Algebra and Trigonometry 2nd edition OpenStax Solutions Manual

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Algebra and Trigonometry 2e.

Chapter 1
Prerequisites
1.1 Real Numbers: Algebra Essentials

Verbal

1. Is $\sqrt{2}$ an example of a rational terminating, rational repeating, or irrational number? Tell why it fits that category.
   Irrational number. The square root of two does not terminate, and it does not repeat a pattern. It can't be written as a quotient of two integers, so it is irrational.
2. What is the order of operations? What acronym is used to describe the order of operations, and what does it stand for?
   The order of operations is a set of rules that prioritize the different operations such that expressions can be evaluated consistently. PEMDAS stands for Parentheses, Exponents, Multiplication and Division, Addition and Subtraction.
3. What do the Associative Properties allow us to do when following the order of operations? Explain your answer.
   The Associative Properties state that the sum or product of multiple numbers can be grouped differently without affecting the result. This is because the same operation is performed (either addition or subtraction), so the terms can be re-ordered.

Numeric

For the following exercises, simplify the given expression.

4. $10+2\times(5-3)$
   $10+2\times 2$
   $10+4$
   14
5. $6\div 2-\left(81\div 3^2\right)$
   $6\div 2-(81\div 9)$
   $6\div 2-(9)$
   $3-9$
   -6
6. $18+\left(6-8\right)^3$
   $18+(-2)^3$
   $18+(-8)$
   $18-8$
   10

7. $-2\times\left[16\div\left(8-4\right)^2\right]^2$

$-2\times\left[16\div\left(4\right)^2\right]^2$

$-2\times\left[16\div16\right]^2$

-2×1^2

-2×1

-2

8. $4-6+2\times7$

$4-6+14$

$-2+14$

12

9. $3\left(5-8\right)$

$3(-3)$

-9

10. $4+6-10\div2$

$4+6-5$

$10-5$

5

11. $12\div\left(36\div9\right)+6$

$12\div\left(4\right)+6$

$3+6$

9

12. $\left(4+5\right)^2\div3$

$(9)^2\div3$

$81\div3$

27

13. $3-12\times2+19$

$3-24+19$

$-21+19$

-2

14. $2+8\times7\div4$

$2+56\div 4$

$2+14$

16

15. $5+(6+4)-11$

$5+(10)-11$

$15-11$

4

16. $9-18\div 3^2$

$9-18\div 9$

$9-2$

7

17. $14\times 3\div 7-6$

$42\div 7-6$

$6-6$

0

18. $9-(3+11)\times 2$

$9-(14)\times 2$

$9-28$

-19

19. $6+2\times 2-1$

$6+4-1$

$10-1$

9

20. $64\div(8+4\times 2)$

$64\div(8+8)$

$64\div(16)$

4

21. $9+4\left(2^2\right)$

$9+4(4)$

$9+16$

25

22. $(12 \div 3 \times 3)^2$

$(4 \times 3)^2$

12^2

144

23. $25 \div 5^2 - 7$

$25 \div 25 - 7$

$1 - 7$

-6

24. $(15 - 7) \times (3 - 7)$

$8 \times (3 - 7)$

8×-4

-32

25. $2 \times 4 - 9(-1)$

$8 - 9(-1)$

$8 + 9$

17

26. $4^2 - 25 \times \frac{1}{5}$

$16 - 25 \times \frac{1}{5}$

$16 - 5$

11

27. $12(3 - 1) \div 6$

$12(2) \div 6$

$24 \div 6$

4

Algebraic

For the following exercises, evaluate the expression using the given value of the variable.

28. $8(x + 3) - 64$ for $x = 2$

-24

29. $4y + 8 - 2y$ for $y = 3$

14

30. $(11a+3)-18a+4$ for $a=-2$

    21

31. $4z-2z(1+4)-36$ for $z=5$

    -66

32. $4y(7-2)^2+200$ for $y=-2$

    0

33. $-(2x)^2+1+3$ for $x=2$

    -12

34. For the $8(2+4)-15b+b$ for $b=-3$

    90

35. $2(11c-4)-36$ for $c=0$

    -44

36. $4(3-1)x-4$ for $x=10$

    76

37. $\frac{1}{4}\left(8w-4^2\right)$ for $w=1$

    -2

For the following exercises, simplify the expression.

38. $4x+x(13-7)$

    $4x+x(13-7)$ Combine terms inside of parenthesis

    $4x+x(6)$ Rewrite

    $4x+6x$ Combine like terms

    $10x$

39. $2y-(4)^2y-11$

    $2y-(4)^2y-11$ Square the 4

    $2y-16y-11$ Combine like terms

    $-14y-11$

40. $\frac{a}{2^3}(64)-12a\div 6$

$\frac{a}{2^3}(64)-12a\div 6$ Cube the 2

$\frac{a}{8}(64)-12a\div 6$ Multiply 64 by 1/8

$a(8)-12a\div 6$ Rewrite and divide 12 by 6

$8a-2a$ Combine like terms

$6a$

41. $8b-4b(3)+1$

$8b-4b(3)+1$ Multiply the coefficients of b

$8b-12b+1$ Combine like terms

$-4b+1$

42. $5l\div 3l\times(9-6)$

$5l\div 3l\times(9-6)$ Combine terms inside of parenthesis

$5l\div 3l\times 3$ Divide 5l by 3l

$\frac{5}{3}\times 3$ Multiply 5/3 by 3

5

43. $7z-3+z\times 6^2$

$7z-3+z\times 6^2$ Square the 6

$7z-3+z\times 36$ Multiply 36 and z

$7z-3+36z$ Combine like terms

$43z-3$

44. $4\times 3+18x\div 9-12$

$4\times 3+18x\div 9-12$ Multiply 4 and 3

$12+18x\div 9-12$ Divide 18x by 9

$12+2x-12$ Combine like terms

$2x$

45. $9(y+8)-27$

$9(y+8)-27$ Distribute the 9

$9y+72-27$ Combine like terms

$9y+45$

46. $\left(\frac{9}{6}t-4\right)2$

$\left(\frac{9}{6}t-4\right)2$ Distribute the 2

$\frac{18}{6}t-8$ Simplify the fraction

$3t-8$

47. $6+12b-3\times 6b$

$6+12b-3\times 6b$ Multiply 3 and 6b

$6+12b-18b$ Combine like terms

$-6b+6$

48. $18y-2(1+7y)$

$18y-2(1+7y)$ Distribute the -2

$18y-2-14y$ Combine like terms

$4y-2$

49. $\left(\frac{4}{9}\right)^2\times 27x$

$\left(\frac{4}{9}\right)^2\times 27x$ Square the expression in parenthesis

$\frac{16}{81}\times 27x$ Note that $27\times 3 = 81$. Simplify

$\frac{16x}{3}$

50. $8(3-m)+1(-8)$

$8(3-m)+1(-8)$ Distribute the 8

$24-8m+1(-8)$ Multiply the 8 and -1

$24-8m-8$ Combine like terms and rewrite

$-8m+16$

51. $9x+4x(2+3)-4(2x+3x)$

$9x+4x(2+3)-4(2x+3x)$ Combine terms inside of the parenthesis

$9x+4x(5)-4(5x)$ Distribute the 4x and the -4

$9x+20x-20x$ Combine like terms

$9x$

52. $5^2 - 4(3x)$

$5^2 - 4(3x)$ Square the 5 and multiply 4 by 3x

$25 - 12x$ Rewrite

$-12x + 25$

Real-World Applications

For the following exercises, consider this scenario: Fred earns $40 at the community garden. He spends $10 on a streaming subscription, puts half of what is left in a savings account, and gets another $5 for walking his neighbor's dog.

53. Write the expression that represents the number of dollars Fred keeps (and does not put in his savings account). Remember the order of operations.

Note that Fred earns 40 then spends 10 before putting half inside of his bank account.

The additional 5 comes after the deposit, so we write the expression:

$\frac{1}{2}(40-10)+5$

54. How much money does Fred keep?

$\frac{1}{2}(40-10)+5$ Parenthesis first

$\frac{1}{2}(30)+5$ Multiplication

$15+5$ Addition last

20

55. According to the U.S. Mint, the diameter of a quarter is 0.955 inches. The circumference of the quarter would be the diameter multiplied by π. Is the circumference of a quarter a whole number, a rational number, or an irrational number?

π is irrational and 0.955 is rational, so their product is an irrational number.

56. Jessica and her roommate, Adriana, have decided to share a change jar for joint expenses. Jessica put her loose change in the jar first, and then Adriana put her change in the jar. We know that it does not matter in which order the change was added to the jar. What property of addition describes this fact?

The commutative property of addition

For the following exercises, consider this scenario: There is a mound of g pounds of gravel in a quarry. Throughout the day, 400 pounds of gravel is added to the mound. Two orders of 600 pounds are sold and the gravel is removed from the mound. At the end of the day, the mound has 1,200 pounds of gravel.

57. Write the equation that describes the situation.

$g + 400 - 2(600) = 1200$

58. Solve for g.

$g + 400 - 2(600) = 1200$ Distribute the 2

$g + 400 - 1200 = 1200$ Add 1200 to both sides

$g + 400 = 2400$ Subtract 400 from both sides

$g = 2000$

59. Ramon runs the marketing department at their company. Their department gets a budget every year, and every year, they must spend the entire budget without going over. If they spend less than the budget, then the department gets a smaller budget the following year. At the beginning of this year, Ramon got $2.5 million for the annual marketing budget.

They must spend the budget such that $2,500,000 - x = 0$. What property of addition tells us what the value of x must be?

inverse property of addition

Technology

For the following exercises, use a graphing calculator to solve for x. Round the answers to the nearest hundredth.

60. $0.5(12.3)^2 - 48x = \frac{3}{5}$

1.56

61. $(0.25 - 0.75)^2 x - 7.2 = 9.9$

68.4

Extensions

62. If a whole number is not a natural number, what must be the number?

0. Recall that the whole numbers are the natural numbers plus zero.

63. Determine whether the statement is true or false: The multiplicative inverse of a rational number is also rational.

true

64. Determine whether the statement is true or false: The product of a rational and irrational number is always irrational.

false. 0 is rational and 0 times an irrational number is 0.

65. Determine whether the simplified expression is rational or irrational: $\sqrt{-18 - 4(5)(-1)}$.

$\sqrt{-18-4(5)(-1)}$ Multiply 4 and 5

$\sqrt{-18-20(-1)}$ Multiply 20 and -1

$\sqrt{-18+20}$ Add -18 and 20

$\sqrt{2}$

irrational

66. Determine whether the simplified expression is rational or irrational: $\sqrt{-16+4(5)+5}$.

$\sqrt{-16+4(5)+5}$ Multiply 4 and 5

$\sqrt{-16+20+5}$ Add 20 and 5

$\sqrt{-16+25}$ Add -16 and 25

$\sqrt{9}$ Evaluate

3

rational

67. The division of two natural numbers will always be what type of number?

rational

68. What property of real numbers would simplify the following expression: $4+7(x-1)$?

distributive property

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Chapter 1
Prerequisites
1.2 Exponents and Scientific Notation

Verbal

1. Is 2^3 the same as 3^2? Explain.
   No, the two expressions are not the same. An exponent tells how many times you multiply the base. So 2^3 is the same as $2\times2\times2$, which is 8. 3^2 is the same as 3×3, which is 9.
2. When can you add two exponents?
   You can add two exponents when the bases are the same and the expressions are being multiplied.
3. What is the purpose of scientific notation?
   It is a method of writing very small and very large numbers.
4. Explain what a negative exponent does.
   A negative exponent switches a numerator to a denominator or a denominator to a numerator. It is the number of times you multiply 1 over the base.

Numeric

For the following exercises, simplify the given expression. Write answers with positive exponents.

5. 9^2
   $9\cdot9$
   81

6. 15^{-2}
   $\frac{1}{15^2}$
   $\frac{1}{15\cdot15}$
   $\frac{1}{225}$

7. $3^2\times3^3$
   $(3\cdot3)\times(3\cdot3\cdot3)$
   9×27
   243

8. $4^4 \div 4$

$(4 \cdot 4 \cdot 4 \cdot 4) \div 4$

$256 \div 4$

64

9. $\left(2^2\right)^{-2}$

$\frac{1}{\left(2^2\right)^2}$

$\frac{1}{4^2}$

$\frac{1}{16}$

10. $(5-8)^0$

$(-3)^0$

1

11. $11^3 \div 11^4$

$(11 \cdot 11 \cdot 11) \div (11 \cdot 11 \cdot 11 \cdot 11)$

$\frac{1}{11}$

12. $6^5 \times 6^{-7}$

$(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6) \times \frac{1}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}$

$\frac{1}{36}$

13. $\left(8^0\right)^2$

1^2

1

14. $5^{-2} \div 5^2$

$\frac{1}{5 \cdot 5} \div (5 \cdot 5)$

$\frac{1}{25} \times \frac{1}{25}$

$\frac{1}{625}$

For the following exercises, write each expression with a single base. Do not simplify further. Write answers with positive exponents.

15. $4^2 \times 4^3 \div 4^{-4}$

$4^5 \div 4^{-4}$

$4^5 \div \frac{1}{4^4}$

$4^5 \times 4^4$

4^9

16. $\frac{6^{12}}{6^9}$

6^{12-9}

6^3

17. $\left(12^3 \times 12\right)^{10}$

$\left(12^4\right)^{10}$

12^{40}

18. $10^6 \div \left(10^{10}\right)^{-2}$

$10^6 \div \frac{1}{\left(10^{10}\right)^2}$

$10^6 \div \frac{1}{10^{20}}$

$10^6 \times 10^{20}$

10^{26}

19. $7^{-6} \times 7^{-3}$

7^{-6+-3}

7^{-9}

$\frac{1}{7^9}$

20. $\left(3^3 \div 3^4\right)^5$

$\left(3^{3-4}\right)^5$

$\left(3^{-1}\right)^5$

3^{-5}

$\frac{1}{3^5}$

For the following exercises, express the decimal in scientific notation.

21. 0.0000314

    3.14×10^{-5}

22. 148,000,000

    1.48×10^{8}

For the following exercises, convert each number in scientific notation to standard notation.

23. 1.6×10^{10}
    16,000,000,000

24. 9.8×10^{-9}
    0.0000000098

Algebraic

For the following exercises, simplify the given expression. Write answers with positive exponents.

25. $\frac{a^3a^2}{a}$

    $\frac{a^5}{a}$

    a^{5-1}

    a^4

26. $\frac{mn^2}{m^{-2}}$

    $m^{1-(-2)}n^2$

    m^3n^2

27. $\left(b^3c^4\right)^2$

b^6c^8

28. $\left(\frac{x^{-3}}{y^2}\right)^{-5}$

$\left(\frac{1}{x^3y^2}\right)^{-5}$

$\left(x^3y^2\right)^5$

$x^{15}y^{10}$

29. $ab^2 \div d^{-3}$

$ab^2 \div \frac{1}{d^3}$

$ab^2 \times d^3$

ab^2d^3

30. $\left(w^0x^5\right)^{-1}$

$\frac{1}{w^0x^5}$

$\frac{1}{x^5}$

31. $\frac{m^4}{n^0}$

$\frac{m^4}{1}$

m^4

32. $y^{-4}\left(y^2\right)^2$

$\frac{1}{y^4}(y^4)$

1

33. $\dfrac{p^{-4}q^{2}}{p^{2}q^{-3}}$

$p^{-4-2}q^{2-(-3)}$

$p^{-6}q^{5}$

$\dfrac{q^{5}}{p^{6}}$

34. $(l\times w)^{2}$

$l^{2}w^{2}$

35. $\left(y^{7}\right)^{3}\div x^{14}$

$y^{21}\div x^{14}$

$\dfrac{y^{21}}{x^{14}}$

36. $\left(\dfrac{a}{2^{3}}\right)^{2}$

$\dfrac{a^{2}}{8^{2}}$

$\dfrac{a^{2}}{64}$

37.

$(25m)\div(5^{0}m)$

$=(25m)\div m$

$=\dfrac{25m}{m}$

$=25$

38. $\dfrac{\left(16\sqrt{x}\right)^{2}}{y^{-1}}$

$\dfrac{256x}{\dfrac{1}{y}}$

$256xy$

39. $\dfrac{2^{3}}{(3a)^{-2}}$

$$\frac{8}{\frac{1}{(3a)^2}}$$

$$8 \cdot 9a^2$$

$$72a^2$$

40. $\left(ma^6\right)^2 \frac{1}{m^3 a^2}$

$$m^2 a^{12} \frac{1}{m^3 a^2}$$

$$m^{2-3} a^{12-2}$$

$$m^{-1} a^{10}$$

$$\frac{a^{10}}{m}$$

41. $\left(b^{-3} c\right)^3$

$$\left(\frac{c}{b^3}\right)^3$$

$$\frac{c^3}{b^9}$$

42. $\left(x^2 y^{13} \div y^0\right)^2$

$$\left(x^2 y^{13} \div 1\right)^2$$

$$\left(x^2 y^{13}\right)^2$$

$$x^4 y^{26}$$

43. $\left(9z^3\right)^{-2} y$

$$\frac{y}{\left(9z^3\right)^2}$$

$$\frac{y}{81z^6}$$

Real-World Applications

44. To reach escape velocity, a rocket must travel 2.2×10^6 feet per minute. Write the rate in standard notation.
2,200,000 feet per minute

45. A dime is the thinnest coin in U.S. currency. A dime's thickness measures 1.35×10^{-3} meters. Write the number in standard notation.
    0.00135 meter
46. The distance between Earth and the Sun (on average) is 92,960,000 miles. Rewrite the distance using scientific notation.

    9.2960×10^{7} miles
47. A terabyte is made of approximately 1,099,500,000,000 bytes. Rewrite in scientific notation.

    1.0995×10^{12}
48. The Gross Domestic Product for the United States in the first quarter of 2014 was

    $\$1.71496\times10^{13}$. Rewrite GDP in standard notation.
    \$17,149,600,000,000
49. One picometer is approximately 3.397×10^{-11} inches. Rewrite the number of inches in a picometer using standard notation.
    0.00000000003397 inches
50. The value of the services sector of the U.S. economy in the first quarter of 2012 was \$10,633.6 billion. Rewrite this amount in scientific notation.

    $\$1.06336\times10^{13}$

Technology

For the following exercises, use a graphing calculator to simplify. Round the answers to the nearest hundredth.

51. $\left(\dfrac{12^{3}m^{33}}{4^{-3}}\right)^{2}$

    $12{,}230{,}590{,}464\,m^{66}$

52. $17^{3}\div15^{2}x^{3}$

    $\dfrac{4913}{225x^{3}}$

Extensions

For the following exercises, simplify the given expression. Write answers with positive exponents.

53. $\left(\frac{3^2}{a^3}\right)^{-2}\left(\frac{a^4}{2^2}\right)^2$

$\left(\frac{a^3}{9}\right)^2\left(\frac{a^8}{4^2}\right)$

$\left(\frac{a^6}{81}\right)\left(\frac{a^8}{16}\right)$

$\frac{a^{14}}{1296}$

54. $\left(6^2-24\right)^2 \div \left(\frac{x}{y}\right)^{-5}$

$\left(36-24\right)^2 \div \left(\frac{y^5}{x^5}\right)$

$12^2 \times \frac{x^5}{y^5}$

$\frac{144x^5}{y^5}$

55. $\frac{m^2n^3}{a^2c^{-3}} \cdot \frac{a^{-7}n^{-2}}{m^2c^4}$

$\frac{m^{2-2}n^{3-2}a^{-7-2}}{c^{-3+4}}$

$\frac{m^0na^{-9}}{c}$

$\frac{n}{a^9c}$

56. $\left(\frac{x^6y^3}{x^3y^{-3}} \cdot \frac{y^{-7}}{x^{-3}}\right)^{10}$

$\left(\frac{x^6 y^{3-7}}{x^{3-3} y^{-3}}\right)^{10}$

$\left(\frac{x^6 y^{-4}}{1 \cdot y^{-3}}\right)^{10}$

$\left(x^6 y^{-4-(-3)}\right)^{10}$

$\left(x^6 y^{-1}\right)^{10}$

$\frac{x^{60}}{y^{10}}$

57. $\left(\frac{\left(ab^2c\right)^{-3}}{b^{-3}}\right)^2$

$\left(\frac{a^{-3}b^{-6}c^{-3}}{b^{-3}}\right)^2$

$\frac{a^{-6}b^{-12}c^{-6}}{b^{-6}}$

$\frac{1}{a^6 b^{12-6} c^6}$

$\frac{1}{a^6 b^6 c^6}$

58. Avagadro's constant is used to calculate the number of particles in a mole. A mole is a basic unit in chemistry to measure the amount of substance. The constant is 6.0221413×10^{23}. Write Avagadro's constant in standard notation.

602,214,130,000,000,000,000,000

59. Planck's constant is another important unit of measure in quantum physics. It describes the relationship between energy and frequency. The constant is written as $6.62606957 \times 10^{-34}$. Write Planck's constant in standard notation.

0.000000000000000000000000000000000062606957

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Chapter 1
Prerequisites
1.3 Radicals and Rational Expressions

Verbal

1. What does it mean when a radical does not have an index? Is the expression equal to the radicand? Explain.

   When there is no index, it is assumed to be 2 or the square root. The expression would only be equal to the radicand if the index were 1.

2. Where would radicals come in the order of operations? Explain why.

   Radicals are another way of writing fractional exponents, so they would come after performing operations within parenthesis but before multiplication and division.

3. Every number will have two square roots. What is the principal square root?

   The principal square root is the nonnegative root of the number.

4. Can a radical with a negative radicand have a real square root? Why or why not?

   A radical with a negative radicand cannot have a real square root. This is because the roots must be congruent, but the product of two negative numbers is positive, so we cannot multiply two real numbers that are the same and get a negative product.

Numeric

For the following exercises, simplify each expression.

5. $\sqrt{256}$

   $\sqrt{256}$ Note that 256 is 16^2

   $\sqrt{16 \cdot 16}$ Evaluate

   16

6. $\sqrt{\sqrt{256}}$

   $\sqrt{\sqrt{256}}$ Note that 256 is 16^2

   $\sqrt{\sqrt{16 \cdot 16}}$ Evaluate inner square root

   $\sqrt{16}$ Evaluate

   4

7. $\sqrt{4(9+16)}$

$\sqrt{4(9+16)}$ Combine terms inside of parenthesis

$\sqrt{4(25)}$ Separate into two different square roots

$\sqrt{4}\sqrt{25}$ Evaluate sqaure roots

$2 \cdot 5$ Multiply

10

8. $\sqrt{289} - \sqrt{121}$

$\sqrt{289} - \sqrt{121}$ Note that 289 is 17^2 and 121 is 11^2

$\sqrt{17 \cdot 17} - \sqrt{11 \cdot 11}$ Evaluate square roots

$17 - 11$ Subtract

6

9. $\sqrt{196}$

$\sqrt{196}$ Note that 196 is 14^2

$\sqrt{14 \cdot 14}$ Evaluate square root

14

10. $\sqrt{1}$

$\sqrt{1}$

$\sqrt{1 \cdot 1}$

1

11. $\sqrt{98}$

$\sqrt{98}$ 98 has the factors 2 and 49

$\sqrt{49 \cdot 2}$ Multiplication property of radicals

$\sqrt{49}\sqrt{2}$ 49 is a square root; 2 is not a perfect square

$7\sqrt{2}$

12. $\sqrt{\frac{27}{64}}$

$\sqrt{\frac{27}{64}}$ Division property of radicals

$\frac{\sqrt{27}}{\sqrt{64}}$ 27 has factors of 9 and 3; 64 is a perfect square

$\frac{\sqrt{3\cdot 9}}{8}$ Multiplication property of radicals

$\frac{\sqrt{3}\sqrt{9}}{8}$ 9 is a perfect square; 3 is not a perfect square

$\frac{3\sqrt{3}}{8}$

13. $\sqrt{\frac{81}{5}}$

$\sqrt{\frac{81}{5}}$ Division property of radicals

$\frac{\sqrt{81}}{\sqrt{5}}$ 81 is a perfect square; rationalize the denominator

$\frac{9}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$ Note that $\sqrt{5^2}=5$ and multiply

$\frac{9\sqrt{5}}{5}$

14. $\sqrt{800}$

$\sqrt{800}$ 800 has factors 400 and 2

$\sqrt{400\cdot 2}$ Muliplication property of radicals

$\sqrt{400}\sqrt{2}$ 400 is a perfect square; 2 is not a perfect square

$20\sqrt{2}$

15. $\sqrt{169}+\sqrt{144}$

$\sqrt{169}+\sqrt{144}$ 169 is 13^2 and 144 is 12^2

$\sqrt{13\cdot 13}+\sqrt{12\cdot 12}$ Evaluate square roots

$13+12$ Add

25

16. $\sqrt{\frac{8}{50}}$

$\sqrt{\frac{8}{50}}$ Division property of radicals

$\frac{\sqrt{8}}{\sqrt{50}}$ 8 has factors 4 and 2; 50 has factors 25 and 2

$\frac{\sqrt{4\cdot 2}}{\sqrt{25\cdot 2}}$ Multiplication property of radicals

$\frac{\sqrt{4}\sqrt{2}}{\sqrt{25}\sqrt{2}}$ 4 is a perfect square; 25 is a perfect square; note that $\frac{\sqrt{2}}{\sqrt{2}}=1$

$\frac{2}{5}$

17. $\frac{18}{\sqrt{162}}$

$\frac{18}{\sqrt{162}}$ 162 has factors 81 and 2

$\frac{18}{\sqrt{81\cdot 2}}$ Multiplication property of addition

$\frac{18}{\sqrt{81}\sqrt{2}}$ 81 is a perfect square; 2 is not a perfect square

$\frac{18}{9\sqrt{2}}$ Divide

$\frac{2}{\sqrt{2}}$ Rationalize the denominator

$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$ Multiply

$\frac{2\sqrt{2}}{2}=\sqrt{2}$

18. $\sqrt{192}$

$\sqrt{192}$ 192 has factors 64 and 3

$\sqrt{64\cdot 3}$ Multiplication property of radicals

$\sqrt{64}\sqrt{3}$ 64 is a perfect square; 3 is not a perfect square

$8\sqrt{3}$

19. $14\sqrt{6}-6\sqrt{24}$

$14\sqrt{6}-6\sqrt{24}$ 24 has factors 4 and 6

$14\sqrt{6}-6\sqrt{4\cdot 6}$ Multiplication property of radicals

$14\sqrt{6}-6\sqrt{4}\sqrt{6}$ 4 is a perfect square; 6 is not a perfect square

$14\sqrt{6}-6(2)\sqrt{6}$ Multiply

$14\sqrt{6}-12\sqrt{6}$ Subtract

$2\sqrt{6}$

20. $15\sqrt{5}+7\sqrt{45}$

$15\sqrt{5}+7\sqrt{45}$ 45 has factors 9 and 5

$15\sqrt{5}+7\sqrt{9\cdot 5}$ Multiplication property of radicals

$15\sqrt{5}+7\sqrt{9}\sqrt{5}$ 9 is a perfect square; 5 is not a perfect square

$15\sqrt{5}+7(3)\sqrt{5}$ Multiply

$15\sqrt{5}+21\sqrt{5}$ Add

$36\sqrt{5}$

21. $\sqrt{150}$

$\sqrt{150}$ 150 has factors 25 and 6

$\sqrt{25\cdot 6}$ Multiplication property of radicals

$\sqrt{25}\sqrt{6}$ 25 is a perfect square; 6 is not a perfect square

$5\sqrt{6}$

22. $\sqrt{\frac{96}{100}}$

$\sqrt{\frac{96}{100}}$ Division property of radicals

$\frac{\sqrt{96}}{\sqrt{100}}$ 96 has factors 16 and 6; 100 is a perfect square

$\frac{\sqrt{16\cdot 6}}{10}$ Multiplication property of radicals

$\frac{\sqrt{16}\sqrt{6}}{10}$ 16 is a perfect square; 6 is not a perfect square

$\frac{4\sqrt{6}}{10}$ Simplify

$\frac{2\sqrt{6}}{5}$

23. $\left(\sqrt{42}\right)\left(\sqrt{30}\right)$

$\left(\sqrt{42}\right)\left(\sqrt{30}\right)$ 42 has factors 7 and 6; 30 has factors 5 and 6

$\left(\sqrt{7\cdot 6}\right)\left(\sqrt{5\cdot 6}\right)$ Multiplication property of radicals

$\sqrt{7\cdot 5\cdot 6\cdot 6}$ Multiplication property of radicals

$\sqrt{7\cdot 5}\sqrt{6\cdot 6}$ Evaluate and rewrite

$6\sqrt{35}$

24. $12\sqrt{3}-4\sqrt{75}$

$12\sqrt{3}-4\sqrt{75}$ 75 has factors 25 and 3

$12\sqrt{3}-4\sqrt{25\cdot 3}$ Multiplication property of radicals

$12\sqrt{3}-4\sqrt{25}\sqrt{3}$ 25 is a perfect square; 3 is not a perfect square

$12\sqrt{3}-4(5)\sqrt{3}$ Multiply

$12\sqrt{3}-20\sqrt{3}$ Subtract

$-8\sqrt{3}$

25. $\sqrt{\frac{4}{225}}$

$\sqrt{\frac{4}{225}}$ Division property of radicals

$\frac{\sqrt{4}}{\sqrt{225}}$ 4 and 225 are perfect squares

$\frac{2}{15}$

26. $\sqrt{\frac{405}{324}}$

$\sqrt{\frac{405}{324}}$ Division property of radicals

$\frac{\sqrt{405}}{\sqrt{324}}$ 405 has factors 81 and 5; 324 is a perfect square

$\frac{\sqrt{81\cdot 5}}{18}$ Multiplication property of radicals

$\frac{\sqrt{81}\sqrt{5}}{18}$ 81 is a perfect square; 5 is not a perfect square

$\frac{9\sqrt{5}}{18}$ Simplify

$\frac{\sqrt{5}}{2}$

27. $\sqrt{\frac{360}{361}}$

$\sqrt{\frac{360}{361}}$ Division property of radicals

$\frac{\sqrt{360}}{\sqrt{361}}$ 360 has factors 36 and 10; 361 is a perfect square

$\frac{\sqrt{36\cdot 10}}{19}$ Multiplication property of radicals

$\frac{\sqrt{36}\sqrt{10}}{19}$ 36 is a perfect square; 10 is not a perfect square

$\frac{6\sqrt{10}}{19}$

28. $\frac{5}{1+\sqrt{3}}$

$\frac{5}{1+\sqrt{3}}$ Multiply top and bottom by the conjugate of the denominator

$\frac{5}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}$ Use the FOIL method in the denominator

$\frac{5-5\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-\sqrt{3^2}}$ Simplfy

$-\frac{5-5\sqrt{3}}{2}$

29. $\dfrac{8}{1-\sqrt{17}}$

$\dfrac{8}{1-\sqrt{17}}$ Multiply top and bottom by the conjugate of the denominator

$\dfrac{8}{1-\sqrt{17}} \cdot \dfrac{1+\sqrt{17}}{1+\sqrt{17}}$ Use the FOIL method in the denominator

$\dfrac{8(1+\sqrt{17})}{1+\sqrt{17}-\sqrt{17}-\sqrt{17^2}}$ Simplify

$-\dfrac{8(1+\sqrt{17})}{16}$ Simplify

$-\dfrac{1+\sqrt{17}}{2}$

30. $\sqrt[4]{16}$

$\sqrt[4]{16}$ 16 is the same as 2^4

$\sqrt[4]{2^4}$ Evaluate

2

31. $\sqrt[3]{128}+3\sqrt[3]{2}$

$\sqrt[3]{128}+3\sqrt[3]{2}$ 128 has factors 64 and 2

$\sqrt[3]{64\cdot 2}+3\sqrt[3]{2}$ Multiplication property of radicals

$\sqrt[3]{64}\sqrt[3]{2}+3\sqrt[3]{2}$ 64 is a perfect cube; 2 is not a perfect cube

$4\sqrt[3]{2}+3\sqrt[3]{2}$ Add

$7\sqrt[3]{2}$

32. $\sqrt[5]{\dfrac{-32}{243}}$

$\sqrt[5]{\dfrac{-32}{243}}$ Division property of radicals

$\dfrac{\sqrt[5]{32}}{\sqrt[5]{243}}$ $32 = 2^5$ and $243 = 3^5$

$\dfrac{2}{3}$

33. $\dfrac{15\sqrt[4]{125}}{\sqrt[4]{5}}$

$\frac{15\sqrt[4]{125}}{\sqrt[4]{5}}$ Division property of radicals

$15\sqrt[4]{\frac{125}{5}}$ Divide

$15\sqrt[4]{25}$ $25 = 5^2$ therefore $5^{\frac{2}{4}} = \sqrt{5}$

$15\sqrt{5}$ $15\sqrt{5}$

34. $3\sqrt[3]{-432} + \sqrt[3]{16}$

$3\sqrt[3]{-432} + \sqrt[3]{16}$ 432 has factors 216 and 2; 16 has factors 8 and 2

$3\sqrt[3]{-216 \cdot 2} + \sqrt[3]{8 \cdot 2}$ Multiplication property of radicals

$3\sqrt[3]{-216}\sqrt[3]{2} + \sqrt[3]{8}\sqrt[3]{2}$ 216 is a perfect cube; 8 is a perfect cube; 2 is not a perfect cube

$3(-6)\sqrt[3]{2} + 2\sqrt[3]{2}$ Multiply

$-18\sqrt[3]{2} + 2\sqrt[3]{2}$ Add

$-16\sqrt[3]{2}$

Algebraic

For the following exercises, simplify each expression.

35. $\sqrt{400x^4}$

$\sqrt{400x^4}$ Multiplication property of radicals

$\sqrt{400}\sqrt{x^4}$ 400 is a perfect square; x^4 is a perfect square

$20x^2$

36. $\sqrt{4y^2}$

$\sqrt{4y^2}$ Multiplication property of radicals

$\sqrt{4}\sqrt{y^2}$ 4 is a perfect square; y^2 is a perfect square

$2y$

37. $\sqrt{49p}$

$\sqrt{49p}$ Multiplication property of radicals

$\sqrt{49}\sqrt{p}$ 49 is a perfect square; p is not a perfect square

$7\sqrt{p}$

38. $\left(144p^2q^6\right)^{\frac{1}{2}}$

$\left(144p^2q^6\right)^{\frac{1}{2}}$ Rewrite with square roots

$\sqrt{144}\sqrt{p^2}\sqrt{q^6}$ 144, p, and q are perfect squares

$12pq^3$

39. $m^{\frac{5}{2}}\sqrt{289}$

$m^{\frac{5}{2}}\sqrt{289}$ Use properties of exponents to rewrite $m^{\frac{5}{2}}$

$m^{\frac{4}{2}}m^{\frac{1}{2}}\sqrt{289}$ $m^{\frac{4}{2}}=m^2$; 289 is a perfect square

$17m^2\sqrt{m}$

40. $9\sqrt{3m^2}+\sqrt{27}$

$9\sqrt{3m^2}+\sqrt{27}$ Multiplication property of radicals; 27 has factors 9 and 3

$9\sqrt{3}\sqrt{m^2}+\sqrt{9\cdot 3}$ m^2 is a perfect square; multiplication property of radicals

$9m\sqrt{3}+\sqrt{9}\sqrt{3}$ 9 is a perfect square; 3 is not a perfect square

$9m\sqrt{3}+3\sqrt{3}$

41. $3\sqrt{ab^2}-b\sqrt{a}$

$3\sqrt{ab^2}-b\sqrt{a}$ Multiplication property of addition

$3\sqrt{a}\sqrt{b^2}-b\sqrt{a}$ b^2 is a perfect square

$3b\sqrt{a}-b\sqrt{a}$ Subtract

$2b\sqrt{a}$

42. $\dfrac{4\sqrt{2n}}{\sqrt{16n^4}}$

$\dfrac{4\sqrt{2n}}{\sqrt{16n^4}}$ Multiplication property of radicals

$\dfrac{4\sqrt{2n}}{\sqrt{16}\sqrt{n^4}}$ 16 and n^4 are perfect squares

$\dfrac{4\sqrt{2n}}{4n^2}$ Simplify

$\dfrac{\sqrt{2n}}{n^2}$

43. $\sqrt{\frac{225x^3}{49x}}$

$\sqrt{\frac{225x^3}{49x}}$ Division property of radicals

$\frac{\sqrt{225x^3}}{\sqrt{49x}}$ Multiplication property of radicals

$\frac{\sqrt{225}\sqrt{x^2}\sqrt{x}}{\sqrt{49}\sqrt{x}}$ 225, 49 and x^2 are perfect squares

$\frac{15x\sqrt{x}}{7\sqrt{x}}$ Simplify

$\frac{15x}{7}$

44. $3\sqrt{44z}+\sqrt{99z}$

$3\sqrt{44z}+\sqrt{99z}$ 44z has factors 4 and 11z; 99z has factors 9 and 11z

$3\sqrt{4\cdot 11z}+\sqrt{9\cdot 11z}$ Multiplication property of radicals

$3\sqrt{4}\sqrt{11z}+\sqrt{9}\sqrt{11z}$ 4 and 9 are perfect squares

$3(2)\sqrt{11z}+3\sqrt{11z}$ Simplify

$9\sqrt{11z}$

45. $\sqrt{50y^8}$

$\sqrt{50y^8}$ Multiplication property of radicals

$\sqrt{50}\sqrt{y^8}$ 50 has factors 25 and 2; y^4 is a perfect square

$\sqrt{25\cdot 2}y^4$ Multiplication property of radicals

$\sqrt{25}\sqrt{2}y^4$ 25 is a perfect square

$5y^4\sqrt{2}$

46. $\sqrt{490bc^2}$

$\sqrt{490bc^2}$ Multiplication property of radicals

$\sqrt{490b}\sqrt{c^2}$ 490b has factors 49 and 10b; c^2 is a perfect square

$c\sqrt{49\cdot 10b}$ Multiplication property of radicals

$c\sqrt{49}\sqrt{10b}$ 49 is a perfect square

$7c\sqrt{10b}$

47. $\sqrt{\frac{32}{14d}}$

$\sqrt{\frac{32}{14d}}$ Simplify

$\sqrt{\frac{16}{7d}}$ Division property of radicals

$\frac{\sqrt{16}}{\sqrt{7d}}$ 16 is a perfect square

$\frac{4}{\sqrt{7d}} \cdot \frac{\sqrt{7d}}{\sqrt{7d}}$ Rationalize the denominator

$\frac{4\sqrt{7d}}{7d}$

48. $q^{\frac{3}{2}}\sqrt{63p}$

$q^{\frac{3}{2}}\sqrt{63p}$ Rewrite $q^{\frac{3}{2}}$; 63p has factors 9 and 7p

$q^{\frac{2}{2}}q^{\frac{1}{2}}\sqrt{9 \cdot 7p}$ Simplify; Multiplication property of radicals

$q\sqrt{q}\sqrt{9}\sqrt{7p}$ 9 is a perfect square; q and 7p are not perfect squares

$3q\sqrt{7p}\sqrt{q}$ Multiplication property of radicals

$3q\sqrt{7pq}$

49. $\frac{\sqrt{8}}{1-\sqrt{3x}}$

$\frac{\sqrt{8}}{1-\sqrt{3x}}$ Multiply top and bottom by the conjugate of the denominator

$\frac{\sqrt{8}}{1-\sqrt{3x}} \cdot \frac{1+\sqrt{3x}}{1+\sqrt{3x}}$ $\sqrt{8} = 2\sqrt{2}$; Use the FOIL method

$\frac{2\sqrt{2}+2\sqrt{2}\sqrt{3x}}{1-\sqrt{3x}+\sqrt{3x}-3x}$ Multiplication property of radicals; simplify

$\frac{2\sqrt{2}+2\sqrt{6x}}{1-3x}$

50. $\sqrt{\frac{20}{121d^4}}$

$\sqrt{\frac{20}{121d^4}}$ Division property of radicals

$\frac{\sqrt{20}}{\sqrt{121d^4}}$ 20 has factors 4 and 5; Multiplication property of radicals

$\frac{\sqrt{4\cdot 5}}{\sqrt{121}\sqrt{d^4}}$ Multiplication property of radicals; 121 and d^4 are perfect squares

$\frac{\sqrt{4}\sqrt{5}}{11d^2}$ 4 is a perfect square

$\frac{2\sqrt{5}}{11d^2}$

51. $w^{\frac{3}{2}}\sqrt{32}-w^{\frac{3}{2}}\sqrt{50}$

$w^{\frac{3}{2}}\sqrt{32}-w^{\frac{3}{2}}\sqrt{50}$ Rewrite $w^{\frac{3}{2}}$

$w^{\frac{2}{2}}w^{\frac{1}{2}}\sqrt{32}-w^{\frac{2}{2}}w^{\frac{1}{2}}\sqrt{50}$ Simplify

$w\sqrt{w}\sqrt{32}-w\sqrt{w}\sqrt{50}$ Multiplication property of radicals

$w\sqrt{32w}-w\sqrt{50w}$ 32w has factors 16 and 2w; 50w has factors 25 and 2w

$w\sqrt{16\cdot 2w}-w\sqrt{25\cdot 2w}$ Multiplication property of radicals

$w\sqrt{16}\sqrt{2w}-w\sqrt{25}\sqrt{2w}$ 16 and 25 are perfect squares

$4w\sqrt{2w}-5w\sqrt{2w}$ Subtract

$-w\sqrt{2w}$

52. $\sqrt{108x^4}+\sqrt{27x^4}$

$\sqrt{108x^4}+\sqrt{27x^4}$ 108 has factors 36 and 3; 27 has factors 9 and 3

$\sqrt{36x^4\cdot 3}+\sqrt{9x^4\cdot 3}$ Multiplication property of radicals

$\sqrt{36}\sqrt{x^4}\sqrt{3}+\sqrt{9}\sqrt{x^4}\sqrt{3}$ 36, 9 and x^4 are perfect squares

$6x^2\sqrt{3}+3x^2\sqrt{3}$ Add

$9x^2\sqrt{3}$

53. $\frac{\sqrt{12x}}{2+2\sqrt{3}}$

$\frac{\sqrt{12x}}{2+2\sqrt{3}}$ 12 has factors 4 and 3

$\frac{\sqrt{4\cdot 3x}}{2+2\sqrt{3}}$ Multiplication property of radicals

$\frac{\sqrt{4}\sqrt{3x}}{2+2\sqrt{3}}$ 4 is a perfect square

$\frac{2\sqrt{3x}}{2+2\sqrt{3}}\cdot\frac{2-2\sqrt{3}}{2-2\sqrt{3}}$ Multiply top and bottom by the conjugate of the denominator

$\frac{4\sqrt{3x}-4\sqrt{9x}}{4-4\sqrt{3}+4\sqrt{3}-12}$ Simplify

$-\frac{4\sqrt{3x}-12\sqrt{x}}{8}$ Simplify

$\frac{3\sqrt{x}-\sqrt{3x}}{2}$

54. $\sqrt{147k^3}$

$\sqrt{147k^3}$ 147 has factors 49 and 3; Rewrite k^3

$\sqrt{49\cdot 3\cdot k^2\cdot k}$ Multiplication property of radicals

$\sqrt{49}\sqrt{k^2}\sqrt{3k}$ 49 and k^2 are perfect squares

$7k\sqrt{3k}$

55. $\sqrt{125n^{10}}$

$\sqrt{125n^{10}}$ 125 has factors 25 and 5

$\sqrt{25n^{10}\cdot 5}$ Multiplication property of radicals

$\sqrt{25n^{10}}\sqrt{5}$ Multiplication property of radicals

$\sqrt{25}\sqrt{n^{10}}\sqrt{5}$ 25 and n^{10} are perfect squares

$5n^5\sqrt{5}$

56. $\sqrt{\frac{42q}{36q^3}}$

$\sqrt{\frac{42q}{36q^3}}$ Division property of radicals

$\frac{\sqrt{42q}}{\sqrt{36q^3}}$ Multiplication property of radicals; rewrite q^3

$\frac{\sqrt{42q}}{\sqrt{36}\sqrt{q^2q}}$ 36 and q^2 are perfect squares

$\frac{\sqrt{42q}}{6q\sqrt{q}}$ Division property of radicals

$\frac{1}{6q}\sqrt{\frac{42q}{q}}$ Simplify

$\frac{\sqrt{42}}{6q}$

57. $\sqrt{\frac{81m}{361m^2}}$

$\sqrt{\frac{81m}{361m^2}}$ Division property of radicals

$\frac{\sqrt{81m}}{\sqrt{361m^2}}$ Multiplication property of radicals

$\frac{\sqrt{81}\sqrt{m}}{\sqrt{361}\sqrt{m^2}}$ 81, 361 and m^2 are perfect squares

$\frac{9\sqrt{m}}{19m}$

58. $\sqrt{72c}-2\sqrt{2c}$

$\sqrt{72c}-2\sqrt{2c}$ 72 has factors 36 and 2

$\sqrt{36\cdot 2c}-2\sqrt{2c}$ Multiplication property of radicals

$\sqrt{36}\sqrt{2c}-2\sqrt{2c}$ 36 is a perfect square

$6\sqrt{2c}-2\sqrt{2c}$ Subtract

$4\sqrt{2c}$

59. $\sqrt{\frac{144}{324d^2}}$

$\sqrt{\frac{144}{324d^2}}$ Division property of radicals

$\frac{\sqrt{144}}{\sqrt{324d^2}}$ 144 is a perfect square; Multiplication property of radicals

$\frac{12}{\sqrt{324}\sqrt{d^2}}$ 324 and d^2 are perfect squares

$\frac{12}{18d}$ Simplify

$\frac{2}{3d}$

60. $\sqrt[3]{24x^6}+\sqrt[3]{81x^6}$

$\sqrt[3]{24x^6}+\sqrt[3]{81x^6}$ Multiplication property of radicals

$\sqrt[3]{24}\sqrt[3]{x^6}+\sqrt[3]{81}\sqrt[3]{x^6}$ x^6 is a perfect cube; 24 has factors 8 and 3; 81 has factors 27 and 3

$x^2\sqrt[3]{8\cdot 3}+x^2\sqrt[3]{27\cdot 3}$ Multiplication property of radicals

$x^2\sqrt[3]{8}\sqrt[3]{3}+x^2\sqrt[3]{27}\sqrt[3]{3}$ 8 and 27 are perfect cubes

$2x^2\sqrt[3]{3}+3x^2\sqrt[3]{3}$ Add

$5x^2\sqrt[3]{3}$

61. $\sqrt[4]{\frac{162x^6}{16x^4}}$

$\sqrt[4]{\frac{162x^6}{16x^4}}$ Division property of radicals

$\frac{\sqrt[4]{162x^6}}{\sqrt[4]{16x^4}}$ Multiplication property of radicals; rewrite x^6

$\frac{\sqrt[4]{162}\sqrt[4]{x^4x^2}}{\sqrt[4]{16}\sqrt[4]{x^4}}$ 162 has factors 81 and 2; simplify; multiplication property of radicals

$\frac{\sqrt[4]{81\cdot 2}\sqrt[4]{x^4}\sqrt[4]{x^2}}{2x}$ Multiplication property of radicals; simplify

$\frac{x\sqrt[4]{81}\sqrt[4]{2}\sqrt[4]{x^2}}{2x}$ $81 = 3^4$

$\frac{3x\sqrt[4]{2x^2}}{2x}$ Simplify

$\frac{3\sqrt[4]{2x^2}}{2}$

62. $\sqrt[3]{64y}$

$\sqrt[3]{64y}$ Multiplication property of radicals

$\sqrt[3]{64}\sqrt[3]{y}$ 64 is a perfect cube

$4\sqrt[3]{y}$

63. $\sqrt[3]{128z^3} - \sqrt[3]{-16z^3}$

$\sqrt[3]{128z^3} - \sqrt[3]{-16z^3}$ Multiplication property of radicals

$\sqrt[3]{128z^3} - \sqrt[3]{-16}\sqrt[3]{z^3}$ 128 has factors 64 and 2; -16 has factors -8 and 2

$\sqrt[3]{64 \cdot 2}\sqrt[3]{z^3} - \sqrt[3]{-8 \cdot 2}\sqrt[3]{z^3}$ Multiplication property of radicals; z^3 is a perfect cube

$z\sqrt[3]{64}\sqrt[3]{2} - z\sqrt[3]{-8}\sqrt[3]{2}$ 64 and -8 are perfect cubes

$4z\sqrt[3]{2} - (-2)z\sqrt[3]{2}$ Simplify and subtract

$6z\sqrt[3]{2}$

64. $\sqrt[5]{1024c^{10}}$

$\sqrt[5]{1024c^{10}}$ Multiplication property of radicals

$\sqrt[5]{1024}\sqrt[5]{c^{10}}$ $1024 = 4^5$ and $\sqrt[5]{c^{10}} = c^{\frac{10}{5}}$

$4c^2$

Real-World Applications

65. A guy wire for a suspension bridge runs from the ground diagonally to the top of the closest pylon to make a triangle. We can use the Pythagorean Theorem to find the length of guy wire needed. The square of the distance between the wire on the ground and the pylon on the ground is 90,000 feet. The square of the height of the pylon is 160,000 feet. So the length of the guy wire can be found by evaluating $\sqrt{90,000 + 160,000}$. What is the length of the guy wire?

$\sqrt{90,000 + 160,000}$ Add

$\sqrt{250,000}$ Note: $250,000 = 500^2$

500 feet

66. A car accelerates at a rate of $6-\frac{\sqrt{4}}{\sqrt{t}}\ \text{m/s}^2$ where t is the time in seconds after the car moves from rest. Simplify the expression.

$6-\frac{\sqrt{4}}{\sqrt{t}}\ \text{m/s}^2$ 4 is a perfect square

$6-\frac{2}{\sqrt{t}}\cdot\frac{\sqrt{t}}{\sqrt{t}}\ \text{m/s}^2$ Rationalize the denominator

$6-\frac{2\sqrt{t}}{t}\ \text{m/s}^2$

Extensions

For the following exercises, simplify each expression.

67. $\frac{\sqrt{8}-\sqrt{16}}{4-\sqrt{2}}-2^{\frac{1}{2}}$

$\frac{\sqrt{8}-\sqrt{16}}{4-\sqrt{2}}-2^{\frac{1}{2}}$ 16 is a perfect square; Note that 8 is $4\cdot 2$

$\frac{2\sqrt{2}-4}{4-\sqrt{2}}\cdot\frac{4+\sqrt{2}}{4+\sqrt{2}}-\sqrt{2}$ Multiply the top and bottom by the conjugate of the denom.

$\frac{8\sqrt{2}+4-16-4\sqrt{2}}{16+4\sqrt{2}-4\sqrt{2}-2}-\sqrt{2}$ Simplify

$\frac{-12+4\sqrt{2}}{14}-\frac{14\sqrt{2}}{14}$ Give $\sqrt{2}$ a common denominator

$\frac{-12-10\sqrt{2}}{14}$ Simplify

$-\frac{6+5\sqrt{2}}{7}$

68. $\dfrac{4^{\frac{3}{2}}-16^{\frac{3}{2}}}{8^{\frac{1}{3}}}$

$\dfrac{4^{\frac{3}{2}}-16^{\frac{3}{2}}}{8^{\frac{1}{3}}}$ Rewrite all fractional exponents

$\dfrac{\left(\sqrt{4}\right)^3-\left(\sqrt{16}\right)^3}{\sqrt[3]{8}}$ 4 and 16 are perfect squares; 8 is a perfect cube

$\dfrac{2^3-4^3}{2}$ Evaluate exponential expressions

$\dfrac{8-64}{2}$ Simplify

-28

69. $\dfrac{\sqrt{mn^3}}{a^2\sqrt{c^{-3}}}\cdot\dfrac{a^{-7}n^{-2}}{\sqrt{m^2c^4}}$

$\dfrac{\sqrt{mn^3}}{a^2\sqrt{c^{-3}}}\cdot\dfrac{a^{-7}n^{-2}}{\sqrt{m^2c^4}}$ Rewrite all negative exponents

$\dfrac{\sqrt{mn^3c^3}}{a^2}\cdot\dfrac{1}{a^7n^2\sqrt{m^2c^4}}$ Multiplication property of radicals; multiply

$\dfrac{\sqrt{m}\sqrt{n^3}\sqrt{c^3}}{a^9n^2\sqrt{m^2}\sqrt{c^4}}$ Evaluate square roots and combine like terms

$\dfrac{nc\sqrt{mnc}}{a^9n^2mc^2}$ Simplify

$\dfrac{\sqrt{mnc}}{a^9mnc}$

70. $\dfrac{a}{a-\sqrt{c}}$

$\dfrac{a}{a-\sqrt{c}}$ Multiply top and bottom by the conjugate of the denominator

$\dfrac{a}{a-\sqrt{c}}\cdot\dfrac{a+\sqrt{c}}{a+\sqrt{c}}$ Multiply; Use the FOIL method

$\dfrac{a^2+a\sqrt{c}}{a^2+a\sqrt{c}-a\sqrt{c}-c}$ Combine like terms

$\dfrac{a^2+a\sqrt{c}}{a^2-c}$

71. $\dfrac{x\sqrt{64y}+4\sqrt{y}}{\sqrt{128y}}$

$\dfrac{x\cdot\sqrt{64}\cdot\sqrt{y}+4\cdot\sqrt{y}}{\sqrt{128}\cdot\sqrt{y}}$

$\dfrac{x\cdot\sqrt{2^2}\cdot\sqrt{2^2}\cdot\sqrt{2^2}+4}{\sqrt{2^2}\cdot\sqrt{2^2}\cdot\sqrt{2^2}\cdot\sqrt{2}}$

$\dfrac{x\cdot 2\cdot 2\cdot 2+2\cdot 2\cdot 1}{2\cdot 2\cdot 2\cdot\sqrt{2}}$

$\dfrac{2x+1}{2\cdot\sqrt{2}}$

72. $\left(\dfrac{\sqrt{250x^2}}{\sqrt{100b^3}}\right)\left(\dfrac{7\sqrt{b}}{\sqrt{125x}}\right)$

$\left(\dfrac{7\sqrt{2x}}{10b}\right)$

73. $\sqrt{\dfrac{\sqrt[3]{64}+\sqrt[4]{256}}{\sqrt{64}+\sqrt{256}}}$

$\dfrac{\sqrt{3}}{3}$

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Chapter 1
Prerequisites
1.4 Polynomials

Verbal

1. Evaluate the following statement: the degree of a polynomial in standard form is the exponent of the leading term. Explain why the statement is true or false.
   The statement is true. In standard form, the polynomial with the highest value exponent is placed first and is the leading term. The degree of a polynomial is the value of the highest exponent, which in standard form is also the exponent of the leading term.
2. Many times, multiplying two binomials with two variables results in a trinomial. This is not the case when there is a difference of two squares. Explain why the product in this case is also a binomial.
   In a difference of two squares, the products are additive inverses of each other so when they are added, their sum is zero.
3. You can multiply polynomials with any number of terms and any number of variables using four basic steps over and over until you reach the expanded polynomial. What are the four steps?
   Use the distributive property, multiply, combine like terms, and simplify.
4. State whether the following statement is true and explain why or why not: a trinomial is always a higher degree than a monomial.
   The statement is false. The degree doesn't have anything to do with the number of terms in a polynomial; it is the highest power of the variable that occurs in the polynomial. So a monomial can have a power of 4 while a trinomial may have a power of 2.

Algebraic

For the following exercises, identify the degree of the polynomial.

5. $7x-2x^2+13$
   2

6. $14m^3+m^2-16m+8$
   3

7. $-625a^8+16b^4$
   8

8. $200p-30p^2m+40m^3$
   3

9. x^2+4x+4

   2

10. $6y^4-y^5+3y-4$

    5

For the following exercises, find the sum or difference.

11. $\left(12x^2+3x\right)-\left(8x^2-19\right)$

    $\left(12x^2+3x\right)-\left(8x^2-19\right)$ Distribute the minus symbol

    $12x^2+3x-8x^2+19$ Combine like terms

    $4x^2+3x+19$

12. $\left(4z^3+8z^2-z\right)+\left(-2z^2+z+6\right)$

    $\left(4z^3+8z^2-z\right)+\left(-2z^2+z+6\right)$ Rewrite

    $4z^3+8z^2-z-2z^2+z+6$ Combine like terms

    $4z^3+6z^2+6$

13. $\left(6w^2+24w+24\right)-\left(3w^2-6w+3\right)$

    $\left(6w^2+24w+24\right)-\left(3w^2-6w+3\right)$ Distribute the minus symbol

    $6w^2+24w+24-3w^2+6w-3$ Combine like terms

    $3w^2+30w+21$

14. $\left(7a^3+6a^2-4a-13\right)+\left(-3a^3-4a^2+6a+17\right)$

    $\left(7a^3+6a^2-4a-13\right)+\left(-3a^3-4a^2+6a+17\right)$ Rewrite

    $7a^3+6a^2-4a-13-3a^3-4a^2+6a+17$ Combine like terms

    $4a^3+2a^2+2a+4$

15. $\left(11b^4-6b^3+18b^2-4b+8\right)-\left(3b^3+6b^2+3b\right)$

    $\left(11b^4-6b^3+18b^2-4b+8\right)-\left(3b^3+6b^2+3b\right)$ Distribute the minus symbol

    $11b^4-6b^3+18b^2-4b+8-3b^3-6b^2-3b$ Combine like terms

    $11b^4-9b^3+12b^2-7b+8$

16. $\left(49p^2-25\right)+\left(16p^4-32p^2+16\right)$

$(49p^2-25)+(16p^4-32p^2+16)$ Rewrite

$49p^2-25+16p^4-32p^2+16$ Combine like terms and rewrite

$16p^4+17p^2-9$

For the following exercises, find the product.

17. $(4x+2)(6x-4)$

$(4x+2)(6x-4)$ Use the FOIL method

$24x^2-16x+12x-8$ Combine like terms

$24x^2-4x-8$

18. $(14c^2+4c)(2c^2-3c)$

$(14c^2+4c)(2c^2-3c)$ Use the FOIL method

$18c^4-42c^3+8c^3-12c^2$ Combine like terms

$28c^4-34c^3-12c^2$

19. $(6b^2-6)(4b^2-4)$

$(6b^2-6)(4b^2-4)$ Use the FOIL method

$24b^4-24b^2-24b^2+24$ Combine like terms

$24b^4-48b^2+24$

20. $(3d-5)(2d+9)$

$(3d-5)(2d+9)$ Use the FOIL method

$6d^2+27d-10d-45$ Combine like terms

$6d^2+17d-45$

21. $(9v-11)(11v-9)$

$(9v-11)(11v-9)$ Use the FOIL method

$99v^2-81v-121v+99$ Combine like terms

$99v^2-202v+99$

22. $(4t^2+7t)(-3t^2+4)$

$(4t^2+7t)(-3t^2+4)$ Use the FOIL method

$-12t^4+16t^2-21t^3+28t$ Rewrite

$-12t^4-21t^3+16t^2+28t$

23. $(8n-4)(n^2+9)$

$(8n-4)(n^2+9)$ Use the FOIL method

$8n^3+72n-4n^2-36$ Rewrite

$8n^3-4n^2+72n-36$

For the following exercises, expand the binomial.

24. $(4x+5)^2$

$(4x+5)^2$ Expand the expression

$(4x+5)(4x+5)$ Use the FOIL method

$16x^2+20x+20x+25$ Combine like terms

$16x^2+40x+25$

25. $(3y-7)^2$

$(3y-7)^2$ Expand the expression

$(3y-7)(3y-7)$ Use the FOIL method

$9y^2-21y-21y+49$ Combine like terms

$9y^2-42y+49$

26. $(12-4x)^2$

$(12-4x)^2$ Expand the expression

$(12-4x)(12-4x)$ Use the FOIL method

$144-48x-48x+16x^2$ Combine like terms and rewrite

$16x^2-96x+144$

27. $(4p+9)^2$

$(4p+9)^2$ Expand the expression

$(4p+9)(4p+9)$ Use the FOIL method

$16p^2+36p+36p+81$ Combine like terms

$16p^2+72p+81$

28. $(2m-3)^2$

$(2m-3)^2$ Expand the expression

$(2m-3)(2m-3)$ Use the FOIL method

$4m^2-6m-6m+9$ Combine like terms

$4m^2-12m+9$

29. $(3y-6)^2$

$(3y-6)^2$ Expand the expression

$(3y-6)(3y-6)$ Use the FOIL method

$9y^2-18y-18y+36$ Combine like terms

$9y^2-36y+36$

30. $(9b+1)^2$

$(9b+1)^2$ Expand the expression

$(9b+1)(9b+1)$ Use the FOIL method

$81b^2+9b+9b+1$ Combine like terms

$81b^2+18b+1$

For the following exercises, multiply the binomials.

31. $(4c+1)(4c-1)$

$(4c+1)(4c-1)$ Use the FOIL method

$16c^2-4c+4c-1$ Combine like terms

$16c^2-1$

32. $(9a-4)(9a+4)$

$(9a-4)(9a+4)$ Use the FOIL method

$81a^2+36a-36a-16$ Combine like terms

$81a^2-16$

33. $(15n-6)(15n+6)$

$(15n-6)(15n+6)$ Use the FOIL method

$225n^2+90n-90n-36$ Combine like terms

$225n^2-36$

34. $(25b+2)(25b-2)$

$(25b+2)(25b-2)$ Use the FOIL method

$625b^2-50b+50b-4$ Combine like terms

$625b^2-4$

35. $(4+4m)(4-4m)$

$(4+4m)(4-4m)$ Use the FOIL method

$16-16m+16m-16m^2$ Combine like terms

$-16m^2+16$

36. $(14p+7)(14p-7)$

$(14p+7)(14p-7)$ Use the FOIL method

$196p^2-98p+98p-49$ Combine like terms

$196p^2-49$

37. $(11q-10)(11q+10)$

$(11q-10)(11q+10)$ Use the FOIL method

$121q^2+110q-110q-100$ Combine like terms

$121q^2-100$

For the following exercises, multiply the polynomials.

38. $(2x^2+2x+1)(4x-1)$

$(2x^2+2x+1)(4x-1)$ Use the distributive property

$8x^3+8x^2+4x-2x^2-2x-1$ Combine like terms

$8x^3+6x^2+2x-1$

39. $(4t^2+t-7)(4t^2-1)$

$(4t^2+t-7)(4t^2-1)$ Use the distributive property

$16t^4+4t^3-28t^2-4t^2-t+7$ Combine like terms

$16t^4+4t^3-32t^2-t+7$

40. $(x-1)(x^2-2x+1)$

$(x-1)(x^2-2x+1)$ Use the distributive property

$x^3-2x^2+x-x^2+2x-1$ Combine like terms

x^3-3x^2+3x-1

41. $(y-2)(y^2-4y-9)$

$(y-2)(y^2-4y-9)$ Use the distributive property

$y^3-4y^2-9y-2y^2+8y+18$ Combine like terms

y^3-6y^2-y+18

42. $(6k-5)(6k^2+5k-1)$

$(6k-5)(6k^2+5k-1)$ Use the distributive property

$36k^3+30k^2-6k-30k^2-25k+5$ Combine like terms

$36k^3-31k+5$

43. $(3p^2+2p-10)(p-1)$

$(3p^2+2p-10)(p-1)$ Use the distributive property

$3p^3-3p^2+2p^2-2p-10p+10$ Combine like terms

$3p^3-p^2-12p+10$

44. $(4m-13)(2m^2-7m+9)$

$(4m-13)(2m^2-7m+9)$ Use the distributive property

$8m^3-28m^2+36m-26m^2+91m-117$ Combine like terms

$8m^3-54m^2+127m-117$

45. $(a+b)(a-b)$

$(a+b)(a-b)$ Use the FOIL method

$a^2-ab+ab-b^2$ Combine like terms

a^2-b^2

46. $(4x-6y)(6x-4y)$

$(4x-6y)(6x-4y)$ Use the FOIL method

$24x^2-16xy-36xy+24y^2$ Combine like terms

$24x^2-52xy+24y^2$

47. $(4t-5u)^2$

$(4t-5u)^2$ Expand the expression

$(4t-5u)(4t-5u)$ Use the FOIL method

$16t^2-20tu-20tu+25u^2$ Combine like terms

$16t^2-40tu+25u^2$

48. $(9m+4n-1)(2m+8)$

$(9m+4n-1)(2m+8)$ Use the distributive property

$18m^2+72m+8mn+32n-2m-8$ Combine like terms

$18m^2+70m+8mn+32n-8$

49. $(4t-x)(t-x+1)$

$(4t-x)(t-x+1)$ Use the distributive property

$4t^2-4tx+4t-tx+x^2-x$ Combine like terms

$4t^2+x^2+4t-5tx-x$

50. $(b^2-1)(a^2+2ab+b^2)$

$(b^2-1)(a^2+2ab+b^2)$ Use the distributive property

$a^2b^2+2ab^3+b^4-a^2-2ab-b^2$ Rewrite

$b^4+2ab^3+a^2b^2-b^2-2ab-a^2$

51. $(4r-d)(6r+7d)$

$(4r-d)(6r+7d)$ Use the FOIL method

$24r^2+28rd-6rd-7d^2$ Combine like terms

$24r^2+22rd-7d^2$

52. $(x+y)(x^2-xy+y^2)$

$(x+y)(x^2-xy+y^2)$ Use the distributive property

$x^3-x^2y+xy^2+x^2y-xy^2+y^3$ Combine like terms

x^3+y^3

Real-World Applications

53. A developer wants to purchase a plot of land to build a house. The area of the plot can be described by the following expression: $(4x+1)(8x-3)$ where x is measured in meters.

Multiply the binomials to find the area of the plot in standard form.

$(4x+1)(8x-3)$ Use the FOIL method

$32x^2-12x+8x-3$ Combine like terms

$32x^2-4x-3$ square meters

54. A prospective buyer wants to know how much grain a specific silo can hold. The area of the floor of the silo is $(2x+9)^2$. The height of the silo is $10x+10$, where x is measured in feet. Expand the square and multiply by the height to find the expression that shows how much grain the silo can hold.

$(2x+9)^2$ Expand the expression

$(2x+9)(2x+9)$ Use the FOIL method

$4x^2+18x+18x+81$ Combine like terms

$4x^2+36x+81(10x+10)$ Multiply by the height using FOIL

$40x^3+40x^2+360x^2+360x+810x+810$ Combine like terms

$40x^3+400x^2+1170x+810$ cubic feet

Extensions

For the following exercises, perform the given operations.

55. $(4t-7)^2(2t+1)-(4t^2+2t+11)$

$(4t-7)^2(2t+1)-(4t^2+2t+11)$ Expand the first expression

$(4t-7)(4t-7)(2t+1)-(4t^2+2t+11)$ Use the FOIL method

$16t^2-28t-28t+49(2t+1)-(4t^2+2t+11)$ Combine like terms

$(16t^2-56t+49)(2t+1)-(4t^2+2t+11)$ Use the distributive property

$32t^3+16t^2-112t^2-56t+98t+49-(4t^2+2t+11)$ Combine like terms

$32t^3-96t^2+42t+49-4t^2-2t-11$ Combine like terms

$32t^3-100t^2+40t+38$

56. $(3b+6)(3b-6)(9b^2-36)$

$(3b+6)(3b-6)(9b^2-36)$ Use the FOIL method

$(9b^2-18b+18b-36)(9b^2-36)$ Combine like terms

$(9b^2-36)(9b^2-36)$ Use the FOIL method

$81b^4-324b^2-324b^2+1296$ Combine like terms

$81b^4-648b^2+1296$

57. $(a^2+4ac+4c^2)(a^2-4c^2)$

$(a^2+4ac+4c^2)(a^2-4c^2)$ Use the distributive property

$a^4-4a^2c^2+4a^3c-16ac^3+4a^2c^2-16c^4$ Combine like terms

$a^4+4a^3c-16ac^3-16c^4$

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Chapter 1

Prerequisites

1.5 Factoring Polynomials

Verbal

1. If the terms of a polynomial do not have a greatest common factor, then does that mean it is not factorable? Explain.

   The terms of a polynomial don't have to have a common factor for the entire polynomial to be factorable. For example, $4x^2$ and $-9y^2$ don't have a common factor, but the whole polynomial is still factorable $4x^2-9y^2=(2x+3y)(2x-3y)$.

2. A polynomial is factorable, but it is not a perfect square trinomial or a difference of two squares. Can you factor the polynomial without finding the greatest common factor?

   Yes, you would just have to use extra steps to factor out multiple common terms.

3. How do you factor by grouping?

   Divide the x term into the sum of two terms, factor each portion of the expression separately, and then factor out the GCF of the entire expression.

Algebraic

For the following exercises, find the greatest common factor.

4. $14x+4xy-18xy^2$

   $2x$

5. $49mb^2-35m^2ba+77ma^2$

   $7m$

6. $30x^3y - 45x^2y^2 + 135xy^3$

$15xy$

7. $200p^3m^3 - 30p^2m^3 + 40m^3$

$10m^3$

8. $36j^4k^2 - 18j^3k^3 + 54j^2k^4$

$18j^2k^2$

9. $6y^4 - 2y^3 + 3y^2 - y$

y

For the following exercises, factor by grouping.

10. $6x^2 + 5x - 4$

$6x^2 + 5x - 4$ Rewrite $5x$ as $8x$ - $3x$
$(6x^2 + 8x) + (-3x - 4)$ Factor out the GCF from each expression
$2x(3x + 4) - (3x + 4)$ Use the distributive property
$(2x - 1)(3x + 4)$

11. $2a^2 + 9a - 18$

$2a^2 + 9a - 18$ Rewrite $9a$ as $12a$ - $3a$
$(2a^2 + 12a) + (-3a - 18)$ Factor out the GCF from each expression
$2a(a + 6) - 3(a + 6)$ Use the distributive property
$(2a - 3)(a + 6)$

12. $6c^2 + 41c + 63$

$6c^2+41c+63$	Rewrite $41x$ as $27x + 14x$
$(6c^2+14c)+(27c+63)$	Factor out the GCF from each expression
$2c(3c+7)+9(3c+7)$	Use the distributive property
$(2c+9)(3c+7)$	

13. $6n^2-19n-11$

$6n^2-19n-11$	Rewrite $-19n$ as $3n - 22n$
$(6n^2+3n)+(-22n-11)$	Factor out the GCF from each expression
$3n(2n+1)-11(2n+1)$	Use the distributive property
$(3n-11)(2n+1)$	

14. $20w^2-47w+24$

$20w^2-47w+24$	Rewrite $-47w$ as $-15w-32w$
$(20w^2-15w)+(-32w+24)$	Factor out the GCF from each expression
$5w(4w-3)-8(4w-3)$	Use the distributive property
$(5w-8)(4w-3)$	

15. $2p^2-5p-7$

$2p^2-5p-7$	Rewrite $-5p$ as $2p-7p$
$(2p^2+2p)+(-7p-7)$	Factor out the GCF of each expression
$2p(p+1)-7(p+1)$	Use the distributive property
$(2p-7)(p+1)$	

For the following exercises, factor the polynomial.

16. $7x^2+48x-7$

$7x^2+48x-7$	Rewrite $48x$ as $49x - x$
$(7x^2+49x)+(-x-7)$	Factor out the GCF of each expression
$7x(x+7)-(x+7)$	Use the distributive property
$(7x-1)(x+7)$	

17. $10h^2 - 9h - 9$

$10h^2 - 9h - 9$	Rewrite $-9h$ as $-15h + 6h$
$(10h^2 - 15h) + (6h - 9)$	Factor out the GCF of each expression
$5h(2h - 3) + 3(2h - 3)$	Use the distributive property
$(5h + 3)(2h - 3)$	

18. $2b^2 - 25b - 247$

$2b^2 - 25b - 247$	Rewrite $-25b$ as $-38b + 13b$
$(2b^2 - 38b) + (13b - 247)$	Factor out the GCF of each expression
$2b(b - 19) + 13(b - 19)$	Use the distributive property
$(2b + 13)(b - 19)$	

19. $9d^2 - 73d + 8$

$9d^2 - 73d + 8$	Rewrite $-73d$ as $-72d - d$
$(9d^2 - 72d) + (-d + 8)$	Factor out the GCF of each expression
$9d(d - 8) - (d - 8)$	Use the distributive property
$(9d - 1)(d - 8)$	

20. $90v^2 - 181v + 90$

$90v^2 - 181v + 90$	Rewrite $-181v$ as $-100v - 81v$
$90v^2 - 100v - 81v + 90$	Factor out the GCF of each expression
$10v(9v - 10) - 9(9v - 10)$	Use the distributive property
$(10v - 9)(9v - 10)$	

21. $12t^2 + t - 13$

$12t^2 + t - 13$	Rewrite t as $13t - 12t$
$(12t^2 + 13t) + (-12t - 13)$	Factor out the GCF of each expression
$t(12t + 13) - (12t + 13)$	Use the distributive property
$(t - 1)(12t + 13)$	

22. $2n^2 - n - 15$

$2n^2 - n - 15$ Rewrite $-n$ as $-6n + 5n$

$(2n^2 - 6n) + (5n - 15)$ Factor out the GCF of each expression

$2n(n-3) + 5(n-3)$ Use the distributive property

$(2n+5)(n-3)$

23. $16x^2 - 100$

$16x^2 - 100$ Rewrite $0x$ as $-40x + 40x$

$(16x^2 - 40x) + (40x - 100)$ Factor out the GCF of each expression

$4x(4x-10) + 10(4x-10)$ Use the distributive property

$(4x+10)(4x-10)$

24. $25y^2 - 196$

$25y^2 - 196$ Rewrite $0x$ as $70x - 70x$

$(25y^2 + 70y) + (-70y - 196)$ Factor out the GCF of each expression

$5y(5y+14) - 14(5y+14)$ Use the distributive property

$(5y-14)(5y+14)$

25. $121p^2 - 169$

$121p^2 - 169$ Rewrite $0x$ as $143p - 143p$

$(121p^2 + 143p) + (-143p - 169)$ Factor out the GCF of each expression

$11p(11p+13) - 13(11p-13)$ Use the distributive property

$(11p+13)(11p-13)$

26. $4m^2 - 9$

$4m^2 - 9$ Rewrite $0m$ as $6m - 6m$

$(4m^2 + 6m) + (-6m - 9)$ Factor out the GCF of each expression

$2m(m+3) - 3(m+3)$ Use the distributive property

$(2m-3)(m+3)$

27. $361d^2 - 81$

$361d^2 - 81$ Rewrite $0d$ as $171d - 171d$

$(361d^2 + 171d) + (-171d - 81)$ Factor out the GCF of each expression

$19d(19d + 9) - 9(19d - 9)$ Use the distributive property

$(19d - 9)(19d + 9)$

28. $324x^2 - 121$

$324x^2 - 121$ Rewrite $0x$ as $198x - 198x$

$(324x^2 + 198x) + (-198x - 121)$ Factor out the GCF of each expression

$18x(18x + 11) - 11(18x - 11)$ Use the distributive property

$(18x - 11)(18x + 11)$

29. $144b^2 - 25c^2$

$144b^2 - 25c^2$ Rewrite $0bc$ as $60bc - 60bc$

$(144b^2 + 60bc) + (-60bc - 25c^2)$ Factor out the GCF of each expression

$12b(12b + 5c) - 5c(12b - 5c)$ Use the distributive property

$(12b - 5c)(12b + 5c)$

30. $16a^2 - 8a + 1$

$16a^2 - 8a + 1$ Rewrite $-8a$ as $-4a - 4a$

$(16a^2 - 4a) + (-4a + 1)$ Factor out the GCF of each expression

$4a(4a - 1) - (4a - 1)$ Use the distributive property

$(4a - 1)(4a - 1)$ Rewrite

$(4a - 1)^2$

31. $49n^2 + 168n + 144$

$49n^2 + 168n + 144$ Rewrite $168n$ as $84n + 84n$

$(49n^2 + 84n) + (84n + 144)$ Factor out the GCF of each expression

$7n(7n + 12) + 12(7n + 12)$ Use the distributive property

$(7n + 12)(7n + 12)$ Rewrite

$(7n + 12)^2$

32. $121x^2 - 88x + 16$

$121x^2-88x+16$	Rewrite $88x$ as $-44x - 44x$
$(121x^2-44x)+(-44x+16)$	Factor out the GCF of each expression
$11x(11x-4)-4(11-4)$	Use the distributive property
$(11x-4)(11x-4)$	Rewrite
$(11x-4)^2$	

33. $225y^2+120y+16$

$225y^2+120y+16$	Rewrite $120y$ as $60y + 60y$
$(225y^2+60y)+(60y+16)$	Factor out the GCF of each expression
$15y(15y+4)+4(15y+4)$	Use the distributive property
$(15y+4)(15y+4)$	Rewrite
$(15y+4)^2$	

34. $m^2-20m+100$

$m^2-20m+100$	Rewrite $-20m$ as $-10m - 10m$
$(m^2-10m)+(-10m+100)$	Factor out the GCF of each expression
$m(m-10)-10(m-10)$	Use the distributive property
$(m-10)(m-10)$	Rewrite
$(m-10)^2$	

35. $25p^2-120p+144$

$25p^2-120p+144$	Rewrite $-120p$ as $-60p - 60p$
$(25p^2-60p)+(-60p+144)$	Factor out the GCF of each expression
$5p(5p-12)-12(5p-12)$	Use the distributive property
$(5p-12)(5p-12)$	Rewrite
$(5p-12)^2$	

36. $36q^2+60q+25$

$36q^2+60q+25$	Rewrite $60q$ as $30q + 30q$
$(36q^2+30q)+(30q+25)$	Factor out the GCF of each expression
$6q(6q+5)+5(6q+5)$	Use the distributive property
$(6q+5)(6q+5)$	Rewrite
$(6q+5)^2$	

For the following exercises, factor the polynomials.

37. x^3+216

x^3+216	Sum of cubes; $216 = 6^3$
$(x+6)(x^2-6x+36)$	

38. $27y^3-8$

$27y^3-8$	Difference of cubes; $27 = 3^3$ and $8 = 2^3$
$(3y-2)(9y^2+6y+4)$	

39. $125a^3+343$

$125a^3+343$	Sum of cubes; $125 = 5^3$ and $343 = 7^3$
$(5a+7)(25a^2-35a+49)$	

40. b^3-8d^3

b^3-8d^3	Difference of cubes; $8 = 2^3$
$(b-2d)(b^2+2bd+4d^2)$	

41. $64x^3-125$

$64x^3-125$	Difference of cubes; $64 = 4^3$ and $125 = 5^3$
$(4x-5)(16x^2+20x+25)$	

42. $729q^3+1331$

$729q^3+1331$ Sum of cubes; $729 = 9^2$ and $1331 = 11^3$

$(9q+11)(81q^2-99q+121)$

43. $125r^3+1728s^3$

$125r^3+1728s^3$ Sum of cubes; $125 = 5^3$ and $1728 = 12^3$

$(5r+12s)(25r^2-60rs+144s^2)$

44. $4x(x-1)^{-\frac{2}{3}}+3(x-1)^{\frac{1}{3}}$

$4x(x-1)^{-\frac{2}{3}}+3(x-1)^{\frac{1}{3}}$ Factor out the GCF of each expression

$(x-1)^{-\frac{2}{3}}[4x+3(x-1)]$ Distribute the 3

$(x-1)^{-\frac{2}{3}}(4x+3x-3)$ Combine like terms

$(x-1)^{-\frac{2}{3}}(7x-3)$

45. $3c(2c+3)^{-\frac{1}{4}}-5(2c+3)^{\frac{3}{4}}$

$3c(2c+3)^{-\frac{1}{4}}-5(2c+3)^{\frac{3}{4}}$ Factor out the GCF of the expression

$(2c+3)^{-\frac{1}{4}}[3c-5(2c+3)]$ Distribute the -5

$(2c+3)^{-\frac{1}{4}}(3c-10c-15)$ Combine like terms

$(2c+3)^{-\frac{1}{4}}(-7c-15)$

46. $3t(10t+3)^{\frac{1}{3}}+7(10t+3)^{\frac{4}{3}}$

$3t(10t+3)^{\frac{1}{3}}+7(10t+3)^{\frac{4}{3}}$ Factor out the GCF of the expression

$(10t+3)^{\frac{1}{3}}[3t+7(10t+3)]$ Distribute the 7

$(10t+3)^{\frac{1}{3}}(3t+70t+21)$ Combine like terms

$(10t+3)^{\frac{1}{3}}(73t+21)$

47. $14x(x+2)^{-\frac{2}{5}}+5(x+2)^{\frac{3}{5}}$

$14x(x+2)^{-\frac{2}{5}}+5(x+2)^{\frac{3}{5}}$ Factor out the GCF of the expression

$(x+2)^{-\frac{2}{5}}[14x+5(x+2)]$ Distribute the 5

$(x+2)^{-\frac{2}{5}}(14x+5x+10)$ Combine like terms

$(x+2)^{-\frac{2}{5}}(19x+10)$

48. $9y(3y-13)^{\frac{1}{5}}-2(3y-13)^{\frac{6}{5}}$

$9y(3y-13)^{\frac{1}{5}}-2(3y-13)^{\frac{6}{5}}$ Factor out the GCF of the expression

$(3y-13)^{\frac{1}{5}}[9y-2(3y-13)]$ Distribute the -2

$(3y-13)^{\frac{1}{5}}(9y-6y+26)$ Combine like terms

$(3y-13)^{\frac{1}{5}}(3y+26)$

49. $5z(2z-9)^{-\frac{3}{2}}+11(2z-9)^{-\frac{1}{2}}$

$5z(2z-9)^{-\frac{3}{2}}+11(2z-9)^{-\frac{1}{2}}$ Factor out the GCF of the expression

$(2z-9)^{-\frac{3}{2}}[5z+11(2z-9)]$ Distribute the 11

$(2z-9)^{-\frac{3}{2}}(5z+22z-99)$ Combine like terms

$(2z-9)^{-\frac{3}{2}}(27z-99)$

50. $6d(2d+3)^{-\frac{1}{6}}+5(2d+3)^{\frac{5}{6}}$

$6d(2d+3)^{-\frac{1}{6}}+5(2d+3)^{\frac{5}{6}}$ Factor out the GCF of the expression

$(2d+3)^{-\frac{1}{6}}[6d+5(2d+3)]$ Distribute the 5

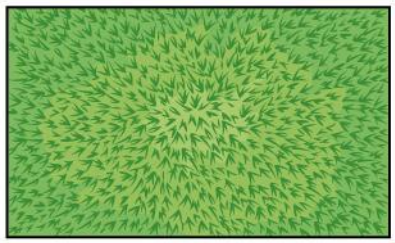
$(2d+3)^{-\frac{1}{6}}(6d+10d+15)$ Combine like terms

$(2d+3)^{-\frac{1}{6}}(16d+15)$

Real-World Applications

For the following exercises, consider this scenario:

Charlotte has appointed a chairperson to lead a city beautification project. The first act is to install statues and fountains in one of the city's parks. The park is a rectangle with an area of $98x^2+105x-27$ square meters, as shown. The length and width of the park are perfect factors of the area.



$l \times w = 98x^2 + 105x - 27$

51. Factor by grouping to find the length and width of the park.

$98x^2+105x-27$	Rewrite 105x as 126x - 21x
$(98x^2+126x)+(-21x-27)$	Factor out the GCF of each expression
$14x(7x+9)-3(7x+9)$	Use the distributive property
$(14x-3)(7x+9)$	

52. A statue is to be placed in the center of the park. The area of the base of the statue is $4x^2+12x+9$. Factor the area to find the lengths of the sides of the statue.

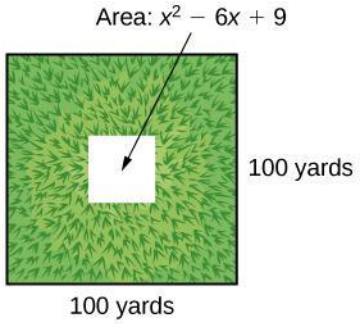
$4x^2+12x+9$	Rewrite 12x as 6x + 6x
$(4x^2+6x)+(6x+9)$	Factor out the GCF of each expression
$2x(2x+3)+3(2x+3)$	Use the distributive property
$(2x+3)(2x+3)$	Rewrite
$(2x+3)^2$	

53. At the northwest corner of the park, the city is going to install a fountain. The area of the base of the fountain is $9x^2-25$. Factor the area to find the lengths of the sides of the fountain.

$9x^2-25$	Rewrite 0x as 15x - 15x
$(9x^2+15x)+(-15x-25)$	Factor out the GCF of each expression
$3x(3x+5)-5(3x+5)$	Use the distributive property
$(3x+5)(3x-5)$	

For the following exercise, consider the following scenario:

A school is installing a flagpole in the central plaza. The plaza is a square with side length 100 yards as shown**.** The flagpole will take up a square plot with area x^2-6x+9 square yards.



54. Find the length of the base of the flagpole by factoring.

x^2-6x+9	Rewrite -6x as -3x - 3x
$x^2-3x-3x+9$	Factor out the GCF of each expression
$x(x-3)-3(x-3)$	Use the distributive property
$(x-3)(x-3)$	Rewrite
$(x-3)^2$	

Extensions

For the following exercises, factor the polynomials completely.

55. $16x^4 - 200x^2 + 625$

$16x^4 - 200x^2 + 625$	Rewrite $-200x^2$ as $-100x^2 - 100x^2$
$(16x^4 - 100x^2) + (-100x^2 + 625)$	Factor out the GCF of each expression
$4x^2(4x^2 - 25) - 25(4x^2 - 25)$	Use the distributive property
$(4x^2 - 25)(4x^2 - 25)$	Rewrite
$(4x^2 - 25)^2$	Rewrite $0x$ as $10x - 10x$
$((4x^2 + 10x) + (-10x - 25))^2$	Factor out the GCF of each expression
$(2x(2x + 5) - 5(2x + 5))^2$	Use the distributive property
$((2x - 5)(2x + 5))^2$	Distribute the exponent
$(2x - 5)^2(2x + 5)^2$	

56. $81y^4 - 256$

$81y^4 - 256$	Rewrite $0y^2$ as $-144y^2 + 144y^2$
$(81y^4 - 144y^2) + (144y^2 - 256)$	Factor out the GCF of each expression
$9y^2(9y^2 - 16) + 16(9y^2 - 16)$	Use the distributive property
$(9y^2 + 16)(9y^2 - 16)$	Rewrite $0y$ as $-12y + 12y$
$(9y^2 + 16)[(9y^2 - 12y) + (12y - 16)]$	Factor out the GCF of each expression
$(9y^2 + 16)[3y(3y - 4) + 4(3y - 4)]$	Use the distributive property
$(9y^2 + 16)(3y + 4)(3y - 4)$	

57. $16z^4 - 2401a^4$

$16z^4 - 2401a^4$	Rewrite $0z^2a^2$ as $-196z^2a^2 + 196z^2a^2$
$(16z^4 - 196z^2a^2) + (196z^2a^2 - 2401a^4)$	Factor out the GCF of each expression
$4z^2(4z^2 - 49a^2) + 49a^2(4z^2 - 49a^2)$	Use the distributive property
$(4z^2 + 49a^2)(4z^2 - 49a^2)$	Rewrite $0za$ as $-14za + 14za$
$(4z^2 + 49a^2)[(4z^2 - 14za) + (14za - 49a^2)]$	Factor out the GCF of each expression
$(4z^2 + 49a^2)[2z(2z - 7a) + 7a(2z - 7a)]$	Use the distributive property
$(4z^2 + 49a^2)(2z + 7a)(2z - 7a)$	

58. $5x(3x + 2)^{-\frac{2}{4}} + (12x + 8)^{\frac{3}{2}}$

$5x(3x+2)^{-\frac{2}{4}}+(12x+8)^{\frac{3}{2}}$	Factor out the GCF of the expression
$5x(3x+2)^{-\frac{1}{2}}+4^{\frac{3}{2}}(3x+2)^{\frac{3}{2}}$	Factor out the GCF of the expressions; Evaluate
$(3x+2)^{-\frac{1}{2}}[5x+8(3x+2)^2]$	Square the expression
$(3x+2)^{-\frac{1}{2}}[5x+8(9x^2+6x+6x+4)]$	Combine like terms
$(3x+2)^{-\frac{1}{2}}[5x+8(9x^2+12x+4)]$	Distribute the 8
$(3x+2)^{-\frac{1}{2}}(5x+72x^2+96x+32)$	Combine like terms
$(3x+2)^{-\frac{1}{2}}(72x^2+101x+32)$	

59. $\left(32x^3+48x^2-162x-243\right)^{-1}$

$\left(32x^3+48x^2-162x-243\right)^{-1}$	Note that $(4x+9)$ is a factor of the expression
$((4x+9)(8x^2-6x-27))^{-1}$	Rewrite $-6x$ as $-18x+12x$
$((4x+9)((8x^2-18x)+(12x-27)))^{-1}$	Factor out the GCF of each expression
$((4x+9)(2x(4x-9)+3(4x-9))^{-1}$	Use the distributive property
$((4x+9)(2x+3)(4x-9))^{-1}$	Rewrite
$\frac{1}{(4x+9)(4x-9)(2x+3)}$	

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Chapter 1
Prerequisites
1.6 Rational Expressions
Chapter 1 Review Exercises
Chapter 1 Practice Test

Verbal

1. How can you use factoring to simplify rational expressions?
   You can factor the numerator and denominator to see if any of the terms can cancel one another out.
2. How do you use the Least Common Denominator to combine two rational expressions?
   You express each denominator as a factor of the LCD and then multiply each side by the other term over itself. This creates a common denominator so the expression can be combined.
3. Tell whether the following statement is true or false and explain why: You only need to find the Least Common Denominator when adding or subtracting rational expressions.
   True, multiplication and division do not require finding the LCD because the denominators can be combined through those operations, whereas addition and subtraction require like terms.

Algebraic

For the following exercises, simplify the rational expressions.

4. $\dfrac{x^2-16}{x^2-5x+4}$

$\dfrac{x^2-16}{x^2-5x+4}$ Difference of squares; rewrite $-5x$ as $-x-4x$

$\dfrac{(x-4)(x+4)}{(x^2-x)+(-4x+4)}$ Factor out the GCF of each expression

$\dfrac{(x-4)(x+4)}{x(x-1)-4(x-1)}$ Use the distributive property

$\dfrac{(x-4)(x+4)}{(x-4)(x-1)}$ Simplify

$\dfrac{x+4}{x-1}$

5. $\dfrac{y^2+10y+25}{y^2+11y+30}$

$\dfrac{y^2+10y+25}{y^2+11y+30}$ Rewrite $10y$ as $5y+5y$; Rewrite $11y$ as $6y+5y$

$\dfrac{(y^2+5y)+(5y+25)}{(y^2+6y)+(5y+30)}$ Factor out the GCF of each expression

$\dfrac{y(y+5)+5(y+5)}{y(y+6)+5(y+6)}$ Use the distributive property

$\dfrac{(y+5)(y+5)}{(y+5)(y+6)}$ Simplify

$\dfrac{y+5}{y+6}$

6. $\dfrac{6a^2-24a+24}{6a^2-24}$

$\dfrac{6a^2-24a+24}{6a^2-24}$ Rewrite $-24a$ as $-12a - 12a$; Rewrite $0a$ as $12a - 12a$

$\dfrac{(6a^2-12a)+(-12a+24)}{(6a^2+12a)+(-12a-24)}$ Factor out the GCF of each expression

$\dfrac{6a(a-2)-12(a-2)}{6a(a+2)-12(a+2)}$ Use the distributive property

$\dfrac{(6a-12)(a-2)}{(6a-12)(a+2)}$ Simplify

$\dfrac{a-2}{a+2}$

7. $\dfrac{9b^2+18b+9}{3b+3}$

$\dfrac{9b^2+18b+9}{3b+3}$ Rewrite $18b$ as $9b + 9b$

$\dfrac{(9b^2+9b)+(9b+9)}{3(b+1)}$ Factor out the GCF of each expression

$\dfrac{9b(b+1)+9(b+1)}{3(b+1)}$ Use the distributive property

$\dfrac{(9b+9)(b+1)}{3(b+1)}$ Factor out the 9

$\dfrac{9(b+1)(b+1)}{3(b+1)}$ Simplify

$3(b+1)=3b+3$

8. $\frac{m-12}{m^2-144}$

$\frac{m-12}{m^2-144}$

Rewrite $0m$ as $-12m + 12m$

$\frac{m-12}{(m^2-12m)+(12m-144)}$

Factor out the GCF of each expression

$\frac{m-12}{m(m-12)+12(m-12)}$

Use the distributive property

$\frac{m-12}{(m+12)(m-12)}$

Simplify

$\frac{1}{m+12}$

9. $\frac{2x^2+7x-4}{4x^2+2x-2}$

$\frac{2x^2+7x-4}{4x^2+2x-2}$

Rewrite $7x$ as $8x - x$; Rewrite $2x$ as $4x - 2x$

$\frac{(2x^2+8x)+(-x-4)}{(4x^2+4x)+(-2x-2)}$

Factor out the GCF of the expressions

$\frac{2x(x-4)-(x+4)}{4x(x+1)-2(x+1)}$

Use the distributive property

$\frac{(2x-1)(x+4)}{(4x-2)(x+1)}$

Factor out the 2

$\frac{(2x-1)(x+4)}{2(2x-1)(x+1)}$

Simplify

$\frac{x+4}{2(x+1)}$

Simplify

$\frac{x+4}{2x+2}$

10. $\dfrac{6x^2+5x-4}{3x^2+19x+20}$

$\dfrac{6x^2+5x-4}{3x^2+19x+20}$ Rewrite $5x$ as $8x - 3x$; Rewrite $19x$ as $15x + 4x$

$\dfrac{(6x^2+8x)+(-3x-4)}{(3x^2+15x)+(4x+20)}$ Factor out the GCF of each expression

$\dfrac{2x(3x+4)-(3x+4)}{3x(x+5)+4(x+5)}$ Use the distirbutive property

$\dfrac{(2x-1)(3x+4)}{(3x+4)(x+5)}$ Simplify

$\dfrac{2x-1}{x+5}$

11. $\dfrac{a^2+9a+18}{a^2+3a-18}$

$\dfrac{a^2+9a+18}{a^2+3a-18}$ Rewrite $9a$ as $6a + 3a$; Rewrite 3a as $6a - 3a$

$\dfrac{(a^2+6a)+(3a+18)}{(a^2+6a)+(-3a-18)}$ Factor out the GCF of the expressions

$\dfrac{a(a+6)+3(a+6)}{a(a+6)-3(a+6)}$ Use the distributive property

$\dfrac{(a+3)(a+6)}{(a-3)(a+6)}$ Simplify

$\dfrac{a+3}{a-3}$

12. $\dfrac{3c^2+25c-18}{3c^2-23c+14}$

$\frac{3c^2+25c-18}{3c^2-23c+14}$ Rewrite $25c$ as $27c - 2c$; Rewrite $-23c$ as $-21c - 2c$

$\frac{(3c^2+27c)+(-2c-18)}{(3c^2-21c)+(-2c+14)}$ Factor out the GCF of each expression

$\frac{3c(c+9)-2(c+9)}{3c(c-7)-2(c-7)}$ Use the distributive property

$\frac{(3c-2)(c+9)}{(3c-2)(c-7)}$ Simplify

$\frac{c+9}{c-7}$

13. $\frac{12n^2-29n-8}{28n^2-5n-3}$

$\frac{12n^2-29n-8}{28n^2-5n-3}$ Rewrite $-29n$ as $3n - 32n$; Rewrite $-5n$ as $-12n + 7n$

$\frac{(12n^2+3n)+(-32n-8)}{(28n^2-12n)+(7n-3)}$ Factor out the GCF of the expressions

$\frac{3n(4n+1)-8(4n+1)}{4n(7n-3)+(7n-3)}$ Use the distributive property

$\frac{(3n-8)(4n+1)}{(4n+1)(7n-3)}$ Simplify

$\frac{3n-8}{7n-3}$

For the following exercises, multiply the rational expressions and express the product in simplest form.

14. $\frac{x^2-x-6}{2x^2+x-6}\cdot\frac{2x^2+7x-15}{x^2-9}$

$$\frac{x^2-x-6}{2x^2+x-6}\cdot\frac{2x^2+7x-15}{x^2-9}$$ Rewrite functions; Difference of squares

$$\frac{(x^2-3x)+(2x-6)}{(2x^2+4x)+(-3x-6)}\cdot\frac{(2x^2+10x)+(-3x-15)}{(x-3)(x+3)}$$ Factor out the GCF of the expressions

$$\frac{x(x-3)+2(x-3)}{2x(x+2)-3(x+2)}\cdot\frac{2x(x+5)-3(x+5)}{(x-3)(x+3)}$$ Use the distributive property

$$\frac{(x+2)(x-3)}{(2x-3)(x+2)}\cdot\frac{(2x-3)(x+5)}{(x-3)(x+3)}$$ Simplify

$$\frac{x+5}{x+3}$$

15. $$\frac{c^2+2c-24}{c^2+12c+36}\cdot\frac{c^2-10c+24}{c^2-8c+16}$$

$$\frac{c^2+2c-24}{c^2+12c+36}\cdot\frac{c^2-10c+24}{c^2-8c+16}$$ Rewrite the expressions

$$\frac{(c^2+6c)+(-4c-24)}{(c^2+6c)+(6c+36)}\cdot\frac{(c^2-6c)+(-4c+24)}{(c^2-4c)+(-4c+16)}$$ Factor out the GCF of the expressions

$$\frac{c(c+6)-4(c+6)}{c(c+6)+6(c+6)}\cdot\frac{c(c-6)-4(c-6)}{c(c-4)-4(c-4)}$$ Use the distributive property

$$\frac{(c-4)(c+6)}{(c+6)(c+6)}\cdot\frac{(c-4)(c-6)}{(c-4)(c-4)}$$ Simplify

$$\frac{c-6}{c+6}$$

16. $$\frac{2d^2+9d-35}{d^2+10d+21}\cdot\frac{3d^2+2d-21}{3d^2+14d-49}$$

$$\frac{2d^2+9d-35}{d^2+10d+21}\cdot\frac{3d^2+2d-21}{3d^2+14d-49}$$ Rewrite the expressions

$$\frac{(2d^2+14d)+(-5d-35)}{(d^2+7d)+(3d+21)}\cdot\frac{(3d^2+9d)+(-7d-21)}{(3d^2+21d)+(-7d-49)}$$ Factor out the GCF of the expressions

$$\frac{2d(d+7)-5(d+7)}{d(d+7)+3(d+7)}\cdot\frac{3d(d+3)-7(d+3)}{3d(d+7)-7(d+7)}$$ Use the distributive property

$$\frac{(2d-5)(d+7)}{(d+3)(d+7)}\cdot\frac{(3d-7)(d+3)}{(3d-7)(d+7)}$$ Simplify

$$\frac{2d-5}{d+7}$$

17. $\dfrac{10h^2-9h-9}{2h^2-19h+24}\cdot\dfrac{h^2-16h+64}{5h^2-37h-24}$

$\dfrac{10h^2-9h-9}{2h^2-19h+24}\cdot\dfrac{h^2-16h+64}{5h^2-37h-24}$	Rewrite the expressions
$\dfrac{(10h^2-15h)+(6h-9)}{(2h^2-16h)+(-3h+24)}\cdot\dfrac{(h^2-8h)+(-8h+64)}{(5h^2-40h)+(3h-24)}$	Factor out the GCF of the expressions
$\dfrac{5h(2h-3)+3(2h-3)}{2h(h-8)-3(h-8)}\cdot\dfrac{h(h-8)-8(h-8)}{5h(h-8)+3(h-8)}$	Use the distributive property
$\dfrac{(5h+3)(2h-3)}{(2h-3)(h-8)}\cdot\dfrac{(h-8)(h-8)}{(5h+3)(h-8)}$	Simplify
1	

18. $\dfrac{6b^2+13b+6}{4b^2-9}\cdot\dfrac{6b^2+31b-30}{18b^2-3b-10}$

$\dfrac{6b^2+13b+6}{4b^2-9}\cdot\dfrac{6b^2+31b-30}{18b^2-3b-10}$	Rewrite the expressions
$\dfrac{(6b^2+9b)+(4b+6)}{(2b-3)(2b+3)}\cdot\dfrac{(6b^2+36b)+(-5b-30)}{(18b^2-15b)+(12b-10)}$	Factor out the GCF from th expressions
$\dfrac{3b(2b+3)+2(2b+3)}{(2b-3)(2b+3)}\cdot\dfrac{6b(b+6)-5(b+6)}{3b(6b-5)+2(6b-5)}$	Use the distributive property
$\dfrac{(3b+2)(2b+3)}{(2b-3)(2b+3)}\cdot\dfrac{(6b-5)(b+6)}{(3b+2)(6b-5)}$	Simplify
$\dfrac{b+6}{2b-3}$	

19. $\dfrac{2d^2+15d+25}{4d^2-25}\cdot\dfrac{2d^2-15d+25}{25d^2-1}$

$$\frac{2d^2+15d+25}{4d^2-25}\cdot\frac{2d^2-15d+25}{25d^2-1}$$ Rewrite the expressions

$$\frac{(2d^2+10d)+(5d+25)}{(2d-5)(2d+5)}\cdot\frac{(2d^2-10d)+(-5d+25)}{(5d-1)(5d+1)}$$ Factor out the GCF in the expressions

$$\frac{2d(d+5)+5(d+5)}{(2d-5)(2d+5)}\cdot\frac{2d(d-5)-5(d-5)}{(5d-1)(5d+1)}$$ Use the distributive property

$$\frac{(2d+5)(d+5)}{(2d-5)(2d+5)}\cdot\frac{(2d-5)(d-5)}{(5d-1)(5d+1)}$$ Simplify

$$\frac{(d+5)(d-5)}{(5d-1)(5d+1)}$$ Multiply

$$\frac{d^2-25}{25d^2-1}$$

20. $$\frac{6x^2-5x-50}{15x^2-44x-20}\cdot\frac{20x^2-7x-6}{2x^2+9x+10}$$

$$\frac{6x^2-5x-50}{15x^2-44x-20}\cdot\frac{20x^2-7x-6}{2x^2+9x+10}$$ Rewrite the expressions

$$\frac{(6x^2-20x)+(15x-50)}{(15x^2-50x)+(6x-20)}\cdot\frac{(20x^2-15x)+(8x-6)}{(2x^2+4x)+(5x+10)}$$ Factor out the GCF of the expressions

$$\frac{2x(3x-10)+5(3x-10)}{5x(3x-10)+2(3x-10)}\cdot\frac{5x(4x-3)+2(4x-3)}{2x(x+2)+5(x+2)}$$ Use the distributive property

$$\frac{(2x+5)(3x-10)}{(5x+2)(3x-10)}\cdot\frac{(5x+2)(4x-3)}{(2x+5)(x+2)}$$ Simplify

$$\frac{4x-3}{x+2}$$

21. $$\frac{t^2-1}{t^2+4t+3}\cdot\frac{t^2+2t-15}{t^2-4t+3}$$

$\frac{t^2-1}{t^2+4t+3}\cdot\frac{t^2+2t-15}{t^2-4t+3}$ Rewrite the expressions; Difference of squares

$\frac{(t-1)(t+1)}{(t^2+3t)+(t+3)}\cdot\frac{(t^2+5t)+(-3t-15)}{(t^2-3t)+(-t+3)}$ Factor out the GCF of the expressions

$\frac{(t-1)(t+1)}{t(t+3)+(t+3)}\cdot\frac{t(t+5)-3(t+5)}{t(t-3)-(t-3)}$ Use the distributive property

$\frac{(t-1)(t+1)}{(t+1)(t+3)}\cdot\frac{(t-3)(t+5)}{(t-1)(t-3)}$ Simplify

$\frac{t+5}{t+3}$

22. $\frac{2n^2-n-15}{6n^2+13n-5}\cdot\frac{12n^2-13n+3}{4n^2-15n+9}$

$\frac{2n^2-n-15}{6n^2+13n-5}\cdot\frac{12n^2-13n+3}{4n^2-15n+9}$ Rewrite the expressions

$\frac{(2n^2-6n)+(5n-15)}{(6n^2+15n)+(-2n-5)}\cdot\frac{(12n^2-9n)+(-4n+3)}{(4n^2-12n)+(-3n+9)}$ Factor out the GCF of the expressions

$\frac{2n(n-3)+5(n-3)}{3n(2n+5)-(2n+5)}\cdot\frac{3n(4n-3)-(4n-3)}{4n(n-3)-3(n-3)}$ Use the distributive property

$\frac{(2n+5)(n-3)}{(3n-1)(2n+5)}\cdot\frac{(3n-1)(4n-3)}{(4n-3)(n-3)}$ Simplify

1

23. $\frac{36x^2-25}{6x^2+65x+50}\cdot\frac{3x^2+32x+20}{18x^2+27x+10}$

$\frac{36x^2-25}{6x^2+65x+50}\cdot\frac{3x^2+32x+20}{18x^2+27x+10}$ Rewrite the expressions

$\frac{(6x-5)(6x+5)}{(6x^2+5x)+(60x+50)}\cdot\frac{(3x^2+30x)+(2x+20)}{(18x^2+15x)+(12x+10)}$ Factor out the GCF of the expressions

$\frac{(6x-5)(6x+5)}{x(6x+5)+10(6x+5)}\cdot\frac{3x(x+10)+2(x+10)}{3x(6x+5)+2(6x+5)}$ Use the distributive property

$\frac{(6x-5)(6x+5)}{(x+10)(6x+5)}\cdot\frac{(3x+2)(x+10)}{(3x+2)(6x+5)}$ Simplify

$\frac{6x-5}{6x+5}$

For the following exercises, divide the rational expressions.

24. $\dfrac{3y^2-7y-6}{2y^2-3y-9} \div \dfrac{y^2+y-2}{2y^2+y-3}$

$\dfrac{3y^2-7y-6}{2y^2-3y-9} \div \dfrac{y^2+y-2}{2y^2+y-3}$ Rewrite the expressions

$\dfrac{(3y^2-9y)+(2y-6)}{(2y^2-6y)+(3y-9)} \div \dfrac{(y^2+2y)+(-y-2)}{(2y^2+3y)+(-2y-3)}$ Factor out the GCF of each expression; divide

$\dfrac{3y(y-3)+2(y-3)}{2y(y-3)+3(y-3)} \cdot \dfrac{y(2y+3)-(2y+3)}{y(y+2)-(y+2)}$ Use the distributive property

$\dfrac{(3y+2)(y-3)}{(2y+3)(y-3)} \cdot \dfrac{(y-1)(2y+3)}{(y-1)(y+2)}$ Simplify

$\dfrac{3y+2}{y+2}$

25. $\dfrac{6p^2+p-12}{8p^2+18p+9} \div \dfrac{6p^2-11p+4}{2p^2+11p-6}$

$\dfrac{6p^2+p-12}{8p^2+18p+9} \div \dfrac{6p^2-11p+4}{2p^2+11p-6}$ Rewrite the expressions; divide

$\dfrac{(6p^2+9p)+(-8p-12)}{(8p^2+12p)+(6p+9)} \cdot \dfrac{(2p^2+12p)+(-p-6)}{(6p^2-3p)+(-8p+4)}$ Factor out the GCF of each expression

$\dfrac{3p(2p+3)-4(2p+3)}{4p(2p+3)+3(2p+3)} \cdot \dfrac{2p(p+6)-(p+6)}{3p(2p-1)-4(2p-1)}$ Use the distributive property

$\dfrac{(3p-4)(2p+3)}{(4p+3)(2p+3)} \cdot \dfrac{(2p-1)(p+6)}{(3p-4)(2p-1)}$ Simplify

$\dfrac{p+6}{4p+3}$

26. $\dfrac{q^2-9}{q^2+6q+9} \div \dfrac{q^2-2q-3}{q^2+2q-3}$

$$\frac{q^2-9}{q^2+6q+9}\div\frac{q^2-2q-3}{q^2+2q-3}$$ Rewrite the expressions; divide

$$\frac{(q-3)(q+3)}{(q^2+3q)+(3q+9)}\cdot\frac{(q^2+3q)+(-q-3)}{(q^2-3q)+(q-3)}$$ Factor out the GCF of the expressions

$$\frac{(q-3)(q+3)}{q(q+3)+3(q+3)}\cdot\frac{q(q+3)-(q+3)}{q(q-3)+(q-3)}$$ Use the distributive property

$$\frac{(q-3)(q+3)}{(q+3)(q+3)}\cdot\frac{(q-1)(q+3)}{(q+1)(q-3)}$$ Simplify

$$\frac{q-1}{q+1}$$

27. $\dfrac{18d^2+77d-18}{27d^2-15d+2}\div\dfrac{3d^2+29d-44}{9d^2-15d+4}$

28. $\dfrac{16x^2+18x-55}{32x^2-36x-11}\div\dfrac{2x^2+17x+30}{4x^2+25x+6}$

$$\frac{16x^2+18x-55}{32x^2-36x-11}\div\frac{2x^2+17x+30}{4x^2+25x+6}$$ Rewrite the expressions; divide

$$\frac{(16x^2+40x)+(-22x-55)}{(32x^2+8x)+(-44x-11)}\cdot\frac{(4x^2+24x)+(x+6)}{(2x^2+12x)+(5x+30)}$$ Factor out the GCF of each expression

$$\frac{8x(2x+5)-11(2x+5)}{8x(4x+1)-11(4x+1)}\cdot\frac{4x(x+6)+(x+6)}{2x(x+6)+5(x+6)}$$ Use the distributive property

$$\frac{(8x-11)(2x+5)}{(8x-11)(4x+1)}\cdot\frac{(4x+1)(x+6)}{(2x+5)(x+6)}$$ Simplify

$$1$$

29. $\dfrac{144b^2-25}{72b^2-6b-10}\div\dfrac{18b^2-21b+5}{36b^2-18b-10}$

$$\frac{144b^2-25}{72b^2-6b-10}\div\frac{18b^2-21b+5}{36b^2-18b-10}$$ Rewrite the expressions; divide

$$\frac{(12b-5)(12b+5)}{(72b^2+24b)+(-30b-10)}\cdot\frac{(36b^2-30b)+(12b-10)}{(18b^2-6b)+(-15b+5)}$$ Factor out the GCF of each expression

$$\frac{(12b-5)(12b+5)}{24b(3b+1)-10(3b+1)}\cdot\frac{6b(6b-5)+2(6b-5)}{6b(3b-1)-5(3b-1)}$$ Use the distributive property

$$\frac{(12b-5)(12b+5)}{(24b-10)(3b+1)}\cdot\frac{(6b+2)(6b-5)}{(6b-5)(3b-1)}$$ Factor

$$\frac{(12b-5)(12b+5)}{2(12b-5)(3b+1)}\cdot\frac{2(3b+1)(6b-5)}{(6b-5)(3b-1)}$$ Simplify

$$\frac{12b+5}{3b-1}$$

30. $$\frac{16a^2-24a+9}{4a^2+17a-15}\div\frac{16a^2-9}{4a^2+11a+6}$$

$$\frac{16a^2-24a+9}{4a^2+17a-15}\div\frac{16a^2-9}{4a^2+11a+6}$$ Rewrite the expressions; divide

$$\frac{(16a^2-12a)+(-12a+9)}{(4a^2+20a)+(-3a-15)}\cdot\frac{(4a^2+8a)+(3a+6)}{(4a-3)(4a+3)}$$ Factor out the GCF of each expression

$$\frac{4a(4a-3)-3(4a-3)}{4a(a+5)-3(a+5)}\cdot\frac{4a(a+2)+3(a+2)}{(4a-3)(4a+3)}$$ Use the distributive property

$$\frac{(4a-3)(4a-3)}{(4a-3)(a+5)}\cdot\frac{(4a+3)(a+2)}{(4a-3)(4a+3)}$$ Simplify

$$\frac{a+2}{a+5}$$

31. $$\frac{22y^2+59y+10}{12y^2+28y-5}\div\frac{11y^2+46y+8}{24y^2-10y+1}$$

$$\frac{22y^2+59y+10}{12y^2+28y-5}\div\frac{11y^2+46y+8}{24y^2-10y+1}$$ Rewrite the expression; divide

$$\frac{(22y^2+55y)+(4y+10)}{(12y^2+30y)+(-2y-5)}\cdot\frac{(24y^2-6y)+(-4y+1)}{(11y^2+44y)+(2y+8)}$$ Factor out the GCF of each expression

$$\frac{11y(2y+5)+2(2y+5)}{6y(2y+5)-(2y+5)}\cdot\frac{6y(4y-1)-(4y-1)}{11y(y+4)+2(y+4)}$$ Use the distributive property

$$\frac{(11y+2)(2y+5)}{(6y-1)(2y+5)}\cdot\frac{(6y-1)(4y-1)}{(11y+2)(y+4)}$$ Simplify

$$\frac{4y-1}{y+4}$$

32. $\frac{9x^2+3x-20}{3x^2-7x+4}\div\frac{6x^2+4x-10}{x^2-2x+1}$

$$\frac{9x^2+3x-20}{3x^2-7x+4}\div\frac{6x^2+4x-10}{x^2-2x+1}$$ Rewrite the expressions; divide

$$\frac{(9x^2+15x)+(-12x-20)}{(3x^2-3x)+(-4x+4)}\cdot\frac{(x^2-x)+(-x+1)}{(6x^2-6x)+(10x-10)}$$ Factor out the GCF of each expression

$$\frac{3x(3x+5)-4(3x+5)}{3x(x-1)-4(x-1)}\cdot\frac{x(x-1)-(x-1)}{6x(x-1)+10(x-1)}$$ Use the distributive property

$$\frac{(3x-4)(3x+5)}{(3x-4)(x-1)}\cdot\frac{(x-1)(x-1)}{(6x+10)(x-1)}$$ Simplify; factor

$$\frac{3x+5}{2(3x+5)}$$ Simplify

$$\frac{1}{2}$$

For the following exercises, add and subtract the rational expressions, and then simplify.

33. $\frac{4}{x}+\frac{10}{y}$

$\frac{4}{x}+\frac{10}{y}$ The LCD is xy

$\frac{4}{x}\cdot\frac{y}{y}+\frac{10}{y}\cdot\frac{x}{x}$ Multiply

$\frac{4y}{xy}+\frac{10x}{xy}$ Add

$\frac{4y+10x}{xy}$

34. $\frac{12}{2q}-\frac{6}{3p}$

$\frac{12}{2q}-\frac{6}{3p}$ The LCD is $6pq$

$\frac{12}{2q}\cdot\frac{3p}{3p}-\frac{6}{3p}\cdot\frac{2q}{2q}$ Multiply

$\frac{36p}{6pq}-\frac{12q}{6pq}$ Subtract

$\frac{36p-12q}{6pq}$ Factor out the GCF of each expression

$\frac{6(6p-2q)}{6(pq)}$ Simplify

$\frac{6p-2q}{pq}$

35. $\frac{4}{a+1}+\frac{5}{a-3}$

$\frac{4}{a+1}+\frac{5}{a-3}$ The LCD is $(a + 1)(a - 3)$

$\frac{4}{a+1}\cdot\frac{a-3}{a-3}+\frac{5}{a-3}\cdot\frac{a+1}{a+1}$ Multiply

$\frac{4(a-3)}{(a+1)(a-3)}+\frac{5(a+1)}{(a+1)(a-3)}$ Distribute and combine fractions

$\frac{4a-12+5a+5}{a^2-3a+a-3}$ Simplify

$\frac{9a-7}{a^2-2a-3}$

36. $\frac{c+2}{3}-\frac{c-4}{4}$

$\frac{c+2}{3}-\frac{c-4}{4}$ The LCD is 12

$\frac{c+2}{3}\cdot\frac{4}{4}-\frac{c-4}{4}\cdot\frac{3}{3}$ Multiply

$\frac{4(c+2)}{12}-\frac{3(c-4)}{12}$ Distribute and combine fractions

$\frac{4c+8-3c+12}{12}$ Simplify

$\frac{c+20}{12}$

37. $\frac{y+3}{y-2}+\frac{y-3}{y+1}$

$\frac{y+3}{y-2}+\frac{y-3}{y+1}$ The LCD is $(y - 2)(y + 1)$

$\frac{y+3}{y-2}\cdot\frac{y+1}{y+1}+\frac{y-3}{y+1}\cdot\frac{y-2}{y-2}$ Multiply

$\frac{(y+3)(y+1)}{(y-2)(y+1)}+\frac{(y-3)(y-2)}{(y+1)(y-2)}$ Use the FOIL method

$\frac{y^2+y+3y+3}{y^2+y-2y-2}+\frac{y^2-2y-3y+6}{y^2+y-2y-2}$ Combine like terms

$\frac{y^2+4y+3+y^2-5y+6}{y^2-y-2}$ Combine like terms

$\frac{2y^2-y+9}{y^2-y-2}$

38. $\frac{x-1}{x+1}-\frac{2x+3}{2x+1}$

$\frac{x-1}{x+1}-\frac{2x+3}{2x+1}$ The LCD is $(x + 1)(2x + 1)$

$\frac{x-1}{x+1}\cdot\frac{2x+1}{2x+1}-\frac{2x+3}{2x+1}\cdot\frac{x+1}{x+1}$ Multiply

$\frac{(x-1)(2x+1)}{(x+1)(2x+1)}-\frac{(2x+3)(x+1)}{(2x+1)(x+1)}$ Use the FOIL method

$\frac{2x^2+x-2x-1}{2x^2+x+2x+1}-\frac{2x^2+2x+3x+3}{2x^2+x+2x+1}$ Combine like terms

$\frac{2x^2-x-1-2x^2-5x-3}{2x^2+3x+1}$ Combine like terms

$-\frac{6x+4}{2x^2+3x+1}$

39. $\frac{3z}{z+1}+\frac{2z+5}{z-2}$

$\frac{3z}{z+1}+\frac{2z+5}{z-2}$ The LCD is $(z + 1)(z - 2)$

$\frac{3z}{z+1}\cdot\frac{z-2}{z-2}+\frac{2z+5}{z-2}\cdot\frac{z+1}{z+1}$ Multiply

$\frac{(3z)(z-2)}{(z+1)(z-2)}+\frac{(2z+5)(z+1)}{(z-2)(z+1)}$ Use the FOIL method

$\frac{3z^2-6z}{z^2-2z+z-2}+\frac{2z^2+2z+5z+5}{z^2-2z+z-2}$ Combine like terms

$\frac{3z^2-6z+2z^2+7z+5}{z^2-z-2}$ Combine like terms

$\frac{5z^2+z+5}{z^2-z-2}$

40. $\frac{4p}{p+1}-\frac{p+1}{4p}$

$\frac{4p}{p+1}-\frac{p+1}{4p}$ The LCD is $4p(p+1)$

$\frac{4p}{p+1}\cdot\frac{4p}{4p}-\frac{p+1}{4p}\cdot\frac{p+1}{p+1}$ Multiply

$\frac{16p^2}{4p(p+1)}-\frac{(p+1)(p+1)}{4p(p+1)}$ Use the FOIL method

$\frac{16p^2}{4p^2+4p}-\frac{p^2+p+p+1}{4p^2+4p}$ Combine like terms

$\frac{16p^2-p^2-2p-1}{4p^2+4p}$ Combine like terms

$\frac{15p^2-2p-1}{4p^2+4p}$

41. $\frac{x}{x+1}+\frac{y}{y+1}$

$\frac{x}{x+1}+\frac{y}{y+1}$ The LCD is $(x+1)(y+1)$

$\frac{x}{x+1}\cdot\frac{y+1}{y+1}+\frac{y}{y+1}\cdot\frac{x+1}{x+1}$ Multiply

$\frac{x(y+1)}{(x+1)(y+1)}+\frac{y(x+1)}{(x+1)(y+1)}$ Use the FOIL method

$\frac{xy+x}{xy+x+y+1}+\frac{xy+y}{xy+x+y+1}$ Combine like terms

$\frac{2xy+x+y}{xy+x+y+1}$

For the following exercises, simplify the rational expression.

$$\frac{\frac{6}{y}-\frac{4}{x}}{y}$$

42.

$$\frac{\frac{6}{y}-\frac{4}{x}}{y}$$ Rewrite the expression

$$\frac{1}{y}\left(\frac{6}{y}-\frac{4}{x}\right)$$ Distribute

$$\frac{6}{y^2}-\frac{4}{xy}$$ The LCD is y^2x

$$\frac{6}{y^2}\cdot\frac{x}{x}-\frac{4}{xy}\cdot\frac{y}{y}$$ Multiply

$$\frac{6x}{y^2x}-\frac{4y}{y^2x}$$ Subtract

$$\frac{6x-4y}{y^2x}$$

$$\frac{\frac{2}{a}+\frac{7}{b}}{b}$$

43.

$$\frac{\frac{2}{a}+\frac{7}{b}}{b}$$ Rewrite the expression

$$\frac{1}{b}\left(\frac{2}{a}+\frac{7}{b}\right)$$ Distribute

$$\frac{2}{ab}+\frac{7}{b^2}$$ The LCD is ab^2

$$\frac{2}{ab}\cdot\frac{b}{b}+\frac{7}{b^2}\cdot\frac{a}{a}$$ Multiply

$$\frac{2b}{ab^2}+\frac{7a}{ab^2}$$ Add

$$\frac{7a+2b}{ab^2}$$

$$\frac{\frac{x}{4}-\frac{p}{8}}{p}$$

44.

$$\frac{\frac{x}{4}-\frac{p}{8}}{p}$$ Rewrite the expression

$$\frac{1}{p}\left(\frac{x}{4}-\frac{p}{8}\right)$$ Distribute

$$\frac{x}{4p}-\frac{p}{8p}$$ Simplify

$$\frac{x}{4p}-\frac{1}{8}$$

$$\frac{\frac{3}{a}+\frac{b}{6}}{\frac{2b}{3a}}$$

45.

$$\frac{\frac{3}{a}+\frac{b}{6}}{\frac{2b}{3a}}$$ Rewrite the expression

$$\frac{3a}{2b}\left(\frac{3}{a}+\frac{b}{6}\right)$$ Distribute

$$\frac{9a}{2ab}+\frac{3ab}{12b}$$ Simplify; The LCD is $4b$

$$\frac{9}{2b}\cdot\frac{2}{2}+\frac{ab}{4b}$$ Multiply

$$\frac{18}{4b}+\frac{ab}{4b}$$ Add

$$\frac{18+ab}{4b}$$

$$\frac{\frac{3}{x+1}+\frac{2}{x-1}}{\frac{x-1}{x+1}}$$

46.

$$\frac{\frac{3}{x+1}+\frac{2}{x-1}}{\frac{x-1}{x+1}}$$ Rewrite the expression

$$\frac{x+1}{x-1}\left(\frac{3}{x+1}+\frac{2}{x-1}\right)$$ Distribute

$$\frac{3(x+1)}{(x-1)(x+1)}+\frac{2(x+1)}{(x-1)(x-1)}$$ Simplify and multiply; The LCD is $(x - 1)(x - 1)$

$$\frac{3}{x-1}\cdot\frac{x-1}{x-1}+\frac{2x+2}{(x-1)(x-1)}$$ Multiply

$$\frac{3x-3}{x^2-x-x+1}+\frac{2x+2}{x^2-x-x+1}$$ Combine like terms

$$\frac{5x-1}{x^2-2x+1}$$

$$\frac{\frac{a}{b}-\frac{b}{a}}{\frac{a+b}{ab}}$$

47.

$$\frac{\frac{a}{b}-\frac{b}{a}}{\frac{a+b}{ab}}$$ Rewrite the expression

$$\frac{ab}{a+b}\left(\frac{a}{b}-\frac{b}{a}\right)$$ Distribute

$$\frac{a^2b}{b(a+b)}-\frac{ab^2}{a(a+b)}$$ Simplify

$$\frac{a^2}{a+b}-\frac{b^2}{a+b}$$ Combine

$$\frac{a^2-b^2}{a+b}$$ Difference of squares

$$\frac{(a-b)(a+b)}{a+b}$$ Simplify

$$a-b$$

48. $\dfrac{\dfrac{2x}{3}+\dfrac{4x}{7}}{\dfrac{x}{2}}$

$\dfrac{\dfrac{2x}{3}+\dfrac{4x}{7}}{\dfrac{x}{2}}$ Rewrite the expression

$\dfrac{2}{x}\left(\dfrac{2x}{3}+\dfrac{4x}{7}\right)$ Distribute

$\dfrac{4x}{3x}+\dfrac{8x}{7x}$ Simplify

$\dfrac{4}{3}\cdot\dfrac{7}{7}+\dfrac{8}{7}\cdot\dfrac{3}{3}$ The LCD is 21

$\dfrac{28}{21}+\dfrac{24}{21}$ Add

$\dfrac{52}{21}$

49. $\dfrac{\dfrac{2c}{c+2}+\dfrac{c-1}{c+1}}{\dfrac{2c+1}{c+1}}$

$$\frac{\frac{2c}{c+2}+\frac{c-1}{c+1}}{\frac{2c+1}{c+1}}$$ Rewrite the expression

$$\frac{c+1}{2c+1}\left(\frac{2c}{c+2}+\frac{c-1}{c+1}\right)$$ Distribute

$$\frac{2c(c+1)}{(2c+1)(c+2)}+\frac{(c+1)(c-1)}{(2c+1)(c+1)}$$ Simplify; The LCD is $(2c + 1)(c + 2)$

$$\frac{2c^2+2c}{(2c+1)(c+2)}+\frac{c-1}{2c+1}\cdot\frac{c+2}{c+2}$$ Multiply

$$\frac{2c^2+2c}{2c^2+4c+c+2}+\frac{c^2+2c-c-2}{2c^2+4c+c+2}$$ Combine like terms

$$\frac{2c^2+2c+c^2+c-2}{2c^2+5c+2}$$ Combine like terms

$$\frac{3c^2+3c-2}{2c^2+5c+2}$$

50. $$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}}$$

$\dfrac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}}$	Rewrite the denominator; the LCD is xy
$\frac{x}{y}\cdot\frac{x}{x}+\frac{y}{x}\cdot\frac{y}{y}$	Multiply
$\frac{x^2}{xy}+\frac{y^2}{xy}$	Add
$\dfrac{\frac{x^2+y^2}{xy}}{\frac{x}{y}-\frac{y}{x}}$	Rewrite the expression with the new denominator
$\dfrac{\frac{x^2+y^2}{xy}}{\frac{x^2+y^2}{xy}}$	Rewrite the expression
$\frac{xy}{x^2+y^2}\left(\frac{x}{y}-\frac{y}{x}\right)$	Distribute
$\frac{x(xy)}{y(x^2+y^2)}-\frac{y(xy)}{x(x^2+y^2)}$	Simplify
$\frac{x^2}{x^2+y^2}-\frac{y^2}{x^2+y^2}$	Subtract
$\frac{x^2-y^2}{x^2+y^2}$	

Real-World Applications

51. Brenda is placing tile on her bathroom floor. The area of the floor is $15x^2-8x-7$ square feet. The area of one tile is x^2-2x+1. To find the number of tiles needed, simplify the rational expression: $\dfrac{15x^2-8x-7}{x^2-2x+1}$.



Area = $15x^2 - 8x - 7$

$\frac{15x^2-8x-7}{x^2-2x+1}$	Rewrite $-8x$ as $-15x + 7x$; Rewrite $-2x$ as $-x - x$
$\frac{(15x^2-15x)+(7x-7)}{(x^2-x)+(-x+1)}$	Factor out the GCF of each expression
$\frac{15x(x-1)+7(x-1)}{x(x-1)-(x-1)}$	Use the distributive property
$\frac{(15x+7)(x-1)}{(x-1)(x-1)}$	Simplify
$\frac{15x+7}{x-1}$	

52. The area of Lijuan's yard is $25x^2 - 625$ square feet. A patch of sod has an area of $x^2 - 10x + 25$ square feet. Divide the two areas and simplify to find how many pieces of sod Lijuan needs to cover her yard.

$\frac{25x^2-625}{x^2-10x+25}$	Difference of squares; Rewrite $-10x$ as $-5x - 5x$
$\frac{(5x-25)(5x+25)}{x^2-5x-5x+25}$	Factor the GCF of each expression
$\frac{25(x-5)(x+5)}{x(x-5)-5(x-5)}$	Use the distributive property
$\frac{25(x-5)(x+5)}{(x-5)(x-5)}$	Simplify
$\frac{25(x+5)}{x-5}$	Distribute
$\frac{25x+125}{x-5}$	

53. Elroi wants to mulch his garden. His garden is $x^2+18x+81$ square feet. One bag of mulch covers x^2-81 square feet. Divide the expressions and simplify to find how many bags of mulch Elroi needs to mulch his garden.

$\frac{x^2+18x+81}{x^2-81}$ Rewrite $18x$ as $9x + 9x$; difference of squares

$\frac{(x^2+9x)+(9x+81)}{(x-9)(x+9)}$ Factor out the GCF of each expression

$\frac{x(x+9)+9(x+9)}{(x-9)(x+9)}$ Use the distributive property

$\frac{(x+9)(x+9)}{(x-9)(x+9)}$ Simplify

$\frac{x+9}{x-9}$

Extensions

For the following exercises, perform the given operations and simplify.

54. $\frac{x^2+x-6}{x^2-2x-3}\cdot\frac{2x^2-3x-9}{x^2-x-2}\div\frac{10x^2+27x+18}{x^2+2x+1}$

$\frac{x^2+x-6}{x^2-2x-3}\cdot\frac{2x^2-3x-9}{x^2-x-2}\div\frac{10x^2+27x+18}{x^2+2x+1}$

Rewrite the expressions; divide

$\frac{x^2+3x-2x-6}{x^2-3x+x-3}\cdot\frac{2x^2-6x+3x-9}{x^2-2x+x-2}\cdot\frac{x^2+x+x+1}{10x^2+15x+12x+18}$

Factor out the GCF of each expression

$\frac{x(x+3)-2(x+3)}{x(x-3)+(x-3)}\cdot\frac{2x(x-3)+3(x-3)}{x(x-2)+(x-2)}\cdot\frac{x(x+1)+(x+1)}{5x(2x+3)+6(2x+3)}$

Use the distributive property

$\frac{(x-2)(x+3)}{(x+1)(x-3)}\cdot\frac{(2x+3)(x-3)}{(x+1)(x-2)}\cdot\frac{(x+1)(x+1)}{(5x+6)(2x+3)}$

Simplify

$\frac{x+3}{5x+6}$

55. $$\frac{\frac{3y^2-10y+3}{3y^2+5y-2}\cdot\frac{2y^2-3y-20}{2y^2-y-15}}{y-4}$$

$$\frac{\frac{3y^2-10y+3}{3y^2+5y-2}\cdot\frac{2y^2-3y-20}{2y^2-y-15}}{y-4}$$

Rewrite the expression

$$\frac{1}{y-4}\left(\frac{3y^2-10y+3}{3y^2+5y-2}\cdot\frac{2y^2-3y-20}{2y^2-y-15}\right)$$

Rewrite the expressions

$$\frac{1}{y-4}\left(\frac{3y^2-9y-y+3}{3y^2+6y-y-2}\cdot\frac{2y^2-8y+5y-20}{2y^2-6y+5y-15}\right)$$

Factor out the GCF of each expression

$$\frac{1}{y-4}\left(\frac{3y(y-3)-(y-3)}{3y(y+2)-(y+2)}\cdot\frac{2y(y-4)+5(y-4)}{2y(y-3)+5(y-3)}\right)$$

Use the distributive property

$$\frac{1}{y-4}\left(\frac{(3y-1)(y-3)}{(3y-1)(y+2)}\cdot\frac{(2y+5)(y-4)}{(2y+5)(y-3)}\right)$$

Simplify

$$\frac{1}{y+2}$$

56. $\dfrac{\dfrac{4a+1}{2a-3}+\dfrac{2a-3}{2a+3}}{\dfrac{4a^2+9}{a}}$

$\dfrac{\dfrac{4a+1}{2a-3}+\dfrac{2a-3}{2a+3}}{\dfrac{4a^2+9}{a}}$ Rewrite the expression

$\dfrac{a}{4a^2+9}\left(\dfrac{4a+1}{2a-3}+\dfrac{2a-3}{2a+3}\right)$ The LCD is $(2a - 3)(2a + 3)$

$\dfrac{a}{4a^2+9}\left(\dfrac{4a+1}{2a-3}\cdot\dfrac{2a+3}{2a+3}+\dfrac{2a-3}{2a+3}\cdot\dfrac{2a-3}{2a-3}\right)$ Multiply

$\dfrac{a}{4a^2+9}\left(\dfrac{8a^2+12a+2a+3}{4a^2-6a+6a-9}+\dfrac{4a^2-6a-6a+9}{4a^2-6a+6a-9}\right)$ Combine like terms

$\dfrac{a}{4a^2+9}\left(\dfrac{8a^2+14a+3+4a^2-12a+9}{4a^2-9}\right)$ Combine like terms

$\dfrac{a}{4a^2+9}\left(\dfrac{12a^2+2a+12}{4a^2-9}\right)$ Distribute and multiply

$\dfrac{12a^3+2a^2+12a}{(4a^2+9)(4a^2-9)}$ Use the FOIL method

$\dfrac{12a^3+2a^2+12a}{16a^4-36a^2+36a^2-81}$ Combine like terms

$\dfrac{12a^3+2a^2+12a}{16a^4-81}$

57. $\dfrac{x^2+7x+12}{x^2+x-6}\div\dfrac{3x^2+19x+28}{8x^2-4x-24}\div\dfrac{2x^2+x-3}{3x^2+4x-7}$

$$\frac{x^2+7x+12}{x^2+x-6}\div\frac{3x^2+19x+28}{8x^2-4x-24}\div\frac{2x^2+x-3}{3x^2+4x-7}$$

Rewrite the expressions; divide

$$\frac{x^2+4x+3x+12}{x^2+3x-2x-6}\cdot\frac{8x^2-16x+12x-24}{3x^2+12x+7x+28}\cdot\frac{3x^2+7x-3x-7}{2x^2+3x-2x-3}$$

Factor out the GCF of each expression

$$\frac{x(x+4)+3(x+4)}{x(x+3)-2(x+3)}\cdot\frac{8x(x-2)+12(x-2)}{3x(x+4)+7(x+4)}\cdot\frac{x(3x+7)-(3x+7)}{x(2x+3)-(2x+3)}$$

Use the distributive property

$$\frac{(x+3)(x+4)}{(x-2)(x+3)}\cdot\frac{(8x+12)(x-2)}{(3x+7)(x+4)}\cdot\frac{(x-1)(3x+7)}{(x-1)(2x+3)}$$

Simplify

$$\frac{4(2x+3)}{2x+3}=4$$

Chapter 1 Review Exercises

Section 1.1

For the following exercises, perform the given operations.

1. $(5-3\cdot 2)^2-6$

$(5-3\cdot 2)^2-6$	Multiply
$(5-6)^2-6$	Subtract inside of parenthesis
$(-1)^2-6$	Square the -1
$1-6$	Subtract
-5	

2. $64\div(2\cdot 8)+14\div 7$

$64\div(2\cdot 8)+14\div 7$	Multiply
$64\div 16+14\div 7$	Divide
$4+2$	Divide
6	Add

3. $2 \cdot 5^2 + 6 \div 2$

$2 \cdot 5^2 + 6 \div 2$ Sqaure the 5

$2 \cdot 25 + 6 \div 2$ Multiply

$50 + 6 \div 2$ Divide

$50 + 3$ Add

53

For the following exercises, solve the equation.

4. $5x + 9 = -11$

$5x + 9 = -11$ Subract 9 from both sides

$5x = -20$ Divide both sides by 5

$x = -4$

5. $2y + 4^2 = 64$

$2y + 4^2 = 64$ Sqaure the 4

$2y + 16 = 64$ Subtract 16 from both sides

$2y = 48$ Divide 2 from both sides

$y = 24$

For the following exercises, simplify the expression.

6. $9(y + 2) \div 3 \cdot 2 + 1$

$9(y + 2) \div 3 \cdot 2 + 1$ Distribute

$(9y + 18) \div 3 \cdot 2 + 1$ Divide

$(3y + 6)2 + 1$ Multiply

$6y + 12 + 1$ Add

$6y + 13$

7. $3m(4 + 7) - m$

$3m(4 + 7) - m$ Add inside of the parenthesis

$3m(11) - m$ Multiply

$33m - m$ Subtract

$32m$

For the following exercises, identify the number as rational, irrational, whole, or natural. Choose the most descriptive answer.

8. 11

   natural

9. 0

   whole

10. $\frac{5}{6}$

    :rational

11. $\sqrt{11}$

    irrational

Section 1.2

For the following exercises, simplify the expression.

12. $2^2 \cdot 2^4$

    $2^2 \cdot 2^4 = 2^{2+4}$

    $2^6 = 64$

13. $\frac{4^5}{4^3}$

    $\frac{4^5}{4^3} = 4^{5-3}$

    $4^2 = 16$

14. $\left(\frac{a^2}{b^3}\right)^4$

    $\frac{a^{2 \cdot 4}}{b^{3 \cdot 4}}$

    $\frac{a^8}{b^{12}}$

15. $\frac{6a^2 \cdot a^0}{2a^{-4}}$

$\frac{6a^2 \cdot 1}{2a^{-4}}$

$\frac{6a^2 a^4}{2}$

$3a^{2+4}$

$3a^6$

16. $\frac{(xy)^4}{y^3} \cdot \frac{2}{x^5}$

$\frac{x^4 y^4}{y^3} \cdot \frac{2}{x^5}$

$\frac{2x^4 y^4}{x^5 y^3}$

$2x^{4-5} y^{4-3}$

$2x^{-1} y$

$\frac{2y}{x}$

17. $\frac{4^{-2} x^3 y^{-3}}{2x^0}$

$\frac{x^3}{(4^2 y^3)(2 \cdot 1)}$

$\frac{x^3}{2(16y^3)}$

$\frac{x^3}{32y^3}$

18. $\left(\frac{2x^2}{y}\right)^{-2}$

$\left(\frac{y}{2x^2}\right)^2$

$\frac{y^2}{4x^4}$

19. $\left(\frac{16a^3}{b^2}\right)\left(4ab^{-1}\right)^{-2}$

$\left(\frac{16a^3}{b^2}\right)\left(\frac{1}{4ab^{-1}}\right)^2$

$\left(\frac{16a^3}{b^2}\right)\left(\frac{b}{4a}\right)^2$

$\left(\frac{16a^3}{b^2}\right)\left(\frac{b^2}{16a^2}\right)$

a

20. Write the number in standard notation: 2.1314×10^{-6}

0.0000021314

21. Write the number in scientific notation: 16,340,000

1.634×10^7

Section 1.3

For the following exercises, find the principal square root.

22. $\sqrt{121}$

11

23. $\sqrt{196}$

14

24. $\sqrt{361}$

19

25. $\sqrt{75}$

$\sqrt{75}$	75 has factors 25 and 3
$\sqrt{25\cdot3}$	Multiplication property of radicals
$\sqrt{25}\sqrt{3}$	25 is a perfect square
$5\sqrt{3}$	

26. $\sqrt{162}$

$\sqrt{162}$ 162 has factors 81 and 2

$\sqrt{81 \cdot 2}$ Multiplication property of radicals

$\sqrt{81}\sqrt{2}$ 81 is a perfect square

$9\sqrt{2}$

27. $\sqrt{\frac{32}{25}}$

$\sqrt{\frac{32}{25}}$ Division property of radicals

$\frac{\sqrt{32}}{\sqrt{25}}$ 32 has factors 16 and 2; 25 is a perfect square

$\frac{\sqrt{16 \cdot 2}}{5}$ Multiplication property of radicals

$\frac{\sqrt{16}\sqrt{2}}{5}$ 16 is a perfect square

$\frac{4\sqrt{2}}{5}$

28. $\sqrt{\frac{80}{81}}$

$\sqrt{\frac{80}{81}}$ Division property of radicals

$\frac{\sqrt{80}}{\sqrt{81}}$ 80 has factors 16 and 5; 81 is a perfect square

$\frac{\sqrt{16 \cdot 5}}{9}$ Multiplicatin property of radicals

$\frac{\sqrt{16}\sqrt{5}}{9}$ 16 is a perfect square

$\frac{4\sqrt{5}}{9}$

29. $\sqrt{\frac{49}{1250}}$

$\sqrt{\frac{49}{1250}}$ Division property of radicals

$\frac{\sqrt{49}}{\sqrt{1250}}$ 49 is a perfect square; 1250 has factors 625 and 2

$\frac{7}{\sqrt{625 \cdot 2}}$ Multiplication property of radicals

$\frac{7}{\sqrt{625}\sqrt{2}}$ 625 is a perfect square

$\frac{7}{25\sqrt{2}}$ Rationalize the denominator

$\frac{7}{25\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ Simplify

$\frac{7\sqrt{2}}{50}$

30. $\frac{2}{4+\sqrt{2}}$

$\frac{2}{4+\sqrt{2}}$ Multiply top and bottom by the conjugate of the denominator

$\frac{2}{4+\sqrt{2}} \cdot \frac{4-\sqrt{2}}{4-\sqrt{2}}$ Multiply

$\frac{2(4-\sqrt{2})}{16-4\sqrt{2}+4\sqrt{2}-2}$ Combine like terms

$\frac{2\left(4-\sqrt{2}\right)}{14}$ Simplify

$\frac{4-\sqrt{2}}{7}$

31. $4\sqrt{3}+6\sqrt{3}$

$10\sqrt{3}$

32. $12\sqrt{5}-13\sqrt{5}$

$-\sqrt{5}$

33. $\sqrt[5]{-243}$

$\sqrt[5]{-243}$ Note that $-243 = -3^5$

$\sqrt[5]{-3^5}$ Evaluate

-3

34. $\dfrac{\sqrt[3]{250}}{\sqrt[3]{-8}}$

$\dfrac{\sqrt[3]{250}}{\sqrt[3]{-8}}$ 250 has factors 125 and 2; Note $-8 = -2^3$

$\dfrac{\sqrt[3]{125\cdot 2}}{\sqrt[3]{-2^3}}$ Multiplication property of radicals; evaluate

$\dfrac{\sqrt[3]{125}\sqrt[3]{2}}{-2}$ Note: $125 = 5^3$

$-\dfrac{5\sqrt[3]{2}}{2}$

Section 1.4

For the following exercises, perform the given operations and simplify.

35. $\left(3x^3+2x-1\right)+\left(4x^2-2x+7\right)$

$\left(3x^3+2x-1\right)+\left(4x^2-2x+7\right)$

$3x^3+2x-1+4x^2-2x+7$

$3x^2+4x^2+6$

36. $\left(2y+1\right)-\left(2y^2-2y-5\right)$

$\left(2y+1\right)-\left(2y^2-2y-5\right)$

$2y+1-2y^2+2y+5$

$-2y^2+4y+6$

37. $\left(2x^2+3x-6\right)+\left(3x^2-4x+9\right)$

$\left(2x^2+3x-6\right)+\left(3x^2-4x+9\right)$

$5x^2-x+3$

38. $\left(6a^2+3a+10\right)-\left(6a^2-3a+5\right)$

$\left(6a^2+3a+10\right)-\left(6a^2-3a+5\right)$

$6a^2+3a+10-6a^2+3a-5$

$6a+5$

39. $(k+3)(k-6)$

$(k+3)(k-6)$

$k^2-6k+3k-18$

$k^2-3k-18$

40. $(2h+1)(3h-2)$

$(2h+1)(3h-2)$

$6h^2-4h+3h-2$

$6h^2-h-2$

41. $(x+1)\left(x^2+1\right)$

$(x+1)\left(x^2+1\right)$

x^3+x^2+x+1

42. $(m-2)\left(m^2+2m-3\right)$

$(m-2)\left(m^2+2m-3\right)$

$m^3+2m^2-3m-2m^2-4m+6$

m^3-7m+6

43. $(a+2b)(3a-b)$

$(a+2b)(3a-b)$

$3a^2-ab+6ab-2b^2$

$3a^2+5ab-2b^2$

44. $(x+y)(x-y)$

$(x+y)(x-y)$

$x^2 - xy + xy - y^2$

$x^2 - y^2$

Section 1.5

For the following exercises, find the greatest common factor.

45. $81p + 9pq - 27p^2q^2$

$9p$

46. $12x^2y + 4xy^2 - 18xy$

$2xy$

47. $88a^3b + 4a^2b - 144a^2$

$4a^2$

For the following exercises, factor the polynomial.

48. $2x^2 - 9x - 18$

$2x^2 - 9x - 18$ Rewrite $-9x$ as $-12x + 3x$

$(2x^2 - 12x) + (3x - 18)$ Factor out the GCF of each expression

$2x(x-6) + 3(x-6)$ Use the distributive property

$(2x+3)(x-6)$

49. $8a^2 + 30a - 27$

$8a^2 + 30a - 27$ Rewrite $30a$ as $36a - 6a$

$(8a^2 + 36a) + (-6a - 27)$ Factor out the GCF of each expression

$4a(2a+9) - 3(2a+9)$ Use the distributive property

$(4a-3)(2a+9)$

50. $d^2 - 5d - 66$

$d^2 - 5d - 66$	Rewrite $-5d$ as $-11d + 6d$
$(d^2 - 11d) + (6d - 66)$	Factor out the GCF of each expression
$d(d - 11) + 6(d - 11)$	Use the distributive property
$(d + 6)(d - 11)$	

51. $x^2 + 10x + 25$

$x^2 + 10x + 25$	Rewrite $10x$ as $5x + 5x$
$(x^2 + 5x) + (5x + 25)$	Factor out the GCF of each expression
$x(x + 5) + 5(x + 5)$	Use the distributive property
$(x + 5)(x + 5)$	Rewrite
$(x + 5)^2$	

52. $y^2 - 6y + 9$

$y^2 - 6y + 9$	Rewrite $-6y$ as $-3y - 3y$
$(y^2 - 3y) + (-3y + 9)$	Factor out the GCF of each expression
$y(y - 3) - 3(y - 3)$	Use the distributive property
$(y - 3)(y - 3)$	Rewrite
$(y - 3)^2$	

53. $4h^2 - 12hk + 9k^2$

$4h^2 - 12hk + 9k^2$	Rewrite $-12hk$ as $-6hk - 6hk$
$4h^2 - 6hk - 6hk + 9k^2$	Factor out the GCF of each expression
$2h(2h - 3k) - 3k(2h - 3k)$	Use the distributive property
$(2h - 3k)(2h - 3k)$	Rewrite
$(2h - 3k)^2$	

54. $361x^2 - 121$

$361x^2 - 121$	Rewrite $0x = 209x - 209x$
$(361x^2 + 209x) + (-209x - 121)$	Factor out the GCF of each expression
$19x(19x + 11) - 11(19x + 11)$	Use the distributive property
$(19x - 11)(19x + 11)$	

55. p^3+216

p^3+216 Sum of cubes

$(p+6)(p^2-6p+36)$

56. $8x^3-125$

$8x^3-125$ Difference of cubes

$(2x-5)(4x^2+10x+25)$

57. $64q^3-27p^3$

$64q^3-27p^3$ Difference of cubes

$(4q-3p)(16q^2+12pq+9p^2)$

58. $4x(x-1)^{-\frac{1}{4}}+3(x-1)^{\frac{3}{4}}$

$4x(x-1)^{-\frac{1}{4}}+3(x-1)^{\frac{3}{4}}$ Factor out the GCF of the expression

$(x-1)^{-\frac{1}{4}}[4x+3(x-1)]$ Distribute the 3

$(x-1)^{-\frac{1}{4}}(4x+3x-3)$ Combine like terms

$(x-1)^{-\frac{1}{4}}(7x-3)$

59. $3p(p+3)^{\frac{1}{3}}-8(p+3)^{\frac{4}{3}}$

$3p(p+3)^{\frac{1}{3}}-8(p+3)^{\frac{4}{3}}$ Factor out the GCF of the expression

$(p+3)^{\frac{1}{3}}[3p-8(p+3)]$ Distribute the -8

$(p+3)^{\frac{1}{3}}(3p-8p-24)$ Combine like terms

$(p+3)^{\frac{1}{3}}(-5p-24)$

60. $4r(2r-1)^{-\frac{2}{3}}-5(2r-1)^{\frac{1}{3}}$

$4r(2r-1)^{-\frac{2}{3}}-5(2r-1)^{\frac{1}{3}}$ Factor out the GCF of the expression

$(2r-1)^{-\frac{2}{3}}\left[4r-5(2r-1)\right]$ Distribute the -5

$(2r-1)^{-\frac{2}{3}}(4r-10r+5)$ Combine like terms

$(2r-1)^{-\frac{2}{3}}(-6r+5)$

Section 1.6

For the following exercises, simplify the expression.

61. $\dfrac{x^2-x-12}{x^2-8x+16}$

$\dfrac{x^2-x-12}{x^2-8x+16}$ Rewrite $-x$ as $-4x+3x$; Rewrite $-8x$ as $-4x-4x$

$\dfrac{(x^2-4x)+(3x-12)}{(x^2-4x)+(-4x+16)}$ Factor out the GCF of each expression

$\dfrac{x(x-4)+3(x-4)}{x(x-4)-4(x-4)}$ Use the distributive property

$\dfrac{(x+3)(x-4)}{(x-4)(x-4)}$ Simplify

$\dfrac{x+3}{x-4}$

62. $\dfrac{4y^2-25}{4y^2-20y+25}$

$\dfrac{4y^2-25}{4y^2-20y+25}$ Rewrite $0x$ as $10y$ - $10y$; rewrite -20y as $-10y$ - $10y$

$\dfrac{(4y^2+10y)+(-10y-25)}{(4y^2-10y)+(-10y+25)}$ Factor out the GCF of each expression

$\dfrac{2y(2y+5)-5(2y+5)}{2y(2y-5)-5(2y-5)}$ Use the distributive property

$\dfrac{(2y-5)(2y+5)}{(2y-5)(2y-5)}$ Simplify

$\dfrac{2y+5}{2y-5}$

63. $\dfrac{2a^2-a-3}{2a^2-6a-8}\cdot\dfrac{5a^2-19a-4}{10a^2-13a-3}$

$\dfrac{2a^2-a-3}{2a^2-6a-8}\cdot\dfrac{5a^2-19a-4}{10a^2-13a-3}$ Rewrite the expressions

$\dfrac{(2a^2+2a)+(-3a-3)}{(2a^2+2a)+(-8a-8)}\cdot\dfrac{(5a^2-20a)+(a-4)}{(10a^2-15a)+(2a-3)}$ Factor out the GCF of each expression

$\dfrac{2a(a+1)-3(a+1)}{2a(a+1)-8(a+1)}\cdot\dfrac{5a(a-4)+(a-4)}{5a(2a-3)+(2a-3)}$ Use the distributive property

$\dfrac{(2a-3)(a+1)}{(2a-8)(a+1)}\cdot\dfrac{(5a+1)(a-4)}{(5a+1)(2a-3)}$ Simplify

$\dfrac{a-4}{2(a-4)}=\dfrac{1}{2}$

64. $\dfrac{d-4}{d^2-9}\cdot\dfrac{d-3}{d^2-16}$

$\dfrac{d-4}{d^2-9}\cdot\dfrac{d-3}{d^2-16}$ Rewrite the expressions

$\dfrac{d-4}{(d^2-3d)+(3d-9)}\cdot\dfrac{d-3}{(d^2-4d)+(4d-16)}$ Factor out the GCF from each expression

$\dfrac{d-4}{d(d-3)+3(d-3)}\cdot\dfrac{d-3}{d(d-4)+4(d-4)}$ Use the distributive property

$\dfrac{d-4}{(d+3)(d-3)}\cdot\dfrac{d-3}{(d+4)(d-4)}$ Simplify

$\dfrac{1}{(d+3)(d+4)}$ Use the FOIL method

$\dfrac{1}{d^2+4d+3d+12}$ Combine like terms

$\dfrac{1}{d^2+7d+12}$

65. $\dfrac{m^2+5m+6}{2m^2-5m-3}\div\dfrac{2m^2+3m-9}{4m^2-4m-3}$

$\dfrac{m^2+5m+6}{2m^2-5m-3}\div\dfrac{2m^2+3m-9}{4m^2-4m-3}$ Rewrite the expressions; divide

$\dfrac{(m^2+3m)+(2m+6)}{(2m^2-6m)+(m-3)}\cdot\dfrac{(4m^2-6m)+(2m-3)}{(2m^2+6m)+(-3m-9)}$ Factor out the GCF of each expression

$\dfrac{m(m+3)+2(m+3)}{2m(m-3)+(m-3)}\cdot\dfrac{2m(2m-3)+(2m-3)}{2m(m+3)-3(m+3)}$ Use the distributive property

$\dfrac{(m+2)(m+3)}{(2m+1)(m-3)}\cdot\dfrac{(2m+1)(2m-3)}{(2m-3)(m+3)}$ Simplify

$\dfrac{m+2}{m-3}$

66. $\dfrac{4d^2-7d-2}{6d^2-17d+10} \div \dfrac{8d^2+6d+1}{6d^2+7d-10}$

$\dfrac{m^2+5m+6}{2m^2-5m-3} \div \dfrac{2m^2+3m-9}{4m^2-4m-3}$ Rewrite the expressions; divide

$\dfrac{(m^2+3m)+(2m+6)}{(2m^2-6m)+(m-3)} \cdot \dfrac{(4m^2-6m)+(2m-3)}{(2m^2+6m)+(-3m-9)}$ Factor out the GCF of each expression

$\dfrac{m(m+3)+2(m+3)}{2m(m-3)+(m-3)} \cdot \dfrac{2m(2m-3)+(2m-3)}{2m(m+3)-3(m+3)}$ Use the distributive property

$\dfrac{(m+2)(m+3)}{(2m+1)(m-3)} \cdot \dfrac{(2m+1)(2m-3)}{(2m-3)(m+3)}$ Simplify

$\dfrac{m+2}{m-3}$

67. $\dfrac{10}{x}+\dfrac{6}{y}$

$\dfrac{10}{x}+\dfrac{6}{y}$ The LCD is xy

$\dfrac{10}{x}\cdot\dfrac{y}{y}+\dfrac{6}{y}\cdot\dfrac{x}{x}$ Multiply

$\dfrac{10y}{xy}+\dfrac{6x}{xy}$ Add

$\dfrac{6x+10y}{xy}$

68. $\dfrac{12}{a^2+2a+1}-\dfrac{3}{a^2-1}$

$\dfrac{12}{a^2+2a+1}-\dfrac{3}{a^2-1}$ Rewrite the expressions

$\dfrac{12}{a^2+a+a+1}-\dfrac{3}{(a-1)(a+1)}$ Factor out the GCF from each expression

$\dfrac{12}{a(a+1)+(a+1)}-\dfrac{3}{(a-1)(a+1)}$ Use the distributive property

$\dfrac{12}{(a+1)(a+1)}-\dfrac{3}{(a-1)(a+1)}$ The LCD is $(a+1)(a+1)(a-1)$

$\dfrac{12}{(a+1)(a+1)}\cdot\dfrac{a-1}{a-1}-\dfrac{3}{(a-1)(a+1)}\cdot\dfrac{a+1}{a+1}$ Multiply

$\dfrac{12a-12}{(a+1)(a+1)(a-1)}-\dfrac{3a+3}{(a+1)(a+1)(a-1)}$ Combine like terms

$\dfrac{9a-15}{(a+1)^2(a-1)}$

69. $\dfrac{\dfrac{1}{d}+\dfrac{2}{c}}{\dfrac{6c+12d}{dc}}$

$\dfrac{\dfrac{1}{d}+\dfrac{2}{c}}{\dfrac{6c+12d}{dc}}$ Rewrite the expression

$\dfrac{dc}{6c+12d}\left(\dfrac{1}{d}+\dfrac{2}{c}\right)$ The LCD is dc

$\dfrac{dc}{6c+12d}\left(\dfrac{1}{d}\cdot\dfrac{c}{c}+\dfrac{2}{c}\cdot\dfrac{d}{d}\right)$ Multiply

$\dfrac{dc}{6c+12d}\left(\dfrac{c}{dc}+\dfrac{2d}{dc}\right)$ Multiply

$\dfrac{dc(c+2d)}{dc(6c+12d)}$ Simplify

$\dfrac{c+2d}{6(c+2d)}=\dfrac{1}{6}$

$$\frac{\frac{3}{x}-\frac{7}{y}}{\frac{2}{x}}$$

70.

$$\frac{\frac{3}{x}-\frac{7}{y}}{\frac{2}{x}}$$ Rewrite the expression

$$\frac{x}{2}\left(\frac{3}{x}-\frac{7}{y}\right)$$ The LCD is xy

$$\frac{x}{2}\left(\frac{3}{x}\cdot\frac{y}{y}-\frac{7}{y}\cdot\frac{x}{x}\right)$$ Multiply

$$\frac{x}{2}\left(\frac{3y}{xy}-\frac{7x}{xy}\right)$$ Multiply

$$\frac{x(3y-7x)}{2xy}$$ Simplify

$$\frac{3y-7x}{2y}$$

Chapter 1 Practice Test

For the following exercises, identify the number as rational, irrational, whole, or natural. Choose the most descriptive answer.

1. -13

   rational

2. $\sqrt{2}$

   irrational

For the following exercises, evaluate the expression.

3. $2(x+3)-12; x=2$

   $2(2+3)-12$ Substitute $x=2$

   $2(5)-12$ Combine

   $10-12$ Distribute the 2

   $=-2$ Combine

4. $y(3+3)^2-26; y=1$

   $1(3+3)^2-26$ Substitute $y=1$

   $(6)^2-26$ Combine

   $36-26$ Expand

   $=10$ Combine

5. Write the number in standard notation: 3.1415×10^6.

   3,141,500

6. Write the number in scientific notation: 0.0000000212.

   2.12×10^{-8}

For the following exercises, simplify the expression.

7. $-2\cdot(2+3\cdot2)^2+144$

   $-2\cdot(2+3\cdot2)^2+144$ Multiply

   $-2\cdot(2+6)^2+144$ Add

   $-2\cdot(8)^2+144$ Square the 8

   $-2\cdot64+144$ Multiply

   $-128+144$ Add

   16

8. $4(x+3)-(6x+2)$

$4(x+3)-(6x+2)$ Distribute the 4 and the minus symbol

$4x+12-6x-2$ Combine like terms

$-2x+10$

9. $3^5 \cdot 3^{-3}$

$3^5 \cdot 3^{-3} = 3^{5-3}$

$3^2 = 9$

10. $\left(\frac{2}{3}\right)^3$

$\frac{2^3}{3^3}$

$\frac{8}{27}$

11. $\frac{8x^3}{(2x)^2}$

$\frac{8x^3}{4x^2}$

$2x^{3-2}$

$2x$

12. $(16y^0)2y^{-2}$

$(16 \cdot 1)2y^{-2}$

$\frac{16(2)}{y^2}$

$\frac{32}{y^2}$

13. $\sqrt{441}$

21

14. $\sqrt{490}$

$\sqrt{490}$ 490 has factors 49 and 10

$\sqrt{49 \cdot 10}$ Multiplication property of radicals

$\sqrt{49}\sqrt{10}$ 49 is a perfect square

$7\sqrt{10}$

15. $\sqrt{\frac{9x}{16}}$

$\sqrt{\frac{9x}{16}}$ Division property of radicals

$\frac{\sqrt{9x}}{\sqrt{16}}$ Multiplication property of radicals; 16 is a perfect square

$\frac{\sqrt{9}\sqrt{x}}{4}$ 9 is a perfect square

$\frac{3\sqrt{x}}{4}$

16. $\frac{\sqrt{121b^2}}{1+\sqrt{b}}$

$\frac{\sqrt{121b^2}}{1+\sqrt{b}}$ Multiplication property of radicals

$\frac{\sqrt{121}\sqrt{b^2}}{1+\sqrt{b}}$ 121 and b^2 are perfect squares

$\frac{11b}{1+\sqrt{b}}\cdot\frac{1-\sqrt{b}}{1-\sqrt{b}}$ Multiply top and bottom by the conjugate of the denominator

$\frac{11b-11b\sqrt{b}}{1-\sqrt{b}+\sqrt{b}-b}$ Combine like terms

$\frac{11-11b\sqrt{b}}{1-b}$

17. $6\sqrt{24}+7\sqrt{54}-12\sqrt{6}$

$6\sqrt{24}+7\sqrt{54}-12\sqrt{6}$ 24 has factors 4 and 6; 54 has factors 9 and 6

$6\sqrt{4\cdot 6}+7\sqrt{9\cdot 6}-12\sqrt{6}$ Multiplication property of radicals

$6\sqrt{4}\sqrt{6}+7\sqrt{9}\sqrt{6}-12\sqrt{6}$ 4 and 9 are perfect squares

$6(2)\sqrt{6}+7(3)\sqrt{6}-12\sqrt{6}$ Multiply

$12\sqrt{6}+21\sqrt{6}-12\sqrt{6}$ Combine like terms

$21\sqrt{6}$

18. $\dfrac{\sqrt[3]{-8}}{\sqrt[4]{625}}$

$\dfrac{\sqrt[3]{-8}}{\sqrt[4]{625}}$ Note: $-8 = -2^3$; Note: $625 = 5^4$

$\dfrac{\sqrt[3]{-2^3}}{\sqrt[4]{5^4}}$ Evaluate

$-\dfrac{2}{5}$

19. $\left(13q^3+2q^2-3\right)-\left(6q^2+5q-3\right)$

$\left(13q^3+2q^2-3\right)-\left(6q^2+5q-3\right)$ Distribute the minus symbol

$13q^3+2q^2-3-6q^2-5q+3$ Combine like terms

$13q^3-4q^2-5q$

20. $\left(6p^2+2p+1\right)+\left(9p^2-1\right)$

$\left(6p^2+2p+1\right)+\left(9p^2-1\right)$

$15p^2+2p$

21. $(n-2)\left(n^2-4n+4\right)$

$(n-2)\left(n^2-4n+4\right)$ Use the distributive property

$n^3-4n^2+4n-2n^2+8n-8$ Combine like terms

$n^3-6n^2+12n-8$

22. $(a-2b)(2a+b)$

$(a-2b)(2a+b)$ Use the FOIL method

$2a^2+ab-4ab-2b^2$ Combine like terms

$2a^2-3ab-2b^2$

For the following exercises, factor the polynomial.

23. $16x^2-81$

$16x^2-81$ Difference of squares

$(4x-9)(4x+9)$

24. $y^2+12y+36$

$y^2+12y+36$	Rewrite 12y as 6y + 6y
$y^2+6y+6y+36$	Factor out the GCF of each expression
$y(y+6)+6(y+6)$	Use the distributive property
$(y+6)(y+6)$	Rewrite
$(y+6)^2$	

25. $27c^3-1331$

$27c^3-1331$	Difference of cubes
$(3c-11)(9c^2+33c+121)$	

26. $3x(x-6)^{-\frac{1}{4}}+2(x-6)^{\frac{3}{4}}$

$3x(x-6)^{-\frac{1}{4}}+2(x-6)^{\frac{3}{4}}$	Factor out the GCF of the expression
$(x-6)^{-\frac{1}{4}}[3x+2(x-6)]$	Distribute the 2
$(x-6)^{-\frac{1}{4}}(3x+2x-12)$	Combine like terms
$(x-6)^{-\frac{1}{4}}(5x-12)$	

For the following exercises, simplify the expression.

27. $\dfrac{2z^2+7z+3}{z^2-9}\cdot\dfrac{4z^2-15z+9}{4z^2-1}$

$\dfrac{2z^2+7z+3}{z^2-9}\cdot\dfrac{4z^2-15z+9}{4z^2-1}$	Rewrite the expressions; difference of squares
$\dfrac{(2z^2+6z)+(z+3)}{(z-3)(z+3)}\cdot\dfrac{(4z^2-12z)+(-3z+9)}{(2z-1)(2z+1)}$	Factor out the GCF of each expression
$\dfrac{2z(z+3)+(z+3)}{(z-3)(z+3)}\cdot\dfrac{4z(z-3)-3(z-3)}{(2z-1)(2z+1)}$	Use the distributive property
$\dfrac{(2z+1)(z+3)}{(z-3)(z+3)}\cdot\dfrac{(4z-3)(z-3)}{(2z-1)(2z+1)}$	Simplify
$\dfrac{4z-3}{2z-1}$	

28. $\frac{x}{y}+\frac{2}{x}$

$\frac{x}{y}+\frac{2}{x}$ The LCD is xy

$\frac{x}{y}\cdot\frac{x}{x}+\frac{2}{x}\cdot\frac{y}{y}$ Multiply

$\frac{x^2}{xy}+\frac{2y}{xy}$ Add

$\frac{x^2+2y}{xy}$

29. $\frac{\frac{a}{2b}-\frac{2b}{9a}}{\frac{3a-2b}{6a}}$

$\frac{\frac{a}{2b}-\frac{2b}{9a}}{\frac{3a-2b}{6a}}$ Rewrite the expression

$\frac{6a}{3a-2b}\left(\frac{a}{2b}-\frac{2b}{9a}\right)$ The LCD is $18ab$

$\frac{6a}{3a-2b}\left(\frac{a}{2b}\cdot\frac{9a}{9a}-\frac{2b}{9a}\cdot\frac{2b}{2b}\right)$ Multiply

$\frac{6a}{3a-2b}\left(\frac{9a^2}{18ab}-\frac{4b^2}{18ab}\right)$ Multiply

$\frac{6a(9a^2-4b^2)}{18ab(3a-2b)}$ Difference of squares

$\frac{6a(3a-2b)(3a+2b)}{18ab(3a-2b)}$ Simplify

$\frac{3a+2b}{3b}$

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