ln b} = n^{\ln b} = \dfrac{1}{n^{-\ln b}}$. This is a p-series, which converges for all b such that $-\ln b > 1 \quad \Leftrightarrow$ $\ln b < -1 \quad \Leftrightarrow \quad b < e^{-1} \quad \Leftrightarrow \quad b < 1/e$ [with $b > 0$].

45. $\lim_{n\to\infty} na_n = \lim_{n\to\infty} \dfrac{a_n}{1/n}$, so we apply the Limit Comparison Test with $b_n = \dfrac{1}{n}$. Since $\lim_{n\to\infty} na_n > 0$ we know that either both series converge or both series diverge, and we also know that $\sum_{n=1}^{\infty} \dfrac{1}{n}$ diverges [p-series with $p = 1$]. Therefore, $\sum a_n$ must be divergent.

46. For the series $\sum_{n=1}^{\infty} \left(\dfrac{c}{n} - \dfrac{1}{n+1}\right)$,

$$s_n = \sum_{i=1}^{n} \left(\frac{c}{i} - \frac{1}{i+1}\right) = \left(\frac{c}{1} - \frac{1}{2}\right) + \left(\frac{c}{2} - \frac{1}{3}\right) + \left(\frac{c}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{c}{n} - \frac{1}{n+1}\right)$$

$$= \frac{c}{1} + \frac{c-1}{2} + \frac{c-1}{3} + \frac{c-1}{4} + \cdots + \frac{c-1}{n} - \frac{1}{n+1} = c + (c-1)\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\right) - \frac{1}{n+1}$$

Thus, $\sum_{n=1}^{\infty} \left(\dfrac{c}{n} - \dfrac{1}{n+1}\right) = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \left[c + (c-1)\sum_{i=2}^{n} \dfrac{1}{i} - \dfrac{1}{n+1}\right]$. Since a constant multiple of a divergent series is divergent, the last limit exists only if $c - 1 = 0$, so the original series converges only if $c = 1$.

8.4 Other Convergence Tests

1. (a) An alternating series is a series whose terms are alternately positive and negative.

(b) An alternating series $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$, where $b_n = |a_n|$, converges if $0 < b_{n+1} \le b_n$ for all n and $\lim_{n\to\infty} b_n = 0$. (This is the Alternating Series Test.)

(c) The error involved in using the partial sum s_n as an approximation to the total sum s is the remainder $R_n = s - s_n$ and the size of the error is smaller than b_{n+1}; that is, $|R_n| \le b_{n+1}$. (This is the Alternating Series Estimation Theorem.)

2. (a) Since $\lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 8 > 1$, part (b) of the Ratio Test tells us that the series $\sum a_n$ is divergent.

(b) Since $\lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 0.8 < 1$, part (a) of the Ratio Test tells us that the series $\sum a_n$ is absolutely convergent (and therefore convergent).

(c) Since $\lim_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test fails and the series $\sum a_n$ might converge or it might diverge.

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