Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

SKU: 9781498776554-SOLUTIONS

Chapter 2. Equations of Motion

1. Determine the lift coefficient of an aircraft with a 180 ft^2 wing area and a mass of 3,200 kg in a cruising flight when flying at sea level with a speed of

a. 80 knot

b. 130 knot.

$S1 := 180 \cdot ft^2$ $m1 := 3200 \cdot kg$ $g = 9.807 m \cdot s^{-2}$ $\rho_o := 1.225 \cdot \frac{kg}{m^3}$

- 80 knot

$V1 := 80 \cdot knot$ $V1 = 41.156 \frac{m}{s}$

$C_L := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V1^2 \cdot S1} = 1.809$ (Equ 2.4)

- 130 knot

$V1 := 130 \cdot knot$ $V1 = 66.878 \frac{m}{s}$

$C_L := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V1^2 \cdot S1} = 0.685$ (Equ 2.4)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

2. An aircraft with a mass of 1,200 kg and a wing area of 14 m^2 is cruising at 3,000 ft altitude. Determine its lift coefficient when the true air speed is 100 knot.

$S1 := 14 \cdot m^2$ $m1 := 1200 \cdot kg$ $g = 9.807 \frac{m}{s^2}$ $h := 3000 \cdot ft$

$\rho_3 := 0.002175 \cdot \frac{slug}{ft^3}$ $\rho_3 = 1.121 \frac{kg}{m^3}$

$V1 := 100 \cdot knot$ $V1 = 51.444 \frac{m}{s}$

$C_L := \frac{2 \cdot m1 \cdot g}{\rho_3 \cdot V1^2 \cdot S1} = 0.567$ (Equ 2.4)

---

3. Assume that the aircraft in problem 2 has drag coefficient of 0.05. How much thrust the engine is producing?

$S1 := 14 \cdot m^2$ $m1 := 1200 kg$ $g = 9.807 \frac{m}{s^2}$ $h := 3000 ft$ $C_D := 0.05$

$\rho_3 := 0.002175 \frac{slug}{ft^3}$ $\rho_3 = 1.121 \frac{kg}{m^3}$

$V1 := 100 \cdot knot$ $V1 = 51.444 \frac{m}{s}$

$D := \frac{1}{2} \cdot \rho_3 \cdot V1^2 \cdot S1 \cdot C_D = 1038.3 N$ (Equ 2.5)

$T1 := D = 1038.3 N$ (Equ 2.17)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

4. Determine lift curve slope (in 1/rad) of a wing with aspect ratio of 12.5. Then, calculate the lift coefficient of this wing when its angle of attack is 5 degrees. Assume that the zero lift angle of attack is zero, and a_o is 2π 1/rad.

$AR := 12.5$ $\alpha := 5 \cdot deg$ $a_o := 2 \cdot \pi$ $\alpha_o := 0$

$$C_{L\alpha} := \frac{a_o}{1 + \frac{a_o}{\pi \cdot AR}} = 5.417 \qquad \text{(Equ 2.13)}$$

$$C_{L\alpha} = \frac{C_L}{(\alpha - \alpha_o)} \qquad C_L := C_{L\alpha} \cdot (\alpha - \alpha_o) = 0.473 \qquad \text{(Equ 2.12)}$$

---

5. Calculate the true and equivalent stall speeds of the aircraft in problem 2, when the maximum lift coefficient is 1.6.

$S1 := 14 \cdot m^2$ $m1 := 1200 \cdot kg$ $g = 9.807 \cdot \frac{m}{s^2}$ $h := 3000 \cdot ft$ $C_{Lmax} := 1.6$

$\rho_3 := 0.002175 \cdot \frac{slug}{ft^3}$ $\rho_3 = 1.121 \cdot \frac{kg}{m^3}$ $\rho_o := 1.225 \cdot \frac{kg}{m^3}$

$$V_s := \sqrt{\frac{2 \cdot m1 \cdot g}{(\rho_3 \cdot C_{Lmax} \cdot S1)}} = 30.616 \frac{m}{s} \qquad V_s = 59.513 knot \qquad \text{(Equ 2.51)} \qquad \text{True}$$

$$\sigma := \frac{\rho_3}{\rho_o} = 0.915 \qquad \text{(Equ 1.25)}$$

$$V_T = \frac{V_E}{\sqrt{\sigma}} \qquad V_E := V_s \cdot \sqrt{\sigma} = 29.287 \frac{m}{s} \qquad V_E = 56.929 knot \qquad \text{(Equ 2.49)} \qquad \text{Equivalent}$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

6. An aircraft is required to climb with a 10 degrees of climb angle. The aircraft has a mass of 30,000 kg and producing 50,000 N of drag. Assume zero angle of attack and zero thrust setting angle.

a. How much lift this aircraft must generate?
b. How much thrust the aircraft engine must produce?

$\gamma := 10 \cdot deg$ $m1 := 30000 \cdot kg$ $D1 := 50000 \cdot N$ $\alpha := 0$ $i_T := 0$

$W1 := m1 \cdot g = 294200 \cdot N$

$L1 := W1 \cdot \cos(\gamma) = 289730 \cdot N$ (Equ 2.33)

$T1 := D1 + W1 \cdot \sin(\gamma) = 101087 \cdot N$ (Equ 2.32)

---

7. An aircraft which is initially at rest is accelerating on a runway with an acceleration of 10 m/sec^2. Consider a moment when other features of this aircraft are:

$S = 30\ m^2$, $m = 6{,}000$ kg, $C_L = 0.7$, $C_D = 0.1$, $V = 60$ knot

Calculate the engine thrust, assuming that the friction force is constant and equal to 2% of the aircraft weight.

$S1 := 30 \cdot m^2$ $m1 := 6000 \cdot kg$ $C_L := 0.7$ $C_D := 0.1$ $a1 := 10 \cdot \frac{m}{s^2}$

$\rho := 1.225 \frac{kg}{m^3}$ $V1 := 60 \cdot knot$ $V1 = 30.9 \frac{m}{s}$

$W1 := m1 \cdot g = 58839.9 \cdot N$

$F_f := 0.02 \cdot W1 = 1176.8 \cdot N$

$D1 := \frac{1}{2} \cdot \rho \cdot (V1)^2 \cdot S1 \cdot C_D = 1750.7 \cdot N$ (Equ 2.5)

$T1 := m1 \cdot a1 + D1 + F_f$ (Equ 2.34)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

8. A cargo aircraft with a weight of 145,000 lb and wing area of 1,318 ft^2 has a maximum lift coefficient of 2.5. Is this aircraft able to cruise at an altitude of 25,000 ft and ISA+15 condition with a speed of 150 KTAS?

$W1 := 145000\,lbf$ $S1 := 1318\,ft^2$ $C_{Lmax} := 2.5$ $h := 25000\,ft$ ISA+15

$V1 := 150\,knot$ $L1 := 2\cdot\frac{K}{1000\,ft}$ Chapter 1 $R1 := 287\cdot\frac{J}{kg\cdot K}$

Sea level: $P_o := 101325\,Pa$ $T_o := (15 + 15 + 273)\cdot K = 303\,K$

25000 ft: $T_{25} := T_o - L1\cdot h = 253\,K$ (Equ 1.6)

Appendix B: $P_{25} := 786.3\frac{lbf}{ft^2}$

$\rho_{25} := \frac{P_{25}}{R1\cdot(T_{25})} = 0.518\frac{kg}{m^3}$ $\rho_{25} = 0.00101\frac{slug}{ft^3}$ (Equ 1.23)

$V_{s15} := \sqrt{\frac{2\cdot W1}{\rho_{25}\cdot S1\cdot C_{Lmax}}} = 90.153\frac{m}{s}$ $V_{s15} = 175.243\,knot$ (Equ 2.51)

This aircraft is not able to cruise at an altitude of 25,000 ft and ISA+15 condition with a speed of 150 KTAS, since this speed is less than the stall speed at that altitude.

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

9. A hang glider (*Nimbus*) has a mass (structure plus pilot) of 138 kg and wing area of 16.2 m^2 and stall speed of 16 knot. What is the maximum lift coefficient?

$S1 := 16.2 \cdot m^2$ $m1 := 138 \cdot kg$ $g = 9.807 \cdot \frac{m}{s^2}$ $\rho_o := 1.225 \cdot \frac{kg}{m^3}$

$V_s := 16 \cdot knot$ $V_s = 8.231 \cdot \frac{m}{s}$

$$C_{Lmax} := \frac{2 \cdot m1 \cdot g}{\rho_o \cdot V_s^2 \cdot S1} = 2.013 \qquad \text{(Equ 2.51)}$$

---

10. Calculate the wing area of a hang glider *Volmer VJ-23*. The aircraft geometry and weight information may be taken from Table 2.2. If the pilot mass is 75 kg, what is the mass of the aircraft structure?

$g = 9.807 \cdot \frac{m}{s^2}$ $\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $m_P := 75 \cdot kg$

From Table 2.2, row 4: $V_s := 13 \cdot knot$ $C_{Lmax} := 2.9$ $m1 := 136 \cdot kg$

$$S1 := \frac{2 \cdot m1 \cdot g}{\left(\rho_o \cdot C_{Lmax} \cdot V_s^2\right)} = 16.616 m^2 \qquad \text{(Equ 2.51)}$$

$m_{st} := m1 - m_P = 61 kg$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

11. The sport aircraft *Butterworth* has the following characteristic

$$m = 635\ kg,\ S = 10.4\ m^2,\ V_s = 56\ knot\ (at\ sea\ level)$$

Assume that the maximum speed of this aircraft at every altitude is 126 knot (TAS). At what altitude maximum true airspeed and stall true airspeed will be the same?

$m1 := 635 \cdot kg$ $S1 := 10.4 \cdot m^2$ $V_s := 56 \cdot knot$ $\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $V_{s_alt} := 126 \cdot knot$

$$C_{Lmax} := \frac{(2 \cdot m1 \cdot g)}{\left(\rho_o \cdot S1 \cdot V_s^2\right)} = 1.178 \qquad \text{(Equ 2.51)}$$

Aircraft CLmax is constant.

$$\rho := \frac{2 \cdot m1 \cdot g}{S1 \cdot C_{Lmax} \cdot V_{s_alt}^2} = 0.242 \frac{kg}{m^3} \qquad \text{(Equ 2.51)}$$

From Appendix A, this air density belongs to an altitude of 13700 m or 45000 ft.

---

12. The cargo aircraft *C-130* has an empty mass of 13,000 kg, a wing area of 85 m^2, and a stall speed of 94 knot (EAS). If maximum lift coefficient is 2.2, determine maximum the mass of payload (cargo and crew) plus fuel to satisfy this stall speed.

$m_E := 13000 \cdot kg$ $S1 := 85 \cdot m^2$ $V_s := 94 \cdot knot$ $\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $C_{Lmax} := 2.2$

$$m \cdot g = \frac{1}{2} \cdot \rho \cdot V_s^2 \cdot S \cdot C_{Lmax} \qquad \text{(Equ 2.51)}$$

$$m_{TO} := \frac{\rho_o \cdot V_s^2 \cdot S1 \cdot C_{Lmax}}{2 \cdot g} = 27312\,kg$$

$$m_{pf} := m_{TO} - m_E = 14312\,kg$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

13. The bomber *B-1B* has a maximum take-off mass of 216,367 kg, a 181 m^2 wing area, and a maximum velocity of Mach 2.2. Assume drag coefficient of this aircraft at cruise is 0.03, how much thrust the four engines are generating for this flight condition?

$m1 := 216367 \cdot kg$ $S1 := 181 \cdot m^2$ $g = 9.807 \cdot \frac{m}{s^2}$ $C_D := 0.0$ $M1 := 2.$

$\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $a := 340 \cdot \frac{m}{s}$ speed of sound at sea level

$V1 := M1 \cdot a = 748 \frac{m}{s}$ (Equ 1.35)

$D := \frac{1}{2} \cdot \rho_o \cdot V1^2 \cdot S1 \cdot C_D = 1860840.4\ N$ (Equ 2.5)

$T1 := D = 1860.8 \cdot kN$ (Equ 2.17)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

14. The trainer aircraft *PC-7* with a mass of 2,700 kg and a wing area of 16.6 m^2 has a cruising speed of 330 km/hr.

a. What is the lift coefficient when cruising at 5,000 m altitude, ISA condition?
b. In a summer day, the temperature at sea level is 42 °C. How much lift coefficient must be increased when cruising at this day and at the same altitude?

$m1 := 2700kg$ $S1 := 16.6m^2$ $h := 5000m$ $V_C := 330\cdot\frac{km}{hr}$ $V_C = 178.186knot$ $V_C = 91.667\frac{m}{s}$

a .... 5000 m, ISA $\rho_5 := 0.7364\frac{kg}{m^3}$

$C_{Lc1} := \frac{2\cdot m1\cdot g}{\rho_5\cdot V_C^2\cdot S1} = 0.516$ (Equ 2.4)

b .....Non-ISA

$L1 := 6.5\frac{K}{1000m}$ Chapter 1 $R1 := 287\cdot\frac{J}{kg\cdot K}$

Sea level: $P_o := 101325Pa$ $T_o := (42 + 273)\cdot K = 315K$

5000 m: $T_5 := T_o - L1\cdot h = 282.5K$ (Equ 1.6)

Appendix B: $P_5 := 54048Pa$

$\rho_5 := \frac{P_5}{R1\cdot T_5} = 0.667\frac{kg}{m^3}$ (Equ 1.23)

$C_{Lc2} := \frac{2\cdot m1\cdot g}{\rho_5\cdot V_C^2\cdot S1} = 0.57$ (Equ 2.4)

$\Delta C_L := C_{Lc2} - C_{Lc1} = 0.054$

$\%\Delta C_L := \frac{C_{Lc2} - C_{Lc1}}{C_{Lc1}} = 10.5\%$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

15. A maneuverable aircraft has a mass of 6,800 kg, a wing area of 32 m^2 and a drag coefficient of 0.02. The aircraft is required to climb vertically with the speed of 100 knot. How much thrust the engine needs to produce?

In a vertical climb, T = D + W

$m1 := 6800 \cdot kg$ $S1 := 32 \cdot m^2$ $C_D := 0.02$ $V1 := 100 \cdot knot$ $V1 = 51.444 \frac{m}{s}$

$\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $a := 340 \cdot \frac{m}{s}$ speed of sound at sea level $g = 9.807 \cdot \frac{m}{s^2}$

$W1 := m1 \cdot g = 66685.2 N$

$D := \frac{1}{2} \cdot \rho_o \cdot V1^2 \cdot S1 \cdot C_D = 1037.4 \cdot N$ (Equ 2.5)

$T1 := D + W1 = 67.7 \cdot kN$ (Equ 2.32)

---

16. Calculate the wing area of the aircraft *EMB-121A1*. The aircraft geometry and weight data may be taken from Table 2.2.

From Table 2.2, row 6: $V_s := 76 \cdot knot$ $C_{Lmax} := 2.16$ $m1 := 5670 \cdot kg$

$\rho_o := 1.225 \cdot \frac{kg}{m^3}$ $g = 9.807 \cdot \frac{m}{s^2}$

$m \cdot g = \frac{1}{2} \cdot \rho \cdot V_s^2 \cdot S \cdot C_{Lmax}$ (Equ 2.51)

$S1 := \frac{2 \cdot m1 \cdot g}{\left(\rho_o \cdot C_{Lmax} \cdot V_s^2\right)} = 27.49 m^2$ (Equ 2.51)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

17. Is fighter aircraft *F-14* able to fly vertically? The aircraft data may be taken from table 2.2. Assume drag coefficient to be 0.03.

From Table 2.2, row 21:

$m1 := 33724 \cdot kg$ $C_{Lmax} := 2.9$ $V_s := 115 \cdot knot$ $V_s = 59.161 \frac{m}{s}$ $T_{max} := 2 \cdot 93 \cdot kN$

$W1 := m1 \cdot g = 330.7kN$

$T_{max} = 186kN$

The maximum engine thrust is less than aircraft weight, so, it is unable, even with the lowest possible speed.

---

18. The dynamic pressure of an aircraft that is cruising at an altitude is 9,000 N/m^2.

a. Determine the altitude, if the aircraft speed is 389 KTAS.
b. Calculate aircraft equivalent airspeed in terms of KEAS.

- Altitude

$q := 9000 \frac{N}{m^2}$ $V_T := 389 \cdot knot$ $V_T = 200.119 \frac{m}{s}$ $\rho_o := 1.225 \frac{kg}{m^3}$

$q = \frac{1}{2} \cdot \rho \cdot V^2$ $\rho := \frac{2 \cdot q}{V_T^2} = 0.449 \frac{kg}{m^3}$ (Equ 2.47)

The altitude for this air density is 9,300 m or 30,500 ft.

- Equivalent airspeed in terms of KEAS

$V_E := V_T \cdot \sqrt{\frac{\rho}{\rho_o}} = 235.63knot$ (Equ 2.49)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

19. A transport aircraft is cruising at 20,000 ft altitude with a speed of Mach 0.5. If a 50 m/sec head-wind is blowing, what is the ground speed and true airspeed in terms of knot?

- **Ground speed**

$h := 20000\,ft$ $M1 := 0.5$ $V_W := 50\,\frac{m}{s}$ $V_W = 97.192\,knot$ $R1 := 287\,\frac{J}{kg \cdot K}$ $\gamma := 1.4$

From Appendix B: $T1 := 447.4\,R$ $T1 = 248.556\,K$

$a1 := \sqrt{\gamma \cdot R1 \cdot T1} = 316.022\,\frac{m}{s}$ (Equ 1.34)

$V1 := M1 \cdot a1$ $V1 = 158.011\,\frac{m}{s}$ (Equ 1.35)

$V_G := V1 - V_W = 108.011\,\frac{m}{s}$ $V_G = 210\,knot$

- **True airspeed**

$V_T := V1 = 307.148\,knot$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

20. The aircraft *Voyager* is able to fly around the globe without refueling. In one mission the aircraft is flying at the equator at an altitude of 15,000 ft with the speed of 110 knot. Assume there is a 15 m/sec wind from West to East all the time.

a. How many days does it take to do this mission if cruising from West to East?

b. How many days does it take to do this mission if cruising from East to West?

Note: The Earth has a diameter of 12,800 km.

- From West to East

$h := 15000\,ft$ $h = 4572\,m$ $V_T := 110\,knot$ $V_T = 56.6\,\frac{m}{s}$ $V_w := 15\,\frac{m}{sec}$ $D_E := 12800\,km$

Total Distance traveled; Circumference: $X := \pi \cdot (D_E + 2 \cdot h) = 40241.1\,km$

$V_{G1} := V_T + V_w = 71.589\,\frac{m}{s}$ $V_{G1} = 139.158\,knot$

$V = \frac{X}{t}$ $t1 := \frac{X}{V_{G1}} = 6.506\,day$ $t1 = 5.621 \times 10^5\,s$ $t1 = 156.1\,hr$

- From East to West

$V_{G2} := V_T - V_w = 41.589\,\frac{m}{s}$ $V_{G2} = 80.842\,knot$

$V = \frac{X}{t}$ $t2 := \frac{X}{V_{G2}} = 11.199\,day$ $t2 = 9.676 \times 10^5\,s$ $t2 = 268.8\,hr$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

21. The aircraft *Cessna Citation II* is climbing with a 3 degrees of angle of attack. The geometry and weight data of this aircraft may be taken from Table 2.2.

a. If the drag coefficient is 0.035, determine its climb angle, when climbing with a speed of 160 knot.
b. Determine the ratio between lift to weight at this climbing flight.

- **Climb angle**

From Table 2.2, row 18: $m3 := 6033\,kg$ $V_s := 82\cdot knot$ $C_{Lmax} := 1.73$ $T3 := 2\cdot 11.1\,kN$

$V_{climb} := 160\cdot knot$ $C_D := 0.035$ $\alpha := 3\cdot deg$ $\rho_o := 1.225\,\frac{kg}{m^3}$

$W3 := m3\,g = 59163.5\,N$

$$S3 := \frac{2\cdot W3}{\rho_o\cdot V_s^{\,2}\cdot C_{Lmax}} = 31.376\,m^2 \qquad \text{(Equ 2.51)}$$

$$D3 := \frac{1}{2}\cdot\rho_o\cdot V_{climb}^{\,2}\cdot S3\cdot C_D = 4557.1\,N \qquad \text{(Equ 2.5)}$$

$$T\cdot\cos(\alpha) = D + W\cdot\sin(\gamma) \qquad \text{(Equ 2.32)}$$

$$\gamma := \operatorname{asin}\left(\frac{T3\cdot\cos(\alpha) - D3}{W3}\right) = 0.302\,rad \qquad \gamma = 17.319\,deg$$

- **Ratio between lift to weight**

$$L = W\cdot\cos(\gamma) - T\cdot\sin(\alpha) \qquad \text{(Equ 2.33)}$$

$$L3 := W3\cdot\cos(\gamma) - T3\cdot\sin(\alpha) = 55319.3\,N$$

$$\frac{L3}{W3} = 0.935$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

22. A transport aircraft with a wing area of 200 m^2 is cruising with a speed of Mach 0.6 at 35,000 ft altitude, ISA condition.

a. determine the mass of aircraft, if lift coefficient is 0.24.

b. determine the engine thrust, if drag coefficient is 0.035.

- **Mass of aircraft**

$S1 := 200 \cdot m^2$ $M1 := 0.6$ $h := 35000 ft$ $C_L := 0.24$ $R1 := 287 \cdot \frac{J}{kg \cdot K}$ $\gamma := 1.4$ $C_D := 0.035$

$g = 9.807 \frac{m}{s^2}$

From Appendix B: $T1 := 394.1 R$ $T1 = 218.944 K$ $\rho_{35} := 0.000738 \frac{slug}{ft^3}$ $\rho_{35} = 0.38 \frac{kg}{m^3}$

$a1 := \sqrt{\gamma \cdot R1 \cdot T1} = 296.601 \frac{m}{s}$ (Equ 1.34)

$V1 := M1 \cdot a1$ $V1 = 177.96 \frac{m}{s}$ $V1 = 345.927 knot$ (Equ 1.35)

$m1 := \frac{\rho_{35} \cdot V1^2 \cdot S1}{2 \cdot g} = 122831.2 kg$ (Equ 2.4)

- **Engine thrust**

$D1 := \frac{1}{2} \cdot \rho_{35} \cdot V1^2 \cdot S1 \cdot C_D = 42159.7 N$ (Equ 2.5)

$T1 := D1 = 4.216 \times 10^4 N$ (Equ 2.17)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

23. A transport aircraft with a wing area of 420 m^2 is cruising with a constant speed of 550 knot (KTAS) at 38,000 ft altitude. The aircraft has a mass of 390,000 kg at the beginning of a cruising flight and consumes 150,000 kg of fuel at the end of the cruise. Determine wing angle of attack at the beginning and at the end of the cruise. Also assume:

$$AR = 8.5;\ a_o = 2\pi\ 1/rad;\ \alpha_o = -1\ deg;$$

- **Wing angle of attack at the beginning of the cruise**

$S1 := 420 \cdot m^2$ $V1 := 550 \cdot knot$ $V1 = 282.944 \frac{m}{s}$ $h := 38000 ft$ $m1 := 390000 kg$

$m_f := 150000 kg$ $AR := 8.5$ $a_o := 2 \cdot \pi \cdot \frac{1}{rad}$ $\alpha_o := -1 \cdot deg$

Appendix B: $\rho_{38} := 6.462910^{-4} \cdot \frac{slug}{ft^3}$ $\rho_{38} = 0.333 \frac{kg}{m^3}$

$$C_{L1} := \frac{2 \cdot m1 \cdot g}{\rho_{38} \cdot (V1)^2 \cdot S1} = 0.683 \qquad \text{(Equ 2.4)}$$

$$C_{L\alpha w} := \frac{a_o}{1 + \frac{a_o}{\pi \cdot AR}} = 5.086 \qquad \text{(Equ 2.13)}$$

$$C_{L\alpha} = \frac{C_L}{(\alpha - \alpha_o)} \qquad \alpha_1 := \frac{C_{L1}}{C_{L\alpha w}} + \alpha_o = 0.117 rad \qquad \alpha_1 = 6.693 deg \qquad \text{(Equ 2.12)}$$

- Wing angle of attack at the end of the cruise

b. $$C_{L2} := \frac{2 \cdot \left[(m1 - m_f) g \right]}{\rho_{38} \cdot (V1)^2 \cdot S1} = 0.42$$

$$\alpha_2 := \frac{C_{L2}}{C_{L\alpha w}} + \alpha_o = 0.065 rad \qquad \alpha_2 = 3.734 deg$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

24. The aircraft *Falco 900* is going to take-off from a runway in a winter day (ISA-20). It starts from rest and after a few seconds, when speed reaches 0.5 V_s, friction force is 1% of aircraft weight, and drag coefficient is 0.1.

a. Determine aircraft acceleration for this moment.
b. How long does it take to come to this point? Assume the acceleration is constant during this period.
Note: Aircraft geometry and weight data may be taken from Table 2.2.

- **Acceleration**

From Table 2.2, row 20: $m1 := 20640\,kg$ $V_s := 82\cdot knot$ $C_{Lmax} := 2.25$ $Th := 3\cdot 20\,kN$

$F_f = 0.01\cdot W1$ $C_D := 0.1$ $V1 := 0.5\cdot V_s$ $g = 9.807\,\frac{m}{s^2}$ $R1 := 287\cdot\frac{J}{kg\cdot K}$ $\rho_o := 1.225\,\frac{kg}{m^3}$

From Appendix B: $P1 := 101325\,Pa$

$T1 := (15 - 20 + 273)\cdot K$ (Equ 2.5)

$W1 := m1\cdot g = 202409.3\,N$

$\rho := \frac{P1}{R1\cdot(T1)} = 1.317\,\frac{kg}{m^3}$ (Equ 1.23)

$S1 := \frac{2\cdot W1}{\rho_o\cdot V_s^{\,2}\cdot C_{Lmax}} = 82.535\,m^2$ (Equ 2.51)

$D1 := \frac{1}{2}\cdot\rho\cdot(0.5\,V_s)^2\cdot S1\cdot C_D = 2418.5\,N$ (Equ 2.5)

$F_f := 0.01\cdot W1 = 2024.1\,N$

$a_{TO} := \frac{Th - D1 - F_f}{m1} = 2.692\,\frac{m}{s^2}$ (Equ 2.34)

- **Duration**

$V_{TO} := 0.5\,V_s = 21.092\,\frac{m}{s}$

$Time := \frac{V_{TO}}{a_{TO}} = 7.836\,s$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

25. Repeat problem 24, assuming the aircraft is taking off in a summer day (ISA+20).

- **Acceleration**

From Table 2.2, row 20: $m1 := 20640\,kg$ $V_s := 82\cdot knot$ $C_{Lmax} := 2.25$ $Th := 3\cdot 20\,kN$

$F_f = 0.01\cdot W1$ $C_D := 0.1$ $V1 := 0.5\cdot V_s$ $g = 9.807\frac{m}{s^2}$ $R1 := 287\cdot\frac{J}{kg\cdot K}$ $\rho_o := 1.225\frac{kg}{m^3}$

From Appendix B: $P1 := 101325\,Pa$

$T1 := (15 + 20 + 273)\cdot K = 308\,K$ (Equ 2.5)

$W1 := m1\cdot g = 202409.3\,N$

$\rho := \frac{P1}{R1\cdot(T1)} = 1.146\frac{kg}{m^3}$ (Equ 1.23)

$S1 := \frac{2\cdot W1}{\rho_o\cdot V_s^{\,2}\cdot C_{Lmax}} = 82.535\,m^2$ (Equ 2.51)

$D1 := \frac{1}{2}\cdot\rho\cdot(0.5\,V_s)^2\cdot S1\cdot C_D = 2104.4\,N$ (Equ 2.5)

$F_f := 0.01\cdot W1 = 2024.1\,N$

$a_{TO} := \frac{Th - D1 - F_f}{m1} = 2.707\frac{m}{s^2}$ (Equ 2.34)

- **Duration**

$V_{TO} := 0.5\,V_s = 21.092\frac{m}{s}$

$Time := \frac{V_{TO}}{a_{TO}} = 7.792\,s$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

26. An aircraft that is initially at rest is accelerating on a runway for a take-off operation. When the aircraft speed is 35 KTAS, the acceleration is 10 m/sec^2. Other features of this aircraft at this time are:

$S = 35\ m^2$, $m = 6400\ kg$, $C_L = 0.8$, $C_D = 0.037$.

If the friction coefficient is 0.02, calculate the engine thrust. Assume sea-level ISA condition.

$S1 := 35 \cdot m^2$ $m1 := 6400 kg$ $C_L := 0.8$ $C_D := 0.037$ $V_{TO} := 35 \cdot knot$ $V_{TO} = 18 \frac{m}{s}$

$a := 10 \cdot \frac{m}{s^2}$ $\mu := 0.02$ $g = 9.807 \frac{m}{s^2}$ $\rho_o := 1.225 \frac{kg}{m^3}$

$W1 := m1 \cdot g = 62762.6N$

$L1 := \frac{1}{2} \cdot \rho_o \cdot (V_{TO})^2 \cdot S1 \cdot C_L = 5560N$ (Equ 2.4)

$D1 := \frac{1}{2} \cdot \rho_o \cdot (V_{TO})^2 \cdot S1 \cdot C_D = 257.2N$ (Equ 2.5)

Normal force: $N1 := W1 - L1 = 57202.5N$

$F_f := \mu \cdot N1 = 1144.1N$

$T1 := m1 \cdot a + D1 + F_f = 65.4kN$ (Equ 2.34)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

27. The following aircraft (Figure 2.24) is descending with a constant airspeed and a descent angle of 20 degrees. Aircraft has an angle of attack of 5 degrees, and engines have 3 degrees of setting angle. Draw forces (weight, aerodynamic forces, and engine thrust) on the aircraft and derive the governing equations for this flight phase.

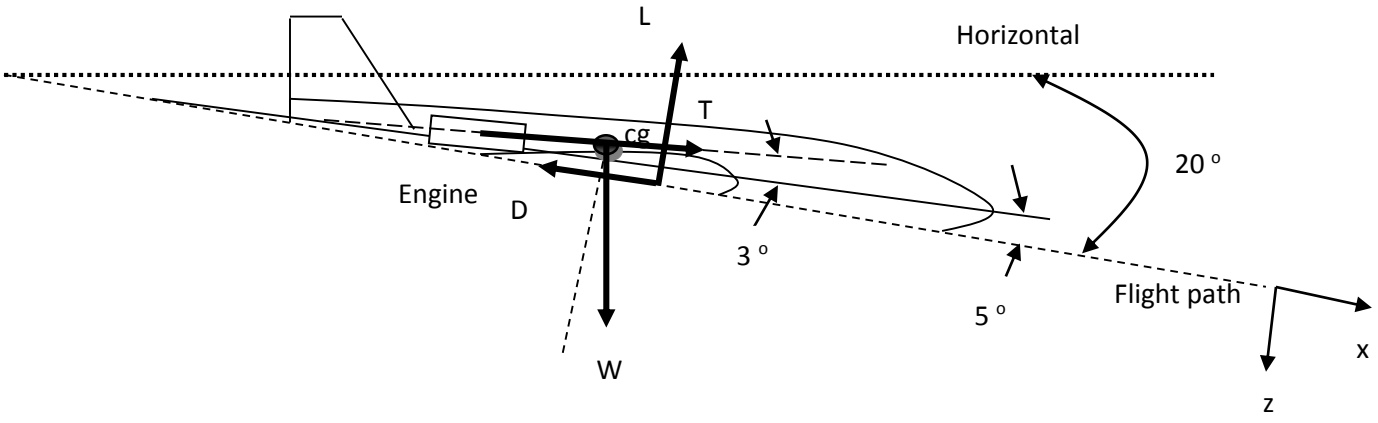


Figure 2.1. An aircraft is descending flight

Governing equations:

$$\sum F_x = 0 \Rightarrow T.\cos(i_t + \alpha) + W.\sin(\gamma) = D$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(i_t + \alpha) = W.\cos(\gamma)$$

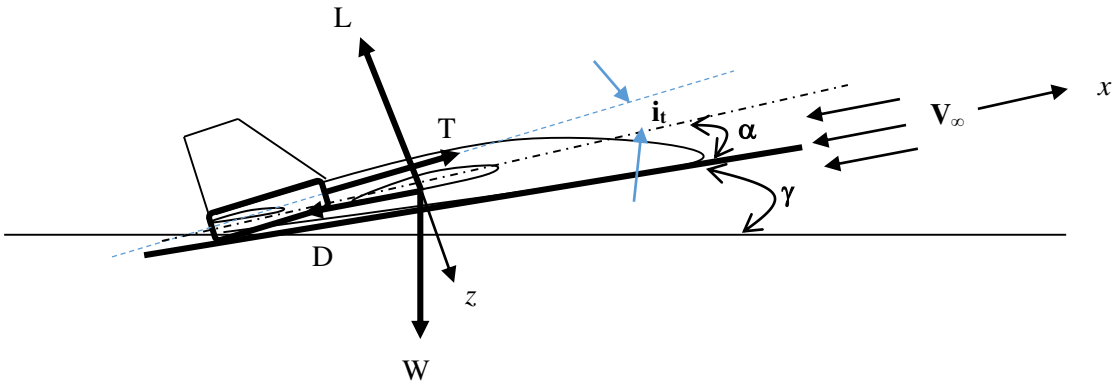
Thus:

$$\sum F_x = 0 \Rightarrow T.\cos(3+5) + W.\sin(20) = D$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(3+5) = W.\cos(20)$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

28. A fighter aircraft is climbing with an arbitrary climb angle. The aircraft has two turbofan engines; both engines have a positive i_t degrees of setting angle. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.



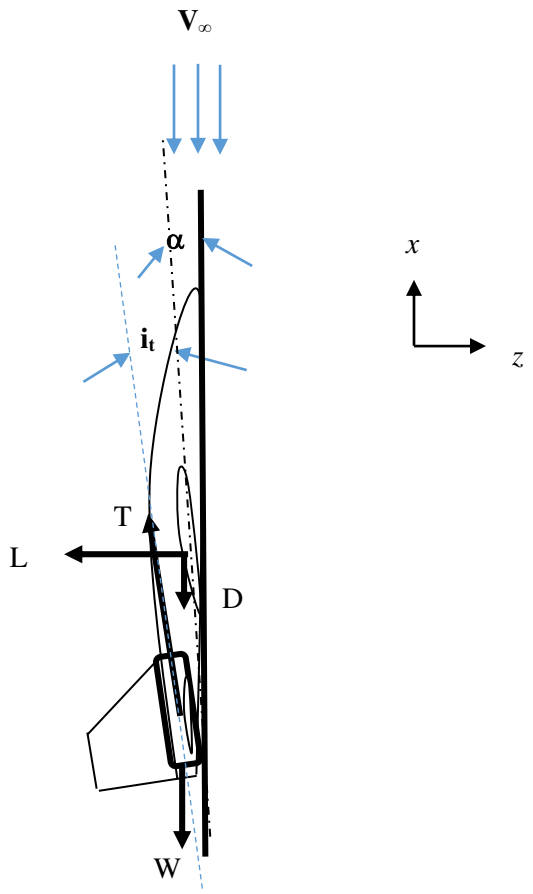
Governing equations:

$$\sum F_x = 0 \Rightarrow T.\cos(i_t + \alpha) = D + W.\sin(\gamma)$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(i_t + \alpha) = W.\cos(\gamma)$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

29. A non-VTOL fighter aircraft is climbing vertically at sea level. The aircraft has two turbofan engines; both engines have a positive i_t degrees of setting angle. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.



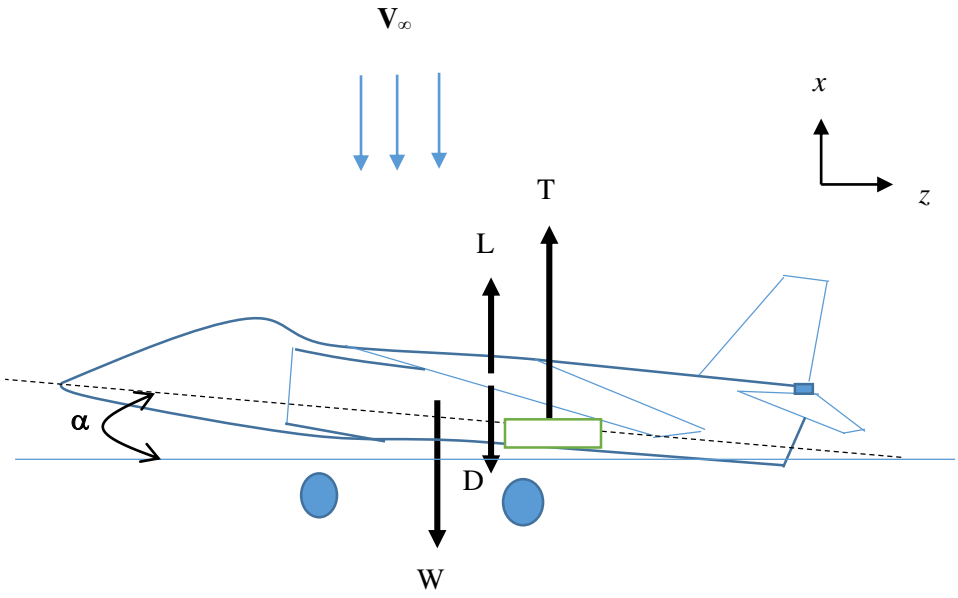
Governing equations:

$$\sum F_x = 0 \Rightarrow T.\cos(i_t + \alpha) = D + W$$

$$\sum F_z = 0 \Rightarrow L + T.\sin(i_t + \alpha) = 0$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

30. A VTOL aircraft is climbing vertically at sea level. The aircraft has two turbofan engines; where during take-off they are arranged such that they produce a thrust which is upward. Draw side-view of the aircraft with an arbitrary angle of attack. Then, derive the governing equations of motion for this climbing flight.



Governing equations:

$$\sum F_x = 0 \Rightarrow T + L = D + W$$

$$\sum F_z = 0$$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

31. A pilot is planning to fly east to west at 5,000 m altitude such that he can watch the sunset for a couple of hours. What must be the flight speed (in Mach number) in order to achieve such objective? Assume there is a 30 knot headwind during this flight. The radius of Earth at sea level is 6400 km.

If the aircraft can fly with the speed of air (i.e., earth) at 5,000 m, the pilot can watch the sunset as long as he/she has fuel to fly.

$h := 5000m$ $V_W := 30 \cdot knot$ $V_W = 15.433 \frac{m}{s}$ $R_E := 6400km$ $\gamma := 1.4$ $R1 := 287 \cdot \frac{J}{kg \cdot K}$

The Earth is revolving around itself in each 24 hours.

$t := 24 \cdot hr$ $t = 8.64 \times 10^4 s$

If the aircraft is flying at sea level, the aircraft velocity should be the same as the earth linear speed at the sea level. For the 5,000 m altitude:

Total Distance traveled; Circumference: $X := 2\pi \cdot (R_E + h) = 40243.8km$

$V_A := \frac{X}{t} = 465.785 \frac{m}{s}$

$V_G := V_A + V_W = 481.218 \frac{m}{s}$

From Appendix A: $T_5 := 255.69K$

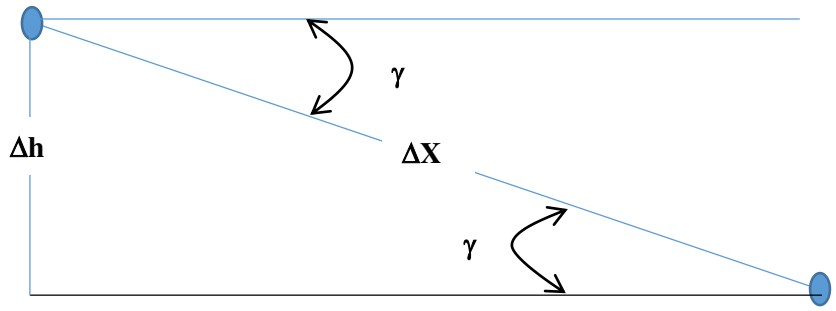
$a1 := \sqrt{\gamma \cdot R1 \cdot T_5} = 320.525 \frac{m}{s}$ (Equ 1.34)

$M1 := \frac{V_G}{a1} = 1.501$ (Equ 1.35)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

32. A transport aircraft Boeing 777 (Figure 7.21) is descending with a velocity of 318 mph at 8,200 ft.

a. If at After one minute, the altitude is 4,000 ft, and the airspeed is 234 mph, determine the average descend angle.
b. If at the touchdown (sea level altitude), the airspeed is 180 mph, determine the average descend angle and deceleration. This phase takes one minute.



- **a……**

$V1 := 318\cdot\frac{mile}{hr}$ $V2 := 234\cdot\frac{mile}{hr}$ $h1 := 8200ft$ $h2 := 4000ft$ $t := 1\cdot min$

$\Delta h := h1 - h2 = 4200ft$ $V_{avg} := \frac{V1 + V2}{2} = 276\frac{mile}{hr}$

Distance traveled $\Delta X := V_{avg}\cdot t = 24288ft$ $\Delta X = 7403m$

$\sin(\gamma) = \frac{\Delta h}{\Delta X}$ $\gamma_{avg} := \operatorname{asin}\left(\frac{\Delta h}{\Delta X}\right)$ $\gamma_{avg} = 9.958deg$

- **b……**

$V1 := 234\cdot\frac{mile}{hr}$ $V2 := 180\cdot\frac{mile}{hr}$ $h1 := 8200ft$ $h2 := 4000ft$ $t := 1\cdot min$

$\Delta h := h1 - h2 = 4200ft$ $V_{avg} := \frac{V1 + V2}{2} = 207\frac{mile}{hr}$

Distance traveled $\Delta X := V_{avg}\cdot t = 18216ft$

$\sin(\gamma) = \frac{\Delta h}{\Delta X}$ $\gamma_{avg} := \operatorname{asin}\left(\frac{\Delta h}{\Delta X}\right)$ $\gamma_{avg} = 13.33deg$

$a := \frac{V1 - V2}{t} = 1.32\frac{ft}{s^2}$ $a = 0.402\frac{m}{s^2}$

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

33. The earth is moving in a circular orbit about the sun, with a radius of 147×10^9 m. The duration of one turn is one year. Determine the velocity of earth in terms of speed of sound (i.e., Mach number) at sea level.

$X_{ES} := 147 \cdot 10^9 \cdot m$ $X_{ES} = 1.47 \times 10^8$ km

$t := 1 \cdot yr$ $t = 3.156 \times 10^7$ s $t = 365.242$ day $t = 8.766 \times 10^3$ hr

Total Distance traveled; Circumference: $X := 2\pi \cdot (X_{ES}) = 9.2 \times 10^8$ km

$V_{Earth} := \frac{X}{t} = 2.927 \times 10^4 \frac{m}{s}$ $V_{Earth} = 29.269 \frac{km}{s}$

Speed of sound: $a := 340 \cdot \frac{m}{s}$

$M1 := \frac{V_{Earth}}{a} = 86.084$

---

34. In 24 Mar 1960, the maximum speed of a Tu-114 – World's fastest propeller-driven aircraft on a 1,000 km closed circuit with payloads of 0 to 25,000 kg was recorded to be 871.38 km/hr. Determine this velocity in terms of Mach number. Assume sea level.

$V_A := 871.38 \frac{km}{hr}$ $V_A = 242.05 \frac{m}{s}$

Speed of sound: $a := 340 \cdot \frac{m}{s}$

$M1 := \frac{V_A}{a} = 0.712$ (Equ 1.35)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

CHAPTER 2

Equations of Motion

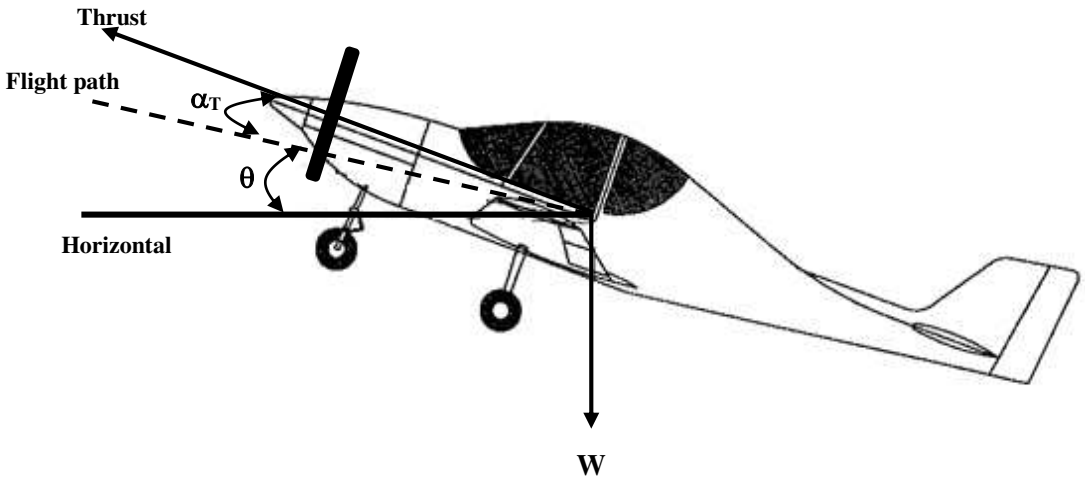


Figure 2.1. An aircraft with two non-aerodynamic forces (weight and thrust)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

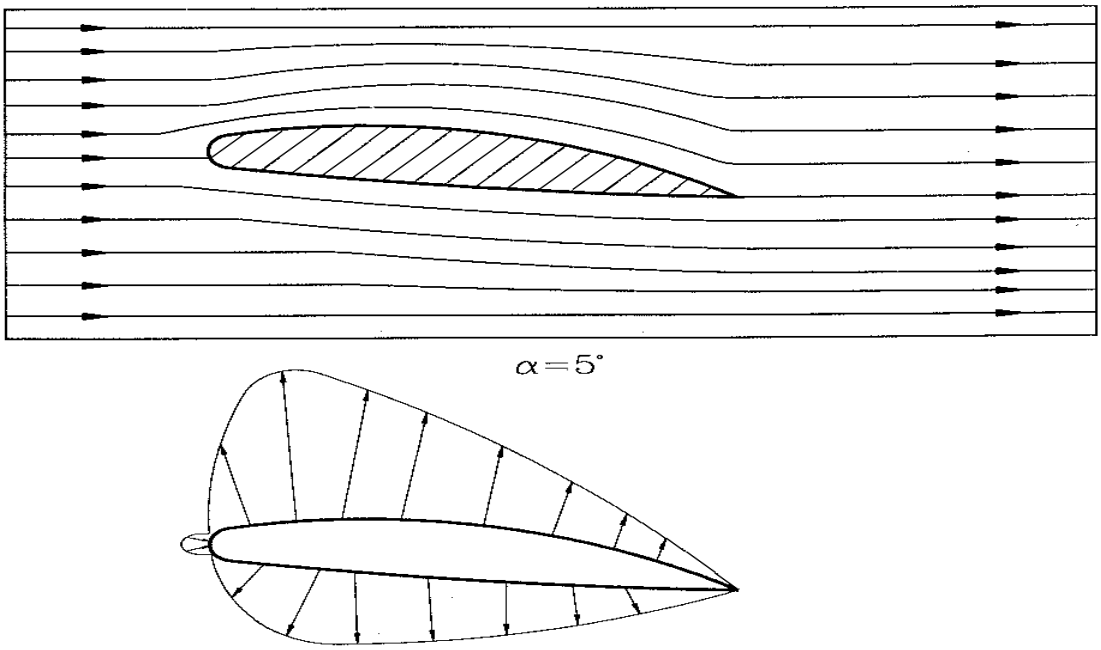


Figure 2.2. A typical pressure distribution over an airfoil with 5 degrees of angle of attack

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

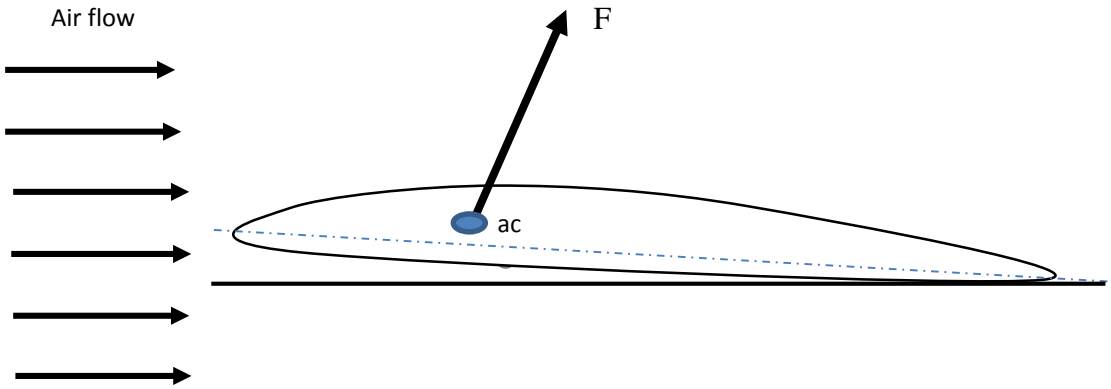
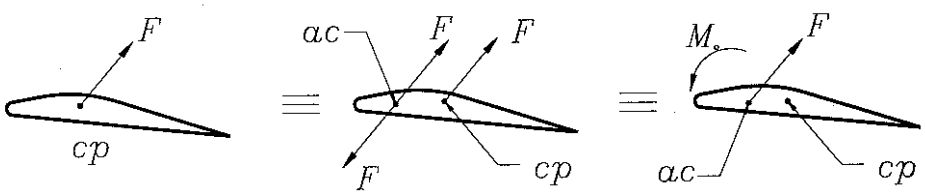


Figure 2.3. The resultant force out of integration of pressure distribution

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual



a. original force b. adding two equal and opposite forces c. resultant force

Figure 2.4. The movement of resultant force to aerodynamic center

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

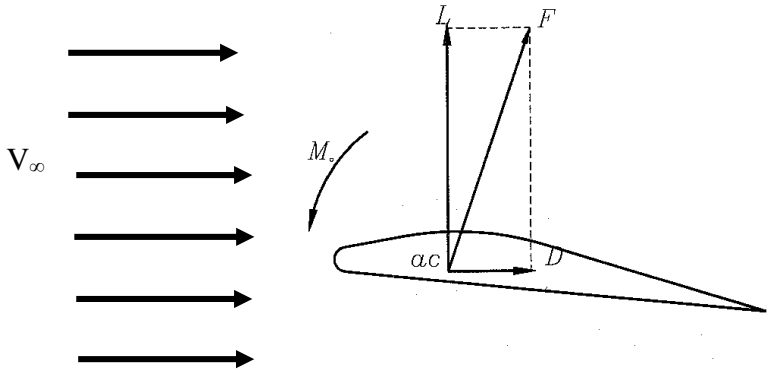


Figure 2.5. The definitions of lift, drag, and pitching moment

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

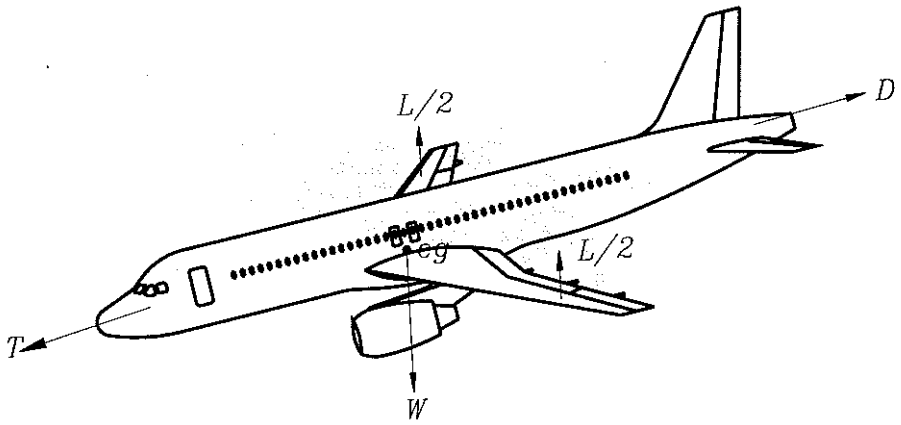


Figure 2.6. The major forces on an airplane

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

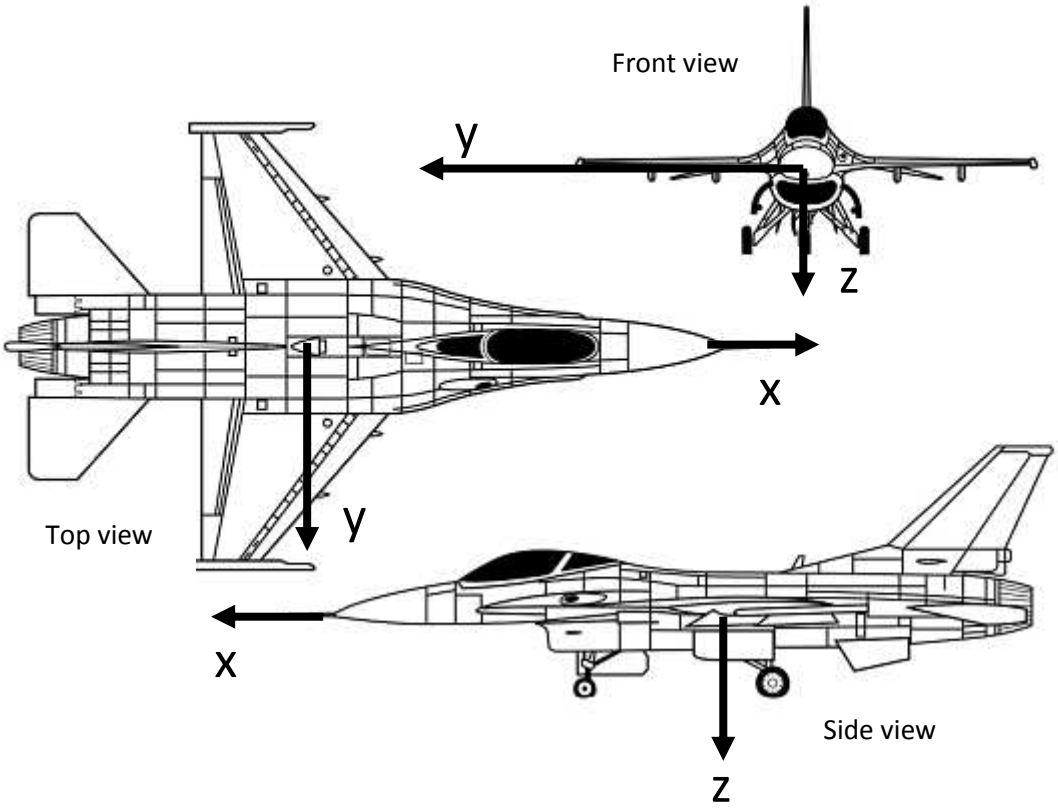


Figure 2.7. Aircraft body-fixed coordinate system (F-16)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

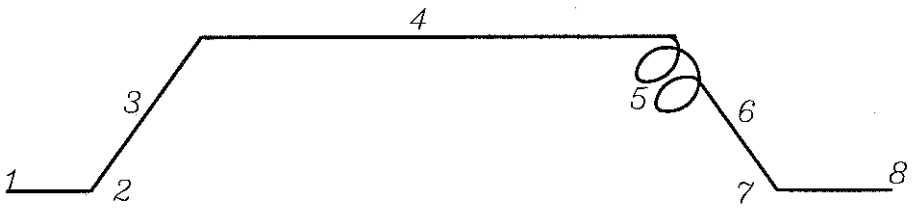


Figure 2.8. A basic and typical flight operation of an aircraft

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

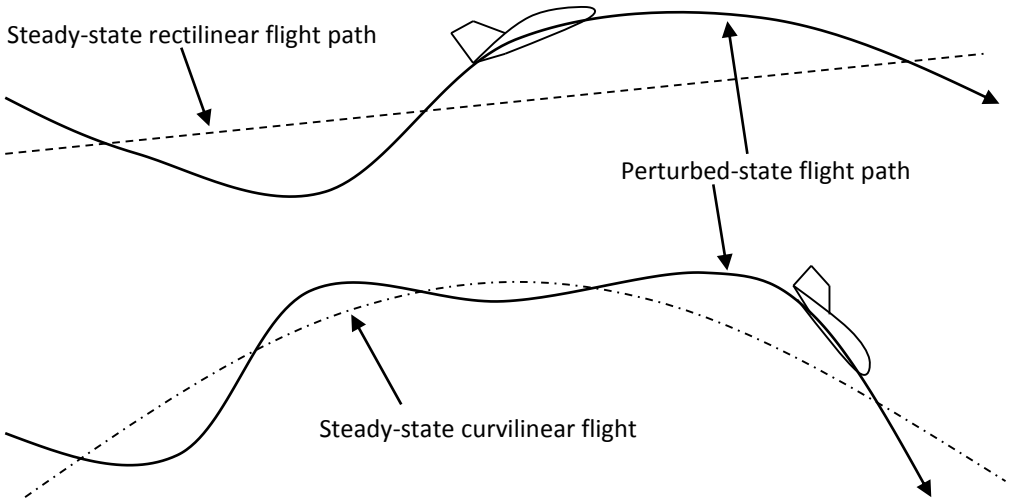
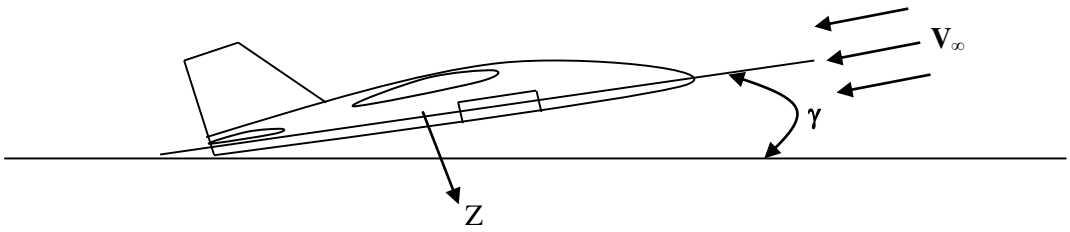
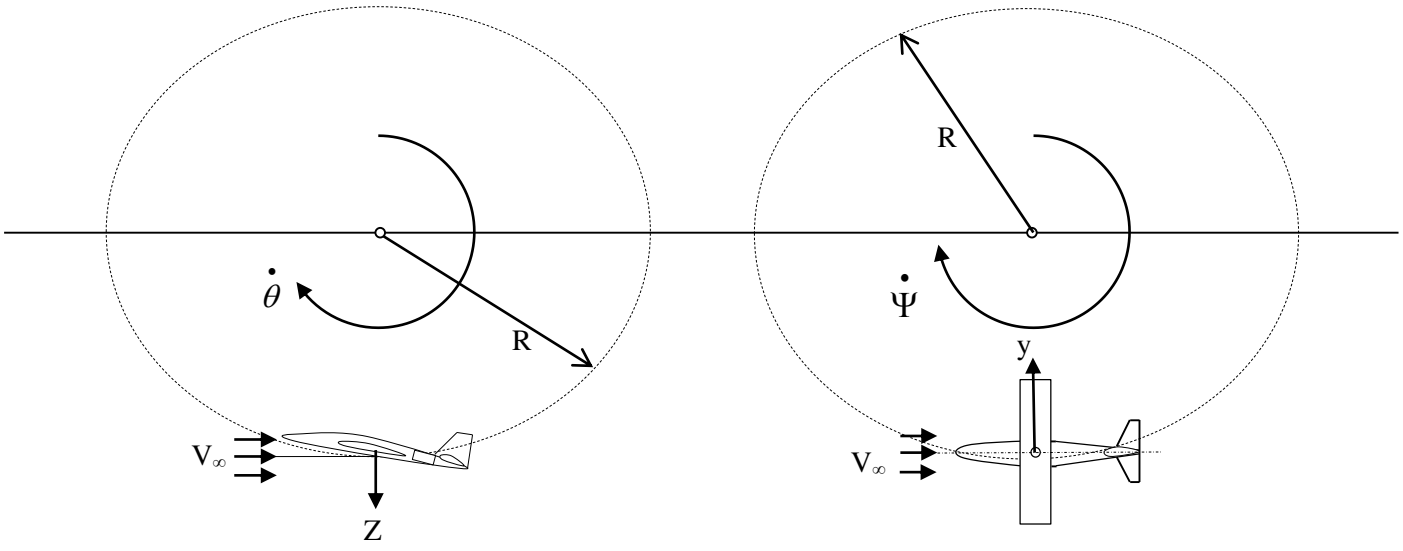


Figure 2.9. Examples of perturbed-state flight paths

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual



a. Rectilinear flight (side-view)



b. Symmetrical pull-up (side-view)

c. Level turn (Top-view)

Figure 2.10. Examples of steady state flight conditions

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual



Figure 2.11. NASA Global Hawk in a cruising flight

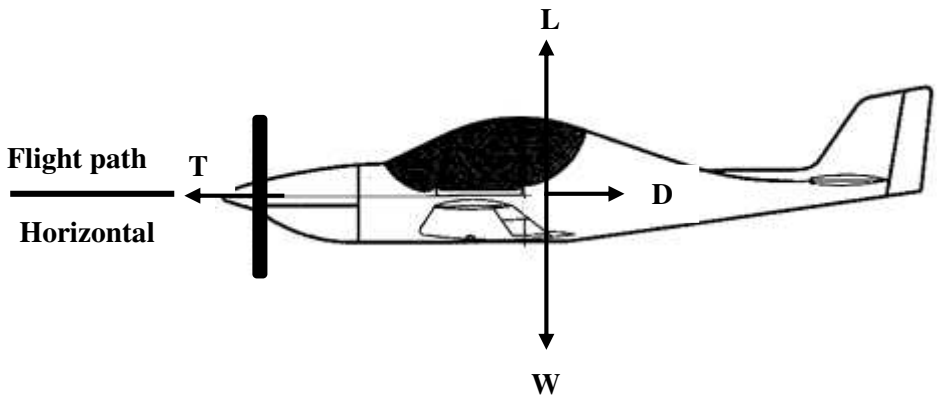


Figure 2.12. Straight line flight

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

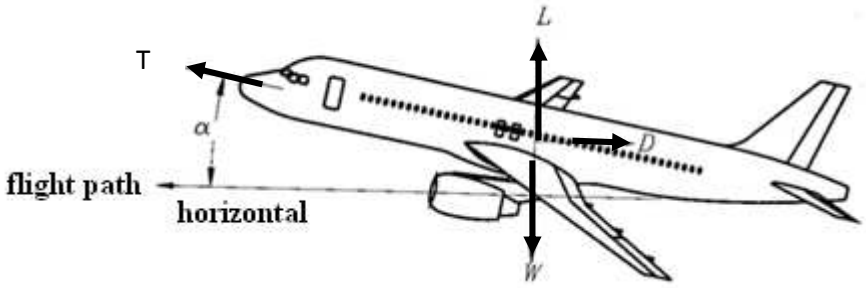


Figure 2.13. An aircraft with an angle of attack in a straight line horizontal flight

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

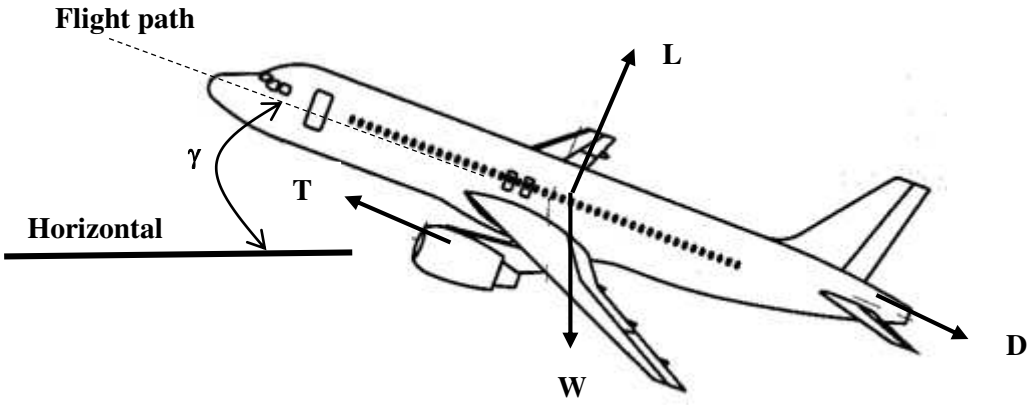


Figure 2.14. An aircraft in climb (assuming zero angle of attack)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

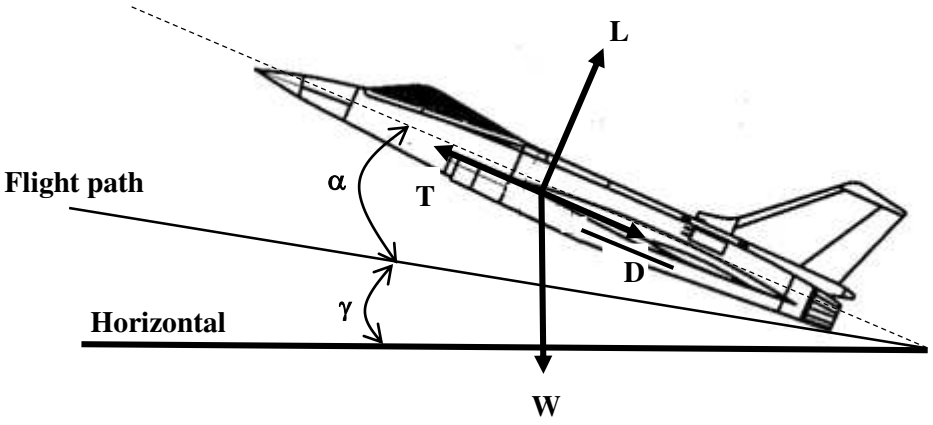


Figure 2.15. An aircraft in a climbing flight with an angle of attack, α

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

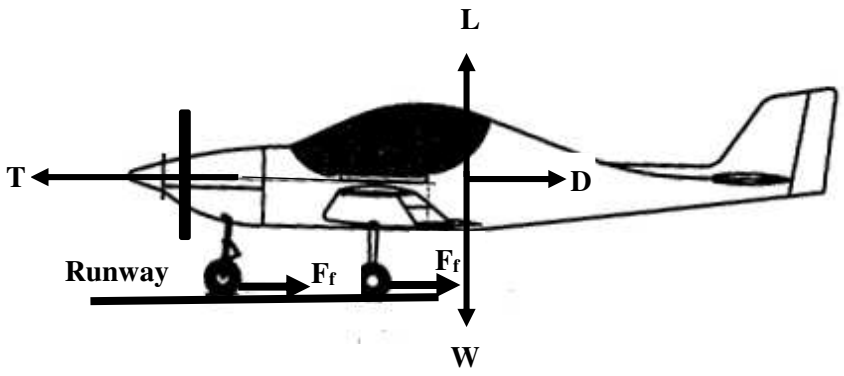
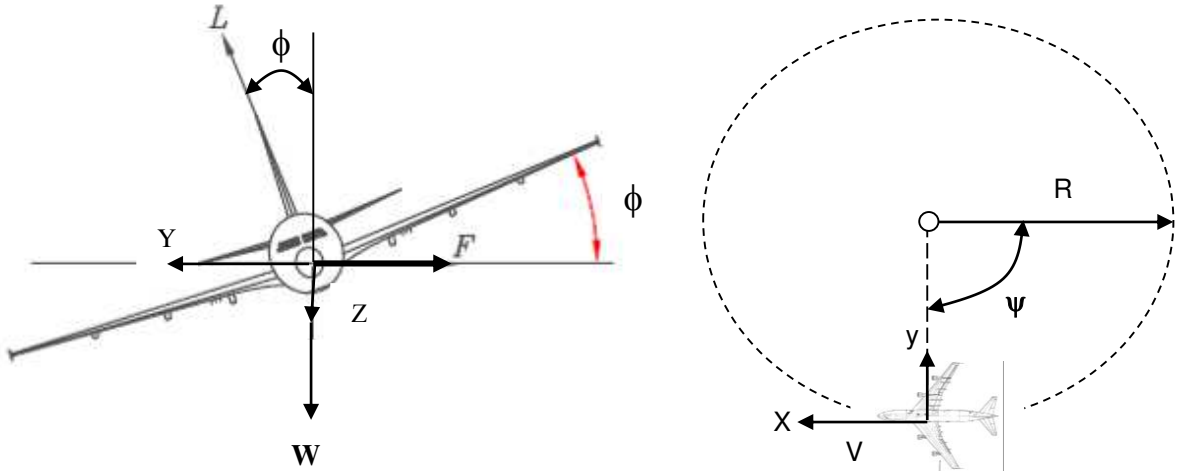


Figure 2.16. An aircraft in take off

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual



a. Front view

b. Top-view

Figure 2.17. An aircraft in a coordinated turning flight

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

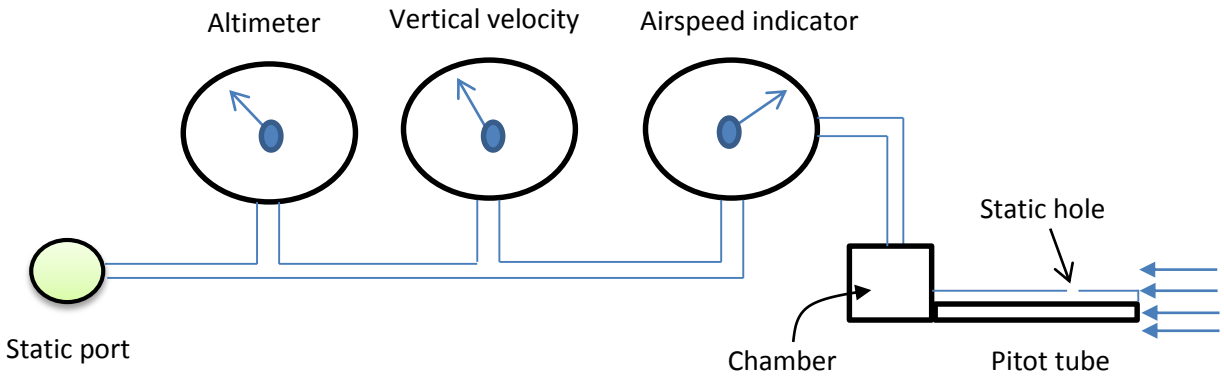


Figure 2.18. Pitot-static measurement device

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

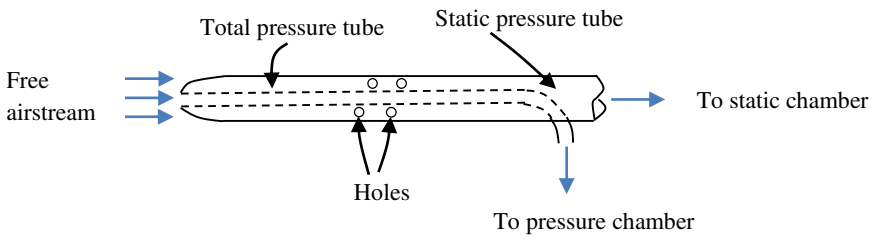
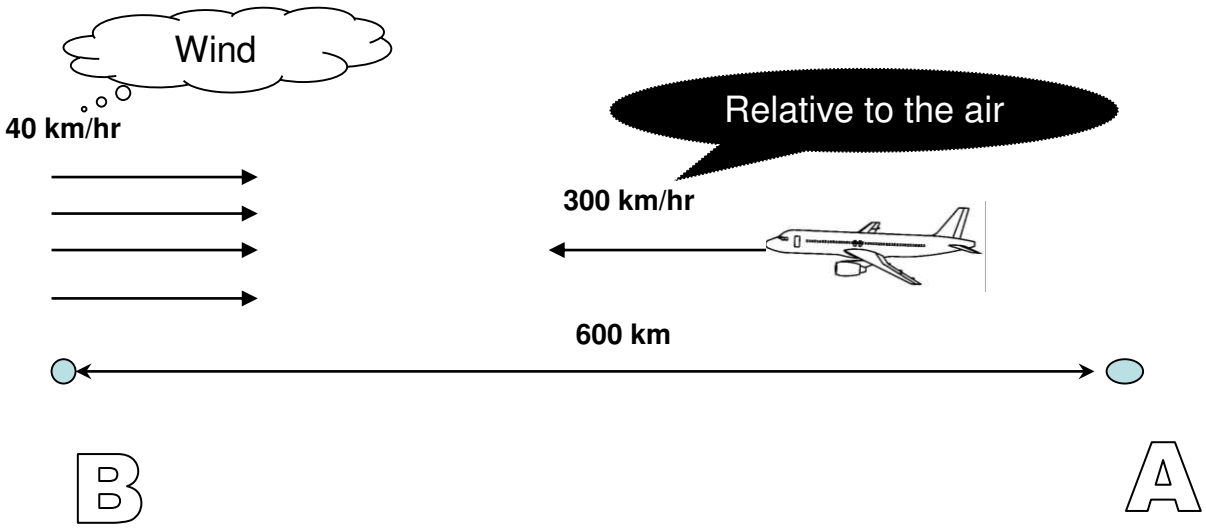
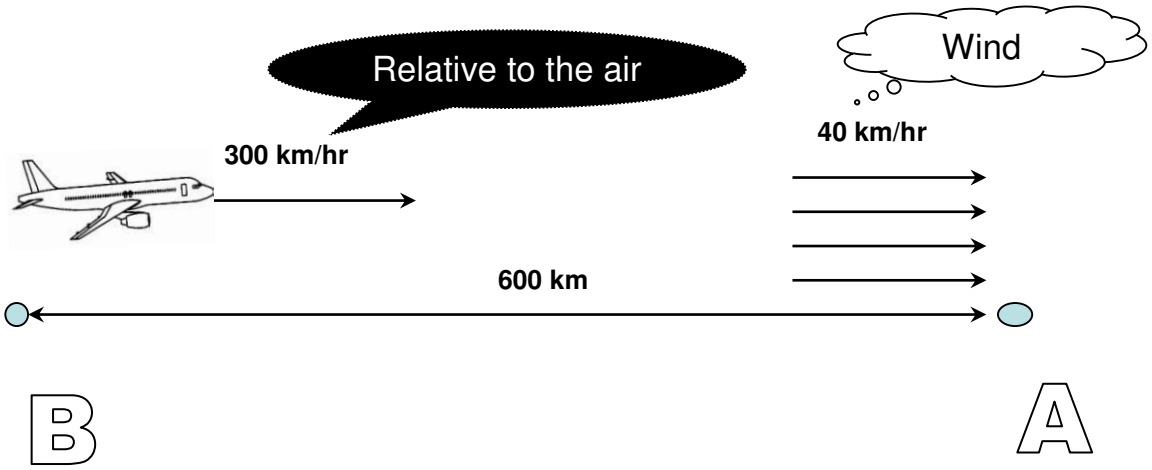


Figure 2.19. A pitot-static tube

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual



a. Headwind



b. Tailwind

Figure 2.20. Airspeed and wind speed

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

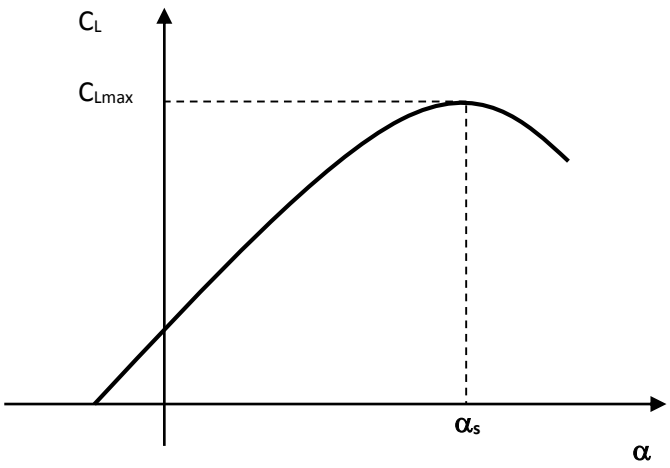


Figure 2.21. Wing lift curve slope

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual



Figure 2.22. Aircraft Raytheon Hawker 800XP with a C_{Lmax} of 2.28 (Courtesy of Gustavo Corujo)



Figure 2.23. Eurofighter EF-2000 Typhoon, a single-seat fighter in an angle beyond stall angle (Courtesy of Fabrizio Capenti)

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual

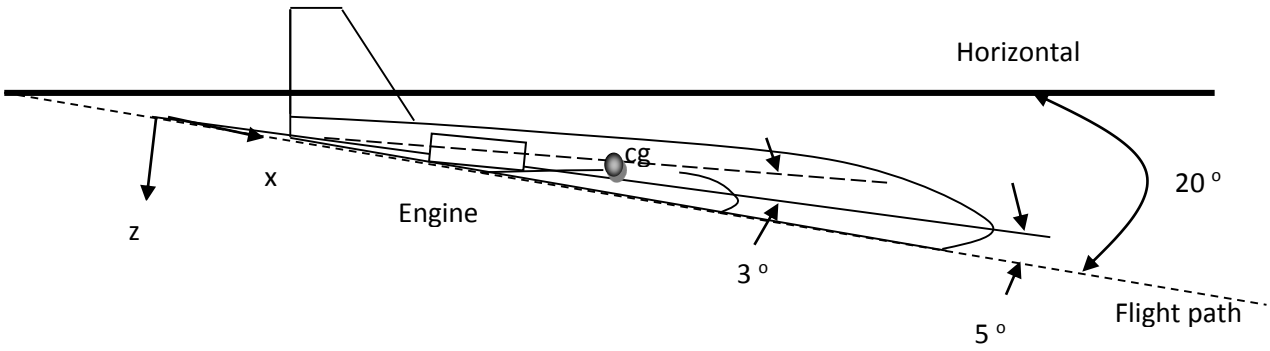


Figure 2.24. An aircraft is a descending flight

Aircraft Performance An Engineering Approach 1st edition Sadraey Solutions Manual