Fundamental Mathematics and Physics of Medical Imaging 1st edition Lancaster Solutions Manual

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OPTIONAL HOMEWORK

Download the CR image (CR.nii.gz) using the link to my dropbox. Mango treats this image as a positive image (high exposure is brightest), but you can invert the greyscale by switching to the Greyscale (inverted) color table if you prefer. Image values do not change. Use Mango's analysis tools with the following problems:

Problem 1. Assuming the maximum exposure in the CR image was 100 mR and the minimum exposure was 0.1 mR, use the maximum and minimum pixel values in the image to estimate C1 and C2 in the equation below. This equation gives the theoretical relationship between pixel values (PVs) in the CR image and the corresponding exposure (E) at the CR detector.

$$PV = C1 \log_{10}(E) + C2$$

Solution:

There are two equations with two unknowns: One equation for PV1 and E1 and the other for PV2 and E2. PV1 = 63,342 is the maximum in the image and E1 = 100 mR. Since this is a positive image increasing exposure leads to increasing PVs unlike film density. Likewise PV2 = 19,044 and R2 = 0.1 mR. Solving the two equations gives

C1 = 14,766
C2 = 33,810

Problem 2. Use the above equation to estimate the exposure for a region of interest associated with fat and a region of interest associated with muscle midway above the knee?

Solution:

Rearranging the above equation leads to an equation for E

$$E = 10^{(PV-C2)/C1}$$

Enter separate ROIs for fat and muscle then use ROI stats to get mean PVs for the ROIs.

For fat (red ROI)	*mean PV = 50019*	*E-fat =*	*12.5… mR*
Fro muscle (green ROI)	*mean PV = 36994*	*E-muscle =*	*1.64… mR*

Problem 3. Measure the widest lateral extent of the knee for the Femur and Tibia bones. Give results in mm.

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Solution:

Femur (blue ROI) *length = 89.0… mm*
Tibia (violet ROI) *length = 82.0… mm*

Lines were drawn using Mango's Trace Line tool. Clicked endpoints only to ensure straight-line distances.

Problem 4. What is the horizontal and vertical field of view (in inches) of the CR image?

Solution:

From Mango's image info the image dimension is 1886x1886 pixels so the FOV is square (ignoring the edge of the collimator), and the spacing is 0.159 mm/pixel. The field of view is therefore

299.874 mm x 299.874 mm
11.8 inches x 11.8 inches *using 25.4 mm/inch*

Document results in a word doc and send them to me before class. Indicate positions of regions of interest used in your pics.

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HOMEWORK

1. The optical density difference between two regions of an x-ray film is 0.7. What is the ratio of the intensity of transmitted light between these two regions? Assuming that the film-screen has a gamma of 0.76, what is the ratio of the radiation exposure between these two regions?

Given: $D_2 - D_1 = 0.7$ $\gamma = 0.76$

Part 1. What is the difference in the intensity of transmitted light in the two regions (D_2 and D_1)?

Solution:

1. *Known* $D = \log (I_0/I)$
2. *So* $D_2 - D_1 = \log (I_0/I_2) - \log (I_0/I_1)$
   $D_2 - D_1 = \log(I_0) - \log(I_2) - \log(I_0) + \log(I_1)$
   $D_2 - D_1 = \log(I_1/I_2)$
3. *Therefore* $0.7 = \log(I_1/I_2)$

$$I_1/I_2 = 10^{0.7}$$
$$I_1/I_2 = 5.01...$$

Discussion: Here D2 is larger than D1 and transmitted light relationship is such that in region 1 the intensity is ~ 5 times that in region 2.

Part 2. What is the relative difference in radiation exposure for the two regions?

Solution:

The important equation is the one that determines the slope or gamma (γ) of the H&D curve. We will assume that D1 and D2 fall within the linear portion of the H&D curve where gamma can be approximated using a two-point estimate.

$$\gamma = \frac{\Delta D}{\Delta Log(E)} \approx \frac{D2-D1}{Log(E_2)-Log(E_1)}.$$

1. $\log(E_2/E_1) = (D_2-D_1)/\gamma$
2. $\log(E_2/E_1) = 0.7/0.76 = 0.921...$
3. $E_2/E_1 = 10^{0.921}$

$$E_2/E_1 = 8.34...$$

Discussion: Though the ratio of exposures was 8.34, the ratio of transmitted light for the two regions was only 5.01.

More generally $\gamma = \dfrac{\log\left(\frac{I_1}{I_2}\right)}{\log\left(\frac{E_2}{E_1}\right)}$ *and if γ=1 then the light intensity ratio matches the exposure ratio.*

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Likewise if $\gamma > 1$ the light intensity ratio is greater than the exposure ratio and vice versa for $\gamma < 1$. The γ parameter determines "transmitted light contrast" relative to the "exposure contrast".

2. Describe the process of latent image generation and recovery for a photostimulable phosphor plate digital radiographic system. Why is the dynamic range of these image receptors much higher than that of film-screen systems?

Solution:

a. A latent image is formed in the PSP plate during exposure by x-ray interactions that promote electrons from the valence band to the conduction band of the phosphor. Some electrons return promptly to the valence band while others are captured in traps between the valence and conduction band. These traps are the result of the impurities (such as Europium) in the screen (Barium Fluorohalide). The residency of electrons in these metastable states is relatively long (half-life >20 min), but eventually all would return to the equilibrium condition. It is the trapped electrons that form the latent image.

The latent image is recovered using a high intensity red laser to stimulate electrons to move from the relatively shallow traps to the conduction band. Many of these will transition to the valence band and in the process give off a shorter wavelength light close to UV (blue light). The intensity of the laser's red light is sufficient to continuously promote those electrons that fall back into traps back to the conduction band where many transition back to the valence band and give off blue light. An optical band-reject filter is used to reject the laser's red light while passing the blue light along to an optical detector where it is converted to an electrical signal and digitized by and ADC. The laser is scanned back and forth across the FOV of the plate and its x-y position along with the digitized signal is used to make a 2-D digital image.

b. Unlike phosphor plates used in PSP systems films have a thin layer of molecules that that can produce density changes following exposure. The non-linearity and limited dynamic range of a film is documented by its H&D curve. The H&D curve shows that film requires a certain level of exposure before entering its linear range. Also, as exposure increases the film reaches a saturation level earlier than the screen in a film-screen system due to it's the limited number of exposable molecules per unit area in the film emulsion.

3. Estimate (without integration) the reduction in scatter produced by (a) 1-mm collimated (fan beam) x-ray scanning system (like in Fig 2-9), with 1cm x 25 cm detector array; (b) conventional broad beam exposure and film-screen detector (no grid and film screen size of 25cm x 25cm), and c) scanned 1-mm collimated fan beam as in part 'a' but with stationary film-screen from part 'b'. Assume that the object is 20x20x20-cm phantom and that the entire phantom is imaged.

Solution:

a. Assume isotropic scatter from within the collimated beam, and most will miss the detector. Estimate fraction of scatter directed toward the detector from a point ½ way into the object at 10 cm away (have to use the farthest point for the area of the sphere = 16 cm).

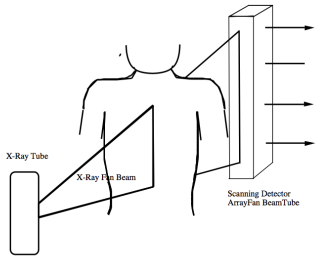


Figure 2-9. Scanning Detector System.

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Scatter fraction toward detector = (area of detector)/(area of sphere at 16 cm)

$$f = \frac{250mm\ x\ 10mm}{4\pi(160mm)^2} = \frac{2500}{321699} \approx 0.008$$

So ~1% of the scatter from the fan beam will reach the detector. This is higher closer to the detector and lower farther away.

b. Again assume isotropic emission of scatter from within the object, but now more will be intercepted by the large area film-screen. We use the same approach where fraction of scatter reaching the film from a position in the center of the object was used as an estimate for locations within the object.

Scatter fraction toward detector = (area of detector)/(area of sphere at 16 cm)

$$f = \frac{(250mm)^2}{4\pi(160mm)^2} = \frac{62500}{321699} \approx 0.19$$

So the fraction of scatter with film-screen and no grid is ~ 20%, which is roughly the ratio of the areas of the detectors. Of course this fraction decreases with increasing distance from the object to the film and increases closer to the film.

c. A point in the fan beam has the same fraction as in part 'b', which is due to the area of the detector, so it is important to have the both a collimated fan beam and a narrow detector as seen in part 'a'.

4. The relationship between pixel value and exposure for a photostimulable phosphor or flat panel detector can be modeled using the following equation:

   Pixel Value = $C_1 \log_{10}(E) + C_2$ where E is the exposure in mR and C_1 & C_2 are constants.

   You make the following measurements in an attempt to determine the relationship between pixel values and exposure:

Pixel Value	1019	738	485	260	7
Exposure (mR)	100	10	1	0.1	0.01

   While the exposure was measured accurately the pixel values were a bit noisy.

   a. Use a least square error fit method to estimate the values of C_1 and C_2.

   b. Plot the raw data and the fitted data on a graph of pixel value vs. $\log_{10}(E)$.

   c. What is the R^2 value for the fit? What is the standard error of the estimate provided by the fit?

Solution: See excel file.

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Chapter 2 Problem 4

c1 = 250.2

c2= 501.8

Exposure (mR)	Log10(Exp)	Pixel Value	Model	error^2
100	2	1019	1002.2	282.24
10	1	738	752.0	196
1	0	485	501.8	282.24
0.1	-1	260	251.6	70.56
0.01	-2	7	1.4	31.36

Using excel trendline with linear fit 862.4 SSE

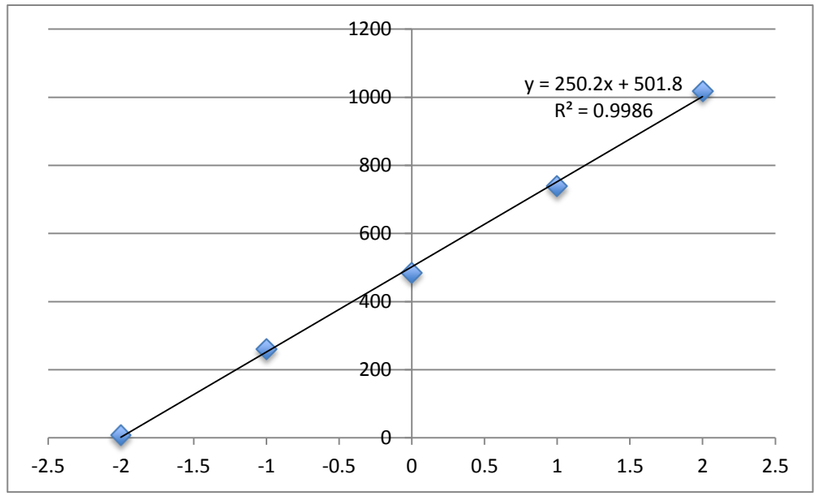
2.4(a) **C1 = 250.2**

C2 = 501.8

2.4c **R^2 = 0.99862** **2.4c**

Std Err of Estimate = 13.1

2.4(b)



$$\sigma_{est} = \sqrt{\frac{\sum (Y - Y')^2}{N}}$$